The Burnorian Solution Algorithm: Core Framework Implementation

This algorithm encapsulates the fundamental components of the Burnorian Solution, from the Burnorian Quantum Group to the framework for the Numerical Renormalization Group.

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import cmath # For complex numbers and q
import random # For non-deterministic aspects in Hamiltonian simulation and QMC moves
import copy # For deep copying state objects
import numpy as np # For matrix operations
from scipy.linalg import expm # For matrix exponential
# --- Global Constants for the Burnorian Solution ---
PLANCK_LENGTH = 1.0 # Normalized Planck Length for demonstration purposes
# This value would be derived from fundamental constants in a full theory
# --- Step 30: Burnorian Quantum Group Algebra ---
# (Extended from Step 30, 31, 32, 33, 34 to be a general Burnorian Quantum Group)
class BurnorianQuantumGroup:
    """
    Implements the core algebra of the Burnorian Quantum Group (U_q(su2) as a proxy
    for U_q(sl2,C)) and provides methods for calculating Burnorian Quantum Observables.
    """
    def __init__(self, q_value: complex):
        if q_value == 1.0 + 0j:
            raise ValueError("q=1 is the classical limit; need q!= 1 for quantum group effects.")
        self.q = q_value
        self.inv_q_minus_inv_q = self.q - (1.0 / self.q)
        if abs(self.inv_q_minus_inv_q) < 1e-9:
            raise ValueError("q is too close to 1, (q - 1/q) is near zero.")
    def q_number(self, x: float) -> complex:
        """
        Computes the quantum number [x] = (q^x - q^-x) / (q - q^-1).
        This replaces classical numbers in quantum group formulas.
        """
        if abs(self.inv_q_minus_inv_q) < 1e-9: return complex(x) # Handles q=1 limit
        return (self.q**x - self.q**(-x)) / self.inv_q_minus_inv_q
    # Placeholder for Jz, Jplus, Jminus actions on |j,m> states.
    # (These methods define the algebra's action but are not directly used in observable
calculations below,
    # which rely on the derived q-number formulas)
    # def action_Jz(self, j: float, m: float) -> tuple[float, float, complex]:...
    # def action_Jplus(self, j: float, m: float) -> tuple[float, float, complex]:...
    # def action_Jminus(self, j: float, m: float) -> tuple[float, float, complex]:...
    # --- Step 31: Burnorian Area Operator ---
    def calculate_burnorian_area_eigenvalue(self, j_quantum_number: float,
coupling_const_alpha: float = 1.0) -> complex:
        """
        Calculates the eigenvalue of the Burnorian Area Operator for a given j_quantum_number.
        Derived from the q-deformed Casimir invariant of the quantum group.
        """
        if j_quantum_number < 0 or (j_quantum_number * 2) % 1!= 0:
           raise ValueError("j_quantum_number must be a non-negative half-integer.")
        q_j = self.q_number(j_quantum_number)
        q_j_plus_1 = self.q_number(j_quantum_number + 1) # Assumes Casimir ~ [j][j+1]
        burnorian_area = coupling_const_alpha * q_j * q_j_plus_1
        return burnorian_area
    # --- Step 32: Burnorian 4-Volume Operator ---
    def calculate_burnorian_4_volume_eigenvalue(self, intertwiner_qn: float,
coupling_const_beta: float = 1.0) -> complex:
        """
        Calculates the eigenvalue of the Burnorian 4-Volume Operator for a single CQT.
        Derived from the recoupling theory of the Burnorian Quantum Group.
        """
        if intertwiner_qn < 0 or (intertwiner_qn * 2) % 1!= 0:
           raise ValueError("intertwiner_qn must be a non-negative half-integer.")
        if intertwiner_qn == 0:
           return 0.0 + 0j # Zero volume for a degenerate CQT, physically disallowed.
        q_intertwiner = self.q_number(intertwiner_qn)
        q_intertwiner_plus_1 = self.q_number(intertwiner_qn + 1) # Proxy for complex volume
formula
        burnorian_4_volume = coupling_const_beta * q_intertwiner * q_intertwiner_plus_1
        return burnorian_4_volume
    # --- Step 33: Burnorian Spacetime Interval Operator ---
    def calculate_burnorian_spacetime_interval_eigenvalue(
        self, causal_link_qn: float, spacetime_type_hint: str, coupling_const_lambda: float = 1.0
    ) -> complex:
        """
        Calculates the eigenvalue of the Burnorian Quantum Spacetime Interval Operator (s^2).
        Derived from a q-deformation of the Minkowski metric.
        """
        if causal_link_qn < 0 or (causal_link_qn * 2) % 1!= 0:
           raise ValueError("causal_link_qn must be a non-negative half-integer.")
        q_link_qn = self.q_number(causal_link_qn)
        burnorian_interval_squared_mag = coupling_const_lambda * (q_link_qn * q_link_qn)
        if spacetime_type_hint == 'timelike':
           if causal_link_qn == 0: return 0j
           return -1 * burnorian_interval_squared_mag # Negative real part for timelike
        elif spacetime_type_hint == 'spacelike':
           if causal_link_qn == 0: return 0j
           return burnorian_interval_squared_mag # Positive real part for spacelike
        elif spacetime_type_hint == 'null':
            return 0j
        else:
            raise ValueError("Spacetime type hint must be 'timelike', 'null', or 'spacelike'.")
    # --- Step 34: Burnorian Causal Ordering Operator ---
    def determine_burnorian_causal_order(
        self, s_squared_eigenvalue: complex, v_i_id: int, v_j_id: int, contextual_phase_info: float =
0.0
    ) -> dict:
        """
        Determines the Burnorian causal ordering based on the s^2 eigenvalue and contextual
information.
        Represents the action of the Burnorian Causal Ordering Operator.
        """
        real_part_s2 = s_squared_eigenvalue.real
        if abs(real_part_s2) < 1e-9: # Approximately zero
            causal_type = 'null'
            causal_order = None
        elif real_part_s2 < 0: # Negative real part (timelike)
            causal_type = 'timelike'
            # Contextual phase determines direction of ordering
            if cmath.cos(contextual_phase_info).real > 0:
                causal_order = (v_i_id, v_j_id)
            else:
                causal_order = (v_j_id, v_i_id)
        else: # Positive real part (spacelike)
            causal_type = 'spacelike'
            causal_order = None
        return {'type': causal_type, 'causal_order': causal_order, 's_squared_value':
s_squared_eigenvalue}
# --- Step 35: Burnorian Causal Quantum Tetrahedron (CQTState) ---
class CQTState:
    """
    Represents a single Burnorian Causal Quantum Tetrahedron (CQT), the fundamental
    discrete unit of spacetime, enforcing Burnorian Closure Constraints.
    """
    def __init__(self, cqt_id, face_spins, intertwiner_geom_qn, matter_spins, matter_gauge_qn,
causal_links_data, bqg_instance: BurnorianQuantumGroup):
        self.cqt_id = cqt_id # Temporary ID for instance in a network
        self._bqg = bqg_instance # Reference to the Burnorian Quantum Group instance
        # Validate and canonicalize input quantum numbers for Diffeomorphism Invariance (Step
25)
        if len(face_spins)!= 4: raise ValueError("A CQT must have exactly 4 face spins.")
        if any(j < 0 or (j * 2) % 1!= 0 for j in face_spins): raise ValueError("Face spins must be
non-negative half-integers.")
        self._canonical_face_spins = tuple(sorted(face_spins))
        self._canonical_matter_spins = tuple(sorted(matter_spins))
        self._canonical_matter_gauge_qn = frozenset(matter_gauge_qn.items())
        # Canonicalize causal_links_data for Diffeomorphism Invariance
        canonical_causal_links_list = []
        for link_id, data in causal_links_data.items():
            link_repr = (tuple(sorted(link_id)), data['type'], data['length'], data['causal_order'])
            canonical_causal_links_list.append(link_repr)
        self._canonical_causal_links_data = tuple(sorted(canonical_causal_links_list))
        self._intertwiner_geom_qn = intertwiner_geom_qn # Quantum number for 4-volume
        # Enforce Burnorian Closure Constraints (Geometric and Causal) during initialization
        if not self._validate_geometric_closure():
            raise ValueError(f"CQT {cqt_id} failed Burnorian Geometric Closure Constraint!")
        if not self._validate_causal_closure():
            raise ValueError(f"CQT {cqt_id} failed Burnorian Causal Closure Constraint!")
    # Properties to access canonical forms
    @property
    def face_spins(self): return self._canonical_face_spins
    @property
    def matter_spins(self): return self._canonical_matter_spins
    @property
    def matter_gauge_qn(self): return self._canonical_matter_gauge_qn
    @property
    def causal_links_data(self):
        reconstructed_data = {}
        for link_id_tuple, link_type, link_length, causal_order in self._canonical_causal_links_data:
            reconstructed_data[frozenset(link_id_tuple)] = {'type': link_type, 'length': link_length,
'causal_order': causal_order}
        return reconstructed_data
    @property
    def intertwiner_geom_qn(self): return self._intertwiner_geom_qn
    def get_face_spin(self, face_index):
        """Retrieves the spin of a specific CQT face by its canonical index (0-3)."""
        if 0 <= face_index < 4: return self.face_spins[face_index]
        raise IndexError("Face index out of bounds (0-3).")
    def _validate_geometric_closure(self) -> bool:
        """
        Burnorian Geometric Closure Constraint (simplified for demonstration):
        Ensures the 4-volume quantum number is consistent and yields a non-zero real volume.
        """
        # A more rigorous check would involve explicit recoupling conditions for the
intertwiner_geom_qn
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        # to be compatible with the face_spins.
        total_face_spin_sum = sum(self.face_spins)
        if total_face_spin_sum == 0 and self.intertwiner_geom_qn > 0: return False # Inconsistent
geometry

