STUDY OF DEAD RECKONING AND SENSOR FUSION

1. INTRODUCTION:

This report deals with the study of the implementation of Sensor fusion of IMU and GPS sensors. The report discusses the Magnetometer calibration, obtaining heading from the Magnetometer using Complementary filter, finding velocity from the accelerometer and finally dead reckoning is done using the velocity and heading estimate from a known starting point.

2. SETUP AND DATA COLLECTION:

To do the dead reckoning using the IMU, two tests were done. The first test was done to calibrate the magnetometer by moving in a circle and the second test was done to collect the data in a set path. To do the testing, IMU and GPS were attached to the vehicle body. The IMU was kept in horizontal position, and it was rigidly attached using electrical tape to the arm rest point at the center of the vehicle. The GPS was attached to the roof of the vehicle so that it could get uninterrupted data. Both sensors were connected to one laptop, a launch file was run, and data was collected in a bag file.

In the first test, the vehicle was rotated in a circle 4-5 times and data was collected. Now while keeping the vehicle in on condition, the vehicle was moved in a set path for 15 minutes and data was again collected for this path.

3. ANALYSIS:

3.1. Magnetometer Calibration:

The IMU data collected while moving in circles was analyzed. The magnetic field in Z axis was plotted. The ideal plot would be a circle with a center at the origin. But the actual plot was an ellipse and shifted from the origin which proves the following error present in the data:

i. **Hard-iron distortion:** Hard iron sources are those which produce their own magnetic field, and it causes a permanent bias, which results in the shifting of center from the origin, but it will not change the shape of the circle.

ii. **Soft-iron distortion:** Soft iron sources are those which do not produce their own magnetic field but show magnetic properties such as ferromagnetic materials (iron, nickel). As the earth magnetic field passes through these materials, the field lines get distorted and thus these distortions result in the distortion of circle to ellipse.

As we can see from fig. 1, it is displaced from the origin and rotated and distorted which confirms the presence of both hard iron and soft iron distortion [1].

So, to remove these errors and calibrate the magnetometer reading, the following three matrices were found from the magnetometer data:

I. **Translation matrix**: It will bring the ellipse to the center.

II. **Rotational matrix**: It will rotate the ellipse to make it parallel to the axis of plot.

III. **Scaling matrix**: It will transform the ellipse to circle.

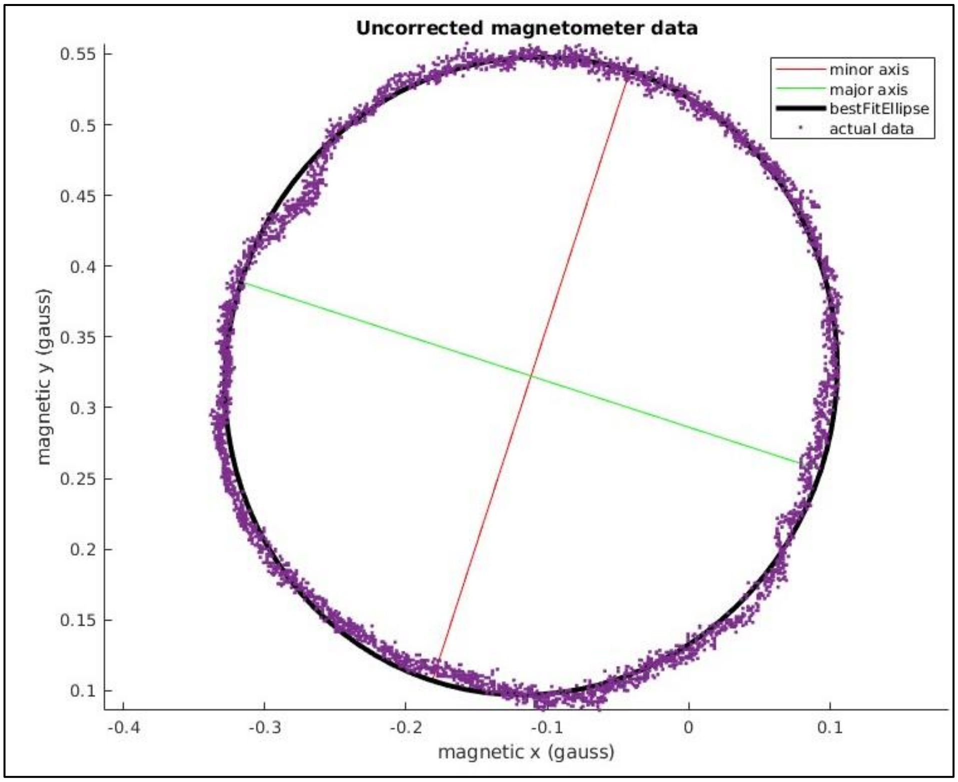
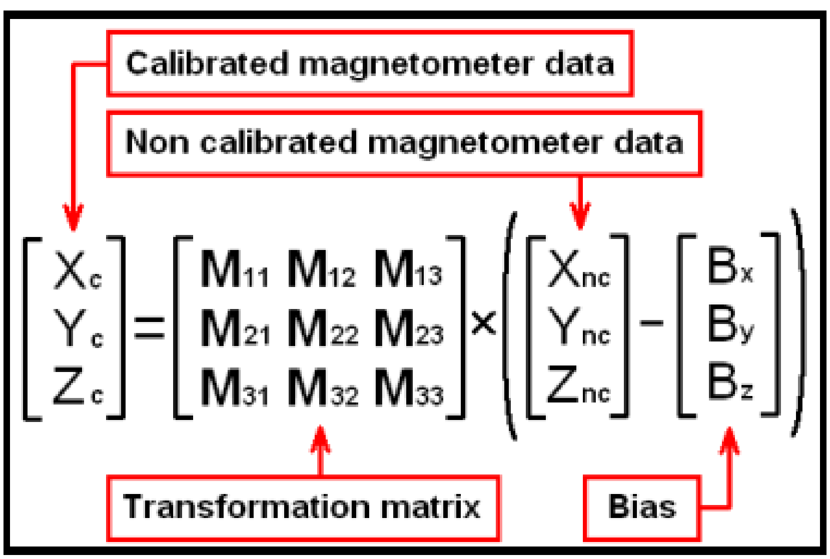


Fig. 1 Uncalibrated Magnetometer data



$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \times \left(\begin{bmatrix} X_{nc} \\ Y_{nc} \\ Z_{nc} \end{bmatrix} - \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \right)$$

Fig. 2 Matrix transformation for removing Hard Iron and Soft Iron distortion

In Figure 2, Bias represents the Translation matrix and Transformation matrix represents the Multiplication of Rotational and Scaling matrix. These matrixes were found from the data and calibrated magnetometer data was calculated.

