**Proposition:** For all $n, m \in \mathbb{N}$ where $m < n$, it follows that $S_n$ : if $2|n$ and $2|m$, then $2|\binom{n}{m}$.

*Proof.* (Strong Induction).
**Basis step.** Suppose $m = 1$ and $n = 2$.
Observe that $\binom{2}{1} = 2$, thus statement is true.
**Inductive step.** Suppose $S_n$ for $n, m \in \mathbb{N}$ where $2 < m, 2 \leq n, m < n$ and $2|n$ but $2 \nmid m$.
We now show $S_n$ implies $S_k$ for all $k \in \mathbb{N}$ where $2|k$ and $m < k$. Observe that

$$\begin{aligned}
\binom{k}{m} &= \binom{k-1}{m} + \binom{k-1}{m-1} && (1)\\
&= \binom{k-2}{m} + \binom{k-2}{m-1} + \binom{k-2}{m-1} + \binom{k-2}{m-2} && \text{(Def. of Pascal's triangle)} \quad (2)\\
&= \binom{k-2}{m} + 2\binom{k-2}{m-1} + \binom{k-2}{m-2} && . \quad (3)
\end{aligned}$$

Since $2|2\binom{k-2}{m-1}$, it follows that the second term is even. Note that $2|k$ implies $2|k-2$, and $2 \nmid m$ implies $2 \nmid m-2$, thus the first and third terms are even. Since the sum of three even terms is even, it follows that $\binom{k}{m}$ is even.
It follows by mathematical induction that $S_n$ for all $n, m \in \mathbb{N}$ where $m < n$. ■