Proposition: Concerning the Fibonacci sequence,
$S_n : \sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$.

Proof. (Induction).
Basis step. Suppose $n = 1$.
Observe that $\sum_{i=1}^{1} F_i^2 = F_1^2 = 1^2 = 1 * 1 = F_1 F_{1+1}$, thus S_1.
Inductive step. Suppose S_k for some $k \in \mathbb{N}$.
We now show S_k implies S_{k+1}. Observe that

$$
\begin{aligned}
\sum_{i=1}^{k+1} F_i^2 &= F_{k+1}^2 + \sum_{i=1}^{k} && (1)\\
&= F_{k+1}^2 + F_k F_{k+1} \qquad \text{(Inductive hypothesis)} && (2)\\
&= F_{k+1}\big(F_{k+1} + F_k\big) && (3)\\
&= F_{k+1} F_{k+2} && (4)\\
&= F_{k+1} F_{(k+1)+1}. && (5)
\end{aligned}
$$

Thus F_{k+1}.
It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■