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SNLP Assignment - I

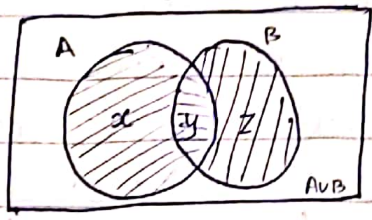
① Mathematical Basics

→ Show that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

· Before we proceed further, lets visualize the above sets

i.e $P(A \cup B) =$



Proof:-

R.H.S ⇒ $P(A) + P(B) - P(A \cap B)$

$= x + y + \cancel{y} + z - \cancel{y}$

$= x + y + z$

$= P(A \cup B)$

③ Bayes Theorem

→ WKT, Conditional probability can be computed using the below formula:

$$\boxed{P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}}$$

Given,

Let, $\bar{P}$ be the event that the person is affected by disease

i.e. $P(\bar{P}) = \frac{1}{100{,}000}$

· If person has disease, Prob of positive result,

$P(P|\bar{P}) = 0.97$

· If person is not affected & the prob of positive result

$P(P|\bar{N}) = 0.007$

Illy,

- $\bar{N}$ be an event, where person is not affected by Disease

$$P(\bar{N}) = (1 - 1/100{,}000) = 0.9999$$

∴ Using all the terms to find out $P(\bar{P}|P)$ i.e probability of person actually being positive when his test results comes out positive

i.e $$P(\bar{P}|P) = \frac{P(P|\bar{P}) \times P(\bar{P})}{P(P|\bar{P}) \times P(\bar{P}) + P(P|\bar{N}) \times P(\bar{N})}$$

$$= \frac{0.97 \times \frac{1}{100{,}000}}{\left(0.97 \times \frac{1}{100{,}000}\right) + (0.007) \times (0.9999)}$$

$$= (1 - 0.001385$$

$$= 0.9984$$

Inference :- Even though only 1 in 100,000 people is affected by the Disease, the probability that person is actually positive ~~not~~ for disease when his test results are positive is 99.84%.

- This basically tells that the test kit we have found is quite accurate & has very low chances of creating false positives/negatives

Question - 2

Total $S = 36$ (two die rolled independently)

A :→ sum of two outcomes is at least 10.

$E(A) \Rightarrow (5,5)\ (4,6)\ (6,4)\ (5,6)\ (6,5)\ (6,6) = 6$

$$\therefore \boxed{P(A) = \frac{E(A)}{S} = \frac{6}{36}}$$

B :→ At least one of the two rolls resulted in 6.

$E(B) = (6,\ 1/2/3/4/5)\ (1/2/3/4/5,\ 6)\ (6,6)$

$5 + 5 + 1 = 11$

$$\therefore \boxed{P(B) = \frac{E(B)}{S} = \frac{11}{36}}$$

C :⇒ At least one of the two rolls resulted in 1.

$E(C) = (1,\ 2/3/4/5/6)\ (2/3/4/5/6,\ 1)\ (1,1)$

$5 + 5 + 1 = 11$

$$\therefore \boxed{P(C) = \frac{E(C)}{S} = \frac{11}{36}}$$

D :→ The outcome of the 2nd roll was higher than the 1st roll.

$E(D) = (1,\ 2/3/4/5/6)\ (2,\ 3/4/5/6)\ (3,\ 4/5/6)\ (4,\ 5/6)\ (5,6)$

$\Rightarrow 5 + 4 + 3 + 2 + 1 \Rightarrow 15$

$$\boxed{P(D) = \frac{E(D)}{S} = \frac{15}{36}}$$

E :→ The difference b/w two rolls outcomes is exactly 1.

$E(E) = (1,2)\ (2,1)\ (2,3)\ (3,2)\ (3,4)\ (4,3)\ (4,5)\ (5,4)\ (5,6)\ (6,5)$

$= 10$

$$\boxed{P(E) = \frac{E(E)}{S} = \frac{10}{36}}$$

→ (a) $P(A) = \frac{6}{36}$ $P(C) = \frac{11}{36}$ $P(E) = \frac{10}{36}$

→ (b) Is event A independent of event B?

or $P(A/B) = P(A)$ (i.e probability of A given B should also be equal to P(A).

condition for independence.

$P(A/B) = \frac{E(A/B)}{S} = \frac{(4,6)\ (6,4)\ (6,5)\ (5,6)\ (6,6)}{36} = \frac{5}{36}$

$\therefore \frac{5}{36} \neq \frac{6}{36}$

$\therefore$ A is dependent on B.

→ (c) event A independent of event C?

~~to prove is~~ $P(A/C) = P(C)$?

$P(A/C) = \frac{E(A/C)}{S} = \frac{0}{36} = 0$

$\therefore 0 \neq \frac{6}{36}$

A is dependent on C

→ (d). Are events D & E independent?

or are $P(D \cap E) = P(D) \cdot P(E)$

$E(D \cap E) = (1,2)\ (2,3)\ (3,4)(4,5)(5,6) = 5$

$\therefore P(D \cap E) = P(D) \cdot P(E)$

$\frac{5}{36} = \frac{15}{36} \cdot \frac{10}{36}$ (not equal).

$\therefore$ D & E are dependent

Q4 **Random Variable:**

x	0	0	1	1
y	0	1	0	1
$P(X=x, Y=y)$	0.32	0.08	0.48	0.12

$\Rightarrow$

	$X=0$	$X=1$	
$Y=0$	0.32	0.48	$P(Y=0) = 0.80$
$Y=1$	0.08	0.12	$P(Y=1) = 0.20$
	$P(X=0) = 0.40$	$P(X=1) = 0.60$	

Two random variables X & Y are independent or independently distributed iff for every x, & y, the events X & Y with X & Y are independent events. i.e. $F_X(x)$ & $F_Y(y)$, are cumulative distribution function independent iff the combined random variable (x, Y) has a joint cumulative distribution function.

Mathematically,

$$\boxed{F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y) \quad \text{for all } x, y}$$

or,

$$P(x) = P(x|y_1) = P(x|y_2)$$

now, for $x=0$,

$$P(x=0) = 0.40$$

$$P(x=0|y=0) = \frac{P(x=0 \cap y=0)}{P(y=0)} = \frac{0.32}{0.80} = 0.4$$

$$P(x=0|y=1) = \frac{P(x=0 \cap y=1)}{P(y=1)} = \frac{0.08}{0.20} = 0.4$$

Here,

$$P(x=0) = P(x=0|y=0) = P(x=0|y=1)$$

now for $x=1$.

$$P(x=1) = 0.60$$

$$P(x=1|y=0) = \frac{P(x=1 \cap y=0)}{P(y=0)} = \frac{0.48}{0.80} = 0.6$$

$$P(x=1|y=1) = \frac{P(x=1 \cap y=1)}{P(y=1)} = \frac{0.12}{0.2} = 0.6$$

Again $P(x=1) = P(x=1|y=0) = P(x=1|y=1)$

$\therefore$ X & Y are independently distributed.