4.

(a) $\sum_{k=0}^{l} \binom{n}{k}\binom{m}{l-k} = \binom{n+m}{l}$

Proof. (TODO)

$$\begin{aligned}
(1+x)^{n+m} &= (1+x)^n(1+x)^m \\
\sum_{i=0}^{n+m} \binom{n+m}{i} x^i &= \Big[\sum_{i=0}^{n} \binom{n}{i} x^i\Big]\Big[\sum_{j=0}^{m} \binom{m}{j} x^j\Big] && \text{(By the binomial theorem)} \\
&= \sum_{i=0}^{n} \Big[\binom{n}{i} \sum_{j=0}^{m} \binom{m}{j} x^{i+j}\Big] && \text{(Distributive property)} \\
&= \sum_{l=0}^{n+m} \Big[x^l \binom{n}{l} \sum_{j=0}^{m} \binom{m}{j}\Big] && \Big(\binom{n}{l} = 0 \text{ for } l > n\Big)
\end{aligned}$$

(b) $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

Proof.

$$\begin{aligned}
\sum_{k=0}^{n} \binom{n}{k}^2 &= \sum_{k=0}^{n} \binom{n}{k}\binom{n}{k} \\
&= \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} && \Big(\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-(n-k))!(n-k)!} = \binom{n}{n-k}\Big) \\
&= \binom{n+n}{n} && \text{By Proposition 4.a} \\
\therefore \sum_{k=0}^{n} \binom{n}{k}^2 &= \binom{2n}{n}.
\end{aligned}$$