

1-9. Soln.

(a)

$$n^2 \gg n \iff n^2 - n \gg n \quad (1)$$

$$\iff (n^2 - n)\frac{1}{2} \gg 6n \quad (2)$$

$$\iff f(n) \gg g(n) \quad (3)$$

Thus $g(n) = O(f(n))$.

(b)

$$n^2 \gg n \gg \sqrt{n} \iff n + \sqrt{n} \ll n^2 \quad (4)$$

$$\iff n + 2\sqrt{n} \ll n^2 \quad (5)$$

$$\iff f(n) \ll g(n) \quad (6)$$

Thus $f(n) = O(g(n))$.

(c) $\sqrt{n} \gg \log n \iff n\sqrt{n}\frac{1}{2} \gg n \log n \iff g(n) \gg f(n)$, thus $f(n) = O(g(n))$.

(d) $n \gg \sqrt{n} \iff \log n + n \gg \sqrt{n} \iff f(n) \gg g(n)$, thus $g(n) = O(f(n))$.

(e) $\log^2 n \gg \log n \iff 2\log^2 n \gg \log n + 1 \iff f(n) \gg g(n)$, thus $g(n) = O(g(n))$.

(f)

$$n \gg \log n \iff n - 1 \gg \log n + 1 \quad (7)$$

$$\iff n(n - 1) \gg n(\log n + 1) \quad (8)$$

$$\iff (n^2 - n)\frac{1}{2} \gg 4n \log n + n \quad (9)$$

$$\iff g(n) \gg f(n) \quad (10)$$

Thus $f(n) = O(g(n))$.