Proposition: Concerning the Fibonacci sequence,
$S_n : F_1 + F_3 + F_5 + ... + F_{2(n-1)-1} + F_{2n-1} = F_{2n}$.

Proof. (Induction).
Basis step. Suppose $n = 1$. Observe that $F_{2n-1} = F_{2(1)-1} = 1 = F_{2(1)} = F_{2n}$. Thus S_1.
Inductive step. Suppose S_k for $k \in \mathbb{N}$.

We now show S_k implies S_{k+1}. Observe that

$$\begin{aligned}
F_1 + F_3 + F_5 + ... + F_{2((k+1)-1)-1} + F_{2(k+1)-1} &= (F_1 + F_3 + F_5 + ... + F_{2k-1}) + F_{2k+1} && (1)\\
&= F_{2k} + F_{2k+1} && (2)\\
&= F_{2k+2} && (3)\\
&= F_{2(k+1)}. && (4)
\end{aligned}$$

Thus S_{k+1}.
It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■