Fig. 3 shows the Uncalibrated and calibrated magnetometer data. It is clear from figure 3 that the hard iron and soft iron error has been removed as the calibrated data is circle and centered at origin.

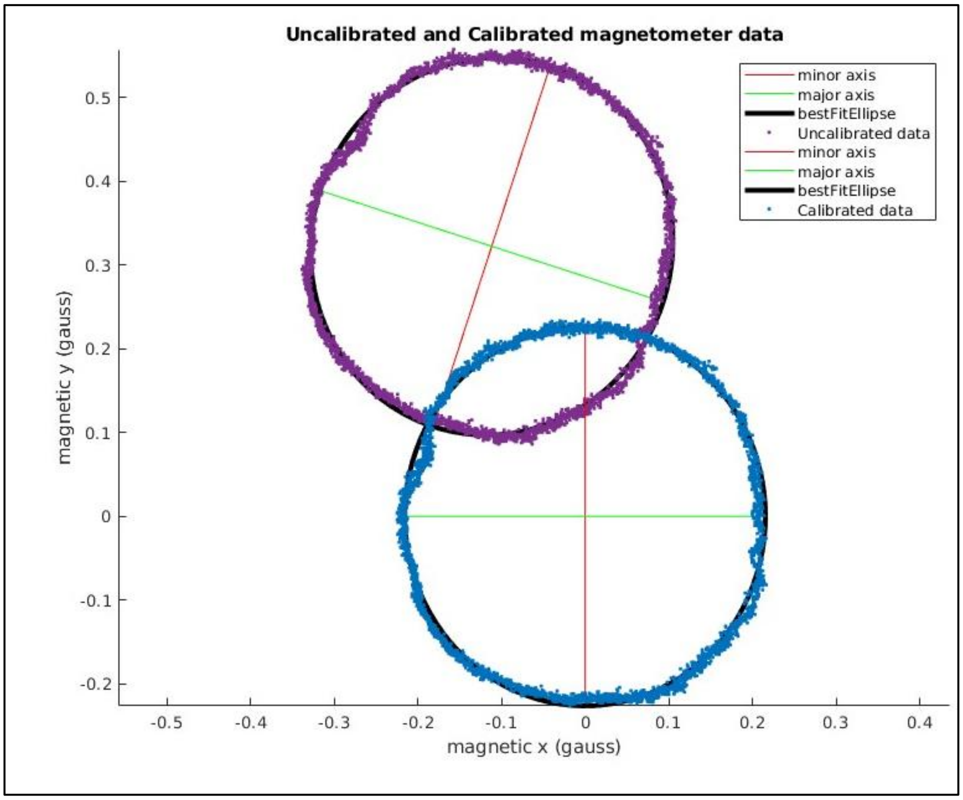


Fig. 3 Uncalibrated and Calibrated magnetometer data

3.2. Complimentary Filter using Magnetometer Yaw and Gyroscope Yaw:

3.2.1. Magnetometer Yaw

This magnetometer data was used to find the yaw using the formulae below [2]:

$$yaw = a\tan 2\left(-\frac{mag(y)}{mag(x)}\right)$$

Where mag(y) represents the magnetic field in y and max(x) represents the magnetic field in x direction.

The magnetometer data that we got while driving during the second test was uncalibrated and before analyzing the magnetometer yaw, data was corrected using the transformation matrix calculated during calibration of magnetometer.

Figure 4 shows the plot of yaw calculated using Calibrated and Uncalibrated Magnetometer data. For further analysis, only calibrated magnetometer yaw is used.

