5.

(a) If $r \in \mathbb{R}, r \neq 1$ and $n \in \mathbb{Z}_0$, then $P(n) : \sum_{i=0}^{n} r^i = \dfrac{1 - r^{n+1}}{1 - r}$

Proof. (Induction).

Let $r \in \mathbb{R}, r \neq 1$.

For the basis case, suppose $n = 0$. Then $r^0 = 1 = \dfrac{1-r}{1-r} = \dfrac{1 - r^{0+1}}{1 - r}$, thus $P(0)$.

Now suppose $P(n)$ for some $n \in \mathbb{Z}_0$. Hence $\sum_{i=0}^{n} r^i = \dfrac{1 - r^{n+1}}{1 - r}$. Observe that

$$\begin{aligned}
\sum_{i=0}^{n+1} r^i &= \sum_{i=0}^{n} r^i + r^{n+1} \\
&= \frac{1 - r^{n+1}}{1 - r} + r^{n+1} \\
&= \frac{1 - r^{n+1} + (1 - r)r^{n+1}}{1 - r} \\
&= \frac{1 - r^{n+1} + r^{n+1} - r^{(n+1)+1}}{1 - r} \\
&= \frac{1 - r^{(n+1)+1}}{1 - r}
\end{aligned}$$

Thus $P(n+1)$. Therefore, by induction, if $r \in \mathbb{R}, r \neq 1$ and $n \in \mathbb{Z}_0$, then $P(n) : \sum_{i=0}^{n} r^i = \dfrac{1 - r^{n+1}}{1 - r}$.

(b)

Let $S = \sum_{i=0}^{n} r^i$. Then

$$\begin{aligned}
S - r * S &= \sum_{i=0}^{n} r^i - r \sum_{i=0}^{n} r^i \\
S(1 - r) &= \sum_{i=0}^{n} r^i - \sum_{i=0}^{n} r^{i+1} \\
&= \sum_{i=0}^{n} r^i - \sum_{i=1}^{n+1} r^i \\
&= \sum_{i=0}^{n} r^i - \left(\sum_{i=0}^{n} r^i - r^0 + r^{n+1} \right) \\
&= 1 - r^{n+1} \\
\therefore S &= \frac{1 - r^{n+1}}{1 - r}.
\end{aligned}$$