

## UNIT-4:

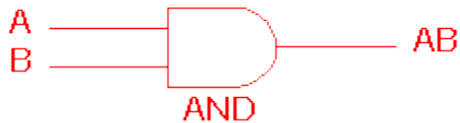
LOGIC GATE  
And  
Boolean Algebra  
K-MAP

# Logic Gate

- Digital systems are said to be constructed by using logic gates.
- These gates are the AND, OR, NOT, NAND, NOR, EXOR and EXNOR gates.
- The basic operations are described with the aid of truth tables.
- Boolean functions practically implemented by using electronic gates
- Generally logic gate have 2 input 1 output
- Gate **INPUTS** are driven by voltages having two nominal values,  
0V logic 0 and 5V logic 1
- The **OUTPUT** of a gate provides two nominal values of voltage only  
0V logic 0 and 5V logic 1

# Logic Gate

## AND gate

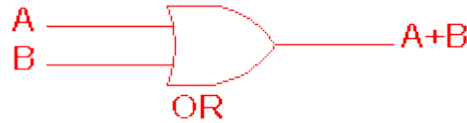


**2 Input AND gate**

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

**high** output (1) only if **all** its inputs are high

## OR gate

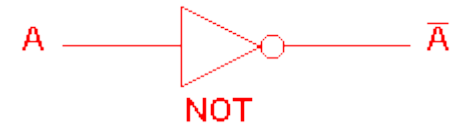


**2 Input OR gate**

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

high output (1) if **one or more** of its inputs are high.

## NOT gate



**NOT gate**

A	$\bar{A}$
0	1
1	0

produces an inverted version of the input at its output.

# Logic Gate

## NAND GATE

AND gate followed by a NOT gate.



2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

**high** output (1) only if **all** its inputs are low

## NOR GATE

OR gate followed by a NOT gate.



2 Input NOR gate		
A	B	$\overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

Low output (0) if **one or more** of its inputs are high.

NAND and NOR gates are called *universal*

# Logic Gate

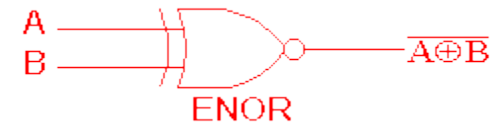
## EXOR GATE/XOR



2 Input EXOR gate		
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

high output (1) for different input

## EXNOR GATE/XNOR

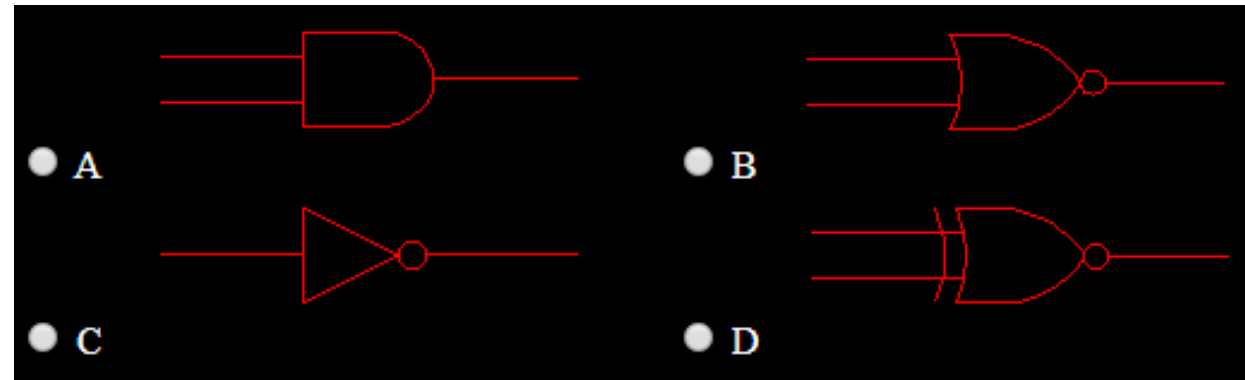


2 Input EXNOR gate		
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

High output (1) for same input

# Practice Questions

Which of the following symbols represents a NOR gate?