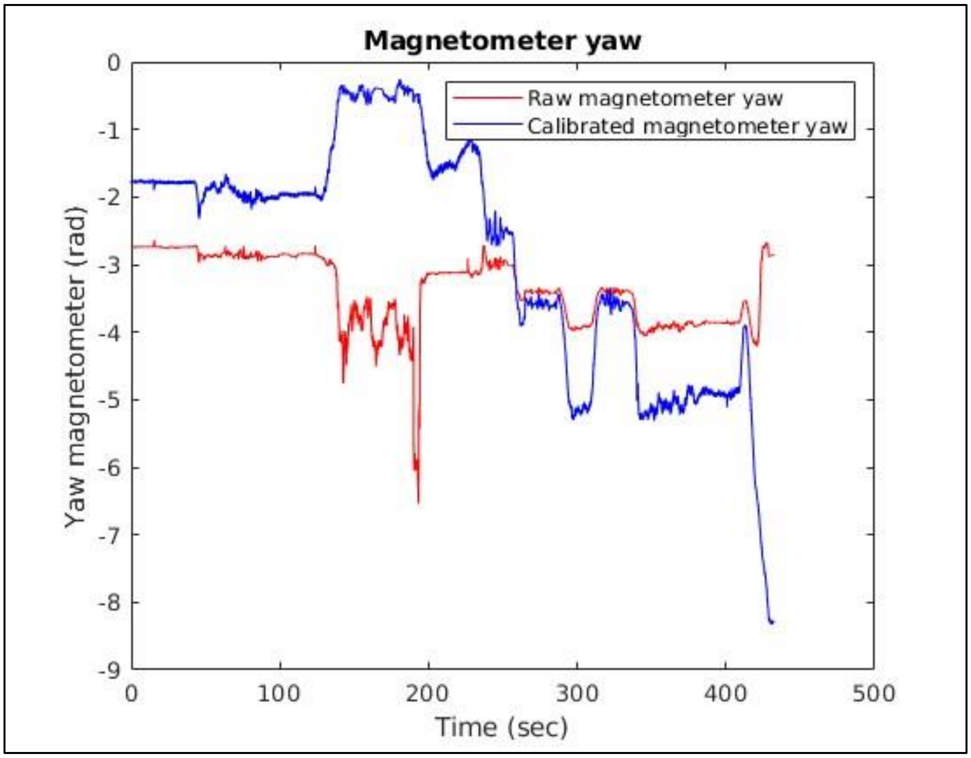


Fig. 4. Magnetometer Yaw from calibrated and Uncalibrated data

3.2.2. Gyroscope Yaw

Also, Yaw can be calculated from Gyroscope Yaw rate by integrating its angular velocity in Z direction.

Figure 5 shows the Yaw calculated using Gyroscope angular velocity. Figure 6 shows the comparison of magnetometer yaw and gyroscope yaw.

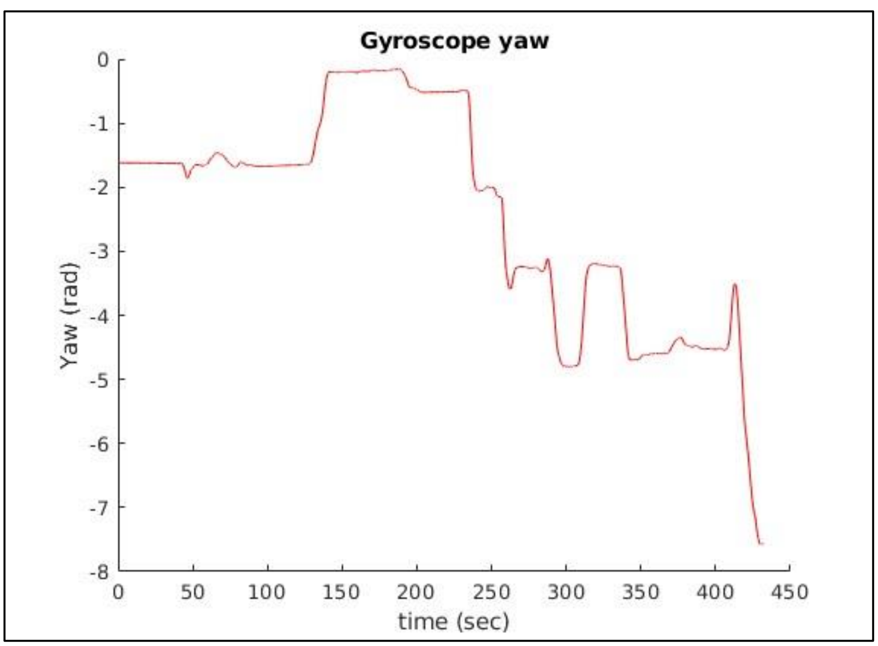


Fig. 5. Gyroscope Z angular velocity integrated yaw

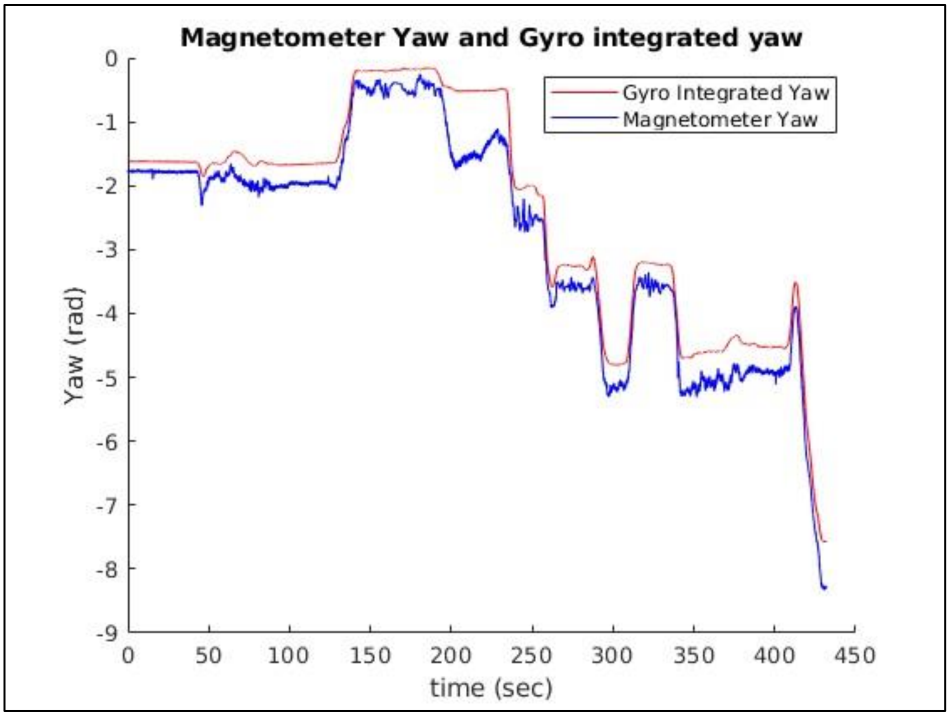


Fig. 6. Comparison of Magnetometer Yaw and Gyroscope Yaw

3.2.3. Complimentary Filter

As we know, yaw angle from magnetometer contains high frequency noises which need to be removed but is a good estimate for long term changes.

On the other hand, Yaw angle from the Gyroscope works well for short time period but the yaw will drift over time because of noise and bias that is present in Gyroscope.

So, Sensor fusion is done in which a Complimentary filter is used to find a better yaw estimate by utilizing both the magnetometer yaw and gyroscope yaw.

