14.

(a) $\sqrt{2}+\sqrt{6}$ is irrational.

Proof. (Contradiction).

Suppose for the sake of contradiction that $\sqrt{2}+\sqrt{6}$ is rational. Hence $\sqrt{2}+\sqrt{6} = \frac{n}{m}$ for some integers n and m. Then $\frac{n^2}{m^2} = (\sqrt{2}+\sqrt{6})^2 = \sqrt{2}^2 + 2\sqrt{2}\sqrt{2*3} + \sqrt{6}^2 = 8 + 2\sqrt{2}^2\sqrt{3} = 8 + 4\sqrt{3}$. Since 4 is rational and $\sqrt{3}$ is irrational (by Proposition 13.a), it follows by Proposition 12.b that $4\sqrt{3}$ is irrational. Further, since 8 is rational, it follows by Proposition 12.a.1 that $8 + 4\sqrt{3}$ is irrational. But $\frac{n^2}{m^2}$ is rational, a contradiction.

(b) $\sqrt{2}+\sqrt{3}$ is irrational.

Proof. (Contradiction).

Suppose for the sake of contradiction that $\sqrt{2}+\sqrt{3}$ is rational. Hence $\sqrt{2}+\sqrt{3} = \frac{n}{m}$ for some integers n and m. Then $\frac{n^2}{m^2} = (\sqrt{2}+\sqrt{3})^2 = \sqrt{2}^2 + 2\sqrt{2}\sqrt{3} + \sqrt{3}^2 = 6 + 2\sqrt{6}$. Since 2 is rational and $\sqrt{6}$ is irrational (by Proposition 13.a), it follows by Proposition 12.b that $2\sqrt{6}$ is irrational. Further, since 6 is rational, it follows by Proposition 12.a.1 that $6 + 2\sqrt{6}$ is irrational. But $\frac{n^2}{m^2}$ is rational, a contradiction.