Which one of the following truth tables represents the behavior a NAND gate?

☐ A

2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	1
0	1	0
1	0	0
1	1	0

☐ B

2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	0
0	1	1
1	0	1
1	1	0

☐ C

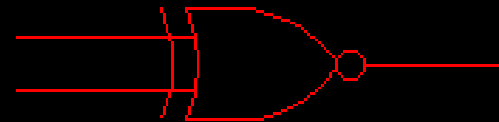
2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	0
0	1	0
1	0	0
1	1	1

☐ D

2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

# Practice Questions

What type of logic gate does this symbol represent?



- ☐ Exclusive OR
- ☐ Exclusive NOR
- ☐ OR
- ☐ NOR

What type of logic gate's behavior does this truth table represent?

?			
A	B	C	?
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- ☐ 2 input OR
- ☐ 3 input OR
- ☐ 3 input EXOR
- ☐ 4 input EXOR

# Practice Questions

The output of an AND gate with three inputs, A, B, and C, is HIGH when \_\_\_\_\_.

- A.         $A = 1, B = 1, C = 0$
- B.         $A = 0, B = 0, C = 0$
- C.         $A = 1, B = 1, C = 1$
- D.         $A = 1, B = 0, C = 1$

If the input to a NOT gate is A and the output is X, then \_\_\_\_\_.

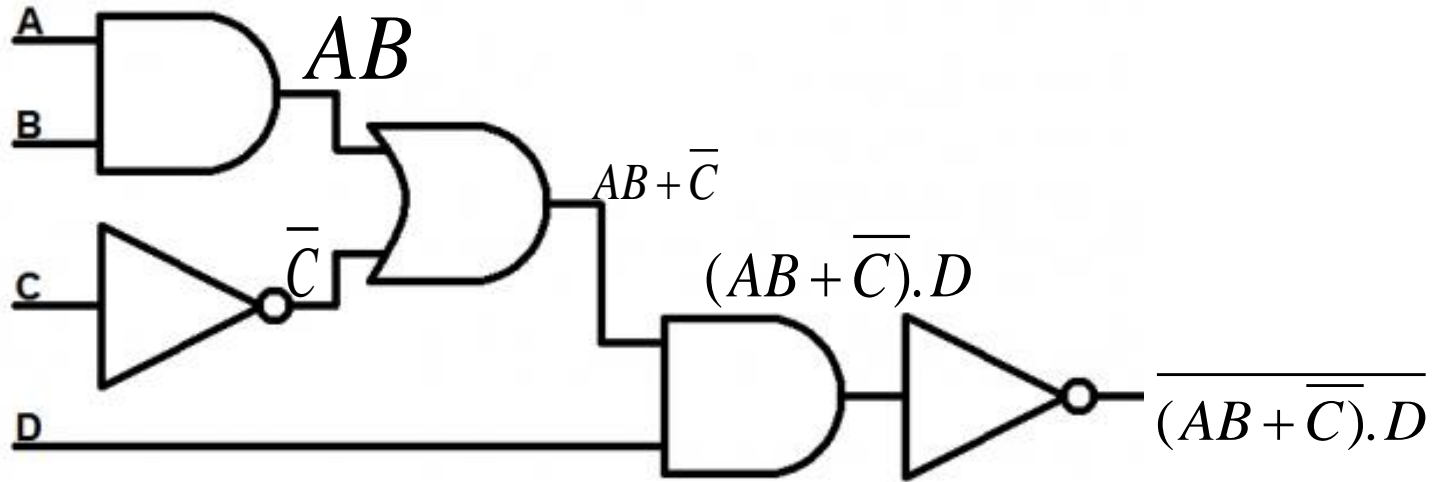
- A.         $X = A$
- B.         $A = \sim A$
- C.         $X = 0$
- D.        none of the above

How many inputs of a four-input AND gate must be HIGH in order for the output of the logic gate to go HIGH?

