Proposition: Concerning the Fibonacci sequence, $S_n : F_n = \sum_{i=0}^{n-1} \binom{n-i-1}{i}$.

Proof. (Induction).
Basis step. Let $n = 1$.
Observe that $\sum_{i=0}^{1-1} \binom{1-i-1}{i} = \binom{0}{0} = 1 = F_1$. Thus S_1.
Inductive step. Let S_k for $k \in \mathbb{N}$.
We now show S_k implies S_{k+1}. Observe that

$$
\begin{aligned}
F_{k+1} &= F_k + F_{k-1} && (1)\\
&= \sum_{i=0}^{k-1} \binom{k-i-1}{i} + \sum_{i=0}^{(k-1)-1} \binom{(k-1)-i-1}{i} && (2)\\
&= \left[\binom{k-1}{0} + \binom{k-2}{1} + ... + \binom{1}{k-2} + \binom{0}{k-1}\right] + \left[\binom{k-2}{0} + \binom{k-3}{1} + ... + \binom{1}{k-3} + \binom{0}{k-2}\right] && (3)\\
&= 0 + \binom{k-1}{0} + \binom{k-2}{0} + \binom{k-2}{1} + \binom{k-3}{1} + ... + \binom{1}{k-3} + \binom{1}{k-2} + \binom{0}{k-2} + \binom{0}{k-1} + 0 && (4)\\
&= \left[\binom{k-1}{-1} + \binom{k-1}{0}\right] + \left[\binom{k-2}{0} + \binom{k-2}{1}\right] + ... + \left[\binom{1}{k-3} + \binom{1}{k-2}\right] + \left[\binom{0}{k-2} + \binom{0}{k-1}\right] + \binom{0}{k} && (5)\\
&= \binom{k}{0} + \binom{k-1}{1} + \binom{k-2}{2} + ... + \binom{2}{k-2} + \binom{1}{k-1} + \binom{0}{k} && (6)\\
&= \sum_{i=0}^{k} \binom{k-i}{i} && (7)\\
&= \sum_{i=0}^{(k+1)-1} \binom{(k+1)-i-1}{i}. && (8)
\end{aligned}
$$

Thus S_{k+1}.
It follows by mathematical induction that S_n for all $n \in \mathbb{N}$. ■