7. Proposition: Let $a, b \in \mathbb{R}$ such that $0 < a < b$. Then $a < \sqrt{ab} < \frac{a+b}{2} < b$.

Proof.

First we find an inequality relating a, b and $\sqrt{ab}$. Observe that

$$\begin{aligned} 0 < a < b &\Longrightarrow \Big((0 < a^2 < ab) \wedge (0 < ab < b^2)\Big) \\ &\Longrightarrow 0 < a^2 < ab < b^2 \\ &\Longrightarrow 0 < \sqrt{a^2} < \sqrt{ab} < \sqrt{b^2} \\ &\Longrightarrow a < \sqrt{ab} < b. \end{aligned}$$

Now we find an inequality relating a, b and $\frac{a+b}{2}$. Observe that

$$\begin{aligned} 0 < a < b &\Longrightarrow \Big((a + b < 2b) \wedge (2a < a + b)\Big) \\ &\Longrightarrow 2a < a + b < 2b \\ &\Longrightarrow a < \frac{a+b}{2} < b. \end{aligned}$$

Finally, we find show that $\sqrt{ab} < \frac{a+b}{2}$. Observe that

$$\begin{aligned} \sqrt{ab} < \frac{a+b}{2} &\Longleftrightarrow 2\sqrt{ab} < a + b \Longleftrightarrow \\ (2\sqrt{ab})^2 &< (a+b)^2 \Longleftrightarrow \\ 4ab &< a(a+b) + b(a+b) \\ &= a^2 + 2ab + b^2 \Longleftrightarrow \\ 0 &< a^2 - 2ab + b^2 \\ &= a(a-b) - b(a-b) \\ &= (a-b)^2 \end{aligned}$$

Since the square of any number is non-negative, and $a < b \Longrightarrow a - b \neq 0$, clearly $0 < (a-b)^2$, which implies $\sqrt{ab} < \frac{a+b}{2}$.
Combining the three inequalities, we get $a < \sqrt{ab} < \frac{a+b}{2} < b$ for $a, b \in \mathbb{R}$.