

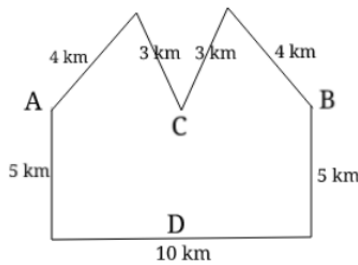
## 1. Finding counterexamples.

**1-1. Soln.** Let  $a = b = -1$ , then  $-1 - 1 = -2 < -1 = \min(-1, -1)$ .

**1-2. Soln.** Let  $a = -1$  and  $b = 2$ , then  $-1 * 2 = -2 < -1 = \min(-1, 2)$ .

**1-3. Soln.** Consider a road network where  $A, B$  and  $C$  are points. The road  $A$  to  $C$  is  $3[km]$ ,  $C$  to  $B$  is  $4[km]$ , and  $A$  to  $B$  is  $5[km]$ . Suppose the roads  $AC$  and  $CB$  let you travel  $2[km/h]$  and the road  $AB$  lets you travel  $1[km/h]$ . Then  $7[km] > 5[km] \iff AC + CB > AB$ , so  $A$  to  $B$  is the shortest route. But  $\frac{7[km]}{2[km/h]} = 3.5[h] < 5[h] = \frac{5[km]}{1[km/h]}$ , so  $A$  to  $C$  to  $B$  is the fastest route.

**1-4. Soln.**



Consider the road network above where  $A, B, C$  and  $D$  are points. Observe that the route  $A$  to  $D$  to  $B$  is  $5 + 10 + 5 = 20[km]$  and has two turns, while the route  $A$  to  $C$  to  $B$  is  $4 + 3 + 3 + 4 = 14[km]$  and has three turns. Thus the former has fewer turns but the latter is shorter.

**1-5. Soln.**

(a) Let  $T = 4$  and  $S = \{1, 2, 3\}$ . The algorithm adds  $\{1, 2\}$  to the knapsack, with 1 unit remaining.

But the subset  $\{1, 3\}$  fills the knapsack completely.

(b) Let  $T = 2$  and  $S = \{1, 2\}$ . The algorithm adds  $\{1\}$  to the knapsack, with 1 unit remaining.

But the subset  $\{2\}$  fills the knapsack completely.

(c) Let  $T = 4$  and  $S = \{3, 2, 2\}$ . The algorithm adds  $\{3\}$  to the knapsack, with 1 unit remaining.

But the subset  $\{2, 2\}$  fills the knapsack completely.

**1-6. Soln.** Let  $U = \{1, 2, 3, 4, 5, 6\}$  and  $S_1 = \{1, 2, 3, 4\}$ ,  $S_2 = \{1, 2, 5\}$  and  $S_3 = \{3, 4, 6\}$ . The algorithm picks  $S_1, S_2$  and  $S_3$ , but observe that  $S_2 \cup S_3 = U$ , thus all elements can be covered with fewer subsets.