8. $P(n)$: Prove that every natural number is either even or odd.

Proof. (Induction).

Suppose $n = 1$, the minimal natural number. Since $1 = 2(0) + 1$, it follows that n is odd. Thus $P(1)$.

Now suppose n is some natural number for which $P(n)$, hence we consider two cases:

1. Let n be even. Then $n = 2x$ for some non-negative integer x. Since $n + 1 = 2x + 1$, it follows that $n + 1$ is odd.

2. Let n be odd. Then $n = 2x + 1$ for some non-negative integer x. Since $n + 1 = (2x + 1) + 1 = 2(x + 1)$, thus $n + 1$ is even.

Thus $P(n + 1)$. Therefore, by induction, it follows that every natural number is either even or odd.