

1-8. Soln.

(a) $f(n) = \log 2^n = 2 \log n \leq 2(\log n + 5) = g(n)$, thus $f(n) \leq c_1 * g(n)$ for $c_1 = 2$. Also $f(n) = 2 \log n \geq \log n + 5$, thus $f(n) \geq c_2 * g(n)$ for $c_2 = 1$. Therefore $f(n) = \Theta(g(n))$.

(b) $\sqrt{n} \gg \log n \iff \sqrt{n} \gg 2 \log n = \log n^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(c) $\log^2 n \gg \log n \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(d) $\sqrt{n} \gg \log n \iff n \gg (\log n)^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(e) $n \gg \log n \iff n(\log n + 1) \gg \log n \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(f) $f(n) = 10 \geq \log 10 = c_1 * g(n)$ when $c_1 = 1$ and $f(n) = 10 \leq 10 \log 10 = c_2 * g(n)$ when $c_2 = 10$, thus $f(n) = \Theta(g(n))$.

(g) $2^n \gg n^2 \iff 2^n \gg 10n^2 \iff f(n) \gg g(n)$, thus $f(n) = \Omega(g(n))$.

(g) $3^n \gg 2^n \iff g(n) \gg f(n)$, thus $f(n) = O(g(n))$.