

**1-6.** Prove that for all  $A_{i \in [1, n]}$ , it follows that  $\max(A)$  returns the maximum value in  $A$ .

*Proof.* Suppose  $n = 1$ . Observe that  $\max(A_1) = A_1$ , which is the maximum value in a single-element array  $A$ . Thus  $\max(A)$  is correct for  $A_1$ . Now suppose  $\max(A_{i \in [1, l]})$  is correct for all  $1 \leq l \leq k$ , and consider  $A_{i \in [1, k+1]}$ . The maximum element of  $A_{i \in [1, k]}$  is  $m$  by our assumption, so we have two cases for  $A_{k+1}$ : If  $A_{k+1} \leq m$ , then the maximum element of  $A_{i \in [1, k+1]}$  is in  $A_{i \in [1, k]}$ , and  $m$  is not updated. If  $A_{k+1} > m$ , then  $A_{k+1}$  is the maximum element of  $A_{i \in [1, k+1]}$ , and the algorithm updates  $m$  to  $m = A_{k+1}$ . Thus  $\max(A)$  is correct for all  $A_{i \in [1, n]}$ .