

11.

(i)  $|x - 3| = 8$

1) Let  $x - 3 \geq 0 \implies x \geq 3$ . Then  $(x - 3 = 8) \implies (x = 11)$ .

2) Let  $x - 3 \leq 0 \implies x \leq 3$ . Then  $(-(x - 3) = 8) \implies (3 - x = 8) \implies (x = -5)$ .

Therefore  $x = 11$  or  $x = -5$ .

(ii)  $|x - 3| < 8$

1) Let  $x - 3 \geq 0 \implies x \geq 3$ . Then  $(x - 3 < 8) \implies (3 \leq x < 11)$ .

2) Let  $x - 3 \leq 0 \implies x \leq 3$ . Then  $(-(x - 3) < 8) \implies (3 - x < 8) \implies (3 \geq x > -5)$ .

Therefore  $3 \geq x > -5$  or  $3 \leq x < 11$  for  $x \in \mathbb{R}$ .

(iii)  $|x + 4| < 2$

1)  $x + 4 \geq 0 \implies x \geq -4$

$$(x + 4 < 2) \implies (-4 \leq x < -2).$$

2)  $x + 4 \leq 0 \implies x \leq -4$

$$(-x - 4 < 2) \implies (-x < 6) \implies (x > -6).$$

Therefore  $(x > -6 \text{ or } -4 \leq x < -2) \implies (-6 < x < -2, x \in \mathbb{R})$ .

(iv)  $|x - 1| + |x - 2| > 1$

1)  $x - 1 \geq 0 \implies x \geq 1$

1.1)  $x - 2 \geq 0 \implies x \geq 2$

$$(x - 1) + (x - 2) > 1 \implies 2x - 3 > 1 \implies x > 2.$$

1.2)  $x - 2 \leq 0 \implies x \leq 2$

$$(x - 1) + (-x + 2) > 1 \implies (x - x) + 1 > 1 \implies \text{No solution.}$$

2)  $x - 1 \leq 0 \implies x \leq 1$

2.1)  $x - 2 \geq 0 \implies x \geq 2$

No solution.

2.2)  $x - 2 \leq 0 \implies x \leq 2$

$$(-x + 1) + (-x + 2) > 1 \implies (-2x > -2) \implies (x < 1).$$

Therefore  $x > 2$  or  $x < 1$  for  $x \in \mathbb{R}$ .

**(v)**  $|x - 1| + |x + 1| < 2$

**1)**  $x - 1 \geq 0 \implies x \geq 1$

**1.1)**  $x + 1 \geq 0 \implies x \geq -1$

$$x - 1 + x + 1 < 2 \implies 2x < 2 \implies x < 1 \implies -1 \leq x < 1.$$

**1.2)**  $x + 1 \leq 0 \implies x \leq -1$

No solution.

**2)**  $x - 1 \leq 0 \implies x \leq 1$

**2.1)**  $x + 1 \geq 0 \implies x \geq -1$

$$(-x + 1) + (x + 1) < 2 \implies (x - x) + 2 < 2 \implies \text{No solution.}$$

**2.2)**  $x + 1 \leq 0 \implies x \leq -1$

$$(-x + 1) + (-x - 1) < 2 \implies -2x < 2 \implies x > -1 \implies \text{No solution.}$$

Therefore  $-1 \leq x < 1, x \in \mathbb{R}$ .

**(vi)**  $|x - 1| + |x + 1| < 1$

**1)**  $x - 1 \geq 0 \implies x \geq 1$

**1.1)**  $x + 1 \geq 0 \implies x \geq -1$

$$x - 1 + x + 1 < 1 \implies 2x < 1 \implies x < \frac{1}{2} \implies -1 \leq x < \frac{1}{2}.$$

**1.2)**  $x + 1 \leq 0 \implies x \leq -1$

No solution.

**2)**  $x - 1 \leq 0 \implies x \leq 1$

**2.1)**  $x + 1 \geq 0 \implies x \geq -1$

$$(-x + 1) + (x + 1) < 1 \implies (x - x) + 2 < 1 \implies \text{No solution.}$$

**2.2)**  $x + 1 \leq 0 \implies x \leq -1$

$$(-x + 1) + (-x - 1) < 1 \implies -2x < 1 \implies x > -\frac{1}{2} \implies \text{No solution.}$$

Therefore  $-1 \leq x < \frac{1}{2}, x \in \mathbb{R}$ .

**(vii)**  $|x - 1| * |x + 1| = 0$

**1)**  $x - 1 \geq 0 \implies x \geq 1$

**1.1)**  $x + 1 \geq 0 \implies x \geq -1$

$(x - 1)(x + 1) = 0 \implies x = 1 \vee x = -1.$

**1.2)**  $x + 1 \leq 0 \implies x \leq -1$

No solution.

**2)**  $x - 1 \leq 0 \implies x \leq 1$

**2.1)**  $x + 1 \geq 0 \implies x \geq -1$

$-(x - 1)(x + 1) = 0 \implies x = 1 \vee x = -1.$

**2.2)**  $x + 1 \leq 0 \implies x \leq -1$

$(-(x - 1))(-(x + 1)) = 0 \implies x = 1 \vee x = -1.$

Therefore  $x = 1 \vee x = -1.$

**(viii)**  $|x - 1| * |x + 2| = 3$

**1)**  $x - 1 \geq 0 \implies x \geq 1$  (1)

**1.1)**  $x + 2 \geq 0 \implies x \geq -2$  (2)

$(x - 1)(x + 2) = 3 \implies (x^2 + x - 5 = 0) \implies \left(x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)}\right) \implies \left(x = \frac{-1 \pm \sqrt{21}}{2}\right).$  (3)

**1.2)**  $x + 2 \leq 0 \implies x \leq -2$  (4)

No solution. (5)

**2)**  $x - 1 \leq 0 \implies x \leq 1$  (6)

**2.1)**  $x + 2 \geq 0 \implies x \geq -2$  (7)

$-(x - 1)(x + 2) = 3 \implies (-x^2 - x - 1 = 0) \implies \left(x = \frac{1 \pm \sqrt{1^2 - 4(-1)(-1)}}{2(-1)}\right) \implies \left(x = \frac{1 \pm \sqrt{-3}}{-2}\right) \implies \text{No solution.}$  (8)

**2.2)**  $x + 2 \leq 0 \implies x \leq -2$  (9)

$(-(x - 1))(-(x + 2)) = (x - 1)(x + 2) = 3 \implies \left(x = \frac{-1 \pm \sqrt{21}}{2}\right)$  by (3). (10)

Therefore  $x = \frac{-1 \pm \sqrt{21}}{2}.$