DATE: _____ NA[redacted]

6. In a microbiology experiment, the bacteria count (in ten thou[redacted] sample of **tap water** left over time **t (days)** is modelled by the function: $f(t) = \frac{4t}{t^2 - 4t + 12}$, while the bacteria count (in ten thousands) in a sample of **pond water** left over time **t (days)** is modelled by the function: $g(t) = \frac{8t}{t^2 - 3t + 20}$

In what **time interval(s)** will the number of bacteria in the **tap water exceed** the bacteria count in the **pond water**?

[10]

$f(t) > g(t)$

$\frac{4t}{t^2-4t+12} > \frac{8t}{t^2-3t+20}$

$\frac{4t(t^2-3t+20)}{(t^2-4t+12)(t^2-3t+20)} > \frac{8t(t^2-4t+12)}{(t^2-4t+12)(t^2-3t+20)}$

$\frac{(4t^3-12t^2+80t)-(8t^3-32t^2+96t)}{(t^2-4t+12)(t^2-3t+20)} > 0$

$\frac{-4t^3+20t^2-16t}{(t^2-4t+12)(t^2-3t+20)} > 0$

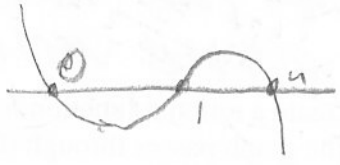
So The tap water has more bacteria in it between the interval of 1 and 4 days.

$\frac{-4t(t^3-5t+4)}{(t^2-4t+12)(t^2-3t+20)} > 0$ $\quad t \in (1,4)$

$\frac{-4t(t-4)(t-1)}{(t^2-4t+12)(t^2-3t+20)} > 0$

always roots, no real

	$(-\infty, 0)$	$(0,1)$	$(1,4)$	$(4,\infty)$
$t^2-4t+13$	+	+	+	+
$t^2-3t+20$	+	+	+	+
$(-4t)$	+	−	−	−
$(t-1)$	−	−	+	+
$(t-4)$	−	−	−	+
TV	-1	1/2	2	5



THINKING

7. $f(x) = \dfrac{ax^2 + bx + e}{cx^2 + dx - 12}$ has the following features:

- the graph passes through (0, 3);
- $x \to \pm\infty, f(x) \to -1$;
- a hole at $x = -2$;
- $x \to 3^-, f(x) \to \infty; x \to 3^+, f(x) \to -\infty$;

Find a, b, c, d, e.

[12]

COMMUNICATION

8. Compare and contrast where a polynomial and rational function can change from positive to negative intervals or vice versa?

[2]

A polynomial changes from positive to negative when it crosses the x-axis it changes from positive to negative at the x-intercepts. However in a rational function a graph can change from positive to negative at the x-intercepts or at the vertical asymptote, a rational function that has two points on either side of a vertical asymptote may have one positive and one negative even though it has no x-intercepts like $f(x) = \frac{1}{x}$

Bonus: Create a rational function for which:

- the graph passes through (0, 4);
- $x \to \pm\infty, f(x) \to 0$;
- $x \to a^{\pm}, f(x)$ does not approach $\pm\infty$ for any value of a in the domain;