10.

(i) $|a+b|-|b|$

Note that $|b| = \begin{cases} b, & b \geq 0 \\ -b, & b \leq 0 \end{cases}$ and $|a+b| = \begin{cases} a+b, & a+b \geq 0 \\ -a-b, & a+b \leq 0 \end{cases} = \begin{cases} a+b, & a \geq -b \\ -a-b, & a \leq -b \end{cases}$. Now simplify $|a+b|-|b|$ by considering each combination of a piece from $|a+b|$ with a piece from $|b|$.

$$|a+b|-|b| = \begin{cases} |a+b|-|b|, & (b \geq 0) \wedge (a \geq -b) \\ |a+b|-|b|, & (b \geq 0) \wedge (a \leq -b) \\ \\ |a+b|-|b|, & (b \leq 0) \wedge (a \geq -b) \\ |a+b|-|b|, & (b \leq 0) \wedge (a \leq -b) \end{cases}$$

$$= \begin{cases} (a+b)-(b), & (b \geq 0) \wedge (a \geq -b) \\ (-a-b)-(b), & (b \geq 0) \wedge (a \leq -b) \\ \\ (a+b)-(-b), & (b \leq 0) \wedge (a \geq -b) \\ (-a-b)-(-b), & (b \leq 0) \wedge (a \leq -b) \end{cases}$$

$$= \begin{cases} a, & (b \geq 0) \wedge (a \geq -b) \\ -a-2b, & (b \geq 0) \wedge (a \leq -b) \\ \\ a+2b, & (b \leq 0) \wedge (a \geq -b) \\ -a, & (b \leq 0) \wedge (a \leq -b) \end{cases}$$

(ii) $|(|x|-1)|$

Note that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$ and $|(|x|-1)| = \begin{cases} |x|-1, & |x|-1 \geq 0 \\ 1-|x|, & |x|-1 \leq 0 \end{cases}$. For each piece of $|(|x|-1)|$, we find the intervals of x on which it applies:

1) $|x|-1 \geq 0 \Longrightarrow |x| \geq 1$

$$\Longrightarrow (x \geq 1) \vee (-x \geq 1) \qquad \text{(Consider each piece of } |x|\text{)}$$

$$\Longrightarrow (x \geq 1) \vee (x \leq -1).$$

2) $|x|-1 \leq 0 \Longrightarrow 0 \leq |x| \leq 1$

$$\Longrightarrow (0 \leq x \leq 1) \vee (0 \leq -x \leq 1)$$

$$\Longrightarrow (0 \leq x \leq 1) \vee (-1 \leq x \leq 0).$$

Thus we can simplify as $|(|x|-1)| = \begin{cases} |x|-1, & (x \geq 1) \vee (x \leq -1) \\ 1-|x|, & (0 \leq x \leq 1) \vee (-1 \leq x \leq 0) \end{cases} = \begin{cases} x-1, & (x \geq 1) \\ -x-1, & (x \leq -1) \\ 1-x, & (0 \leq x \leq 1) \\ 1+x, & (-1 \leq x \leq 0) \end{cases}$.

(iii) $|x| - |x^2|$

Note that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$, and $(x^2 \geq 0) \implies (|x| - |x^2| = |x| - x^2)$.

Now we simplify $|x| - x^2$ by considering each piece of $|x|$:

$$\begin{aligned} |x| - x^2 &= \begin{cases} |x| - x^2, & x \geq 0 \\ |x| - x^2, & x \leq 0 \end{cases} \\ &= \begin{cases} x - x^2, & x \geq 0 \\ -x - x^2, & x \leq 0 \end{cases}. \end{aligned}$$

(iv) $a - |(a - |a|)|$

Note that $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a \leq 0 \end{cases}$, and $|(a-|a|)| = \begin{cases} a-|a|, & a-|a| \geq 0 \\ |a|-a, & a-|a| \leq 0 \end{cases}$. For each piece of $|(a-|a|)|$, we find the intervals of x on which it applies:

$$\begin{aligned} \mathbf{1)} a - |a| \geq 0 &\implies a \geq |a| \\ &\implies (a \geq a) \vee (a \geq -a) && \text{(Consider each piece of } |a|\text{)} \\ &\implies (a \geq 0). && (a \geq a \text{ is redundant}) \\ \mathbf{2)} a - |a| \leq 0 &\implies a \leq |a| \\ &\implies (a \leq a) \vee (a \leq -a) \\ &\implies (a \leq 0). \end{aligned}$$

Now we simplify $a - |(a - |a|)|$ by considering cases **1)** and **2)** separately:

$$\begin{aligned} a - |(a-|a|)| &= \begin{cases} a - (a - |a|), & a \geq 0 \\ a - (|a| - a), & a \leq 0 \end{cases} \\ &= \begin{cases} a - (a - (a)), & a \geq 0 \\ a - ((-a) - a), & a \leq 0 \end{cases} \\ &= \begin{cases} a, & a \geq 0 \\ 3a, & a \leq 0 \end{cases} \end{aligned}$$