19. Let $h \in \mathbb{R}, h > -1$. Then for all $n \in \mathbb{N}$, it follows that $P(n) : (1+h)^n \geq 1 + nh$.

Proof. (Induction).

For the basis case, suppose $n = 1$. Observe that $(1+h)^1 = 1 + h = 1 + (1)h$, thus $P(1)$.

Now suppose $P(n)$ for some $n \in \mathbb{N}$. Hence $(1+h)^n \geq 1 + nh$. Now we consider two cases. Let $h \geq 0$. Then

$$\begin{aligned} h + 1 &\geq 1 \\ (h+1)^n &\geq 1^n \\ h(h+1)^n &\geq h. \end{aligned}$$

Now let $0 > h > -1$. Similarly,

$$\begin{aligned} 0 > h &> -1 \\ 1 > h + 1 &> 0 \\ 1^n > (h+1)^n &> 0 \\ h < h(h+1)^n &< 0 \end{aligned}$$

Combining the inequaity for both cases, we get $h(h+1)^n \geq h$ for all $h > -1$. Adding this inequality to our assumption, $(1+h)^n \geq 1 + nh$, we get $(1+h)^n + h(h+1)^n \geq 1 + nh + h$. Then $(h+1)^n(1+h) = (h+1)^{n+1} \geq 1 + h(n+1)$, thus $P(n+1)$. Therefore, if $h \in \mathbb{R}, h > -1$, then for all $n \in \mathbb{N}$, it follows that $(1+h)^n \geq 1 + nh$ by induction.