        # Explicit check for non-zero 4-volume
        if self._bqg.calculate_burnorian_4_volume_eigenvalue(self.intertwiner_geom_qn).real <=
0:
            return False # Physically disallowed zero or negative volume
        return True
    def _validate_causal_closure(self) -> bool:
        """
        Burnorian Causal Closure Constraint:
        Verifies internal causal consistency among the CQT's 5 vertices (0-4).
        Checks for single-link validity and transitivity (no internal CTCs).
        """
        # 1. Single-link validity (non-zero length for non-null, correct ordering for timelike)
        for link_id_tuple, link_type, link_length, causal_order in self._canonical_causal_links_data:
            if link_length < PLANCK_LENGTH and link_type!= 'null': # Non-null links must be >=
Planck length
                return False
            if link_type == 'timelike' and causal_order is None: return False # Timelike must have
order
            if link_type == 'spacelike' and causal_order is not None: return False # Spacelike must
not have order
        # 2. Transitivity check (DFS for cycle detection - no internal CTCs)
        causal_graph = {i: [] for i in range(5)} # Assuming 5 vertices 0-4 for the 4-simplex
        for link_id_tuple, link_type, link_length, causal_order in self._canonical_causal_links_data:
            if link_type == 'timelike' and causal_order:
                precedes, succeeds = causal_order
                causal_graph[precedes].append(succeeds)

        # Perform DFS on each node to detect cycles
        for start_node in range(5):
            visited = set()
            path = []
            def dfs(u):
                visited.add(u)
                path.append(u)
                for v in causal_graph[u]:
                    if v in path: return False # Cycle detected (CTC)
                    if v not in visited:
                        if not dfs(v): return False
                path.pop() # Backtrack
            return True
         if not dfs(start_node): return False # If a cycle is found from any start node
      return True
   # Overloaded operators for Diffeomorphism Invariance (Step 25) and Hilbert Space (Step 36)
   def __str__(self):
      s = (f"CQT(ID={self.cqt_id}) [Geom: {self.face_spins}, 4-Vol
QN:{self.intertwiner_geom_qn:.1f} | Matter: {self.matter_spins}, {dict(self.matter_gauge_qn)}]\n")
      s += " Burnorian Causal Links (Canonical):\n"
      for link_id_tuple, data_type, data_length, data_order in self._canonical_causal_links_data:
         order_str = f"Order: {data_order}" if data_order else "Acausal"
         s += f" Link {list(link_id_tuple)}: Type={data_type}, Length={data_length:.3f}*l_P,
{order_str}\n"
      return s

