20. Let $a_1 = 1, a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3, n \in \mathbb{N}$. Then $P(n) : a_n = \dfrac{\frac{(1+\sqrt{5})^n}{2^n} - \frac{(1-\sqrt{5})^n}{2^n}}{\sqrt{5}}$.

Proof. (Strong induction).

Suppose $n = 1$. Then $a_1 = 1 = \dfrac{\frac{1-1+2\sqrt{5}}{2}}{\sqrt{5}} = \dfrac{\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}}{\sqrt{5}}$. Thus $P(1)$.

Now suppose $P(k)$ for some n and all $k \leq n$, where $k, n \in \mathbb{N}$. Hence $a_k = \dfrac{\frac{(1+\sqrt{5})^k}{2^k} - \frac{(1-\sqrt{5})^k}{2^k}}{\sqrt{5}}$. Observe that

$$
\begin{aligned}
a_n n + 1 &= a_n + a_{n-1} \\
&= \frac{\frac{(1+\sqrt{5})^n}{2^n} - \frac{(1-\sqrt{5})^n}{2^n}}{\sqrt{5}} + \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}} - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^n}{2^n} + \frac{(1+\sqrt{5})^{n-1}}{2^{n-1}} - \frac{(1-\sqrt{5})^n}{2^n} - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{1+\sqrt{5}}{2} + 1\right) - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{1-\sqrt{5}}{2} + 1\right)}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{2}{2}(\frac{1+\sqrt{5}}{2} + \frac{2}{2})\right) - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{2}{2}(\frac{1-\sqrt{5}}{2} + \frac{2}{2})\right)}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{6+2\sqrt{5}}{2^2}\right) - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{6-2\sqrt{5}}{2^2}\right)}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{1^2+2\sqrt{5}+\sqrt{5}^2}{2^2}\right) - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{1^2-2\sqrt{5}+\sqrt{5}^2}{2^2}\right)}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{(1+\sqrt{5}^2)}{2^2}\right) - \frac{(1-\sqrt{5})^{n-1}}{2^{n-1}}\left(\frac{(1-\sqrt{5})^2}{2^2}\right)}{\sqrt{5}} \\
&= \frac{\frac{(1+\sqrt{5})^{n+1}}{2^{n+1}} - \frac{(1-\sqrt{5})^{n+1}}{2^{n+1}}}{\sqrt{5}}.
\end{aligned}
$$

Thus $P(n+1)$. Therefore, for all $n \in \mathbb{N}$ it follows by induction that $P(n) : a_n = \dfrac{\frac{(1+\sqrt{5})^n}{2^n} - \frac{(1-\sqrt{5})^n}{2^n}}{\sqrt{5}}$.