**17.**

**(a)** $2x^2 - 3x + 4$

$$\begin{aligned}
2x^2 - 3x + 4 &= 2(x^2 - \frac{3}{2}x) + 4 \\
&= 2(x^2 - \frac{3}{4}x - \frac{3}{4}x + \frac{9}{16} - \frac{9}{16}) + 4 \\
&= 2(x^2 - \frac{3}{4}x - \frac{3}{4}x + \frac{9}{16}) - \frac{18}{16} + 4 \\
&= 2(x(x - \frac{3}{4}) - \frac{3}{4}(x - \frac{3}{4})) - \frac{18}{16} + \frac{64}{16} \\
&= 2(x - \frac{3}{4})^2 - \frac{46}{16} \\
&= 2(x - \frac{3}{4})^2 - \frac{23}{8}.
\end{aligned}$$

The minimum value of $(x - \frac{3}{4})^2 \geq 0$ is $(x - \frac{3}{4})^2 = 0$ when $x = \frac{3}{4}$. Since there are no more variable terms, the minimum value of the full expression is $2(0) - \frac{23}{8} = -\frac{23}{8}$.

**(b)** $x^2 - 3x + 2y^2 + 4y + 2$

$$\begin{aligned}
x^2 - 3x + 2y^2 + 4y + 2 &= (x^2 - 3x) + 2(y^2 + 2y) + 2 \\
&= (x^2 - \frac{3}{2}x - \frac{3}{2}x + \frac{9}{4} - \frac{9}{4}) + 2(y^2 + y + y + 1 - 1) + 2 \\
&= (x^2 - \frac{3}{2}x - \frac{3}{2}x + \frac{9}{4}) + 2(y^2 + y + y + 1) + 2 - 2 - \frac{9}{4} \\
&= (x - \frac{3}{2})^2 + 2(y + 1)^2 - \frac{9}{4}.
\end{aligned}$$

The minimum value of $(x - \frac{3}{2})^2 \geq 0$ and $2(y + 1)^2 \geq 0$ is $0$ when $x = \frac{3}{2}$ and $y = -1$. Thus the minimum value of the full expression is $0 + 0 - \frac{9}{4} = -\frac{9}{4}$.

**(c)** $x^2 + 4xy + 5y^2 - 4x - 6y + 7$

First we factor the full expression:

$$\begin{aligned}
x^2 + 4xy + 5y^2 - 4x - 6y + 7 &= (x^2 + 4xy - 4x) + 5y^2 - 6y + 7 \\
&= (x^2 + 4x(y - 1)) + 5y^2 - 6y + 7 \\
&= (x^2 + 2x(y - 1) + 2x(y - 1) + 4(y - 1)^2 - 4(y - 1)^2) + 5y^2 - 6y + 7 \\
&= (x^2 + 2x(y - 1) + 2x(y - 1) + 4(y - 1)^2) - 4(y - 1)^2 + 5y^2 - 6y + 7 \\
&= \Big(x(x + 2(y - 1)) + 2(y - 1)(x + 2(y - 1))\Big) - 4(y^2 - 2y + 1) + 5y^2 - 6y + 7 \\
&= (x + 2(y - 1))^2 - 4y^2 + 8y - 4 + 5y^2 - 6y + 7 \\
&= (x + 2(y - 1))^2 + y^2 + 2y + 3 \\
&= (x + 2(y - 1))^2 + (y^2 + y + y + 1 - 1) + 3 \\
&= (x + 2(y - 1))^2 + (y^2 + y + y + 1) - 1 + 3 \\
&= (x + 2(y - 1))^2 + (y + 1)^2 + 2.
\end{aligned}$$

Now we determine for which $x$ the values of $(x+2(y-1))^2 \geq 0$ are minimal:

$$\begin{aligned}(x+2(y-1))^2 &= 0\\ x+2(y-1) = x+2y-2 &= 0\\ x &= -2y+2.\end{aligned}$$

Note that $(y+1)^2 \geq 0$ is minimal when $(y+1)^2 = 0$, at $y = -1$. Thus the value of both variable terms is 0 when $y = -1$ and $x = -2(-1) + 2 = 4$. Therefore the minimal value of the full expression is $(0) + (0) + 2 = 2$.