1-3. *Soln.* Observe that

$$\begin{aligned}
\text{prestiferous}(n) &= \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{i+j}\sum_{l=1}^{i+j-k} 1 && (1)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=0}^{i}(i+j-k) && (2)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{i}\left(\sum_{k=0}^{i}(i+j)-\sum_{k=0}^{i}k\right) && (3)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{i}\left((i+1)(i+j)-\frac{i(i+1)}{2}\right) && \text{(Def. of } i\text{th triangular number)} \quad (4)\\
&= \sum_{i=1}^{n}\left((i+1)\sum_{j=1}^{i}(i+j)-\frac{i(i+1)}{2}\sum_{j=1}^{i}1\right) && (5)\\
&= \sum_{i=1}^{n}\left((i+1)\Big(\sum_{j=1}^{i}i+\sum_{j=1}^{i}j\Big)-\frac{i^2(i+1)}{2}\right) && (6)\\
&= \sum_{i=1}^{n}\left((i+1)(i^2+\frac{i(i+1)}{2})-\frac{i^2(i+1)}{2}\right) && (7)\\
&= \sum_{i=1}^{n} i(i+1)(i+\frac{i+1}{2}-\frac{i}{2}) && (8)\\
&= \sum_{i=1}^{n} i(i+1)(i+\frac{1}{2}) && (9)\\
&= \sum_{i=1}^{n} i^2(i+1)+\sum_{i=1}^{n}\frac{i(i+1)}{2} && (10)\\
&= \sum_{i=1}^{n} i^3+\sum_{i=1}^{n} i^2+\frac{n(n+1)(n+2)}{6} && \text{(Def. of } n\text{th tetrahedral number)} \quad (11)\\
&= \sum_{i=1}^{n} i^3+\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)(n+2)}{6} && \text{(Def. of } n\text{th square pyramidal number)} \quad (12)\\
&= \frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)(n+2)}{6} && \text{(Def. of } n\text{th squared triangular number)} \quad (13)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{n(n+1)}{2}\Big(\frac{n(n+1)}{2} + \frac{2n+1}{3} + \frac{n+2}{3}\Big) && (14)\\
&= \frac{n(n+1)}{2}\Big(\frac{n(n+1)}{2} + \frac{3n+3}{3}\Big) && (15)\\
&= \frac{n(n+1)}{2}\Big(\frac{n(n+1)}{2} + (n+1)\Big) && (16)\\
&= \frac{n(n+1)}{2}\,\frac{n(n+1)+2(n+1)}{2} && (17)\\
&= \frac{n(n+1)}{2}\,\frac{(n+1)(n+2)}{2} && (18)\\
&= \frac{n(n+1)^2(n+2)}{4}. && (19)
\end{aligned}
$$

Then presitferous$(n) = \frac{1}{4}n(n+1)^2(n+2) = \frac{1}{4}(n^4 + 4n^3 + 5n^2 + 2n) = O(n^4) + O(n^3) + O(n^2) + O(n) = O(n^4)$.