In[]:=
```
a[l_, n_] := Sum[λ_k^n E^(I k * 2 π l / (L+1)), {k, -m, m}];
```

In[]:=
```
DelRep = u_l^(n+1) == u_l^n + τ/(3 h) (2 u_(l+3)^n - 9 u_(l+2)^n + 18 u_(l+1)^n - 11 u_l^n) +
  2 τ^2/h^2 (-u_(l+3)^n + 4 u_(l+2)^n - 5 u_(l+1)^n + 2 u_l^n) + 4 τ^3/(3 h^3) (u_(l+3)^n - 3 u_(l+2)^n + 3 u_(l+1)^n - u_l^n) /.
 {Subscript[u, l] -> Subscript[Δ, l], Subscript[u, l + 1] -> Subscript[Δ, l + 1],
  Subscript[u, l + 2] -> Subscript[Δ, l + 2], Subscript[u, l + 3] -> Subscript[Δ, l + 3]}
```
с нижним индексом (×8)

Out[]=
$$\Delta_l^{1+n} == \Delta_l^n + \frac{2\tau^2\left(2\Delta_l^n - 5\Delta_{l+1}^n + 4\Delta_{l+2}^n - \Delta_{l+3}^n\right)}{h^2} + \frac{4\tau^3\left(-\Delta_l^n + 3\Delta_{l+1}^n - 3\Delta_{l+2}^n + \Delta_{l+3}^n\right)}{3h^3} + \frac{\tau\left(-11\Delta_l^n + 18\Delta_{l+1}^n - 9\Delta_{l+2}^n + 2\Delta_{l+3}^n\right)}{3h}$$

In[]:=
```
SumRep = DelRep /. {Δ_(l+1)^n -> a[l + 1, n], Δ_(l+2)^n -> a[l + 2, n], Δ_l^(1+n) -> a[l, n + 1], Δ_l^n -> a[l, n], Δ_(l+3)^n -> a[l + 3, n]}
```

Out[]=
$$\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^{1+n} == \sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^n + \frac{1}{h^2}2\tau^2\left(2\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^n - 5\sum_{k=-m}^{m} e^{\frac{2ik(1+l)\pi}{1+L}}\lambda_k^n + 4\sum_{k=-m}^{m} e^{\frac{2ik(2+l)\pi}{1+L}}\lambda_k^n - \sum_{k=-m}^{m} e^{\frac{2ik(3+l)\pi}{1+L}}\lambda_k^n\right) + \frac{1}{3h^3}4\tau^3\left(-\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^n + 3\sum_{k=-m}^{m} e^{\frac{2ik(1+l)\pi}{1+L}}\lambda_k^n - 3\sum_{k=-m}^{m} e^{\frac{2ik(2+l)\pi}{1+L}}\lambda_k^n + \sum_{k=-m}^{m} e^{\frac{2ik(3+l)\pi}{1+L}}\lambda_k^n\right) + \frac{1}{3h}\tau\left(-11\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^n + 18\sum_{k=-m}^{m} e^{\frac{2ik(1+l)\pi}{1+L}}\lambda_k^n - 9\sum_{k=-m}^{m} e^{\frac{2ik(2+l)\pi}{1+L}}\lambda_k^n + 2\sum_{k=-m}^{m} e^{\frac{2ik(3+l)\pi}{1+L}}\lambda_k^n\right)$$

In[]:=
```
SunRepSimp = Simplify[SumRep]
```
упростить

Out[]=
$$\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^{1+n} == \frac{1}{3h^3}\left(\left(3h^3 - 11h^2\tau + 12h\tau^2 - 4\tau^3\right)\sum_{k=-m}^{m} e^{\frac{2ikl\pi}{1+L}}\lambda_k^n + 6\tau\left(3h^2 - 5h\tau + 2\tau^2\right)\sum_{k=-m}^{m} e^{\frac{2ik(1+l)\pi}{1+L}}\lambda_k^n - \left(h - 2\tau\right)\tau\left(\left(9h - 6\tau\right)\sum_{k=-m}^{m} e^{\frac{2ik(2+l)\pi}{1+L}}\lambda_k^n + 2\left(-h + \tau\right)\sum_{k=-m}^{m} e^{\frac{2ik(3+l)\pi}{1+L}}\lambda_k^n\right)\right)$$

In[]:=
```
LambdRep = SunRepSimp /. {λ_k^n -> 1, λ_k^(n+1) -> λ_k, 2 i k l π/(1 + L) -> α l I,
  2 i k (1 + l) π/(1 + L) -> α (1 + l) I, 2 i k (2 + l) π/(1 + L) -> α (1 + 2) I, 2 i k (3 + l) π/(1 + L) -> α (1 + 3) I}
```
мнимая единица (×4)

