**9.**

**(i)** $|\sqrt{2}+\sqrt{3}-\sqrt{5}+\sqrt{7}|$

$$\begin{aligned} 7 > 5 &\implies \sqrt{7} > \sqrt{5} \\ &\implies -\sqrt{5}+\sqrt{7} > 0 \\ &\implies 0 < \sqrt{2}+\sqrt{3}-\sqrt{5}+\sqrt{7}. \end{aligned}$$

Thus $|\sqrt{2}+\sqrt{3}-\sqrt{5}+\sqrt{7}| = \sqrt{2}+\sqrt{3}-\sqrt{5}+\sqrt{7}$.

**(ii)** $|(|a+b|-|a|-|b|)|$

Note that $|a+b| \leq |a|+|b|$ by Theorem 1. Then

$$\begin{aligned} |a+b| \leq |a|+|b| &\implies |a+b|-|a|-|b| \leq 0 \\ &\implies 0 \leq -(|a+b|-|a|-|b|) \\ &\implies 0 \leq |a|+|b|-|a+b|. \end{aligned}$$

Thus $|(|a+b|-|a|-|b|)| = |a|+|b|-|a+b|$.

**(iii)** $|(|a+b|+|c|-|a+b+c|)|$

$$|(a+b)+c| \leq |a+b|+|c| \implies 0 \leq |a+b|+|c|-|a+b+c|.$$

Thus $|(|a+b|+|c|-|a+b+c|)| = |a+b|+|c|-|a+b+c|$.

**(iv)** $|x^2-2xy+y^2|$

$$\begin{aligned} x^2-2xy+y^2 &= x(x-y)-y(x-y) \\ &= (x-y)^2 \geq 0. \end{aligned}$$

Thus $|x^2-2xy+y^2 = x^2-2xy+y^2$.

**(v)** $|(|\sqrt{2}+\sqrt{3}|-|\sqrt{5}-\sqrt{7}|)|$

$$\begin{aligned} (0<3 \land 0<2) &\implies (0<\sqrt{2} \land 0<\sqrt{3}) \\ &\implies 0 < \sqrt{2}+\sqrt{3}. \end{aligned}$$

Thus $|(|\sqrt{2}+\sqrt{3}|-|\sqrt{5}-\sqrt{7}|)| = |(\sqrt{2}+\sqrt{3}-|\sqrt{5}-\sqrt{7}|)|$.