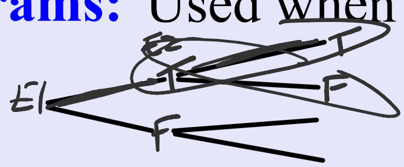
Organized Counting with Venn Diagrams

September 13, 2018 1:17 PM

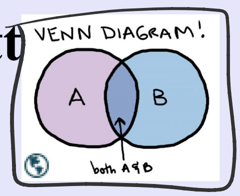
Venn diagrams...

5.1 - Organized Counting with Venn Diagrams

Tree Diagrams: Used when order of items matters.



Venn Diagrams: Used when **order of items does not matt**



A **set** is a collection of *distinct* objects, denoted by an uppercase letter.

Eg: Set of all *skiers*, **S**.

$\{x \in \underline{R}\}$ $\{x \notin N\}$

The objects in a set are called **elements** (*or members of the set*).

Finite sets

Ex: Set A

$A = \{2, 4, 6, 8\}$

Ex: Set, colours of rainbow

$X = \{R, O, Y, G, B, I, V\}$

Ex: Set A

A = {2, 4, 6, 8}

Ex: Set, colours of rainbow

X = {R, O, Y, G, B, I, V}

Infinite sets

Ex: Set of all even positive integers { }

E = {2 , 4, 6, 8, 10, ...}

The order in a set does not matter.

{a, b, c, d} = {b, a, c, d} = ... etc ...

Cardinality (size) of a set:

Number of elements in a set

n(A) = # of elements in set A

Ex: Set A

A = {2, 4, 6, 8} n(A) = 4

Ex: Set, colours of rainbow

X = {R, O, Y, G, B, I, V} n(X) = 7

Ex: Even positive integers

E = {2 , 4, 6, 8, 10, ...} n(E) = ∞

$E = \{2, 4, 6, 8, 10, ...\}$ $n(E) = \infty$

Set Equality:

Two sets A and B are **equal** *if and only if* they have the same elements.

Subsets - sets within sets:

If all the elements of set B are also elements
of Set A, then *B is a subset of set A*.

eg. $X = \{a, b, c, d\}$
$Z = \{b, c, d, a\}$ identical sets

$A = \{1, 2, 3, 4, 5, 6\}$ $B \subseteq A$

$B = \{2, 4, 6\}$ "set B is a subset of A"

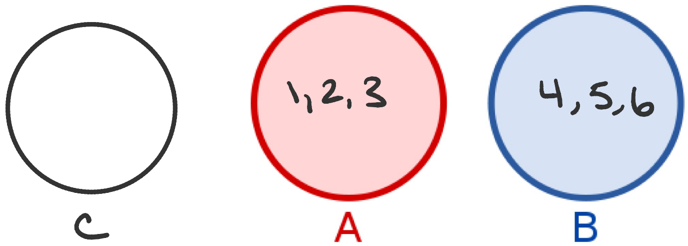
An **Empty set** is a set with no elements

$\emptyset$

{ } - zero elements $\emptyset$ $\{\}$

If two sets have no elements in common, they are **disjoint sets**. In other words

If two sets have no elements in common, they are **disjoint sets**. In other words, they are mutually exclusive



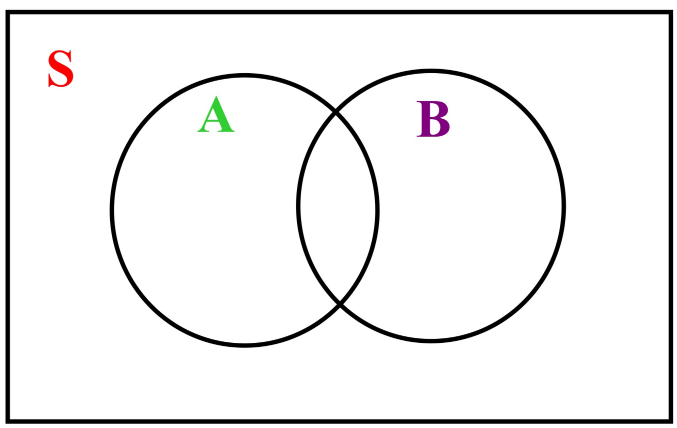
$A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$
$C = \{\}$

Venn Diagrams

Used to organized information and illustrate the relationship between sets and their subsets

Illustrate combinations (the order of elements does not matter)

S = **Universal Set - "universe - everything"**

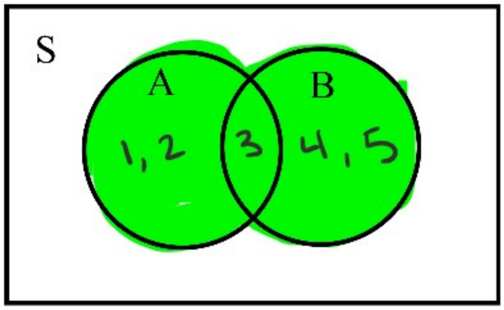


What are the subsets here?

