1. Program analysis

2-1. *Soln.*

Note the partial sum of the first $k-1$ pronic numbers is defined by $\sum_{i=1}^{k-1} i(i+1) = \dfrac{k(k-1)(k+1)}{3}$. Observe that

$$
\begin{aligned}
\text{mystery}(n) &= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{j} 1 && (1)\\
&= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} j && (2)\\
&= \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n} j - \sum_{j=1}^{i} j \right) && (3)\\
&= \sum_{i=1}^{n-1} \left(\frac{n(n+1)}{2} - \frac{i(i+1)}{2} \right) && \text{(Def. of triangular number)} \quad (4)\\
&= \sum_{i=1}^{n-1} \frac{n(n+1)}{2} - \sum_{i=1}^{n-1} \frac{i(i+1)}{2} && (5)\\
&= \frac{n(n-1)(n+1)}{2} - \frac{1}{2} \sum_{i=1}^{n-1} i(i+1) && (6)\\
&= \frac{n(n-1)(n+1)}{2} - \frac{1}{2} \frac{n(n-1)(n+1)}{3} && \text{(Def. of pronic number)} \quad (7)\\
&= \frac{1}{2} \left(n(n-1)(n+1) - \frac{n(n-1)(n+1)}{3} \right) && (8)\\
&= \frac{1}{2} \frac{3n(n-1)(n+1) - n(n-1)(n+1)}{3} && (9)\\
&= \frac{1}{2} \frac{2n(n-1)(n+1)}{3} && (10)\\
&= \frac{n(n-1)(n+1)}{3}. && (11)
\end{aligned}
$$

Thus $\frac{1}{3}n(n-1)(n+1) = \frac{1}{3}(n^3 + n^2 - n^2 - n) = \frac{1}{3}(n^3 - n) = O(n^3 - n) \implies \text{mystery}(n) = O(n^3)$.