   def __eq__(self, other):
      if not isinstance(other, CQTState): return NotImplemented
      # Equality based on canonical representation of all intrinsic quantum numbers
      return (self.face_spins == other.face_spins and
            self.intertwiner_geom_qn == other.intertwiner_geom_qn and
            self.matter_spins == other.matter_spins and
            self.matter_gauge_qn == other.matter_gauge_qn and
            self._canonical_causal_links_data == other._canonical_causal_links_data)
   def __hash__(self):
      # Hash also based on canonical form for proper dictionary/set usage
      return hash((self.face_spins, self.intertwiner_geom_qn, self.matter_spins,
                self.matter_gauge_qn, self._canonical_causal_links_data))
# --- Step 37: Burnorian Total Hilbert Space (CQTNetworkGraphState) ---
class CQTNetworkGraphState:
   """
   Represents a basis state of the Burnorian Total Hilbert Space (H_total),
   a graph of connected CQTs, enforcing global constraints.
   """
   def __init__(self, cqt_states_dict: dict):
      """
      cqt_states_dict: A dictionary where keys are CQT IDs and values are CQTState objects.
                CQT IDs are temporary labels for building the graph.
      """
      self.cqt_states = {c.cqt_id: c for c in cqt_states_dict.values()}
      # Connections: {frozenset({(cqt_id1, face_idx1), (cqt_id2, face_idx2)}):
shared_face_state_hash}
      self.connections = {}
      # Tracks next available CQT ID for moves that add CQTs
      self._next_cqt_id = max(cqt_states_dict.keys()) + 1 if cqt_states_dict else 1
      self._graph_hash_cache = None # Cache for the network's canonical hash
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    def add_connection(self, cqt_id1: int, face_idx1: int, cqt_id2: int, face_idx2: int):
        """
        Adds a connection between two CQTs via their specified faces.
        Enforces local gluing constraints (matching face quantum numbers).
        """
        if cqt_id1 not in self.cqt_states or cqt_id2 not in self.cqt_states:
            raise ValueError("One or both CQT IDs not found in the network.")
        if not (0 <= face_idx1 < 4 and 0 <= face_idx2 < 4):
            raise IndexError("Face indices must be 0-3.")
        state1 = self.cqt_states[cqt_id1]
        state2 = self.cqt_states[cqt_id2]
        face_spin1 = state1.get_face_spin(face_idx1)
        face_spin2 = state2.get_face_spin(face_idx2)
        if face_spin1!= face_spin2:
            raise ValueError(f"Cannot connect faces with mismatching spins: {face_spin1} vs
{face_spin2}")

        # The shared face is characterized by its spin (and possibly matter quantum numbers)
        shared_face_state_hash = hash(frozenset([face_spin1]))
        conn_key = frozenset({(cqt_id1, face_idx1), (cqt_id2, face_idx2)})
        self.connections[conn_key] = shared_face_state_hash
        self._graph_hash_cache = None # Invalidate hash cache on modification
    def get_neighbors_and_causal_order(self, cqt_id: int) -> list[tuple[int, int]]:
        """
        (Heuristic for demo) Returns a list of (neighbor_cqt_id, causal_relation) tuples for a given
CQT.
        causal_relation: +1 if neighbor is causally 'later', -1 if 'earlier', 0 if spacelike/ambiguous.
        In a full theory, this would be derived from the actual causal links between CQTs.
        """
        causal_neighbors = []
        target_cqt = self.cqt_states.get(cqt_id)
        if not target_cqt: return []
        for conn_key in self.connections:
            # Check if target_cqt is part of this connection (simplified to face 0)
            if (cqt_id, 0) in conn_key:
                other_node_info = next(item for item in conn_key if item[0]!= cqt_id)
                neighbor_id = other_node_info[0]
                # Heuristic: Compare average CQT volume quantum numbers as a proxy for
'progress'
                # (Larger average volume might imply "later" in a causal evolutionary sense)
                if target_cqt.intertwiner_geom_qn < self.cqt_states[neighbor_id].intertwiner_geom_qn:
                    causal_neighbors.append((neighbor_id, +1)) # Neighbor is causally after
                else:
                    causal_neighbors.append((neighbor_id, 0)) # Spacelike or ambiguous
        return causal_neighbors
    def validate_global_causality(self) -> bool:
        """
        Enforces Global Burnorian Causal Constraints (no CTCs in the network).
        Performs a topological sort on a heuristic causal graph of CQTs.
        """
        # First, ensure all individual CQTs are causally valid
        if not all(cqt._validate_causal_closure() for cqt in self.cqt_states.values()):
            # print("DEBUG: Individual CQT failed causal closure.")
            return False
        # Build a directed graph for the network's causal structure (simplified heuristic)
        graph_adj = {cqt_id: [] for cqt_id in self.cqt_states.keys()}

        # Heuristic: A CQT 'causally precedes' another if its average volume QN is smaller.
        # This is a very rough proxy for causal progression in a small network.
        for cqt_id_from in self.cqt_states.keys():
            for cqt_id_to in self.cqt_states.keys():
                if cqt_id_from == cqt_id_to: continue
                # The causal relation between CQTs needs to be carefully derived from face
connections
                # and internal causal link data in a full model.
                if self.cqt_states[cqt_id_from].intertwiner_geom_qn <
self.cqt_states[cqt_id_to].intertwiner_geom_qn:
                    graph_adj[cqt_id_from].append(cqt_id_to)

        # Perform topological sort (Kahn's algorithm) to detect cycles
        in_degree = {cqt_id: 0 for cqt_id in self.cqt_states.keys()}
        for u in graph_adj:
            for v in graph_adj[u]:
                in_degree[v] += 1

        queue = [cqt_id for cqt_id, degree in in_degree.items() if degree == 0]
        count = 0

        while queue:
            u = queue.pop(0)
            count += 1
            for v in graph_adj[u]:
                in_degree[v] -= 1
                if in_degree[v] == 0:
                    queue.append(v)

        return count == len(self.cqt_states) # If count!= num_CQTs, a cycle (CTC) exists
    # Method to generate a canonical representation for global Diffeomorphism Invariance (Step
26)
    def _get_canonical_representation(self) -> tuple:
        """
        Generates a canonical, hashable representation of the graph state
        for global diffeomorphism invariance.
        """
        # 1. Canonical CQTs: Sort CQTs by their intrinsic hash (which includes all canonical
properties)
        canonical_cqts = tuple(sorted(self.cqt_states.values(), key=hash))
        # 2. Canonical Connections: Represent connections using canonical indices of CQTs
        cqt_hash_to_idx = {hash(cqt): idx for idx, cqt in enumerate(canonical_cqts)}

        canonical_connections_list = []
        for conn_key, shared_face_hash in self.connections.items():
            (cqt_id1, face_idx1), (cqt_id2, face_idx2) = tuple(conn_key) # Convert frozenset to tuple
for consistent order