- A.        any one of the inputs
- B.        any two of the inputs
- C.        any three of the inputs
- D.        all four inputs



# Circuit with Logic Gate



Draw circuit for  $Y = AB + \bar{A}C$

# Boolean Algebra

Analyze and simplify the digital (logic) circuits

## Commutative law

$$(i) A.B = B.A$$

$$(ii) A + B = B + A$$

## Associative law

$$(i) (A.B).C = A.(B.C)$$

$$(ii) (A + B) + C = A + (B + C)$$

## Distributive law

$$A.(B + C) = A.B + A.C$$

## AND law

$$(i) A.0 = 0$$

$$(ii) A.1 = A$$

$$(iii) A.A = A$$

$$(iv) A.\overline{A} = 0$$

## OR law

$$(i) A + 0 = A$$

$$(ii) A + 1 = 1$$

$$(iii) A + A = A$$

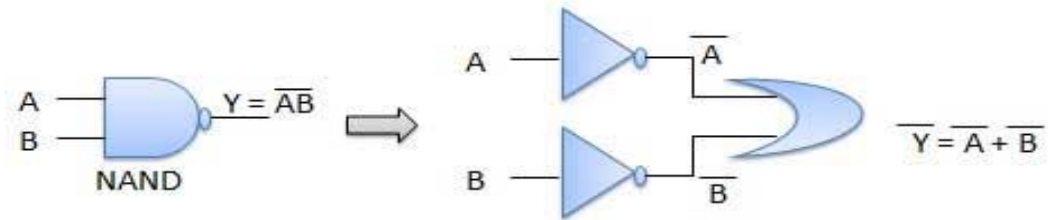
$$(iv) A + \overline{A} = 1$$

## INVERSION law

$$\overline{\overline{A}} = A$$

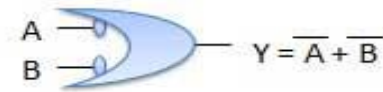
# De Morgan Law

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$



NAND = Bubbled OR

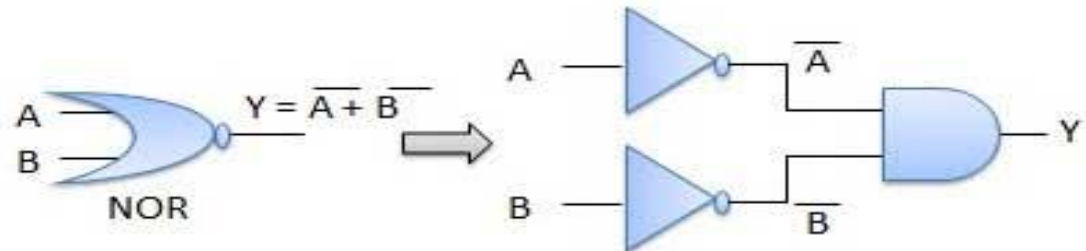
NAND  $\equiv$  Bubbled OR



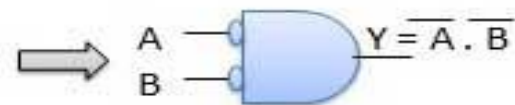
Bubbled OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

