26. Prove that the stack of n rings in the ring puzzle can be moved onto spindle 3 in a minimum of $2^n - 1$ moves.

Proof. (Induction).

Suppose $n = 1$. Then we move the only ring from spindle 1 to spindle 3 in $1 = 2^1 - 1$ move.

Now suppose we can move the stack of $n \in \mathbb{N}$ rings from spindle 1 to spindle 3 in a minimum of $2^n - 1$ moves, and suppose we have a stack of $n + 1$ rings on spindle 1 with spindles 2 and 3 empty. We can symmetrically move a stack of n rings to spindle 2 or 3 in a minimum of $2^n - 1$ moves, hence we move the top stack of n rings to spindle 2. Note that spindle 1 contains the $n + 1$st ring, and spindle 3 is empty (*). Next we move the $n + 1$st ring to spindle 3 in a minimum of 1 move. Now spindle 1 is empty and spindle 3 contains the $n + 1$st ring, a symmetrical situation to (*). Thus move the stack of n rings from spindle 2 to spindle 3 in a minimum of $2^n - 1$ moves. Hence we have $n + 1$ rings on spindle 3 with spindles 1 and 2 empty. Thus we moved $n+1$ rings from spindle 1 to spindle 3 in the minimum total of $(2^n - 1) + 1 + (2^n - 1) = 2 * 2^n - 1 = 2^{n+1} - 1$ moves.

Therefore, by induction, the stack of n rings in the ring puzzle can be moved onto spindle 3 in a minimum of $2^n - 1$ moves.