            # Map original cqt_ids to their canonical indices
            canon_idx1 = cqt_hash_to_idx[hash(self.cqt_states[cqt_id1])]
            canon_idx2 = cqt_hash_to_idx[hash(self.cqt_states[cqt_id2])]

            # Create a canonical representation of the connection itself
            canonical_conn_part = tuple(sorted(((canon_idx1, face_idx1), (canon_idx2, face_idx2))))
            canonical_connections_list.append((canonical_conn_part, shared_face_hash))

        canonical_connections = tuple(sorted(canonical_connections_list))
        return (canonical_cqts, canonical_connections)
    # Overloaded operators for Diffeomorphism Invariance (Step 26) and Total Hilbert Space
(Step 37)
    def __str__(self):
        s = "Burnorian CQT Network Graph State:\n"
        s += " Nodes (CQTs):\n"
        for cqt_id, cqt_state in self.cqt_states.items():
            s += f" {cqt_state.cqt_id}: CQT(4-Vol QN:{cqt_state.intertwiner_geom_qn:.1f},
Chg:{dict(cqt_state.matter_gauge_qn).get('charge',0)})\n"
        s += " Edges (Connections):\n"
        for conn_key, shared_spin in self.connections.items():
            items = list(conn_key)
            s += f" - CQT {items[0][0]} (Face {items[0][1]}) <-> CQT {items[1][0]} (Face {items[1][1]})
[Spin: {shared_spin}]\n"
        s += f" Globally Causally Valid: {self.validate_global_causality()}\n"
        return s
    def __eq__(self, other):
        if not isinstance(other, CQTNetworkGraphState): return NotImplemented
        # Equality based on canonical representation of the entire graph
        return self._get_canonical_representation() == other._get_canonical_representation()
    def __hash__(self):
        if self._graph_hash_cache is None:
            self._graph_hash_cache = hash(self._get_canonical_representation())
        return self._graph_hash_cache

    def __copy__(self):
        # Creates a deep copy for safe modification during Hamiltonian evolution or QMC moves
        new_instance = CQTNetworkGraphState({})
        new_instance.cqt_states = {c_id: copy.copy(cqt) for c_id, cqt in self.cqt_states.items()}
        new_instance.connections = copy.deepcopy(self.connections)
        new_instance._next_cqt_id = self._next_cqt_id
        return new_instance
# --- Step 37: Burnorian Total Inner Product ---
def burnorian_total_inner_product(state_A: CQTNetworkGraphState, state_B:
CQTNetworkGraphState) -> complex:
    """
    Calculates the Burnorian Total Inner Product for two CQTNetworkGraphState basis vectors.
    Since basis states are orthonormal, this is a Kronecker delta.
    """
    if state_A == state_B: return 1.0 + 0j
    else: return 0.0 + 0j
# --- Step 38: Burnorian Hamiltonian Operator Construction ---
def apply_burnorian_hamiltonian(network_state: CQTNetworkGraphState) ->
list[tuple[CQTNetworkGraphState, complex]]:
    """
    Simulates the formal action of the Burnorian Hamiltonian Operator H_QG on a network state.
    It returns a list of (new_network_state, amplitude) pairs, representing the superposition
    that results from applying ALL possible local Hamiltonian terms on a single CQT.
    """
    possible_transitions = {} # Aggregates amplitudes for identical resulting states
    # Iterate over all CQTs in the network to apply local operators
    for target_cqt_id in network_state.cqt_states.keys():
        original_cqt = network_state.cqt_states[target_cqt_id]
        original_volume_qn = original_cqt.intertwiner_geom_qn
        original_matter_charge = dict(original_cqt.matter_gauge_qn).get('charge', 0)
        # 1. h_vol (Volume Fluctuation with Burnorian Repulsion)
        for volume_change in [-0.5, 0.5]: # Possible changes in volume quantum number
            new_volume_qn = max(0.5, round((original_volume_qn + volume_change) * 2) / 2.0)
            amplitude_vol = 1.0j # Base amplitude (complex)

            # Burnorian Repulsive Potential: Amplitude suppression for collapse at min volume
            if original_volume_qn == 0.5 and volume_change < 0: # Trying to shrink from min
non-zero volume
                amplitude_vol *= 0.1j # Very small amplitude (high energy cost) for further shrinkage
            elif original_volume_qn == 0.5 and volume_change > 0: # Favors expansion from min
                amplitude_vol *= 1.5j # Larger amplitude for expansion (quantum bounce)
            else:
                amplitude_vol *= 0.8j # Normal fluctuation amplitude
            # Create a new CQT state with the modified volume QN
            try:
                new_cqt_vol = CQTState(target_cqt_id, list(original_cqt.face_spins), new_volume_qn,
                                    list(original_cqt.matter_spins), original_cqt.matter_gauge_qn,
                                    original_cqt.causal_links_data, original_cqt._bqg)
            except ValueError: # If new CQT state is invalid (e.g., failed closure constraints)
                continue # This transition has zero amplitude

            # Create a new network state with the modified CQT
            new_network_vol = copy.copy(network_state)
            new_network_vol.cqt_states[target_cqt_id] = new_cqt_vol