For getting a yaw through Complimentary filter, the magnetometer yaw data is passed through a Low-Pass filter which filters out the short-term fluctuations (high frequency noise) and provides a smooth curve with long-term changes only and the Gyroscope yaw is passed through a High-Pass filter which allow short term duration signal to pass while filters out signals that are steady over time (low frequency noise), thus cancels out the drift. Then the Low-Pass and High-Pass output are added to find the Yaw estimate from the Complimentary filter. The following figures illustrate the working of Complimentary filter:

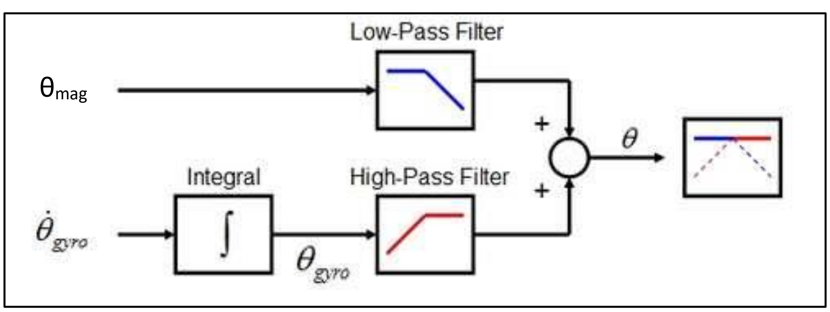


Fig. 7. Working of Complimentary Filter

The cutoff frequency used for the Low-Pass and the High-Pass filter is 0.5 Hz. Figure 8 shows the plots of yaw after Low-Pass and High-Pass filter.

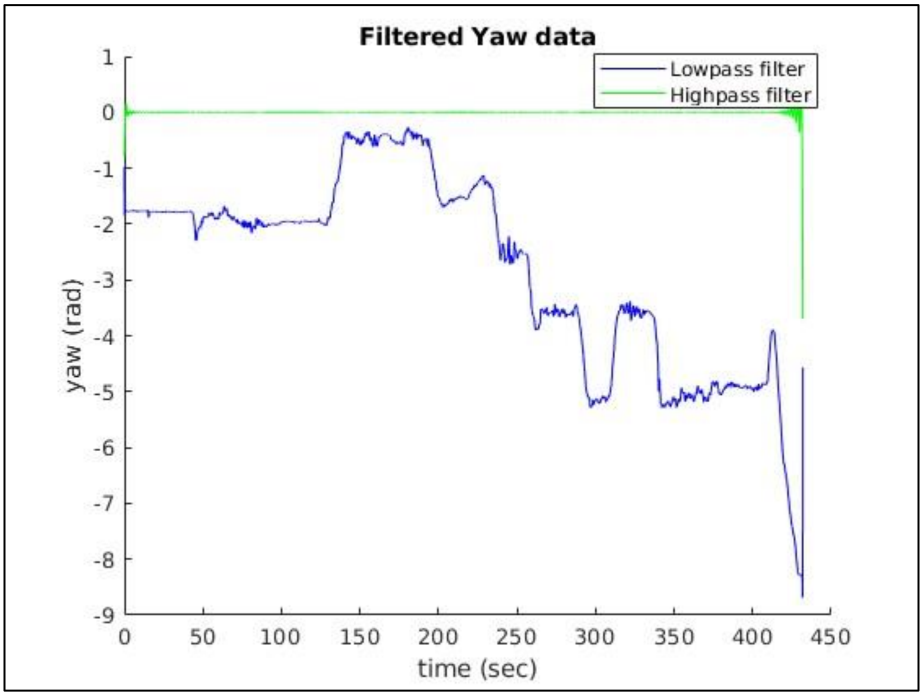


Fig. 8. Yaw plot after using Lowpass and Highpass filter

Figure 9 shows the Yaw data from the Complimentary Filter and Yaw data from IMU which is calculated using the Kalman filter.

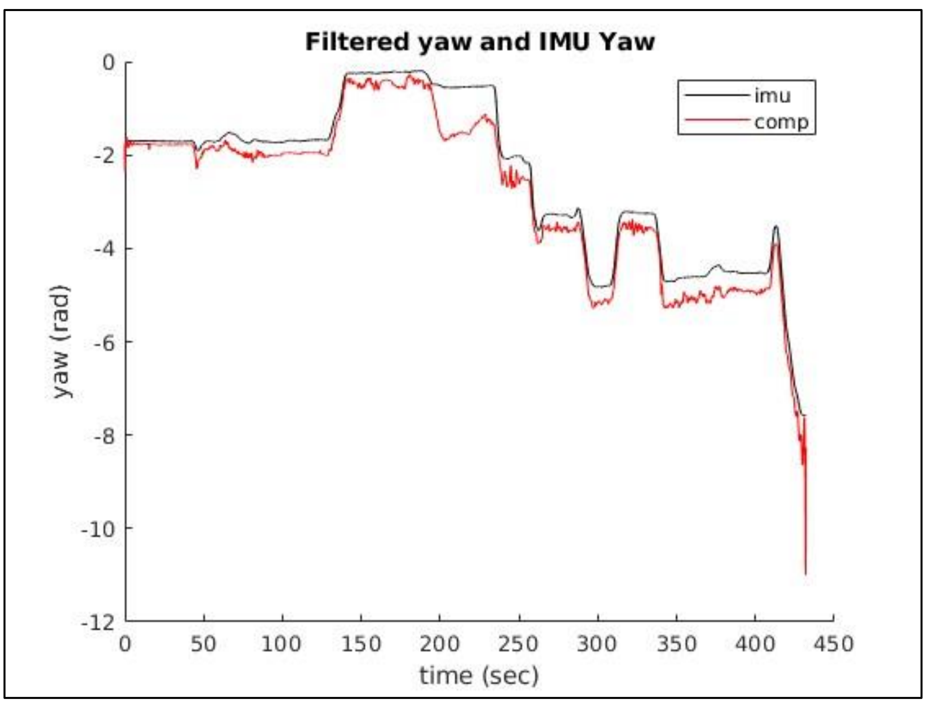


Fig. 9. Comparison between Complimentary Filtered Yaw and IMU Yaw

The yaw computed by magnetometer is more reliable than yaw computed by gyroscope and thus is more trusted for navigation as gyroscope yaw drifts over time and cannot be trusted. With the help of Sensor fusion of Magnetometer and Gyroscope, the Complimentary filtered yaw will be much more accurate than the individual yaw from each sensor.

