Solutions to Exercises of Appendix 1.

1. Yes. This follows because the utility defined by (1) is linear on $\mathcal{P}^*$ in the sense that $u(\lambda p_1 + (1-\lambda)p_2) = \lambda u(p_1) + (1-\lambda)u(p_2)$. A1 is satisfied because $\lambda p_1 + (1-\lambda)q \preceq \lambda p_2 + (1-\lambda)q$ if and only if $u(\lambda p_1 + (1-\lambda)q) \le u(\lambda p_2 + (1-\lambda)q)$, if and only if $\lambda u(p_1) + (1-\lambda)u(q) \le \lambda u(p_2) + (1-\lambda)u(q)$, if and only if $\lambda u(p_1) \le \lambda u(p_2)$, if and only if $p_1 \preceq p_2$ (since $\lambda > 0$). Similarly for A2, if $u(p_1) < u(p_2)$ and $u(q)$ is any given number, we can find a $\lambda > 0$ sufficiently small so that $u(p_1) < \lambda u(q) + (1-\lambda)u(p_2)$.

2. For $\mathcal{P} = \{P_1, P_2\}$, define a preference on $\mathcal{P}^*$ to be

$$(1-\theta)P_1 + \theta P_2 \prec (1-\theta')P_1 + \theta' P_2 \qquad \text{if and only if} \qquad \theta < \theta' \text{ and } \theta' \ge 1/2.$$

This is the preference of a person who prefers P_2 to P_1 but has no preference between lotteries that give probability less than $1/2$ to P_2. A1 is not satisfied since, taking $q = P_1$, $p_1 = P_1$ and $p_2 = P_2$, we have $p_1 \prec p_2$ but $\lambda p_1 + (1-\lambda)P_1 \simeq \lambda p_2 + (1-\lambda)P_1$ if $\lambda < 1/2$. A2 is still satisfied since if $p_1 \prec p_2$ and q is any other element of $\mathcal{P}^*$, we can take λ sufficiently small so that $p_1 \prec \lambda q + (1-\lambda)p_2$ and $\lambda q + (1-\lambda)p_1 \prec p_2$.

3. For $\mathcal{P} = \{P_1, P_2, P_3\}$, we may use $(\theta_1, \theta_2, \theta_3)$ to represent the element $\theta_1 P_1 + \theta_2 P_2 + \theta_3 P_3$, where $\theta_1 \ge 0$, $\theta_2 \ge 0$, $\theta_3 \ge 0$, and $\theta_1 + \theta_2 + \theta_3 = 1$. Define a preference on $\mathcal{P}^*$ to be

$$(\theta_1, \theta_2, \theta_3) \prec (\theta_1', \theta_2', \theta_3') \qquad \text{if and only if} \qquad \theta_1 > \theta_1' \text{ or } (\theta_1 = \theta_1' \text{ and } \theta_3 < \theta_3')$$

This is the preference of the person for whom it is the overriding consideration to avoid P_1 (death), but if two lotteries give the same probability to P_1, then the one that gives higher probability to P_3 is preferred. Then clearly A2 is not satisfied for $q = P_1$, $p_1 = P_2$ and $p_2 = P_3$. Checking A1 for $p_1 = (\theta_1, \theta_2, \theta_3)$ and $p_2 = (\theta_1', \theta_2', \theta_3')$ reduces to showing that both sides of (4) are equivalent to $\theta_1 > \theta_1'$ or $(\theta_1 = \theta_1'$ and $\theta_3 < \theta_3')$, for all q and $\lambda > 0$. This is checked by straightforward analysis.