

1.  $A = \{a, b\}$   
 $\{\{a\}, \{b\}\}$  of  $R = \{(a, a), (b, b)\}$   
 $\{\{a, b\}\}$  of  $R = \{(a, a), (b, b), (a, b), (b, a)\}$

2.  $A = \{a, b, c\}$   
 $\{\{a\}, \{b\}, \{c\}\}$  of  $R = \{(a, a), (b, b), (c, c)\}$   
 $\{\{a, b\}, \{c\}\}$  of  $R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$   
 $\{\{a\}, \{b, c\}\}$  of  $R = \{(a, a), (b, b), (c, c), (b, c), (c, b)\}$   
 $\{\{a, c\}, \{b\}\}$  of  $R = \{(a, a), (b, b), (c, c), (a, c), (c, a)\}$   
 $\{\{a, b, c\}\}$  of  $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (c, a), (c, b), (b, c)\}$

3. The four subsets of the partition of  $\mathbb{Z}$  resulting from the equivalence relation  $\equiv (\text{mod } 4)$  are:

$[0] = [4] = [8] = \dots = [4a]$  for  $a \in \mathbb{Z}$ ,  
 $[1] = [5] = [9] = \dots = [4b + 1]$  for  $b \in \mathbb{Z}$ ,  
 $[2] = [6] = [10] = \dots = [4c + 2]$  for  $c \in \mathbb{Z}$ ,  
 $[3] = [7] = [11] = \dots = [4d + 3]$  for  $d \in \mathbb{Z}$ .

**4.1. Proposition** Suppose  $P$  is a partition of set  $A$ . The relation  $R = \{(x, y) \in A \times A : x, y \in X, (X \in P)\}$  on  $A$  is an equivalence relation.

*Proof.*

For any  $x, y \in A$ , it follows that  $x, y \in X$  for some  $X \in P$  by def. of a partition. Thus  $xRx$ , meaning  $R$  is reflexive, and  $yRx$ , meaning  $R$  is symmetric. Now suppose  $xRy$  and  $yRz$  for some  $x, y, z \in A$ . Then  $x, y \in X$  and  $y, z \in Y$  for some  $X, Y \in P$ . Since  $y \in X \cap Y$ , it follows that  $X = Y$  by def. of a partition. Then  $x, z \in X$ , which implies  $xRz$ , and thus  $R$  is transitive. Therefore the relation  $R$  on  $A$  is an equivalence relation. ■

5.  $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x \equiv y (\text{mod } 2)\}$ .

6.  $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x - y = 0\}$