NOR = Bubbled AND



NOR  $\equiv$  Bubbled AND



Bubbled AND

# Simplification

*Simplify*  $C + \overline{BC}$

$$C + (\overline{B} + \overline{C})$$

$$(C + \overline{C}) + [\overline{B}]$$

$$1 + \overline{B}$$

$$1$$

*Simplify*  $F = ABC + A + A\overline{BC}$

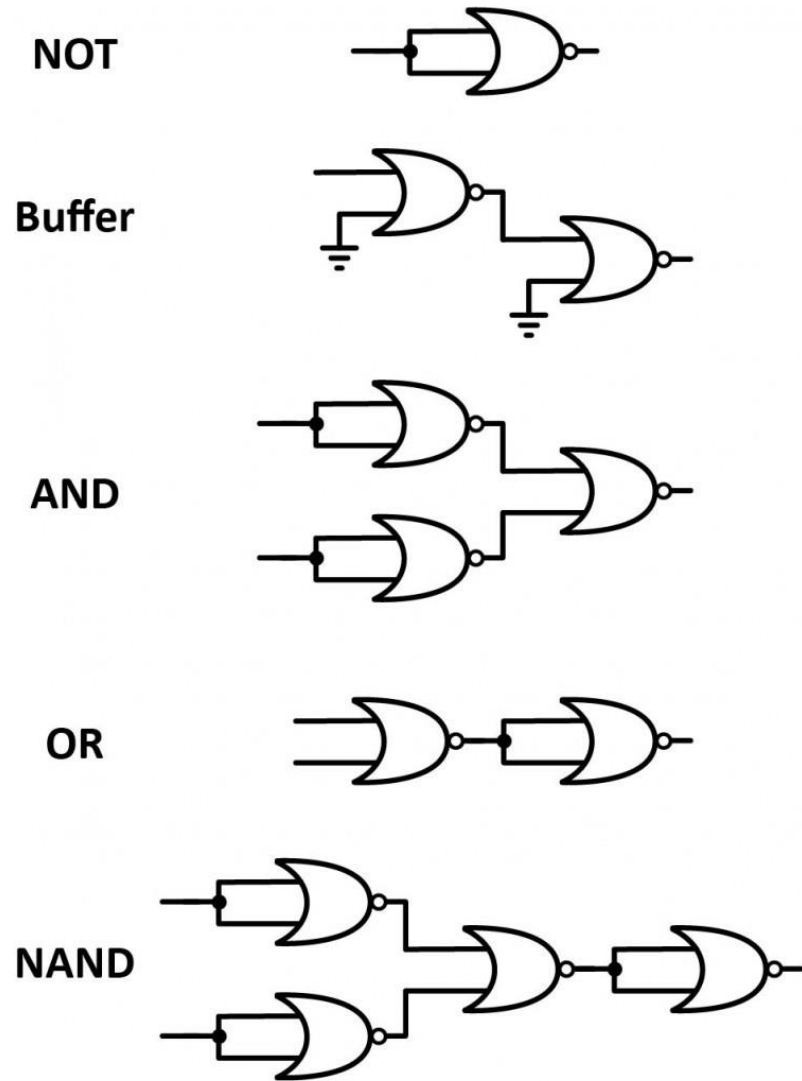
$$AC(B + \overline{B}) + A$$

$$AC + A$$

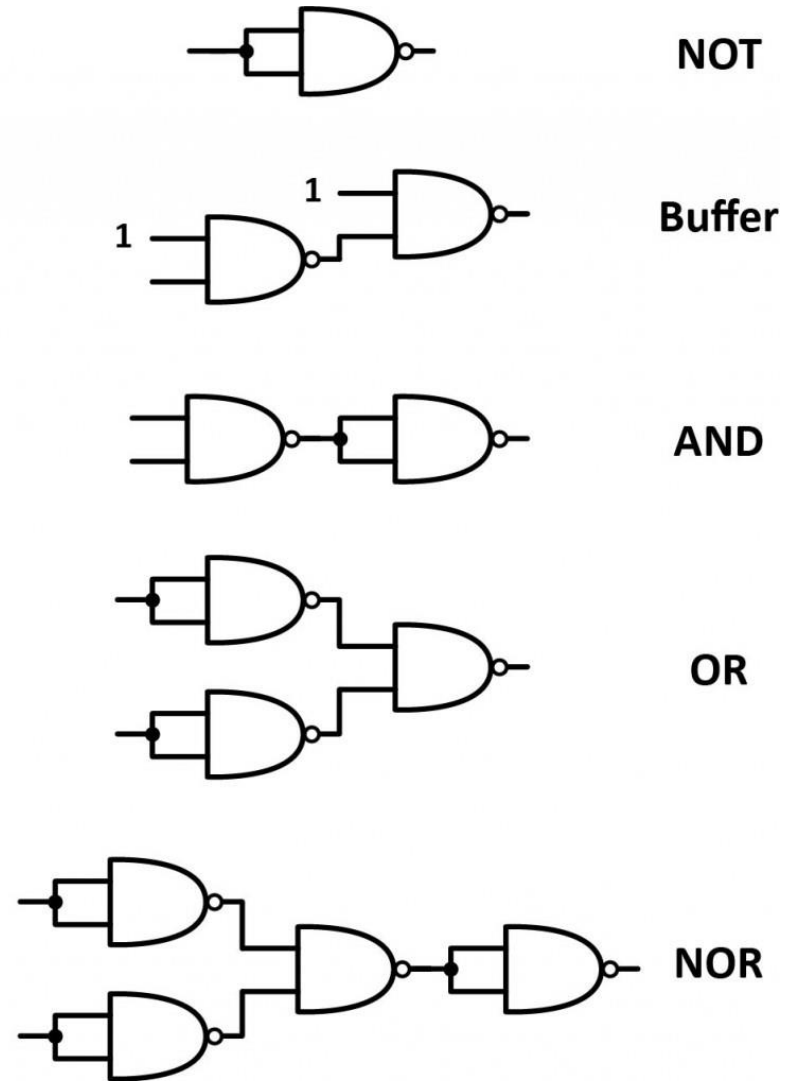
$$A(C + 1)$$

$$A$$

# Logic Gate Implement with NAND-NOR



**NOR**



**NAND**

# SOP-POS

Boolean function is an algebraic form of Boolean expression

Sum-of-Products (**SOP**) - variables are operated by AND (product) are OR(sum) together

Product-of-sums (**POS**) - variables are operated by OR (sum) are AND (product) together

## SOP Expression

- Write AND term for each input combination produces HIGH
- Write the input variables for 1 and compliment for 0.
- OR the AND terms to obtain the output function.

$$F(\text{SOP}) = A'BC + AB'C + ABC' + ABC$$

## POS Expression

- Write OR term for each input combination produces LOW
- Write the input variables for 0 and complement for 1
- AND the OR terms to obtain the output function

$$F(\text{POS}) = F = (A + B + C) (A + B + C') (A + B' + C) (A' + B + C)$$