            # Only add transition if the resulting network state is globally causally valid
            if new_network_vol.validate_global_causality():
                possible_transitions[new_network_vol] = possible_transitions.get(new_network_vol,
0j) + amplitude_vol
        # 2. h_matter (Matter Fluctuation - e.g., charge flip)
        new_matter_charge_flipped = -original_matter_charge
        new_matter_qn_flipped = {'charge': new_matter_charge_flipped} if
new_matter_charge_flipped!= 0 else {}
        amplitude_matter_flip = 0.3j
        try:
            new_cqt_matter = CQTState(target_cqt_id, list(original_cqt.face_spins),
original_volume_qn,
                                    list(original_cqt.matter_spins), new_matter_qn_flipped,
                                    original_cqt.causal_links_data, original_cqt._bqg)
        except ValueError:
            continue

        new_network_matter = copy.copy(network_state)
        new_network_matter.cqt_states[target_cqt_id] = new_cqt_matter;
        if new_network_matter.validate_global_causality():
            possible_transitions[new_network_matter] =
possible_transitions.get(new_network_matter, 0j) + amplitude_matter_flip
        # 3. h_conn (Connectivity Evolution - Burnorian Splitting/Merging/Rewiring)
        # These operations change the graph topology itself and are highly complex to implement
generally.
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        # For this demonstration, they are conceptual.
        # A full implementation would involve graph rewriting rules that propose new
CQTs/connections
        # and checking their validity (face spin matching, causal consistency).
        # Example: if random.random() < 0.01: # Small chance of a topological change
        # # new_network_topology = perform_burnorian_topological_change(network_state,
target_cqt_id)
        # # if new_network_topology and new_network_topology.validate_global_causality():
        # # amplitude_conn = 0.05j # Amplitude for topological change
        # # possible_transitions[new_network_topology] =
possible_transitions.get(new_network_topology, 0j) + amplitude_conn
    return list(possible_transitions.items())
def construct_burnorian_hamiltonian_matrix(basis_states: list[CQTNetworkGraphState]) ->
np.ndarray:
    """
    Constructs the Burnorian Hamiltonian Operator H_QG as a matrix
    in the given basis of Burnorian CQTNetworkGraphStates.
    """
    dim = len(basis_states)
    hamiltonian_matrix = np.zeros((dim, dim), dtype=complex)
    state_to_index = {state: i for i, state in enumerate(basis_states)}
    for j, state_B in enumerate(basis_states): # Loop over columns (initial states)
        transitions_from_B = apply_burnorian_hamiltonian(state_B)
        for next_state, transition_amplitude in transitions_from_B:
            if next_state in state_to_index:
                i = state_to_index[next_state] # Row index (final state)
                hamiltonian_matrix[i, j] += transition_amplitude
    return hamiltonian_matrix
# --- Step 39: Burnorian Path Integral as Matrix Product ---
def state_to_vector(state: CQTNetworkGraphState, basis: list[CQTNetworkGraphState]) ->
np.ndarray:
    """
    Converts a given CQTNetworkGraphState into its vector representation in the chosen basis.
    Returns a one-hot vector if the state is in the basis, or a zero vector otherwise.
    """
    dim = len(basis)
    vec = np.zeros(dim, dtype=complex)
    if state in basis:
        idx = basis.index(state)
        vec[idx] = 1.0 + 0j
    return vec
def execute_burnorian_path_integral_matrix(
    hamiltonian_matrix: np.ndarray,
    basis_states: list[CQTNetworkGraphState],
    initial_state: CQTNetworkGraphState,
    final_state: CQTNetworkGraphState,
    num_steps: int,
    delta_t: float,
    hbar: float = 1.0
) -> complex:
    """
    Computes the Burnorian Path Integral amplitude using matrix exponentiation
    of the time evolution operator.
    """
    #... (Implementation as in Step 39)...
    dim = len(basis_states)
    initial_vector = state_to_vector(initial_state, basis_states)
    final_vector = state_to_vector(final_state, basis_states)
    if np.all(initial_vector == 0) or np.all(final_vector == 0):
        # print("Warning: Initial or final state not found in basis. Amplitude will be 0.")
        return 0.0 + 0j
    # Time evolution operator U(dt) = exp(-i * H * dt / hbar)
    time_evolution_operator = expm(-1j * hamiltonian_matrix * delta_t / hbar)
    # Total evolution operator U_total = (U(dt))^N
    total_evolution_operator = np.linalg.matrix_power(time_evolution_operator, num_steps)
    # Transition amplitude: <final | U_total | initial>
    amplitude = np.vdot(final_vector, total_evolution_operator @ initial_vector)
    return amplitude
# --- Step 40: Burnorian Observables and Expectation Values ---
def get_avg_volume_qn(network_state: CQTNetworkGraphState) -> float:
    """
    Returns the average 4-volume quantum number for a given network state.
    This is the eigenvalue of the Burnorian Average Volume Operator for this state.
    """
    if not network_state.cqt_states: return 0.0
    total_vol_qn = sum(cqt.intertwiner_geom_qn for cqt in network_state.cqt_states.values())
    return total_vol_qn / len(network_state.cqt_states)
def get_avg_charge_qn(network_state: CQTNetworkGraphState) -> float:
    """
    Returns the average matter charge quantum number for a given network state.
    This is the eigenvalue of the Burnorian Average Matter Charge Operator for this state.
    """
    if not network_state.cqt_states: return 0.0
    total_charge_qn = sum(dict(cqt.matter_gauge_qn).get('charge', 0) for cqt in
network_state.cqt_states.values())
    return total_charge_qn / len(network_state.cqt_states)
def construct_burnorian_observable_matrix(
    observable_func, # Function that takes a state and returns its observable value
    basis_states: list[CQTNetworkGraphState]
) -> np.ndarray:
    """
    Constructs the diagonal matrix representation of a Burnorian Observable
    in the given basis.
    """
    dim = len(basis_states)
    observable_matrix = np.zeros((dim, dim), dtype=complex)
    for i, state in enumerate(basis_states):
        observable_matrix[i, i] = observable_func(state) # Diagonal matrix (eigenvalues on
diagonal)
    return observable_matrix
def compute_burnorian_expectation_value(
    hamiltonian_matrix: np.ndarray,
    observable_matrix: np.ndarray,
    basis_states: list[CQTNetworkGraphState],
    initial_state: CQTNetworkGraphState,
    num_steps: int,
    delta_t: float,
    hbar: float = 1.0
) -> complex:
    """
    Computes the expectation value of a Burnorian Observable after num_steps.
    <O(N)> = <Psi(N) | O | Psi(N)> where |Psi(N)> = U^N |initial_state>.
    """
    dim = len(basis_states)
    initial_vector = state_to_vector(initial_state, basis_states)
    if np.all(initial_vector == 0): raise ValueError("Initial state not found in basis.")
    time_evolution_operator = expm(-1j * hamiltonian_matrix * delta_t / hbar)
    total_evolution_operator = np.linalg.matrix_power(time_evolution_operator, num_steps)
    evolved_state_vector = total_evolution_operator @ initial_vector

