3.

(i) Proposition: Let $a, b, c \in \mathbb{R}$ and $b, c \neq 0$. Then $\dfrac{a}{b} = \dfrac{ac}{bc}$

Proof. Note that $c^{-1}c = 1 \implies c^{-1} = \dfrac{1}{c}$. Observe that $\dfrac{a}{b} = (1)\dfrac{a}{b} = (c^{-1}c)\dfrac{a}{b} = \dfrac{ac}{ab}$.

(ii) Proposition: Let $a, b, c, d \in \mathbb{R}$ and $b, d \neq 0$. Then $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

Proof.

$$\begin{aligned}\frac{a}{b} + \frac{c}{d} &= \frac{ad}{bd} + \frac{bc}{bd} \\ &= \frac{1}{bd}(ad + bc) \\ &= \frac{ad + bc}{bd}.\end{aligned}$$

(iii) Proposition: Let $a, b \in \mathbb{R}$ and $a, b \neq 0$. Then $(ab)^{-1} = a^{-1}b^{-1}$.

Proof. Note that $c^{-1}c = 1 \implies c^{-1} = \dfrac{1}{c}$ for any $c \in \mathbb{R}$. Observe that

$$(ab)^{-1} = \frac{1}{ab} = \frac{1}{a} * \frac{1}{b} = a^{-1}b^{-1}.$$

(iv) Proposition: Let $a, b, c, d \in \mathbb{R}$ and $b, d \neq 0$. Then $\dfrac{a}{b} * \dfrac{c}{d} = \dfrac{ac}{bd}$.

Proof. Observe that $\dfrac{a}{b} * \dfrac{c}{d} = (a * \dfrac{1}{b})(b * \dfrac{1}{c}) = (ac) * (\dfrac{1}{bd}) = \dfrac{ac}{bd}$.

(v) Proposition: Let $a, b, c, d \in \mathbb{R}$ and $b, c, d \neq 0$. Then $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ad}{bc}$.

Proof.

$$\begin{aligned}\frac{a}{b} \div \frac{c}{d} &= \frac{(\frac{a}{b})}{(\frac{c}{d})} \\ &= \frac{ab^{-1}}{cd^{-1}} \\ &= \frac{a}{c}(\frac{b}{d})^{-1} \\ &= \frac{a}{c}(bd^{-1})^{-1} \\ &= \frac{a}{c}(b^{-1}d) \\ &= \frac{ad}{cb}.\end{aligned}$$

(vi) Proposition: Let $a, b, c, d \in \mathbb{R}$ and $b, d \neq 0$. Then $\dfrac{a}{b} = \dfrac{c}{d}$ if and only if $ad = bc$.

Proof.

$$\begin{aligned}\frac{a}{b} = \frac{c}{d} &\iff (bd)\frac{a}{b} = (bd)\frac{c}{d} \\ &\iff (ad)(b^{-1}b) = (bc)(d^{-1}d) \\ &\iff ad = bc.\end{aligned}$$

It follows that $\dfrac{a}{b} = \dfrac{b}{a} \iff a^2 = b^2$.