6. Interview problems

1-29. *Soln.* Select 5 groups of 5 horses per group, let those groups be A, B, C, D and E. **Run 5 races**, 1 per group, and sort each group by a horse's rank in ascending order. **Now run 1 race** for horses A_1, B_1, C_1, D_1 and E_1, which ranked first in their respective groups. Let $F = (A, B, C, D, E)$ be sorted by a horse's rank in the second race in ascending order. Note that A_1 is the global first place, so it remains to find the global second and third place.

$$
\begin{array}{ccccc}
\textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
A1 & B1 & C1 & D1 & E1 \\
A2 & B2 & C2 & D2 & E2 \\
A3 & B3 & C3 & D3 & E3 \\
A4 & B4 & C4 & D4 & E4 \\
A5 & B5 & C5 & D5 & E5 \\
\end{array}
$$

Observe that the second and third places must be in $S = \{A_2, A_3, B_1, B_2, C_1\}$ (see graphic). Suppose for the sake of contradiction that there exists some second or third placed horse $X \notin S$. We sorted A, B, C, D, E and F in ascending order of ranks, so X ranked better (lower) or equal to A_3 or B_2 or C_1, which means X must be one of the horses in S, a contradiction.
Now run 1 last race for the horses in S and sort it by a horse's rank in ascending order to find the second and third places. Then we ran **7 races in total** to determine the top 3 horses.

1-30. *Soln.* Suppose the world population is about $7.8 * 10^9$, 1 in 5,000 people are professional musicians, and 1 in 25 of them are piano tuners. So there are $7.8 * 10^9 * \dfrac{1}{5,000} * \dfrac{1}{25} = 62,400$ piano tuners in the world.

1-31. *Soln.* Suppose there are 20 gas stations per city on average. There are 317 cities, so 20*317=6,340 gas stations.

1-32. *Soln.* Suppose the rink is 20 [m] by 8 [m] by 0.1 [m], and ice has a mass of 919 [kg/m^3].
Then the total mass is $m = 20[m] * 8[m] * 0.1[m] * 919[kg/m^3] = 14,704[kg]$. Note that the gravitational acceleration is $g = 9.81[m/s^2]$, and weight is given by $W = mg$. So the weight of the rink is $W = 14,704[kg] * 9.81[m/s^2] = 144,246[N]$.

1-33. *Soln.* Suppose each state has 25,000 [km] of road on average. There are 50 states, so 50*25,000[km]=1,250,000[km] of road.

1-34. *Soln.* Suppose the phone book has 400 pages, has 4 columns per page, and 200 entires per column. Then it has 4*200*2=1,600 entries per two pages. Assuming you search two pages each time you open the book, the probability of finding a page is 2 in 400. So on average you will open the book 200 times to find a specific entry.