POS is compliment of SOP

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

# Min Term –Max Term

Variables			Min terms	Max terms
A	B	C	$m_i$	$M_i$
0	0	0	$A' B' C' = m\ 0$	$A + B + C = M\ 0$
0	0	1	$A' B' C = m\ 1$	$A + B + C' = M\ 1$
0	1	0	$A' B C' = m\ 2$	$A + B' + C = M\ 2$
0	1	1	$A' B C = m\ 3$	$A + B' + C' = M\ 3$
1	0	0	$A B' C' = m\ 4$	$A' + B + C = M\ 4$
1	0	1	$A B' C = m\ 5$	$A' + B + C' = M\ 5$
1	1	0	$A B C' = m\ 6$	$A' + B' + C = M\ 6$
1	1	1	$A B C = m\ 7$	$A' + B' + C' = M\ 7$

Write SOP expression for min term  $F(A, B, C) = \sum m(1, 2, 3)$

In binary 01 10 11

$$= \bar{A}B + A\bar{B} + AB$$

Write POS expression for min term  $F(A, B, C) = \pi M(1, 2, 3)$

In binary = 01 10 11

$$(A + \bar{B}).(\bar{A} + B).(\bar{A} + \bar{B})$$

Write SOP expression for min term  $F(A, B) = \sum m(1, 2, 3)$

$$\begin{aligned} \text{In binary} \quad & 01 \quad 10 \quad 11 \\ & = \overline{A}B + A\overline{B} + AB \end{aligned}$$

Write SOP expression for  $F(A, B, C) = \sum m(2, 4, 6, 7)$

$$\begin{aligned} \text{In binary} \quad & 010 \quad 100 \quad 110 \quad 111 \\ & \overline{A}B\overline{C} + A\overline{B}\overline{C} + AB\overline{C} + ABC \end{aligned}$$



