Determining the coeffecients of quartic polynomial related to ball collision

with(*LinearAlgebra*) :

$$R := \begin{bmatrix} \cos(\text{theta}) & -\sin(\text{theta}) \\ \sin(\text{theta}) & \cos(\text{theta}) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{1}$$

1. Case #1: Sliding

$$r_B := \begin{bmatrix} V_{Bx} \cdot t - \frac{1}{2} \cdot \mu_s \cdot g \cdot u_{Bx} \cdot t^2 \\ V_{By} \cdot t - \frac{1}{2} \cdot \mu_s \cdot g \cdot u_{By} \cdot t^2 \end{bmatrix}$$

$$\begin{bmatrix} V_{Bx}\, t - \frac{1}{2}\, \mu_s\, g\, u_{Bx}\, t^2 \\ V_{By}\, t - \frac{1}{2}\, \mu_s\, g\, u_{By}\, t^2 \end{bmatrix} \tag{2}$$

$$r_T := \langle r_x, r_y \rangle + Multiply(R, r_B)$$

$$\begin{bmatrix} r_x + \cos(\theta) \left(V_{Bx}\, t - \frac{1}{2}\, \mu_r\, g\, u_{Bx}\, t^2 \right) - \sin(\theta) \left(V_{By}\, t - \frac{1}{2}\, \mu_r\, g\, u_{By}\, t^2 \right) \\ r_y + \sin(\theta) \left(V_{Bx}\, t - \frac{1}{2}\, \mu_r\, g\, u_{Bx}\, t^2 \right) + \cos(\theta) \left(V_{By}\, t - \frac{1}{2}\, \mu_r\, g\, u_{By}\, t^2 \right) \end{bmatrix} \tag{3}$$

First we consider x:

$$r_x + \cos(\theta) \left(V_{Bx}\, t - \frac{1}{2}\, \mu_s\, g\, u_{Bx}\, t^2 \right) - \sin(\theta) \left(V_{By}\, t - \frac{1}{2}\, \mu_s\, g\, u_{By}\, t^2 \right) \overset{\text{collect w.r.t. t}}{=}$$

$$\left(-\frac{1}{2}\, \mu_r\, g\, u_{Bx} \cos(\theta) + \frac{1}{2} \sin(\theta)\, \mu_r\, g\, u_{By} \right) t^2 + \left(V_{Bx} \cos(\theta) - \sin(\theta)\, V_{By} \right) t + r_x$$

ax:

$$-\frac{1}{2}\, \mu_s\, g\, u_{Bx} \cos(\theta) + \frac{1}{2} \sin(\theta)\, \mu_s\, g\, u_{By} \overset{\text{simplify}}{=} -\frac{1}{2}\, \mu_r\, g \left(-\sin(\theta)\, u_{By} + u_{Bx} \cos(\theta) \right)$$

bx:

$$V_{Bx} \cos(\theta) - \sin(\theta)\, V_{By} \xrightarrow{\text{simplify symbolic}} V_{Bx} \cos(\theta) - \sin(\theta)\, V_{By}$$

cx:

r_x

Then we consider y:

$$r_y + \sin(\theta)\left(V_{Bx}\,t - \frac{1}{2}\,\mu_s\,g\,u_{Bx}\,t^2\right) + \cos(\theta)\left(V_{By}\,t - \frac{1}{2}\,\mu_s\,g\,u_{By}\,t^2\right) \overset{\text{collect w.r.t. t}}{=}$$

$$\left(-\frac{1}{2}\sin(\theta)\,\mu_r\,g\,u_{Bx} - \frac{1}{2}\cos(\theta)\,\mu_r\,g\,u_{By}\right)t^2 + \left(\sin(\theta)\,V_{Bx} + \cos(\theta)\,V_{By}\right)t + r_y$$

ay:

$$\left(-\frac{1}{2}\sin(\theta)\,\mu_s\,g\,u_{Bx} - \frac{1}{2}\cos(\theta)\,\mu_s\,g\,u_{By}\right) \xrightarrow{\text{simplify symbolic}} -\frac{1}{2}\,\mu_r\,g\left(u_{Bx}\sin(\theta) + \cos(\theta)\,u_{By}\right)$$

by:

$$\sin(\theta)\,V_{Bx} + \cos(\theta)\,V_{By} \overset{\text{simplify}}{=} \sin(\theta)\,V_{Bx} + \cos(\theta)\,V_{By}$$

cy:

r_y

Therefore...