    # Normalize the evolved state vector (important for expectation value calculation)
    norm_squared = np.vdot(evolved_state_vector, evolved_state_vector).real
    if norm_squared < 1e-12: # Check for near-zero norm (state decayed or invalid path)
        return 0.0 + 0j
    normalized_evolved_state_vector = evolved_state_vector / cmath.sqrt(norm_squared)
    # Compute expectation value: <Psi(N) | O | Psi(N)>
    expectation_value = np.vdot(normalized_evolved_state_vector, observable_matrix @
normalized_evolved_state_vector)
    return expectation_value
# --- Step 41: Burnorian Coarse-Graining Transformation ---
def perform_burnorian_coarse_graining(fine_grained_network: CQTNetworkGraphState,
bqg_instance: BurnorianQuantumGroup) -> CQTNetworkGraphState:
    """
    Simulates a Burnorian Coarse-Graining transformation step on a network ensemble member.
    This is a simplified blocking/decimation heuristic for demonstration purposes.
    Groups connected CQTs into 'blocks' and replaces each block with a single effective CQT.
    """
    coarse_grained_cqts = {}
    if not fine_grained_network.cqt_states: return CQTNetworkGraphState({})
    processed_cqt_ids = set()
    current_coarse_cqt_id = 1 # Start ID for new coarse CQTs
    # Iteratively find connected pairs and combine them
    for cqt_id_1 in sorted(fine_grained_network.cqt_states.keys()):
        if cqt_id_1 in processed_cqt_ids: continue
        cqt_1 = fine_grained_network.cqt_states[cqt_id_1]

        # Find a suitable neighbor to block with
        best_neighbor_id = None
        for conn_key in fine_grained_network.connections:
            if (cqt_id_1, 0) in conn_key: # Check if cqt_id_1 is involved in this connection (simplified)
                other_node_info = next(item for item in conn_key if item[0]!= cqt_id_1)
                neighbor_id = other_node_info[0]
                if neighbor_id not in processed_cqt_ids:
                    best_neighbor_id = neighbor_id
                    break # Found a neighbor to block with
        if best_neighbor_id is not None:
            cqt_2 = fine_grained_network.cqt_states[best_neighbor_id]

            # --- Perform Blocking: Coarse-grained CQT properties are averaged/summed ---
            new_intertwiner_geom_qn = (cqt_1.intertwiner_geom_qn + cqt_2.intertwiner_geom_qn)
/ 2
            new_intertwiner_geom_qn = max(0.5, round(new_intertwiner_geom_qn * 2) / 2.0) #
Ensure valid QN
            new_charge = dict(cqt_1.matter_gauge_qn).get('charge', 0) +
dict(cqt_2.matter_gauge_qn).get('charge', 0)
            new_matter_gauge_qn = {'charge': new_charge}

            new_face_spins = list(cqt_1.face_spins) # Retain representative or average
            new_causal_links_data = copy.deepcopy(cqt_1.causal_links_data) # Simplified copy
            try:
                effective_cqt = CQTState(current_coarse_cqt_id, new_face_spins,
new_intertwiner_geom_qn,
                                         [], new_matter_gauge_qn, new_causal_links_data, bqg_instance)
                coarse_grained_cqts[effective_cqt.cqt_id] = effective_cqt
                processed_cqt_ids.add(cqt_id_1)
                processed_cqt_ids.add(best_neighbor_id)
                current_coarse_cqt_id += 1
            except ValueError: # If coarse-grained CQT is invalid, it is effectively decimated
                pass # This block contributes nothing to the coarse-grained network
        else:
            # If CQT has no unprocessed neighbors, keep it as is (no change in scale) or decimate
            if cqt_id_1 not in processed_cqt_ids:
                coarse_grained_cqts[current_coarse_cqt_id] = copy.copy(cqt_1)
                coarse_grained_cqts[current_coarse_cqt_id].cqt_id = current_coarse_cqt_id # Re-ID
                processed_cqt_ids.add(cqt_id_1)
                current_coarse_cqt_id += 1

    # Re-establish connections in the coarse-grained network (highly simplified - no connections
preserved here)
    coarse_network = CQTNetworkGraphState(coarse_grained_cqts)
    return coarse_network
# --- Step 43: Burnorian Quantum Monte Carlo (QMC) Simulation ---
# (Architectural outline, with energy proxy and move proposals)
class BurnorianQMC:
    """
    Architectural design for a Burnorian Quantum Monte Carlo simulation.
    Generates an ensemble of 4D Burnorian Causal Spacetime CQT Networks.
    """
    def __init__(self, bqg_instance: BurnorianQuantumGroup, initial_network:
CQTNetworkGraphState):
        self.bqg_instance = bqg_instance
        self.current_network = initial_network # The 4D CQT network
        self.hbar = 1.0 # Normalized Planck constant
        self.energy_proxy = self._calculate_network_energy_proxy(initial_network)
        # causal_links_data_template_global is needed for add_cqt move
        self._causal_links_template = self._extract_causal_links_template(initial_network)
    def _extract_causal_links_template(self, network: CQTNetworkGraphState) -> dict:
        if network.cqt_states:
            return copy.deepcopy(next(iter(network.cqt_states.values())).causal_links_data)
        return {} # Default empty if no CQTs
    def _calculate_network_energy_proxy(self, network_state: CQTNetworkGraphState) -> float:
        """
        Calculates a proxy for the Euclidean action/energy (S_E[P]) of the 4D CQT network.
        Includes the Burnorian Repulsive Potential.
        """
        total_energy = 0.0
        for cqt in network_state.cqt_states.values():
            vol_qn = cqt.intertwiner_geom_qn
            if vol_qn < 0.51: # Close to minimum volume
```

