Proposition: For all $n \in \mathbb{N}$, it follows that

$$S_n : 3^1 + 3^2 + 3^3 + ... + 3^k = \frac{3^{k+1} - 3}{2}.$$

Proof. (Induction).

Basis step. Suppose $n = 1$. Observe that $3^1 = \frac{9-3}{2} = \frac{3^{1+1}-3}{2}$. Thus S_1.

Inductive step. Suppose S_k for $n \in \mathbb{N}$.

We now show S_k implies S_{k+1}. Observe that

$$\begin{align}
3^1 + 3^2 + 3^3 + ... + 3^k + 3^{k+1} &= 3(3^0 + 3^1 + 3^2 + ... + 3^{k-1} + 3^k) \tag{1} \\
&= 3\left(3^0 + \frac{3^{k+1} - 3}{2}\right) \tag{2} \\
&= 3 + \frac{3(3^{k+1} - 3)}{2} \tag{3} \\
&= \frac{3(2)}{2} + \frac{3^{k+2} - 9}{2} \tag{4} \\
&= \frac{3^{k+2} + 6 - 9}{2} \tag{5} \\
&= \frac{3^{(k+1)+1} - 3}{2}. \tag{6}
\end{align}$$

Thus S_{k+1}.

It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■