18.

(a) Suppose x satisfies $x^n + a_{n-1}x^{n-1} + ... + a_0 = 0$ for some integers $a_{n-1}, ..., a_0$. If x is not an integer, then x is irrational.

Proof. (Contradiction).

Suppose x is not an integer but x is rational. Then $x = \frac{b}{c}$ for some integers b and $c > 1$ which share no common factor

$$\begin{aligned}
0 &= x^n + a_{n-1}x^{n-1} + ... + a_0 \\
&= (\frac{b}{c})^n + a_{n-1}(\frac{b}{c})^{n-1} + ... + a_0 \\
(\frac{b}{c})^n &= -a_{n-1}(\frac{b}{c})^{n-1} - ... - a_0 \\
b^n &= -a_{n-1}b^{n-1}c - ... - a_0c^n && \text{(Multiplying both sides by } c^n\text{.)} \\
&= c(-a_{n-1}b^{n-1} - ... - a_0c^{n-1})
\end{aligned}$$

Thus $c|b^n$. But we assumed b and c share no common factor, so b^n and c cannot share a common factor, a contradiction.