$A \subseteq S$ $\quad$ $A \subseteq A$

$B \subseteq S$ $\quad$ $B \subseteq B$

Union of two sets



$A = \{1,2,3\}$

$B = \{3,4,5\}$

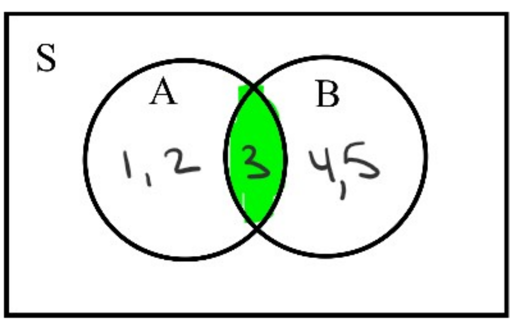
$\cup$

$A \cup B = \{1,2,3,4,5\}$

$A \cup B$ = **union** of A and B
= "A **or** B"

Elements in ***<u>either</u>*** *set A or set B*

Intersection of two sets



$A = \{1,2,3\}$

$\cap$

$B = \{3,4,5\}$

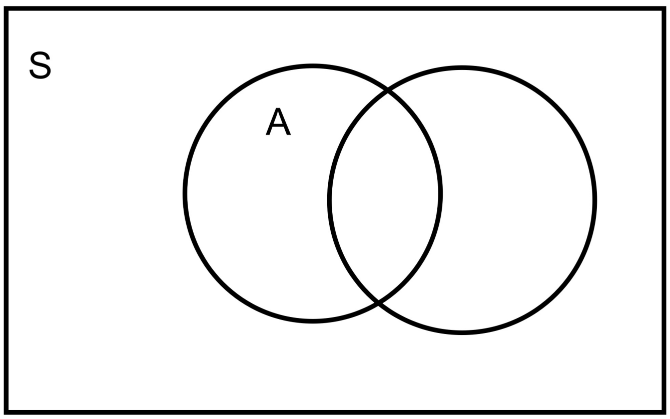
$A \cap B = \{3\}$

$A \cap B$ = **intersection** of A and B
= "A **and** B" $n(A \cap B) = 1$

Elements in ***<u>both</u>*** *set A and set B*

Empty Set $A \cap B$ = disjoint /mutually exclusive.

Complement of a set



A' ← NOT

A' ← prime

Notation A' - everything not included in set A

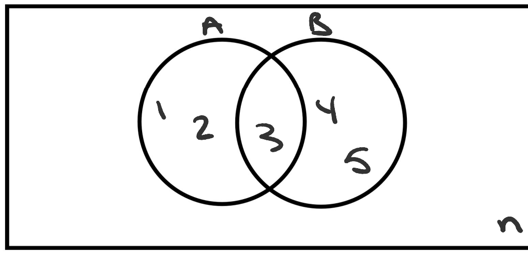
Principle of Inclusion and Exclusion

The relationship bewteen the union and intersection of two sets is shown in the following formula:

following formula:

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Remove duplicate items



$n(A) = 3$

$n(B) = 3$

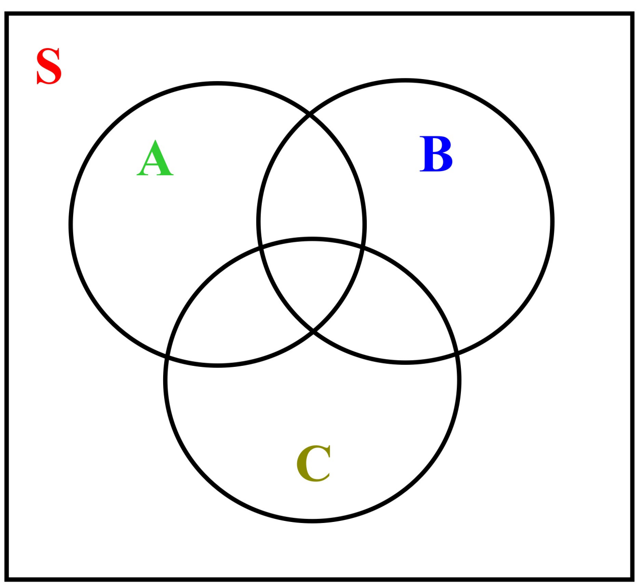
$n(A \cap B) = 1$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 3 + 3 - 1$$
$$= 5$$

Working with 3 sets: Set A, B, C

We can extend our knowledge of set theory to work with three sets, A, B, & C.

