Proposition: Concerning the Fibonacci sequence,
$S_n : \sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$.

Proof. (Induction).
Basis step. Suppose $n = 1$.

Observe that $\sum_{i=1}^{n} F_i^2 = F_1^2 = 1^2 = 1 * 1 = F_1 F_2 = F_1 F_{n+1}$. Thus S_1.

Inductive step. Suppose S_k for $k \in \mathbb{N}$.

We now show S_k implies S_{k+1}. Observe that

$$\begin{aligned}
\sum_{i=1}^{k+1} F_i^2 &= \sum_{i=1}^{k} F_i^2 + F_{k+1}^2 && (1)\\
&= F_k F_{k+1} + F_{k+1}^2 && (2)\\
&= F_k F_{k+1} + (F_k + F_{k-1})^2 && (3)\\
&= F_k F_{k+1} + F_k^2 + F_k F_{k-1} + F_k F_{k-1} + F_{k-1}^2 && (4)\\
&= F_k F_{k+1} + F_k^2 + F_k F_{k-1} + F_{k-1}(F_k + F_{k-1}) && (5)\\
&= F_k F_{k+1} + F_k^2 + F_k F_{k-1} + F_{k-1} F_{k+1} && (6)\\
&= F_k(F_{k+1} + F_k) + F_{k-1}(F_{k+1} + F_k) && (7)\\
&= (F_k + F_{k-1})(F_{k+1} + F_k) && (8)\\
&= F_{k+1} F_{k+2} && (9)\\
&= F_{k+1} F_{(k+1)+1}. && (10)
\end{aligned}$$

Thus S_{k+1}.

It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■