Proposition: Concerning the Fibonacci sequence,

$$S_n : F_2 + F_4 + F_6 + ... + F_{2n} = F_{2n+1} - 1.$$

Proof. (Induction).
Basis step. Suppose $n = 1$.
Observe that $F_{2n} = F_{2(1)} = 1 = 2 - 1 = F_{2(1)+1} - 1 = F_{2n+1} - 1$. Thus S_1.
Inductive step. Suppose S_k for $k \in \mathbb{N}$.
We now show S_k implies S_{k+1}. Observe that

$$\begin{aligned} F_2 + F_4 + F_6 + ... + F_{2k} + F_{2(k+1)} &= (F_{2k+1} - 1) + F_{2(k+1)} && (1) \\ &= F_{2k+2} + F_{2k+1} - 1 && (2) \\ &= F_{2k+3} - 1 && (3) \\ &= F_{2(k+1)+1} - 1. && (4) \end{aligned}$$

Thus S_{k+1}.
It follows by mathemtical induction that S_n for all $n \in \mathbb{N}$. ∎