3.3. Forward velocity from Accelerometer and GPS

3.3.1. Before Adjustment of Accelerometer data

The accelerometer in IMU provides the forward acceleration data which is shown by figure 10. To find the forward velocity, the acceleration data is integrated and is plotted in figure 11.

Also, GPS provides the Latitude and Longitude which is converted into UTM coordinates to get the east and north position. With the help of east and north position, the velocity is calculated using the formulae below:

$$vel = \frac{\sqrt{\mathrm{dx}^2 + dy^2}}{dt}$$

Where dx and dy represent the change in east and change in north respectively for time change dt.

vel will give the speed during time change dt.

After calculating the velocity from GPS data, it is plotted along with velocity calculated from acceleration to compare the result in figure 11.

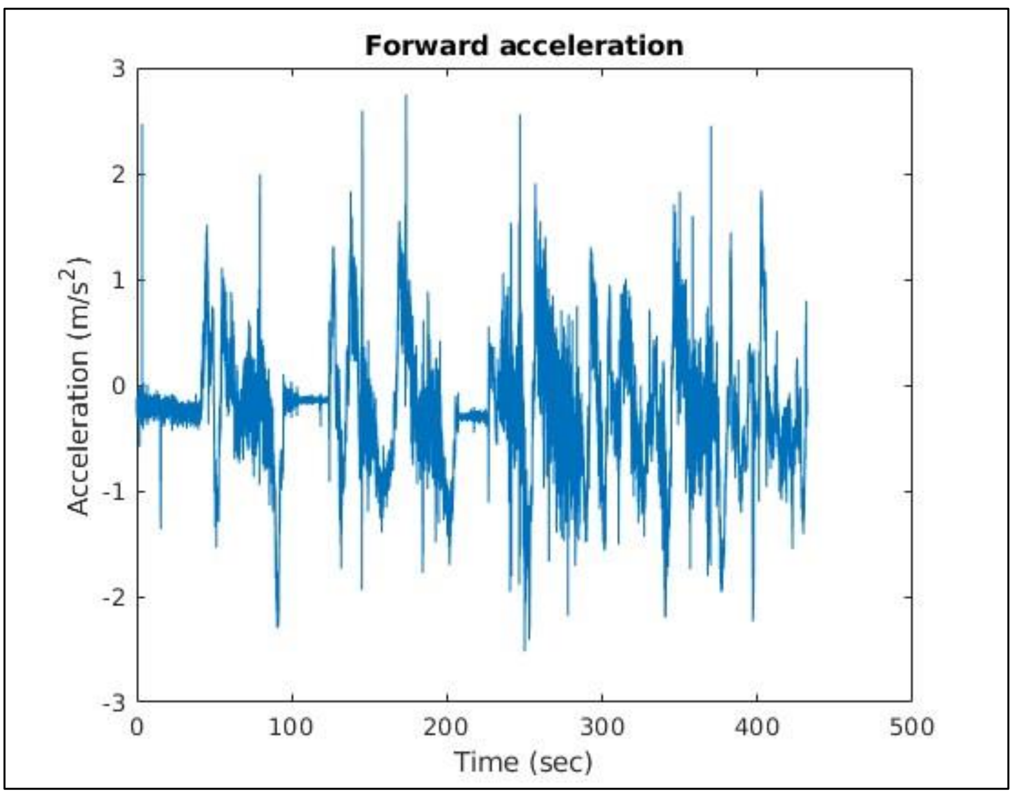


Fig. 10. Forward acceleration of IMU

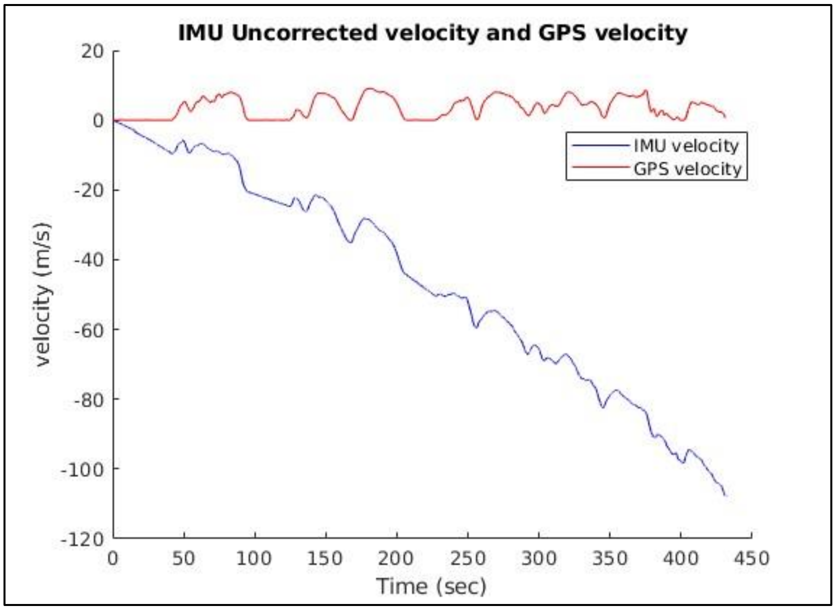


Fig. 11. Velocity calculated from IMU accelerometer data and velocity from GPS data

It is observed from figure 10 that bias is present in the acceleration data which can inferred from the fact that even for the time intervals when the vehicle was stopped, the accelerometer shows a negative acceleration. This has affected the velocity plot and due to presence of negative bias, after integration, it gives a continuously decreasing velocity (fig. 11) which again confirms the presence of negative bias.

