13.

Proposition 1: $max(x,y) = \dfrac{x+y+|y-x|}{2}$

Proof.

We consider two cases. Suppose $max(x,y) = y$. Then $(y \geq x) \implies (y - x \geq 0)$. Thus

$$\begin{aligned}\frac{x+y+|y-x|}{2} &= \frac{x+y+(y-x)}{2} \\ &= \frac{2y}{2} \\ &= y \\ &= max(x,y).\end{aligned}$$

Now suppose $max(x,y) = x$. Then $(y \leq x) \implies (y - x \leq 0)$. Thus

$$\begin{aligned}\frac{x+y+|y-x|}{2} &= \frac{x+y-(y-x)}{2} \\ &= \frac{2x}{2} \\ &= x \\ &= max(x,y).\end{aligned}$$

Therefore $max(x,y) = \dfrac{x+y+|y-x|}{2}$.

Proposition 2: $min(x,y) = \dfrac{x+y-|y-x|}{2}$

Proof.

We consider two cases. Suppose $min(x,y) = x$. Then $(y \geq x) \implies (y - x \geq 0)$. Thus

$$\begin{aligned}\frac{x+y-|y-x|}{2} &= \frac{x+y-(y-x)}{2} \\ &= \frac{2x}{2} \\ &= x \\ &= min(x,y).\end{aligned}$$

Now Suppose $min(x,y) = y$. Then $(y \leq x) \implies (y - x \leq 0)$.

$$\begin{aligned}\frac{x+y-|y-x|}{2} &= \frac{x+y+(y-x)}{2} \\ &= \frac{2y}{2} \\ &= y \\ &= min(x,y).\end{aligned}$$

Therefore $min(x,y) = \dfrac{x+y-|y-x|}{2}$.

Proposition:

$$max(x,y,z) = \frac{2x+y+z+|z-y|+\Big|y+z+|z-y|-2x\Big|}{4}$$
$$\text{and } min(x,y,z) = \frac{2x+y+z-|z-y|-\Big|y+z-|z-y|-2x\Big|}{4}$$

Proof.
Consider two cases. WLOG, let $m = min(y,z)$. Then $min(x,y,z) = min(x,m)$. Now let $m = max(y,z)$, WLOG. Then $max(x,y,z) = max(x,m)$. In either case, combining the expressions for $max(x,m)$ and $min(x,m)$ from Propositions 1 and 2, respectively,

$$\begin{aligned}
\frac{x+m\pm|m-x|}{2} &= \frac{1}{2}\Big(x+m\Big) \pm \frac{1}{2}\Big|\Big(m-x\Big)\Big| \\
&= \frac{1}{2}\Big(x+\frac{y+z\pm|z-y|}{2}\Big) \pm \frac{1}{2}\Big|\Big(\frac{y+z\pm|z-y|}{2}-x\Big)\Big| \\
&= \frac{1}{2}\Big(\frac{2x+y+z+|z-y|}{2}\Big) \pm \frac{1}{2}\Big|\Big(\frac{y+z\pm|z-y|-2x}{2}\Big)\Big| \\
&= \frac{1}{4}\Big(2x+y+z\pm|z-y|\Big) \pm \frac{1}{4}\Big|\Big(y+z\pm|z-y|-2x\Big)\Big| \\
&= \frac{2x+y+z\pm|z-y|\pm\Big|y+z\pm|z-y|-2x\Big|}{4}.
\end{aligned}$$

$$\text{Therefore } max(x,y,z) = \frac{2x+y+z+|z-y|+\Big|y+z+|z-y|-2x\Big|}{4}$$
$$\text{and } min(x,y,z) = \frac{2x+y+z-|z-y|-\Big|y+z-|z-y|-2x\Big|}{4}$$