Proposition: For all $n \in \mathbb{N}$, it follows that S_n : The number of n sized binary strings without consecutive 1's is the Fibonacci number F_{n+2}.

Proof. (Induction).
Basis step. Suppose $n \in \mathbb{N}$ where $n \leq 2$.
Let $n = 1$ and observe that there are two such 1 sized strings, namely 0 and 1.
Since $2 = 1 + 1 = F_1 + F_2 = F_{(1)+2}$, it follows that S_1.
Let $n = 2$ and observe that there are three such 2 sized strings, namely 00, 10, and 01.
Since $3 = 2 + 1 = F_3 + F_2 = F_{(2)+2}$, it follows that S_2.
Inductive step. Suppose S_n for $n \in \mathbb{N}$ where $n \geq 2$.
Let A be the set of all such n sized strings where $A_{i_n} = 1$ for $i \in [1, |A|]$, and B be such set where $B_{j_n} = 0$ for $j \in [1, |B|]$.
Then $F_{n+2} = F_{n+1} + F_n = |A| + |B|$ by the addition principle.
Observe that $B_{j_{n-1}} = 0$ or $B_{j_{n-1}} = 1$, and $A_{i_{n-1}} = 0$ by our supposition.
Thus $|B| > |A|$, so $F_{n+1} = |B|$.
We now show S_n implies S_{n+1}.
Let $C = D \cup E$ be the set of all such $n + 1$ sized strings, where $D = \{X : C_{X_{n+1}} = 0\}$ and $E = \{Y : C_{Y_{n+1}} = 1\}$.
Then $X_n = 0$ or $X_n = 1$ for all $X \in D$ by our supposition, thus $|D| = |A| + |B|$.
Also $Y_n = 0$ for all $Y \in E$ by our supposition, thus $|E| = |B|$.
Then $|C| = |D| + |E| = (|A| + |B|) + |B| = F_{n+2} + F_{n+1} = F_{(n+1)+1}$, thus S_{n+1}.
It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■