1.

(i) For all $n \in \mathbb{N}$ such that $n \geq 2$, $P(n) : 1^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

Proof.

Let $n = 2$. Clearly $1^2 + 2^2 = 5 = \dfrac{2 * 3 * 5}{6} = \dfrac{2 * (2+1) * (2(2)+1)}{6}$, thus $P(2)$. Now let n be some number such that $n \in \mathbb{N}$ and $n \geq 2$, and suppose $P(n)$. Hence $1^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$. Observe that

$$\begin{aligned}
1^2 + ... + (n+1)^2 &= (1^2 + ... + n^2) + (n+1)^2 \\
&= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \\
&= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} \\
&= \frac{(n+1)\Big(n(2n+1) + 6(n+1)\Big)}{6} \\
&= \frac{(n+1)\Big(2n^2 + 1n + 6n + 6\Big)}{6} \\
&= \frac{(n+1)\Big(2n^2 + 4n + 3n + 6\Big)}{6} \\
&= \frac{(n+1)\Big(2n(n+2) + 3(n+2)\Big)}{6} \\
&= \frac{(n+1)\Big(n+2\Big)\Big(2n+3\Big)}{6} \\
&= \frac{(n+1)\Big((n+1)+1\Big)\Big(2(n+1)+1\Big)}{6}.
\end{aligned}$$

Thus $P(n+1)$. By induction, it follows that for all $n \in \mathbb{N}$ such that $n \geq 2$, $P(n) : 1^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

(ii) For all $n \in \mathbb{N}$ such that $n \geq 2$, $P(n) : 1^3 + ... + n^3 = (1 + ... + n)^2$.

Proof.

Let $n = 2$. Clearly, $1^3 + 2^3 = 9 = (1+2)^2$. Thus $P(2)$. Now let n be some number such that $n \in \mathbb{N}$ and $n \geq 2$, and suppose

$P(n)$. Hence $1^3 + ... + n^3 = (1 + ... + n)^2$. Observe that

$$\begin{aligned}
1^3 + ... + (n+1)^3 &= (1^3 + ... + n^3) + (n+1)^3 \\
&= (1 + ... + n)^2 + (n+1)^3 \\
&= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 \\
&= \frac{n^2(n+1)^2}{4} + (n+1)^3 \\
&= \frac{n^2(n+1)^2 + 4(n+1)^3}{4} \\
&= \frac{(n+1)^2(n^2 + 4(n+1))}{4} \\
&= \frac{(n+1)^2(n^2 + 2n + 2n + 4)}{4} \\
&= \frac{(n+1)^2(n+2)^2}{4} \\
&= \left(\frac{(n+1)(n+2)}{2}\right)^2 \\
&= \left(\frac{(n+1)((n+1)+1)}{2}\right)^2 \\
&= (1 + ... + (n+1))^2.
\end{aligned}$$

Thus $P(n+1)$. By induction, it follows that for all $n \in \mathbb{N}$ such that $n \geq 2$, $P(n) : 1^3 + ... + n^3 = (1 + ... + n)^2$.