16.

(a).1 $(x+y)^2 = x^2 + y^2$ only when $x = 0$ or $y = 0$.

$$\begin{aligned} x^2 + y^2 &= (x+y)^2 \\ &= x^2 + 2xy + y^2 \\ 0 &= 2xy \\ 0 &= xy. \end{aligned}$$

Thus $x = 0$ or $y = 0$.

(a).2 $(x+y)^3 = x^3 + y^3$ only when $x = 0$ or $y = 0$ or $x = -y$.

$$\begin{aligned} x^3 + y^3 &= (x+y)^3 \\ &= x^3 + 3x^2y + 3y^2x + y^3 \\ 0 &= 3x^2y + 3y^2x \\ &= 3xy(x+y) \\ &= xy(x+y). \end{aligned}$$

Thus $x = 0$ or $y = 0$ or $(x+y) = 0 \implies x = -y$.

(b) Let $x \neq 0$ or $y \neq 0$. Then $4x^2 + 6xy + 4y^2 > 0$.
Note that $x^2 + xy + y^2 > 0 \implies -x^2 - xy - y^2 < 0$ by Exercise 15. Then

$$\begin{aligned} (x+y)^2 \geq 0 > -x^2 - xy - y^2 \\ (x+y)^2 > -x^2 - xy - y^2 \\ x^2 + 2xy + y^2 > -x^2 - xy - y^2 \\ 2x^2 + 3xy + 2y^2 > 0 \\ 4x^2 + 6xy + 4y^2 > 0. \end{aligned}$$

(c) $(x+y)^4 = x^4 + y^4$ only when $x = 0$ or $y = 0$.

$$\begin{aligned} x^4 + y^4 &= (x+y)^4 \\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \\ 0 &= 4x^3y + 6x^2y^2 + 4xy^3 \\ 0 &= xy(4x^2 + 6xy + 4y^2). \end{aligned}$$

Thus $x = 0$ or $y = 0$. Note that $4x^2 + 6xy + 4y^2 = 0$ only when $x = 0$ and $y = 0$ by Exercise 16.b.
Combining the solutions, $(x+y)^4 = x^4 + y^4$ only when $x = 0$ or $y = 0$.

(d) $(x+y)^5 = x^5 + y^5$ only when $x = 0$ or $y = 0$ or $x = -y$.

$$\begin{aligned} x^5 + y^5 &= (x+y)^5 \\ &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \\ 0 &= 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 \\ &= 5xy(x^3 + 2x^2y + 2xy^2 + y^3). \end{aligned}$$

Thus $x = 0$ or $y = 0$ or $x^3 + 2x^2y + 2xy^2 + y^3 = 0$. For the latter, observe that

$$\begin{aligned} 0 &= x^3 + 2x^2y + 2xy^2 + y^3 \\ &= (x^3 + 3x^2y + 3xy^2 + y^3) - (x^2y + xy^2) \\ &= (x + y)^3 - xy(x + y) \\ 0 &= (x + y)\Big((x + y)^2 - xy\Big). \end{aligned}$$

Thus $(x + y) = 0 \implies x = -y$, or $0 = (x + y)^2 - xy = x^2 + 2xy + y^2 - xy = x^2 + xy + y^2$, meaning $x = 0$ and $y = 0$ by Exercise 15.

Combining the solutions, $(x + y)^5 = x^5 + y^5$ only when $x = 0$ or $y = 0$ or $x = -y$.