

11. Prove the principle of complete induction from the principle of ordinary induction.

Proof.

Suppose the property $A(x)$ holds for all $n_0 \leq x \leq k$, where $x, n_0, k \in \mathbb{N}$, and suppose $A(k)$ implies $A(k+1)$. Then $A(x)$ for all $n_0 \leq x \leq k+1$. By Proposition 10, it follows that $A(n)$ for all $n_0 \leq n$ where $n \in \mathbb{N}$, and consequently $A(x)$ for all $n_0 \leq x \leq n$ where $x \in \mathbb{N}$.