20. If $|x - x_0| < \frac{\epsilon}{2}$ and $|y - y_0| < \frac{\epsilon}{2}$, then $|(x + y) - (x_0 + y_0)| < \epsilon$.

Proof.

Suppose $|x - x_0| < \frac{\epsilon}{2}$ and $|y - y_0| < \frac{\epsilon}{2}$. Then by Proposition 14.b, $-\frac{\epsilon}{2} < x - x_0 < \frac{\epsilon}{2}$ and $-\frac{\epsilon}{2} < y - y_0 < \frac{\epsilon}{2}$. Adding the two inequalities,

$$
\begin{aligned}
-\frac{\epsilon}{2} - \frac{\epsilon}{2} &< x - x_0 + y - y_0 < \frac{\epsilon}{2} + \frac{\epsilon}{2} \\
-\epsilon &< (x + y) - (x_0 + y_0) < \epsilon \\
\therefore\ &|(x + y) - (x_0 + y_0)| < \epsilon.
\end{aligned}
$$

Note also that $-\frac{\epsilon}{2} < y - y_0 < \frac{\epsilon}{2}$ implies $\frac{\epsilon}{2} > -y + y_0 > -\frac{\epsilon}{2}$. Adding this inequality to the inequality with x,

$$
\begin{aligned}
-\frac{\epsilon}{2} - \frac{\epsilon}{2} &< x - x_0 - y + y_0 < \frac{\epsilon}{2} + \frac{\epsilon}{2} \\
-\epsilon &< (x - y) - (x_0 - y_0) < \epsilon \\
\therefore\ &|(x - y) - (x_0 - y_0)| < \epsilon.
\end{aligned}
$$