25.

(a).i $\mathbb{R}$ is inductive.

Proof. Note that $\mathbb{N} \subseteq \mathbb{R}$, thus for any number in the set $\mathbb{N}$, this number is also in the set $\mathbb{R}$. Since $\mathbb{N}$ is inductive, it follows that $\mathbb{R}$ is inductive.

(a).ii The set of positive real numbers is inductive.

Proof. Note the proof 25.a.i, and consider that every number in the set $\mathbb{N}$ is a positive number. Thus the set of positive real numbers is inductive.

(a).iii The set of positive real numbers unequal to $\frac{1}{2}$ is inductive.

Proof. Note the proof 25.a.ii, and note that $\frac{1}{2}$ is not an integer, thus $\frac{1}{2} \notin \mathbb{N}$ but $\mathbb{N}$ is inductive. Therefore the set of positive real numbers unequal to $\frac{1}{2}$ is inductive.

(a).iv The set of positive real numbers unequal to 5 is not inductive.

Proof. Note that $4 \in \mathbb{R} - \{5\}$ but $4 + 1 = 5 \notin \mathbb{R} - \{5\}$, thus $\mathbb{R} - \{5\}$ is not inductive.

(a).v If A and B are both inductive, then the set C of real numbers which are in both A and B is also inductive.

Disproof. Consider the following counterexample: Suppose $A = \mathbb{R}$ and $B = \mathbb{R}$, and note that A and B are inductive. Let $C = \{\frac{1}{2}\}$. Then $C \subseteq A$ and $C \subseteq B$, but C is not inductive because $1 \notin C$. Therefore the statement is false.

(b).i 1 is a natural number.

Proof. By definition of an inductive set of real numbers A, $1 \in A$. Since A is inductive, it follows by definition of natural numbers that 1 is a natural number.

(b).ii $k + 1$ is a natural number if k is a natural number.

Proof. Suppose A is an inductive set of real numbers and suppose $k \in A$. Since A is inductive, $k \in A \Longrightarrow k + 1 \in A$. Thus, by definition of a natural number, $k + 1$ is a natural number. Therefore $k + 1$ is a natural number if k is a natural number.