3.3.2. After Adjustment of Accelerometer data

To adjust the velocity estimate, bias to be found in the acceleration data, but bias was not constant but keeps on changing and bias removal was employed. To do so, the acceleration plot and the jerk plot (fig. 12), which is the derivative of acceleration, were analyzed and time intervals were found out where the bias was changing. Then, mean was calculated in each interval and same was subtracted from the acceleration for each interval and bias was removed up to an extent.

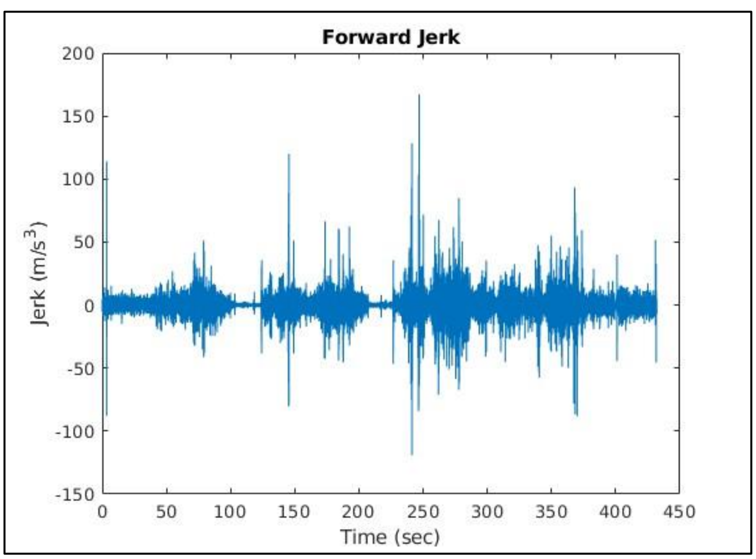


Fig. 12. Plot of Jerk

After adjusting the acceleration, by removing the bias at different intervals, the corrected velocity is again calculated by integrating the adjusted acceleration.

Figure 13 shows the comparison of corrected IMU velocity and the GPS velocity graph. Due to the presence of noise and time varying bias in the acceleration, even after correcting the bias to an extent, the velocity estimate is slightly different from the GPS velocity. The IMU data provides updated data at a higher rate, but integrated acceleration is susceptible to bias and noise (100% errors cannot be removed) and the GPS data update data at slower rate and is also imperfect due to tropospheric and ionospheric delay, satellite clock errors, orbit errors and multipath errors.

So, there will always be discrepancy between acceleration integrated velocity and GPS velocity due to their own errors as explained above.

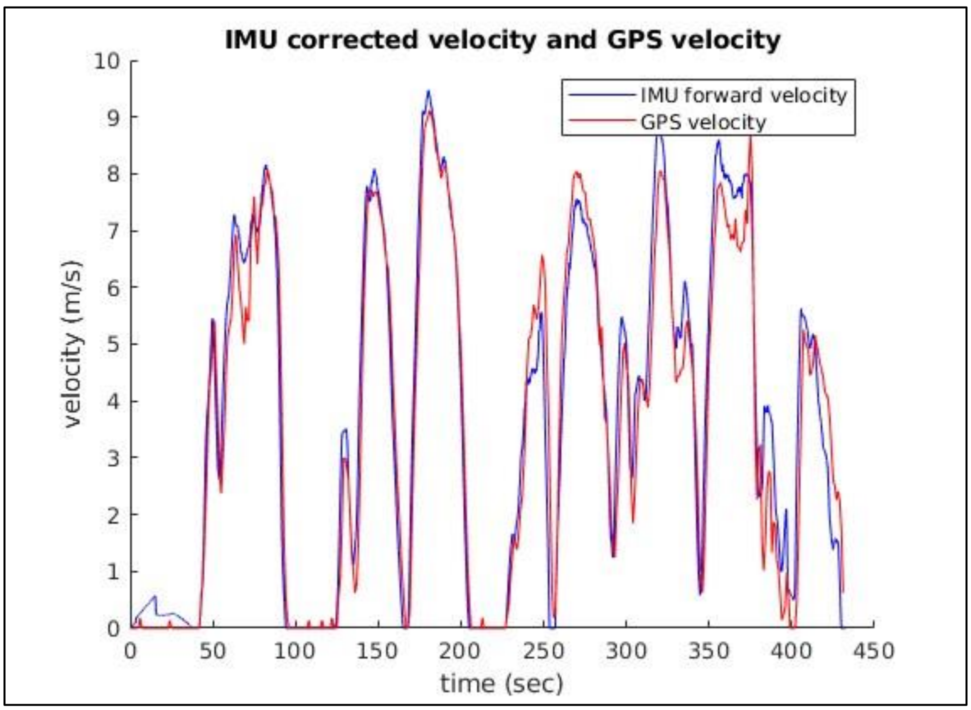


Fig. 13. Comparison of corrected IMU velocity and GPS velocity

3.4. Dead Reckoning with IMU

3.4.1. IMU Displacement and GPS Displacement

The velocity obtained through integration of acceleration, was again integrated to obtain the displacement of the IMU.

Also, the UTM coordinates obtained from the GPS were used to plot the GPS displacement. Figure 14 shows the comparison between IMU displacement and GPS displacement. The IMU displacement closely follows the GPS displacement but after a long time starts to drift away.

