15.

(a/b) Let p and q be rational, and m be any natural number. Then $P(m) : x = p \pm \sqrt{q}$ implies $x^m = a \pm b\sqrt{q}$ for some rational a and b.

Proof. (Induction).

For the basis case, let $m = 1$ and suppose $x = p \pm \sqrt{q}$. Then $x^1 = (p \pm \sqrt{q})^1 = a \pm b\sqrt{q}$ where $a = p$ and $b = 1$. Thus $P(1)$.

Now let m be some natural number, $x = p \pm \sqrt{q}$, and suppose $P(m)$. Hence $x^m = a \pm b\sqrt{q}$ for some rational a and b. Then

$$\begin{aligned}
x^{m+1} &= x * x^m \\
&= (p \pm \sqrt{q})(a \pm b\sqrt{q}) \\
&= pa + p(\pm b\sqrt{q}) + (\pm\sqrt{q})a + (\pm\sqrt{q})(\pm b\sqrt{q}) \\
&= pa \pm pb\sqrt{q} \pm a\sqrt{q} + b\sqrt{q}^2 \\
&= (pa + bq) \pm (pb + a)\sqrt{q}.
\end{aligned}$$

Since p, q, a, b are all rational, $(pa + bq)$ and $(pb + a)$ are rational. Thus $P(m + 1)$.

It follows by induction that if p and q are rational, and m is any natural number, then $P(m) : x = p \pm \sqrt{q}$ implies $x^m = a \pm b\sqrt{q}$ for some rational a and b.