$$a_x^{(slide)} = -\frac{1}{2}\,\mu_s\,g\left(u_{Bx}\cos(\theta) - u_{By}\sin(\theta)\right)$$
$$a_y^{(slide)} = -\frac{1}{2}\,\mu_s\,g\left(u_{Bx}\sin(\theta) + u_{By}\cos(\theta)\right)$$
$$b_x^{(slide)} = V_0\cos(\theta)$$
$$b_y^{(slide)} = V_0\sin(\theta)$$
$$c_x^{(slide)} = r_x$$
$$c_y^{(slide)} = r_y$$

2. Case #2: Rolling

$$r = \langle r_x, r_y\rangle + \langle V_0\cos(\theta), V_0\sin(\theta)\rangle\cdot t - \frac{1}{2}\cdot\mu_r\cdot g\cdot\langle\cos(\theta), \sin(\theta)\rangle\cdot t^2$$

$$r = \begin{bmatrix} r_x + t\,V_0\cos(\theta) - \frac{1}{2}\,\mu_r\,g\,t^2\cos(\theta) \\ r_y + t\,V_0\sin(\theta) - \frac{1}{2}\,\mu_r\,g\,t^2\sin(\theta) \end{bmatrix} \tag{4}$$

Therefore...

$$a_x^{(roll)} = -\frac{1}{2}\,\mu_r\, g\cos(\theta)$$
$$a_y^{(roll)} = -\frac{1}{2}\,\mu_r\, g\sin(\theta)$$
$$b_x^{(roll)} = V_0\cos(\theta)$$
$$b_y^{(roll)} = V_0\sin(\theta)$$
$$c_x^{(roll)} = r_x$$
$$c_y^{(roll)} = r_y$$

3. Case #3: Still (Stationary/Spinning)

$$a_x^{(roll)} = 0$$
$$a_y^{(roll)} = 0$$
$$b_x^{(roll)} = 0$$
$$b_y^{(roll)} = 0$$
$$c_x^{(roll)} = r_x$$
$$c_y^{(roll)} = r_y$$

GREAT! ---------------------------

So how does this work in practice? Basically, for a given pair of balls, we can find their 6 coeffecients. Then, we define 6 new coeffecients defined as

$$A_x = \mathrm{a}_x^{(2)} - a_x^{(1)}$$
$$A_y = \mathrm{a}_y^{(2)} - a_y^{(1)}$$
$$B_x = b_x^{(2)} - b_x^{(1)}$$
$$B_y = b_y^{(2)} - b_y^{(1)}$$
$$C_x = c_x^{(2)} - c_x^{(1)}$$
$$C_y = c_y^{(2)} - c_y^{(1)}$$

$$\left\langle Ax\cdot t^2 + Bx\cdot t^2 + Cx,\; Ay\cdot t^2 + By\cdot t^2 + Cy \right\rangle$$

$$\begin{bmatrix} Ax\, t^2 + Bx\, t^2 + Cx \\ Ay\, t^2 + By\, t^2 + Cy \end{bmatrix} \qquad \textbf{(5)}$$

$$\xrightarrow{\text{Euclidean-norm}}$$

$$\sqrt{|Ax\,t^2 + Bx\,t^2 + Cx|^2 + |Ay\,t^2 + By\,t^2 + Cy|^2} \tag{6}$$

$$(Ax^2 + Ay^2)\cdot t^4 + (2\cdot Ax\cdot Bx + 2\cdot Ay\cdot By)\cdot t^3 + (Bx^2 + 2\,Ax\,Cx + 2\,Ay\,Cy + By^2)\cdot t^2 + Cx^2 + Cy^2 - 4\,Rad^2 = 0$$

$$(Ax^2 + Ay^2)\,t^4 + (2\,Ax\,Bx + 2\,Ay\,By)\,t^3 + (Bx^2 + 2\,Ax\,Cx + 2\,Ay\,Cy + By^2)\,t^2 + Cx^2 + Cy^2 - 4\,Rad^2 = 0 \tag{7}$$

The real, non-negative solutions to Equation (7) yields the time for the collision to occur.