Proposition: For all $n, k \in \mathbb{Z}$ where $n, k \geq 0$, it follows that
$$S_k : \binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + ... + \binom{n+k}{k} = \binom{n+k+1}{k}.$$

Proof. (Induction).
Basis step. Suppose $k = 0$. Observe that $\binom{n}{0} = 1 = \binom{n+(0)+1}{0}$, thus S_0.
Inductive step. Suppose S_k for $n, k \in \mathbb{Z}$ where $n, k \geq 0$.

We now show S_k implies S_{k+1}. Observe that

$$\begin{aligned} \binom{n+(k+1)+1}{k+1} &= \binom{n+k+1}{k} + \binom{n+k+1}{k+1} && \text{(Def. of Pascal's triangle)} \quad (1) \\ &= \binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + ... + \binom{n+k}{k} + \binom{n+(k+1)}{k+1} && \text{(Inductive hypothesis).} \quad (2) \end{aligned}$$

Thus S_{k+1}.
It follows by mathematical induction that S_k for all $n, k \in \mathbb{Z}$ where $n, k \geq 0$. ■