

1-2. Soln. Observe that

$$\text{pesky}(n) = \sum_{i=1}^n \sum_{j=1}^i \sum_{k=j}^{i+j} 1 \quad (1)$$

$$= \sum_{i=1}^n \sum_{j=1}^i \sum_{k=0}^i 1 \quad (2)$$

$$= \sum_{i=1}^n \sum_{j=1}^i (i+1) \quad (3)$$

$$= \sum_{i=1}^n \left(\sum_{j=1}^i i + \sum_{j=1}^i 1 \right) \quad (4)$$

$$= \sum_{i=1}^n (i^2 + i) \quad (5)$$

$$= \sum_{i=1}^n i^2 + \sum_{i=1}^n i \quad (6)$$

$$= \sum_{i=1}^n i^2 + \frac{n(n+1)}{2} \quad (\text{Def. of } n\text{th triangular number}) \quad (7)$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \quad (\text{Def. of sum of the first } n \text{ square numbers}) \quad (8)$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1)}{6} \quad (9)$$

$$= \frac{n(n+1)((2n+1) + 3)}{6} \quad (10)$$

$$= \frac{n(n+1)2(n+2)}{6} \quad (11)$$

$$= \frac{n(n+1)(n+2)}{3}. \quad (12)$$

Thus $n(n+1)(n+2)\frac{1}{3} = (n^3 + 3n^2 + 2n)\frac{1}{3} \leq 2n^3\frac{1}{3} \implies \text{mystery}(n) = O(n^3)$.