```
                total_energy += 1000.0 # High energy cost for very small volumes (Burnorian
Repulsion)
            else:
                total_energy += vol_qn * vol_qn * 0.1 # Energy grows with volume (e.g., gravitational
energy proxy)

            charge = dict(cqt.matter_gauge_qn).get('charge', 0)
            total_energy += charge * charge * 0.05 # Small energy from matter charge

        total_energy += len(network_state.connections) * 0.01 # Connectivity energy (small
penalty)
        if not network_state.validate_global_causality():
            return float('inf') # Infinite energy for non-causal networks
        return total_energy
    def _propose_burnorian_move(self) -> tuple[CQTNetworkGraphState, float]:
        """
        Proposes a random local Burnorian move on the 4D CQT network.
        Returns: (proposed_network, proposal_ratio_q_prime_q)
        """
        proposed_network = copy.copy(self.current_network) # Start with current network

        move_type = random.choice(['modify_cqt_vol_charge', 'add_cqt', 'remove_cqt',
'rewire_connection'])

        if not proposed_network.cqt_states and move_type!= 'add_cqt':
             move_type = 'add_cqt' # Force adding a CQT if empty
        if move_type == 'modify_cqt_vol_charge':
            if not proposed_network.cqt_states: return self.current_network, 0.0
            target_cqt_id = random.choice(list(proposed_network.cqt_states.keys()))
            original_cqt = proposed_network.cqt_states[target_cqt_id]

            change_vol = random.choice([-0.5, 0.5])
            new_vol_qn = max(0.5, round((original_cqt.intertwiner_geom_qn + change_vol) * 2) /
2.0)

            new_charge = dict(original_cqt.matter_gauge_qn).get('charge', 0)
            if random.random() < 0.5: new_charge = -new_charge # Flip charge

            try:
                modified_cqt = CQTState(target_cqt_id, list(original_cqt.face_spins), new_vol_qn,
                                        list(original_cqt.matter_spins), {'charge': new_charge},
                                        original_cqt.causal_links_data, self.bqg_instance)
                proposed_network.cqt_states[target_cqt_id] = modified_cqt
            except ValueError: return self.current_network, 0.0 # Invalid CQT after modification
            proposal_ratio = 1.0

        elif move_type == 'add_cqt':
            new_id = proposed_network._next_cqt_id
            try:
                new_cqt_state = CQTState(new_id, [0.5, 0.5, 0.5, 0.5], 0.5, [], {},
                                         self._causal_links_template, self.bqg_instance) # Smallest valid CQT
                proposed_network.cqt_states[new_id] = new_cqt_state
                proposed_network._next_cqt_id += 1
            except ValueError: return self.current_network, 0.0
            proposal_ratio = 1.0

        elif move_type == 'remove_cqt':
            if not proposed_network.cqt_states: return self.current_network, 0.0
            target_cqt_id = random.choice(list(proposed_network.cqt_states.keys()))
            del proposed_network.cqt_states[target_cqt_id]
            # Remove connections involving the deleted CQT
            proposed_network.connections = {k:v for k,v in proposed_network.connections.items()
                                            if target_cqt_id not in [item[0] for item in k]}
            proposal_ratio = 1.0

        elif move_type == 'rewire_connection':
            if len(proposed_network.cqt_states) < 2: return self.current_network, 0.0

            cqt_ids = list(proposed_network.cqt_states.keys())
            id1, id2 = random.sample(cqt_ids, 2)
            face1 = random.randint(0,3)
            face2 = random.randint(0,3)
            try:
                proposed_network.add_connection(id1, face1, id2, face2)
            except ValueError: # If face spins mismatch or invalid connection
                pass # Connection attempt fails, network unchanged for this proposal
            proposal_ratio = 1.0

        # Validate global causal consistency after the move
        if not proposed_network.validate_global_causality():
            return self.current_network, 0.0 # Proposed network is invalid, move rejected

        return proposed_network, proposal_ratio
    def run_simulation(self, num_sweeps: int, thermalization_sweeps: int) ->
list[CQTNetworkGraphState]:
        """
        Executes the Burnorian Quantum Monte Carlo simulation.
        Generates an ensemble of Burnorian Causal Spacetime CQT Networks.
        """
        ensemble = []
        for sweep in range(num_sweeps + thermalization_sweeps):
            proposed_network, proposal_ratio = self._propose_burnorian_move()

            current_energy = self.energy_proxy
            proposed_energy = self._calculate_network_energy_proxy(proposed_network)

            acceptance_prob = min(1.0, proposal_ratio * cmath.exp(-(proposed_energy -
current_energy) / self.hbar).real)

            if random.random() < acceptance_prob:
                self.current_network = proposed_network
                self.energy_proxy = proposed_energy

            if sweep >= thermalization_sweeps:
                ensemble.append(self.current_network)

        return ensemble
    def compute_ensemble_average(self, ensemble: list[CQTNetworkGraphState],
observable_func) -> float:
        """
        Computes the ensemble average of a given observable function over the QMC ensemble.
        """
        if not ensemble: return 0.0
        sum_obs = 0.0
        for network in ensemble:
            sum_obs += observable_func(network)
        return sum_obs / len(ensemble)
# --- Step 44: Burnorian Numerical Renormalization Group (NRG) Algorithm ---
class BurnorianNRG:
    """
    Implements the Burnorian Numerical Renormalization Group algorithm.
    It takes QMC data, iteratively coarse-grains, and tracks effective couplings.
    """
    def __init__(self, bqg_instance: BurnorianQuantumGroup, qmc_initial_network_seed:
CQTNetworkGraphState):
        self.bqg_instance = bqg_instance
        self.qmc_initial_network_seed = qmc_initial_network_seed
        self.rg_flow_data = {
            'scale_factor': [],
            'G_eff': [],
            'Lambda_eff': [],
            'C1_eff': [] # Higher-order curvature coupling (R^2 term)
        }
    def _calculate_effective_couplings(self, ensemble: list[CQTNetworkGraphState],
current_scale_factor: float) -> tuple[float, float, float]:
        """
        Calculates the effective gravitational (G_eff), cosmological (Lambda_eff),
        and higher-order (C1_eff) couplings from the QMC ensemble.
        This simulates the 'fitting' process to a parametrized effective action.
        """
        if not ensemble: return 0.0, 0.0, 0.0
        # In a real NRG, this fitting would involve complex calculations of correlation functions
        # and matching them to an effective action. Here, it's a heuristic based on scale.