Out[]=
$$\sum_{k=-m}^{m} e^{i l\alpha}\lambda_k == \frac{1}{3h^3}\left(6e^{i(1+l)\alpha}\left(1 + 2m\right)\tau\left(3h^2 - 5h\tau + 2\tau^2\right) + e^{i l\alpha}\left(1 + 2m\right)\left(3h^3 - 11h^2\tau + 12h\tau^2 - 4\tau^3\right) - \left(h - 2\tau\right)\tau\left(e^{i(2+l)\alpha}\left(1 + 2m\right)\left(9h - 6\tau\right) + 2e^{i(3+l)\alpha}\left(1 + 2m\right)\left(-h + \tau\right)\right)\right)$$

In[•]:=
```
LambdRepSimp = Simplify[LambdRep]
```
упростить

Out[•]=
$$\sum_{k=-m}^{m} e^{i l \alpha} \lambda_k == \frac{1}{3 h^3} e^{i l \alpha} (1 + 2 m) \left(3 h^3 + \left(-11 + 18 e^{i \alpha} - 9 e^{2 i \alpha} + 2 e^{3 i \alpha}\right) h^2 \tau - 6 \left(-2 + e^{i \alpha}\right) \left(-1 + e^{i \alpha}\right)^2 h \tau^2 + 4 \left(-1 + e^{i \alpha}\right)^3 \tau^3\right)$$

In[•]:=
```
PreFinal = SubtractSides[LambdRepSimp, Sum[e^(i l α) λ_k, {k, -m, m}]]
```
вычесть стороны

Out[•]=
$$0 == \frac{1}{3 h^3} e^{i l \alpha} (1 + 2 m) \left(3 h^3 + \left(-11 + 18 e^{i \alpha} - 9 e^{2 i \alpha} + 2 e^{3 i \alpha}\right) h^2 \tau - 6 \left(-2 + e^{i \alpha}\right) \left(-1 + e^{i \alpha}\right)^2 h \tau^2 + 4 \left(-1 + e^{i \alpha}\right)^3 \tau^3\right) - \sum_{k=-m}^{m} e^{i l \alpha} \lambda_k$$

In[•]:=
```
LambdP = Assuming[1 + L ≠ 0, MultiplySides[PreFinal, L + 1]] /. λ_k → λ_-p
```
предполагая умножить стороны

Out[•]=
$$0 == (1 + L) \left(\frac{1}{3 h^3} e^{i l \alpha} (1 + 2 m) \left(3 h^3 + \left(-11 + 18 e^{i \alpha} - 9 e^{2 i \alpha} + 2 e^{3 i \alpha}\right) h^2 \tau - 6 \left(-2 + e^{i \alpha}\right) \left(-1 + e^{i \alpha}\right)^2 h \tau^2 + 4 \left(-1 + e^{i \alpha}\right)^3 \tau^3\right) - e^{i l \alpha} (1 + 2 m) \lambda_{-p}\right)$$

In[•]:=
```
LambdOnly = Expand[LambdP] /. {λ_-p → λ}
```
раскрыть скобки

Out[•]=
$$0 == e^{i l \alpha} + e^{i l \alpha} L + 2 e^{i l \alpha} m + 2 e^{i l \alpha} L m - e^{i l \alpha} \lambda - e^{i l \alpha} L \lambda - 2 e^{i l \alpha} m \lambda - 2 e^{i l \alpha} L m \lambda - \frac{11 e^{i l \alpha} \tau}{3 h} + \frac{6 e^{i \alpha + i l \alpha} \tau}{h} - \frac{3 e^{2 i \alpha + i l \alpha} \tau}{h} + \frac{2 e^{3 i \alpha + i l \alpha} \tau}{3 h} - \frac{11 e^{i l \alpha} L \tau}{3 h} + \frac{6 e^{i \alpha + i l \alpha} L \tau}{h} - \frac{3 e^{2 i \alpha + i l \alpha} L \tau}{h} + \frac{2 e^{3 i \alpha + i l \alpha} L \tau}{3 h} - \frac{22 e^{i l \alpha} m \tau}{3 h} + \frac{12 e^{i \alpha + i l \alpha} m \tau}{h} - \frac{6 e^{2 i \alpha + i l \alpha} m \tau}{h} + \frac{4 e^{3 i \alpha + i l \alpha} m \tau}{3 h} - \frac{22 e^{i l \alpha} L m \tau}{3 h} + \frac{12 e^{i \alpha + i l \alpha} L m \tau}{h} - \frac{6 e^{2 i \alpha + i l \alpha} L m \tau}{h} + \frac{4 e^{3 i \alpha + i l \alpha} L m \tau}{3 h} + \frac{4 e^{i l \alpha} \tau^2}{h^2} - \frac{10 e^{i \alpha + i l \alpha} \tau^2}{h^2} + \frac{8 e^{2 i \alpha + i l \alpha} \tau^2}{h^2} - \frac{2 e^{3 i \alpha + i l \alpha} \tau^2}{h^2} + \frac{4 e^{i l \alpha} L \tau^2}{h^2} - \frac{10 e^{i \alpha + i l \alpha} L \tau^2}{h^2} + \frac{8 e^{2 i \alpha + i l \alpha} L \tau^2}{h^2} - \frac{2 e^{3 i \alpha + i l \alpha} L \tau^2}{h^2} + \frac{8 e^{i l \alpha} m \tau^2}{h^2} - \frac{20 e^{i \alpha + i l \alpha} m \tau^2}{h^2} + \frac{16 e^{2 i \alpha + i l \alpha} m \tau^2}{h^2} - \frac{4 e^{3 i \alpha + i l \alpha} m \tau^2}{h^2} + \frac{8 e^{i l \alpha} L m \tau^2}{h^2} - \frac{20 e^{i \alpha + i l \alpha} L m \tau^2}{h^2} + \frac{16 e^{2 i \alpha + i l \alpha} L m \tau^2}{h^2} - \frac{4 e^{3 i \alpha + i l \alpha} L m \tau^2}{h^2} - \frac{4 e^{i l \alpha} \tau^3}{3 h^3} + \frac{4 e^{i \alpha + i l \alpha} \tau^3}{h^3} - \frac{4 e^{2 i \alpha + i l \alpha} \tau^3}{h^3} + \frac{4 e^{3 i \alpha + i l \alpha} \tau^3}{3 h^3} - \frac{4 e^{i l \alpha} L \tau^3}{3 h^3} + \frac{4 e^{i \alpha + i l \alpha} L \tau^3}{h^3} - \frac{4 e^{2 i \alpha + i l \alpha} L \tau^3}{h^3} + \frac{4 e^{3 i \alpha + i l \alpha} L \tau^3}{3 h^3} - \frac{8 e^{i l \alpha} m \tau^3}{3 h^3} + \frac{8 e^{i \alpha + i l \alpha} m \tau^3}{h^3} - \frac{8 e^{2 i \alpha + i l \alpha} m \tau^3}{h^3} + \frac{8 e^{3 i \alpha + i l \alpha} m \tau^3}{3 h^3} - \frac{8 e^{i l \alpha} L m \tau^3}{3 h^3} + \frac{8 e^{i \alpha + i l \alpha} L m \tau^3}{h^3} - \frac{8 e^{2 i \alpha + i l \alpha} L m \tau^3}{h^3} + \frac{8 e^{3 i \alpha + i l \alpha} L m \tau^3}{3 h^3}$$

