1-7. *Soln.*

(a) Let $2^{n+1} \leq c * 2^n$ for some constant c. Observe that $2^{n+1} \leq 2 * 2^n = c * 2^n$ for $c = 2$, thus $2^{n+1} = O(2^n)$.

(b) For all $n \in \mathbb{N}$, it follows that $2n > n$, which implies $2^{2n} \gg 2^n$. Thus there exists no constant c for which $2^{2n} \leq c * 2^n$. Therefore $2^{2n} \neq O(2^n)$.

1-8. *Soln.*

(a) $f(n) = \log 2^n = 2\log n \leq 2(\log n + 5) = g(n)$, thus $f(n) \leq c_1 * g(n)$ for $c_1 = 2$. Also $f(n) = 2\log n \geq \log n + 5$, thus $f(n) \geq c_2 * g(n)$ for $c_2 = 1$. Therefore $f(n) = \Theta(g(n))$.

(b) $\sqrt{n} \gg logn \iff \sqrt{n} \gg 2logn = logn^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(c) $log^2 n \gg logn \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(d) $\sqrt{n} \gg logn \iff n \gg (logn)^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(e) $n \gg logn \iff n(logn + 1) \gg logn \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(f) $f(n) = 10 \geq log10 = c_1 * g(n)$ when $c_1 = 1$, and $f(n) = 10 \leq 10log10 = c_2 * g(n)$ when $c_2 = 10$, thus $f(n) = \Theta(g(n))$.

(g) $2^n \gg n^2 \iff 2^n \gg 10n^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(g) $3^n \gg 2^n \iff g(n) \gg f(n)$, thus $f(n) = O(g(n))$.

1-9. *Soln.*

(a)

$$
\begin{aligned}
n^2 \gg n &\iff n^2 - n \gg n && (1)\\
&\iff (n^2 - n)\frac{1}{2} \gg 6n && (2)\\
&\iff f(n) \gg g(n) && (3)
\end{aligned}
$$

Thus $g(n) = O(f(n))$.

(b)

$$
\begin{aligned}
n^2 \gg n \gg \sqrt{n} &\iff n + \sqrt{n} \ll n^2 && (4)\\
&\iff n + 2\sqrt{n} \ll n^2 && (5)\\
&\iff f(n) \ll g(n) && (6)
\end{aligned}
$$

Thus $f(n) = O(g(n))$.

(c) $\sqrt{n} \gg logn \Longleftrightarrow n\sqrt{n}\frac{1}{2} \gg nlogn \Longleftrightarrow g(n) \gg f(n)$, thus $f(n) = O(g(n))$.

(d) $n \gg \sqrt{n} \Longleftrightarrow logn + n \gg \sqrt{n} \Longleftrightarrow f(n) \gg g(n)$, thus $g(n) = O(f(n))$.

(e) $log^2 n \gg logn \Longleftrightarrow 2log^2 n \gg logn + 1 \Longleftrightarrow f(n) \gg g(n)$, thus $g(n) = O(g(n))$.

(f)

$$
\begin{aligned}
n \gg logn &\Longleftrightarrow n - 1 \gg logn + 1 && (7)\\
&\Longleftrightarrow n(n-1) \gg n(logn + 1) && (8)\\
&\Longleftrightarrow (n^2 - n)\frac{1}{2} \gg 4nlogn + n && (9)\\
&\Longleftrightarrow g(n) \gg f(n) && (10)
\end{aligned}
$$

Thus $f(n) = O(g(n))$.

1-10. *Soln.* $n^3 - 3n^2 - n + 1 \geq c_1 * n^3$ for $c_1 = \frac{1}{2}$ and $n \geq 10$. Also $n^3 - 3n^2 - n_1 \leq c_2 * n^3$ where $c_2 = 1$. Thus $n^3 - 3n^2 - n + 1 = \Theta(n^3)$.

1-11. *Soln.* $2^n \gg n^2$, thus $n^2 = O(2^n)$.