14.

(a) Proposition: $|a| = -|a|$

Proof.

$|-a| = |(-1)a| = |-1| * |a| = |a|$.

(b) Proposition: $-b \leq a \leq b$ if and only if $|a| \leq b$.

Proof.

1) First we let $-b \leq a \leq b$ and show that $|a| \leq b$. Note that $-b \leq b$ implies $0 \leq b$. Consider two cases for a. If $0 \leq a \leq b$, then $|a| = a$, so $|a| \leq b$. If $-b \leq a < 0$, then $-|a| = a$. Then $-b \leq -|a|$, so $b \geq |a|$. Thus $-b \leq a \leq b$ implies $|a| \leq b$.

2) Now we let $|a| \leq b$ and show that $-b \leq a \leq b$. Consider two cases for a. If $0 \leq a$, then $|a| = a$, so $a \leq b$. If $0 > a$, then $|a| = -a$. Then $-a \leq b$, so $a \geq -b$. Combining the results, we get $-b \leq a \leq b$.

Therefore $-b \leq a \leq b$ if and only if $|a| \leq b$.

(c) Proposition: $|a + b| \leq |a| + |b|$

Proof. By Proposition (b), $-|a| \leq a \leq |a|$ and $-|b| \leq b \leq |b|$. Adding the two inequalities,

$$
\begin{aligned}
-|a| - |b| \leq& a + b \leq |a| + |b| \\
-(|a| + |b|) \leq& a + b \leq (|a| + |b|) \\
-(|a| + |b|) \leq& |a + b| \leq (|a| + |b|) && \text{By Proposition (b)}
\end{aligned}
$$

Therefore $|a + b| \leq |a| + |b|$.