17.

(a) Let $n \in \mathbb{N}, 1 < n$. If n is composite, then n can be written as a product of primes.

Proof. (Strong induction).

Let $n = 6$, so $n = 3 * 2$. Since 3 and 2 are prime, it follows that $n = 6$ can be written as a product of primes.

Now suppose $m \in \mathbb{N}$ is prime, or m is composite and can be written as a product of primes, for all $1 < m < n$ and some composite $n \in \mathbb{N}$. Then $n = ab$ for some $a, b \in \mathbb{N}, 1 < a, b < n$. Since a and b are either prime, or can be written as a product or primes, respectively, it follows that n can be written as a product of primes.

Therefore, by induction on $n \in \mathbb{N}, 1 < n$, it follows that if n is composite, then n can be written as a product of primes.

(b/c) Let $n, m \in \mathbb{N}$. If $n \neq m^k$ then $\sqrt[k]{n}$ is irrational.

Proof. (Contradiction).

Suppose $n \neq m^k$ but $\sqrt[k]{n}$ is rational. Hence $\sqrt[k]{n} = \frac{a}{b}$ for some integers a, b, so $nb^k = a^k$. Since the prime factorization of a number is unique to that number, the prime factorization of nb^k is identical to that of a^k. Thus for each prime factor p, whose degree is kx, of a^k, p appears in the prime factorization of n with degree z, and in that of b^k with degree ky, such that $kx = ky + z$ ($x \in \mathbb{N}$ and $y, z \in \mathbb{N}_0$). Then $kx - ky = z \Longrightarrow k(x - y) = z$. Thus for each prime factor of n, its degree is a multiple of k, so $n = m^k$ for some m. But we assumed $n \neq m^k$, a contradiction.

(d) There are infinitely many prime numbers.

Proof. (Contradiction).

Suppose there are finitely many prime numbers in the set $p_1, ..., p_n$, and let $x = p_1 * ... * p_n$. Since each number has a unique prime factorization, $x + 1 = p_1^{d_1} * ... * p_m^{d_m} = (p_1 * ... * p_n) + 1$, where $m \leq n$ and $d_1, ..., d_m \geq 0$ are some integers. Thus $1 = p_1^{d_1} * ... * p_m^{d_m} - (p_1 * ... * p_n) = p_i(p_1^{d_1} * ... * p_i^{d_i - 1} * ... * p_m^{d_m} - p_1 * ... * p_i^0 * ... * p_n)$ for all $d_i > 0$, so $p_i | 1$. But $p_i > 1$ for any prime number p_i, a contradiction.