        # Simulate convergence to an IR fixed point
        G_IR = 0.5001 # Target value for G_eff (dimensionless)
        Lambda_IR = 0.0012 # Target value for Lambda_eff (dimensionless)
        C1_IR = 0.00005 # Target value for C1_eff (very small in IR)
        # Heuristic flow towards fixed point values
        alpha_rg = 0.5 # RG flow strength/speed

        current_G_eff = G_IR - (G_IR - 0.1) * cmath.exp(-current_scale_factor * 0.05).real # Flows
from 0.1 to G_IR
        current_Lambda_eff = Lambda_IR + (10.0 - Lambda_IR) * cmath.exp(-current_scale_factor
* 0.05).real # Flows from 10.0 to Lambda_IR
        current_C1_eff = C1_IR + (5.0 - C1_IR) * cmath.exp(-current_scale_factor * 0.2).real #
Flows from 5.0 to C1_IR (rapid decay)
        return current_G_eff, current_Lambda_eff, current_C1_eff
    def run_nrg(self, num_rg_steps: int, qmc_sweeps_per_rg_step: int,
qmc_thermalization_sweeps: int):
        """
        Executes the Burnorian Numerical Renormalization Group (NRG) flow.
        """
        # Run initial QMC to get the ensemble at the finest scale (UV)
        initial_qmc = BurnorianQMC(self.bqg_instance, self.qmc_initial_network_seed)
        current_ensemble = initial_qmc.run_simulation(qmc_sweeps_per_rg_step,
qmc_thermalization_sweeps)

        initial_avg_cqts = initial_qmc.compute_ensemble_average(current_ensemble, lambda net:
len(net.cqt_states))

        initial_G, initial_Lambda, initial_C1 =
self._calculate_effective_couplings(current_ensemble, initial_avg_cqts)

        self.rg_flow_data['scale_factor'].append(initial_avg_cqts)
        self.rg_flow_data['G_eff'].append(initial_G)
        self.rg_flow_data['Lambda_eff'].append(initial_Lambda)
        self.rg_flow_data['C1_eff'].append(initial_C1)

        print(f"RG Step 0 (Avg CQTs={initial_avg_cqts:.2f}): G_eff={initial_G:.4f},
Lambda_eff={initial_Lambda:.4f}, C1_eff={initial_C1:.4f}")
        current_avg_cqts = initial_avg_cqts

        for i in range(num_rg_steps):
            if current_avg_cqts < 2.0: # Stop if network is too small to coarse-grain meaningfully
                print("RG flow terminated: Network too small to coarse-grain further.")
                break
            coarse_grained_ensemble = []
            for net in current_ensemble:
                coarse_net = perform_burnorian_coarse_graining(net, self.bqg_instance)
                if coarse_net.cqt_states:
                    coarse_grained_ensemble.append(coarse_net)

            if not coarse_grained_ensemble or len(coarse_grained_ensemble) <
len(current_ensemble) / 2: # Check for effective coarse-graining
                print("RG flow terminated: Coarse-graining produced insufficient valid networks.")
                break

            current_ensemble = coarse_grained_ensemble

            # Re-run QMC for this coarser ensemble to get statistically robust data at new scale
            # (In full NRG, this would be a re-sampling or direct evaluation of new effective
Hamiltonian)
            temp_qmc = BurnorianQMC(self.bqg_instance, current_ensemble[0]) # Start new QMC
from a member of coarse ensemble
            current_ensemble = temp_qmc.run_simulation(qmc_sweeps_per_rg_step,
qmc_thermalization_sweeps)
            current_avg_cqts = temp_qmc.compute_ensemble_average(current_ensemble, lambda
net: len(net.cqt_states))
            if current_avg_cqts < 2.0: # Re-check after re-sampling
                 print("RG flow terminated: Coarse-grained network too small after re-sampling.")
                 break
            G_eff, Lambda_eff, C1_eff = self._calculate_effective_couplings(current_ensemble,
current_avg_cqts)

            self.rg_flow_data['scale_factor'].append(current_avg_cqts)
            self.rg_flow_data['G_eff'].append(G_eff)
            self.rg_flow_data['Lambda_eff'].append(Lambda_eff)
            self.rg_flow_data['C1_eff'].append(C1_eff)
            print(f"RG Step {i+1} (Avg CQTs={current_avg_cqts:.2f}): G_eff={G_eff:.4f},
Lambda_eff={Lambda_eff:.4f}, C1_eff={C1_eff:.4f}")

        print("--- Burnorian NRG Flow Finished ---")
# --- Global Causal Links Data Template (needed for QMC moves) ---
# This would represent a canonical template for a simple CQT structure.
causal_links_data_template_global = {
    frozenset({0, 1}): {'type': 'timelike', 'length': 1.5 * PLANCK_LENGTH, 'causal_order': (0, 1)},
    frozenset({1, 2}): {'type': 'spacelike', 'length': 1.1 * PLANCK_LENGTH, 'causal_order': None},
# Example
    frozenset({0, 2}): {'type': 'timelike', 'length': 2.0 * PLANCK_LENGTH, 'causal_order': (0, 2)},
    frozenset({0, 3}): {'type': 'timelike', 'length': 2.2 * PLANCK_LENGTH, 'causal_order': (0, 3)},
    frozenset({1, 3}): {'type': 'spacelike', 'length': 1.0 * PLANCK_LENGTH, 'causal_order': None},
    frozenset({2, 3}): {'type': 'timelike', 'length': 1.2 * PLANCK_LENGTH, 'causal_order': (2, 3)},
    frozenset({0, 4}): {'type': 'timelike', 'length': 3.0 * PLANCK_LENGTH, 'causal_order': (0, 4)},
    frozenset({1, 4}): {'type': 'spacelike', 'length': 2.5 * PLANCK_LENGTH, 'causal_order': None},
    frozenset({2, 4}): {'type': 'timelike', 'length': 1.7 * PLANCK_LENGTH, 'causal_order': (2, 4)},
    frozenset({3, 4}): {'type': 'timelike', 'length': 1.0 * PLANCK_LENGTH, 'causal_order': (3, 4)},
}
```