Proposition: For all $n, m, p \in \mathbb{Z}$ where $n, m, p \geq 0$, it follows that $S_n : \sum_{i=0}^{m} \binom{m}{i}\binom{n}{p+i} = \binom{m+n}{m+p}$.

Proof. (Induction).
Basis step. Suppose $n = 0$ for any $m \in \mathbb{Z}$ where $m \geq 0$. We consider two cases for p.
Let $p = 0$. Observe that $\sum_{i=0}^{m} \binom{m}{i}\binom{0}{0+i} = \binom{m}{0}\binom{0}{0} = 1 = \binom{m+0}{m+0}$.
Let $p \in \mathbb{N}$. Observe that $\sum_{i=0}^{m} \binom{m}{i}\binom{0}{p+i} = \sum_{i=0}^{m} \binom{m}{i}(0) = 0 = \binom{m+0}{m+p}$.
Thus S_0.
Inductive step. Suppose S_n for any $n, m, p \in \mathbb{Z}$ where $n, m, p \geq 0$.
We now show S_n implies S_{n+1}. Observe that

$$\begin{aligned}
\binom{m+n+1}{m+p} &= \binom{m+n}{m+p} + \binom{m+n}{m+p-1} && \text{(Def. of Pascal's triangle)} \quad (1)\\
&= \sum_{i=0}^{m} \binom{m}{i}\binom{n}{p+i} + \sum_{i=0}^{m} \binom{m}{i}\binom{n}{p+i-1} && \text{(Inductive hypothesis)} \quad (2)\\
&= \sum_{i=0}^{m} \left[\binom{m}{i}\binom{n}{p+i} + \binom{m}{i}\binom{n}{p+i-1}\right] && (3)\\
&= \sum_{i=0}^{m} \binom{m}{i}\left(\binom{n}{p+i} + \binom{n}{p+i-1}\right) && (4)\\
&= \sum_{i=0}^{m} \binom{m}{i}\binom{n+1}{p+i}. && (5)
\end{aligned}$$

Thus S_{n+1}.
It follows by mathematical induction that S_n for all $n, m, p \in \mathbb{Z}$ where $n, m, p \geq 0$. ■