# SOP-POS Conversion

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

- The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

- There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

# MCQ

A small circle on the output of a logic gate is used to represent

- A) NOT
- B) BUF

Output will be a LOW for any case when one or more inputs are zero

- A) AND
- B) OR

How many two-input AND and OR gates are required to realize  $Y = CD + EF + G$

- A) 2,3
- B) 2,2

Which is XNOR gate equation

A)  $AB + (\sim A)(\sim B)$

B)  $\sim AB + A(\sim B)$

If one input of XOR gate is connected to high terminal, equivalent to

A) NOT

B) BUF

Which is not correct

A)  $A.1=1$

B)  $A+A=A$

C)  $A+1=1$

D)  $A.A=A$

POS is compliment of SOP

- A) True
- B) False

min term when  $x=0$ ,  $y=0$  and  $z=1$

- A)  $x'y'z$
- B)  $x+y+z'$

$Y=AB+BC+AC$  is

- A) SOP
- B) POS

$Y=(A+B)(B+C)(C+A)$  shows

- A) POS
- B) SOP

# K-MAP

Karnaugh map is a tool for simplification of Boolean algebra

K-Map diagram is made up of squares

K-map is a graphical representation of SOP (Minterm)

K-Map extensively reduce the calculation and provides best minimized solution

K-map solve the expression with grouping of neighbor cells

Y \ X	0	1
0		
1		

2 variable

Y Z \ X	0	1
00		
01		
11		
10		

3 variable

Z \ XY	00	01	11	10
0				
1				

3 variable

Y Z \ WX	00	01	11	10
00				
01				
11				
10				

4 variable

# Kmap Simplification Rule

- 1) Construct k-map and place 1's in the squares according to the truth table.
- 2) Groupings can contain only 1s
- 3) Groups can be formed only at right angles; diagonal groups are not allowed.
- 4) The number of 1's in a group must be a power of 2
- 5) The groups must **be made as large** as possible.
- 6) Groups can overlap and wrap around the sides of the Kmap.
- 7) Every group puts a term in the solution

## Optimized Solution

Minimum number of group

Each group covers maximum possible squares

# Example

$$\text{Out} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C$$

A \ BC	00	01	11	10
0	1	1		
1				

$$\text{Out} = \overline{A}\overline{B}$$

$$\text{Out} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC$$

A \ BC	00	01	11	10
0	1	1	1	1
1				

$$\text{Out} = \overline{A}$$

A \ BC	00	01	11	10
0			1	
1		1	1	1

$$\text{Output} = AB + BC + AC$$

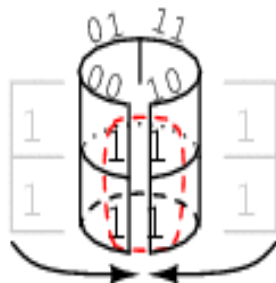
$$\text{Out} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + ABC + AB\overline{C}$$

A \ BC	00	01	11	10
0	1	1	1	1
1			1	1

$$\text{Out} = \overline{A} + B$$

$$\text{Out} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC$$

A \ BC	00	01	11	10
0	1			1
1	1			1



$$\text{Out} = \overline{C}$$

$$\text{Out} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C$$

A \ BC	00	01	11	10
0	1	1	1	1
1	1			1

$$\text{Out} = \overline{A} + \overline{C}$$



$$\begin{aligned}
 \text{Out} = & \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD \\
 & + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD \\
 & + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD
 \end{aligned}$$

$$f(A, B, C, D) = \sum m(0, 1, 3, 4, 5, 7, 12, 13, 15)$$

A \ B	C D			
	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	0	0	0

$$\bar{A}\bar{C} + \bar{A}D + B\bar{C} + BD$$

$$f(A, B, C, D) = \prod M(2, 6, 8, 9, 10, 11, 14)$$

A \ B	C D			
	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	0	0	0

$$f(A, B, C, D) = (\bar{A} + B)(\bar{C} + D)$$