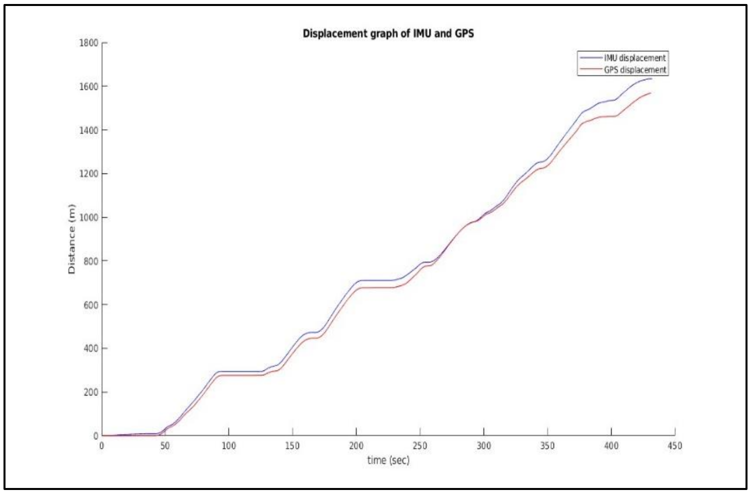


Fig. 14. Displacement graph of IMU and GPS

3.4.2. Linear Acceleration in Y (Lateral acceleration)

IMU sensor is attached to the vehicle and hence its acceleration in inertial frame is given below:

$$\ddot{x}_{obs} = \ddot{X} - \omega\dot{Y} - \omega^2 x_c$$
$$\ddot{y}_{obs} = \ddot{Y} + \omega\dot{X} + \dot{\omega}x_c$$

Here x_c represents the x position of IMU wrt to the center of mass (CM) of vehicle.

ω represents the angular velocity of vehicle about vertical axis.

($\ddot{X}$,$\ddot{Y}$) represents vehicle acceleration and ($\ddot{x}_{obs}$,$\ddot{y}_{obs}$) represents the IMU acceleration.

With the assumption that x_c=0 I.e. IMU was placed at CM and $\ddot{Y}$= 0 (Vehicle is not skidding), then

$$\ddot{y}_{obs} = \omega\dot{X}$$

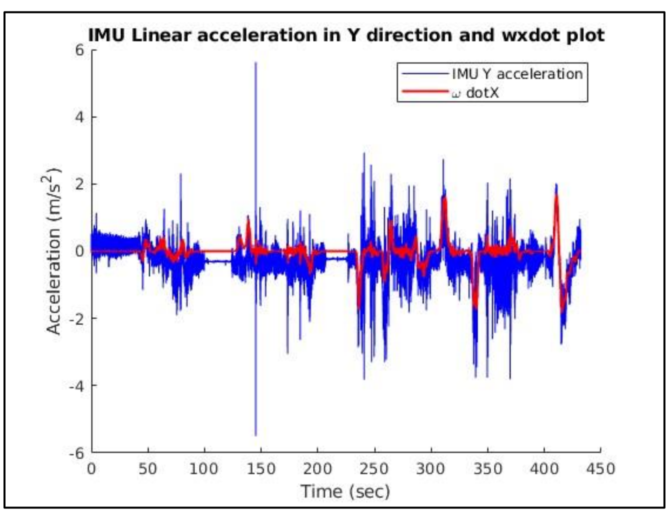


Fig. 15. Comparison of Linear acceleration in Y direction and $\omega\dot{X}$ acceleration

Figure 15 shows the comparison between the linear acceleration in Y direction that is observed by IMU, and acceleration obtained by multiplying Z angular velocity and vehicle forward velocity. Both plots look similar, but they have an offset in Y direction which can be due to the presence of bias in the acceleration in Y direction and because of assumption that IMU is placed exactly at the CM but in actual there will be some acceleration due to offset of IMU from CM of vehicle.

The difference can be minimized by adjusting the Y acceleration data through bias removal.

3.4.3. Dead Reckoning

The trajectory of the vehicle can be estimated using the forward velocity that is calculated in section 3.3.2 and the heading of the vehicle that is calculated in section 3.2.3 using complimentary filter. The forward velocity can be divided into (v_e,v_n), where v_e represents the velocity in east and v_n represents the velocity in north direction using heading (yaw angle) with the help of below equation:

$$v_e = -1 \cdot V \cdot \cos(yaw)$$

$$v_n = V \cdot \sin(yaw)$$

Now, upon integrating these velocities, the displacement estimates in east and north is obtained and same in plotted in figure 16. Also, the GPS trajectory is also plotted in figure 16 to compare the Dead Reckoning estimate with the GPS plot.

