

Assumption 1: Rumor spreads such that if some prof b made an error, then every other prof a ($a \neq b$) knows about b 's error (but b does not know). Further, all profs know assumption 1.

27. Let P be the set of all n profs of University B. Prove that all profs will resign.

Proof.

Consider any $a \in P$, and let $Q \subseteq P$ be the set of profs who have not made an error, by a 's reasoning. Initially, $Q = \emptyset$. a knows that he did not make an error, hence let $a \in Q$, and thinks that some $b \in P - \{a\}$ made the error. Further, each $k \in P - Q$ knows that none of $c \in Q$ made an error, and that he did not make an error, hence let $k \in Q$. Then some $d \in P - Q$ made the error. Repeating this reasoning until $Q = P$, prof a finds that some prof $z \in P$ knows no profs who have made an error. By assumption 1, z knows he is the only prof who may have made an error, and must thus resign. Thus, prof a waits for some other prof z to resign the following year. However, since each prof repeated a 's reasoning, all profs wait for some other prof to resign. Then one year later, since a sees that no other prof had resigned, a knows $a = z$ and resigns. Therefore, all profs resign.

28. How does prof X's statement change the situation?

By assumption 1, prof z certainly knows that no profs besides himself had made an error, but can only assume that he made no error. However, prof X states that there exists some prof who made an error, and since z knows that he is the only one who could have made an error, he will resign.