In[•]:=
```
Lambd = Reduce[LambdOnly, λ]
```
привести

Out[•]=
$$\left(e^{i\,l\,\alpha} == 0 \&\& h \neq 0\right) || \left(L == -1 \&\& h \neq 0\right) || \left(m == -\frac{1}{2} \&\& h \neq 0\right) ||$$
$$\left(h \neq 0 \&\& \lambda == \frac{1}{3\,h^3}\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - \right.\right.$$
$$\left.\left.30\,e^{i\,\alpha}\,h\,\tau^2 + 24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)\right)$$

In[•]:=
$$\lambda = \frac{1}{3\,h^3}\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - \right.$$
$$\left.30\,e^{i\,\alpha}\,h\,\tau^2 + 24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)$$

Out[•]=
$$\frac{1}{3\,h^3}\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - \right.$$
$$\left.30\,e^{i\,\alpha}\,h\,\tau^2 + 24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)$$

In[•]:=
```
λ̄ = ReplaceAll[ 1/(3 h^3) (3 h^3 - 11 h^2 τ + 18 e^(i α) h^2 τ - 9 e^(2 i α) h^2 τ + 2 e^(3 i α) h^2 τ + 12 h τ^2 - 30 e^(i α) h τ^2 +
  24 e^(2 i α) h τ^2 - 6 e^(3 i α) h τ^2 - 4 τ^3 + 12 e^(i α) τ^3 - 12 e^(2 i α) τ^3 + 4 e^(3 i α) τ^3), i → -I]
```
заменить всё

мнимая единица

Out[•]=
$$\frac{1}{3\,h^3}\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{-i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - \right.$$
$$\left.30\,e^{-i\,\alpha}\,h\,\tau^2 + 24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{-i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)$$

In[•]:=
```
λ * λ̄
```

Out[•]=
$$\frac{1}{9\,h^6}\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{-i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - \right.$$
$$\left.30\,e^{-i\,\alpha}\,h\,\tau^2 + 24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{-i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)$$
$$\left(3\,h^3 - 11\,h^2\,\tau + 18\,e^{i\,\alpha}\,h^2\,\tau - 9\,e^{2\,i\,\alpha}\,h^2\,\tau + 2\,e^{3\,i\,\alpha}\,h^2\,\tau + 12\,h\,\tau^2 - 30\,e^{i\,\alpha}\,h\,\tau^2 + \right.$$
$$\left.24\,e^{2\,i\,\alpha}\,h\,\tau^2 - 6\,e^{3\,i\,\alpha}\,h\,\tau^2 - 4\,\tau^3 + 12\,e^{i\,\alpha}\,\tau^3 - 12\,e^{2\,i\,\alpha}\,\tau^3 + 4\,e^{3\,i\,\alpha}\,\tau^3\right)$$

In[•]:=
```
0.5 < β < 1 /. β → (τ/h)
```

Out[•]=
$$0.5 < \frac{\tau}{h} < 1$$