

Proposition: S_n : n (infinitely long) straight lines lie on a plane in such a way that no two of the lines are parallel, and no three lines intersect a single point.

This arrangement divides the plane into $\frac{n^2 + n + 2}{2}$.

Proof. (Induction).

Basis step. Suppose $n = 1$.

Observe that 1 line divides the plane into 2 regions. $2 = \frac{1^2 + 1 + 2}{2}$, thus S_1 .

Inductive step. Suppose S_n for some $n \in \mathbb{N}$.

We now show S_n implies S_{n+1} .

Observe that any 1 line intersects n lines once in any such arrangement of $n + 1$ lines, dividing $n + 1$ of the $\frac{n^2 + n + 2}{2}$ regions in the valid arrangement of n lines. Then

$$\frac{n^2 + n + 2}{2} + (n + 1) = \frac{n^2 + n + 2 + 2(n + 1)}{2} \quad (1)$$

$$= \frac{(n^2 + 2n + 1) + (n + 1) + 1}{2} \quad (2)$$

$$= \frac{(n + 1)^2 + (n + 1) + 1}{2}. \quad (3)$$

Thus S_{n+1} .

It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■