UNIVERSITÄT
DES
SAARLANDES

Exercise Sheet I

Submission Deadline: May 15th, 23:59

Probability Theory Review

1) **Mathematical Basics (2 points)**
   Use set theory and the definition of probability functions to show that:

   $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2) **Indepedence of Events (8 points)**
   Consider a fair 6-sided die whose sides are numbered from 1 to 6 and each die roll is independent of the other rolls. In an experiment that consists of rolling the die twice, the following events can be defined

   $A :$ The sum of the two outcomes is at least 10
   $B :$ At least one of the two rolls resulted in 6
   $C :$ At least one of the two rolls resulted in 1
   $D :$ The outcome of the 2nd roll was higher than the 1st roll
   $E :$ The difference between the two roll outcomes is exactly 1

   (a) Compute the probabilities $\mathbf{P}(A)$, $\mathbf{P}(C)$, and $\mathbf{P}(E)$.
   (b) Is event A independent of event B?
   (c) Is event A independent of event C?
   (d) Are events D and E independent?

3) **Bayes Theorem (4 points)**
   Suppose we are interested in a test to detect a disease which affects one in $100,000$ people on average. A lab has developed a test which works but is not perfect. If a person has the disease, it will give a positive result with probability 0.97; if they do not, the test will be positive with probability 0.007. You took the test, and it gave a positive result. What is the probability that you actually have the disease?

4) **Random Variables (6 points)**
   Are X and Y, as defined in the following table, independently distributed? How did you check?

x	0	0	1	1
y	0	1	0	1
$p(X = x, Y = y)$	0.32	0.08	0.48	0.12

   Justify your answers using the laws of probability and the definition of probabilistic independence.

Submission Instructions

The following instructions are mandatory. Please read them carefully. If you do not follow these instructions, the tutors can decide not to correct your exercise solutions.

- You have to submit the solutions of this exercise sheet as a team of 2 students.

- If you submit source code along with your assignment, please use Python unless otherwise agreed upon with your tutor.

- NLTK modules are not allowed, and not necessary, for the assignments unless otherwise specified.

- Make a single ZIP archive file of your solution with the following structure

  - A source_code directory that contains your well-documented source code and a README file with instructions to run the code and reproduce the results.

  - A PDF report with your solutions, figures, and discussions on the questions that you would like to include. You may also upload scans or photos of high quality.

  - A README file with group member names, matriculation numbers and emails.

- Rename your ZIP submission file in the format

  exercise02_id#1_id#2.zip

  where id#n is the matriculation number of every member in the team.

- Your exercise solution must be uploaded by only one of your team members under *Assignments* in the *General* channel on Microsoft Teams.

- If you have any problems with the submission, contact your tutor before the deadline.