10. Prove the principle of mathematical induction with the well-ordering principle.

Proof.

Note that the set of natural numbers has a least member by the well-ordering principle. Thus let n_0 be the minimum natural number for which a property $A(n_0)$ holds, and suppose $A(k)$ in implies $A(k+1)$ for some natural number $k \geq n_0$. Then by Proposition 9, it follows that $A(n)$ for all natural numbers $n \geq n_0$. In other words, we can prove that the property $A(n)$ holds for all natural numbers $n \geq n_0$ by proving that $A(n_0)$, and that $A(n)$ implies $A(n+1)$.