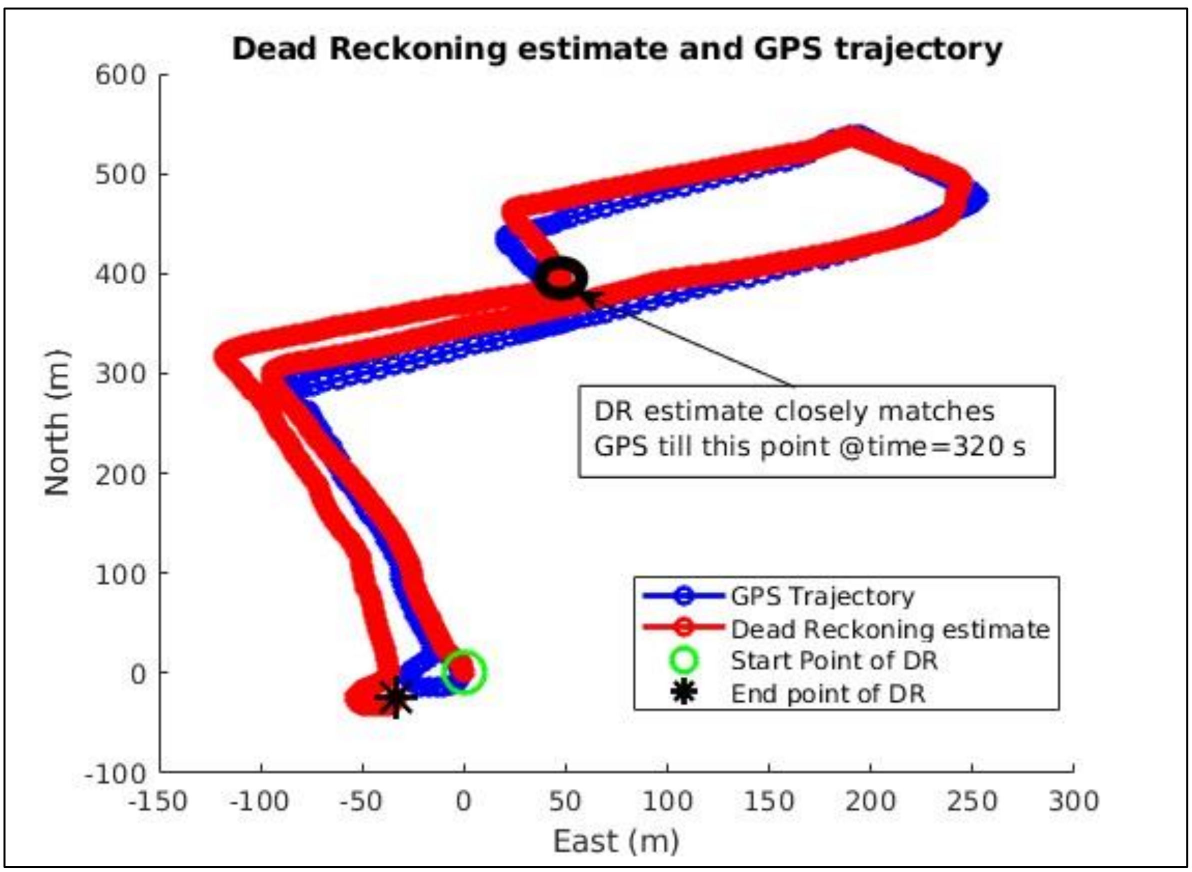


Fig. 16. Dead Reckoning estimate tracking the GPS trajectory

The heading was adjusted by 3.2 radians to maintain the initial orientation of the Dead reckoning estimate with the GPS plot.

No scaling factor is used for comparing both trajectories.

The Dead Reckoning estimate worked quite well and estimating the path closely to GPS at the start but with time it starts drifting away from the GPS trajectory.

With initial adjustment in the heading, the Dead Reckoning matches the GPS coordinates closely till 320 seconds after that it starts drifting a little and at the end of 430 seconds, the DR estimate is off by 30 meters.

It is bound to happen that Dead Reckoning will eventually drift from the actual path after some time as DR is dependent on the Velocity and Heading estimate. Since these estimates are susceptible to drift due to the presence of bias and noise in the magnetometer, accelerometer and the Gyroscope data and these biases/noises are not eliminated completely and thus the drifting of Dead Reckoning is expected and continuous removal of biases/noise will improve its accuracy.

3.5. Estimate x_c

The general equation of motion for IMU is defined as below:

$$\ddot{x} = \dot{v} + \omega \times v = \ddot{X} + \dot{\omega} \times r + \omega \times \dot{X} + \omega \times (\omega \times r)$$

Where $\ddot{x}$ represents the acceleration of IMU in 3D space,

$\ddot{X}$ represents the acceleration of vehicle in 3D space,

$\dot{X}$ represents the velocity of vehicle in 3D space,

ω represents the angular velocity of vehicle in 3D space,

and r represents the position of IMU wrt Center of Mass (CM) of vehicle.

On taking the X and Y component of above equation in the vehicle frame gives the below two equations:

$$\ddot{x}_{obs} = \ddot{X} - \omega\dot{Y} - \omega^2 x_c$$
$$\ddot{y}_{obs} = \ddot{Y} + \omega\dot{X} + \dot{\omega} x_c$$

Here x_c represents the x position of IMU wrt to the center of mass (CM) of vehicle.

ω represents the angular velocity of vehicle about vertical axis.

($\ddot{X}$,$\ddot{Y}$) represents vehicle acceleration and ($\ddot{x}_{obs}$,$\ddot{y}_{obs}$) represents the IMU acceleration.

Taking the assumption that vehicle is not moving sideways or skidding, then $\dot{Y}$=0 and $\ddot{Y}$=0.

Now the equation reduces to:

$$\ddot{x}_{obs} = \ddot{X} - \omega^2 x_c \quad (i)$$

$$\ddot{y}_{obs} = \omega\dot{X} + \dot{\omega} x_c \quad (ii)$$

($\ddot{x}_{obs}$,$\ddot{y}_{obs}$) is known from the IMU acceleration measurement in X and Y direction and ω is known from the gyroscope yaw rate.

$\ddot{X}$, $\dot{X}$ and x_c are unknown. One approach is to differentiate the (ii) equation to find the $\ddot{X}$ and replace this in equation (i), then we will have an equation only containing x_c which can be solved.

On differentiating equation (ii), we get

$$\ddot{X} = \frac{\omega\dddot{y}_{obs} - \dot{\omega}\ddot{y}_{obs} - \omega\ddot{\omega}x_c + \dot{\omega}^2 x_c}{\omega^2} \qquad (iii)$$

On substituting equation (iii) in equation (i), we get

$$x_c\ (\dot{\omega}^2 - \omega\ddot{\omega} - \omega^4) = \omega^2\ddot{x}_{obs} - \omega\dddot{y}_{obs} + \dot{\omega}\ddot{y}_{obs}$$

This equation is in the form of Ax = B, which can be solved using MATLAB.

Matrix A and B is calculated using the IMU X and Y direction acceleration data and Gyroscope Z angular velocity data.

Then, MATLAB function linsolve is used to find the value of x_c and is mentioned below:

$$x_c = -0.15\ m$$

This value looks justifiable as the IMU was placed approximately near to the CM of the vehicle only.

REFERENCES

[1] H. Ghanbarpourasl, G. Gopan, M. Shafi, S. M. Shalik Ershad and M. Sathiyanarayanan, "Integration of Sensor Fusion for Enhancing GPS Navigation," 2021 5th International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT), 2021, pp. 602-607, doi: 10.1109/ISMSIT52890.2021.9604755.

[2] "Implementing a Tilt-Compensated eCompass using Accelerometer and Magnetometer Sensors, Rev. 4.0" Freescale Semiconductor, Inc.

[3] C. S. Raveena, R. S. Sravya, R. V. Kumar and A. Chavan, "Sensor Fusion Module Using IMU and GPS Sensors for Autonomous Car," 2020 IEEE International Conference for Innovation in Technology (INOCON), 2020, pp. 1-6, doi: 10.1109/INOCON50539.2020.9298316.

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