6.

(a) If $x_1, ..., x_n$ are distinct numbers, find a polynomial f_i of degree $n-1$ which is 1 at some x_i, and 0 at x_j for all $j \neq i$.
Consider two cases. Suppose $f_i(x) = 0$. Then $x = x_j$ for any $j \neq i$, so $x - x_j = 0$. Thus $0 = \prod_{j=1, i\neq j}^{n}(x - x_j) = \prod_{j=1, i\neq j}^{n} m(x - x_j)$ (for any number m). Suppose now that $f_i(x) = 1$, then $x = x_i$ for some (unique) i, so $1 = \dfrac{x_i - x_j}{x_i - x_j} = \dfrac{x - x_j}{x_i - x_j}$. Thus $1 = \prod_{j=1, i\neq j}^{n} \dfrac{x - x_j}{x_i - x_j}$. Letting $m = \dfrac{1}{x_i - x_j}$, we combine the results from both cases to obtain $f_i(x) = \prod_{j=1, i\neq j}^{n} \dfrac{x - x_j}{x_i - x_j}$.

(b) Find a polynomial function f of degree $n-1$ such that $f(x_i) = a_i$ $(i \leq n)$, where $a_1, ..., a_n$ are given numbers.
For some unique $i \leq n$, note that $a_i = \sum_{j=1, j\neq i}^{n} a_j * 0 + a_i * 1 = \sum_{j=1, j\neq i}^{n} a_j * f_j(x_i) + \sum_{j=i}^{1} a_j * f_j(x_i)$, thus $f(x) = \sum_{j=1}^{n} a_j * f_j(x)$.