The same concepts extend to 3 sets.



Union

$A \cup B$

$A \cup C$

$B \cup C$

$A \cup B \cup C$

Intersection

$A \cap B$

$A \cap C$

$B \cap C$

$A \cap B \cap C$

5.1 Venn Diagrams Problems

Consider the following sets:

A = {1, 3, 5, 7, 9 }
B = {2, 4, 6, 8, 10}
C = {4, 6, 8 }
D = {4, 5, 7}

Determine the following

a) $A \cup B$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

b) $A \cap B$ { } $\varnothing$

d) $B \cap C$ {4, 6, 8}

e) List any subsets

C is a subset of B $C \subseteq B$

List all the subsets of A={1,2,3}

{ } {1} {1,2} {1,2,3}

{2} {1,3}

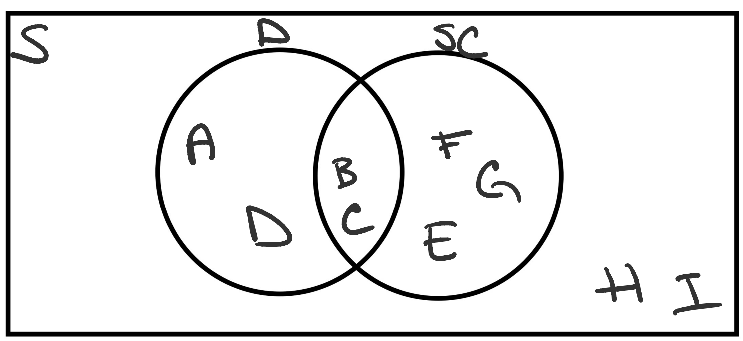
{3} {2,3}

Total # of subsets = 8

Total # of subsets = 8

Investigation

A group of students meet regularly to plan the dances at Vennville High School. Amar, Belinda, Charles, and Danica are on the dance committee, and Belinda, Charles, Edith, Franco, and Geoff are on the students' council. Hans and Irena are not members of either group, but they attend meetings as reporters for the school newspaper.



S =

D = {A, B, C, D} SC = {B, C, E, F, G}

9 people attended the meeting!!

There are 15 girls on the Juvenile Skating team. Some of the skaters were also on the Novice Skating team which has 14

$\{a, b, c, d, e, f, g, h, i\}$

There are 15 girls on the Juvenile Skating team. Some of the skaters were also on the Novice Skating team which has 14 skaters total. When purchasing warm up suits, the manager onl bought 22 track suits.

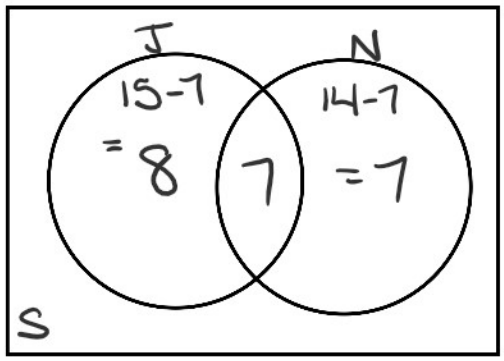
a) Illustrate this situation using a VENN diagram
b) Determine the total number of skaters who are on both teams.
c) Determine the number of skaters who skate on Juvenile onl

$n(A \cup B) = n(A) + n(B) - n($

$22 = 15 + 14 - n($

$22 = 29 - n(A \cap B)$

$n(A \cap B) = 7$



$8 + 7 + 7 = 22$

Ex: Working with 2 sets

At the start of the flu season, Dr. Singh has 50 patients, with the following symptoms:

- *30 have a headache;* $n(H)$
- *24 have a cold;* $n(C)$
- *12 have neither.*

$n(HUC$

(a) Draw a Venn Diagram.

(b) Determine how many have *both* a headache and a cold.

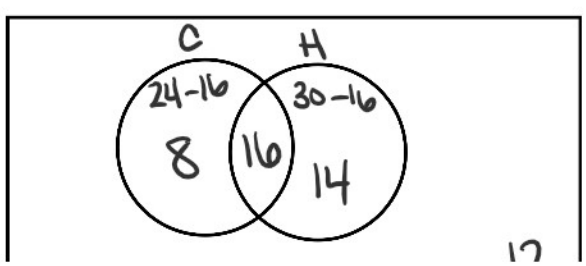
$n(H \cup C) = n($

$38 = 30 + 2$

$38 = 54 - $

$-16 = -n($

$16 = n(H$



$A \cap B)$

$A \cap B)$

$)$

$C) = 50-12$

$= 38$

$H) + n(C) - n(H \cap C)$

$4 - n(H \cap C)$

$n(H \cap C)$

$H \cap C)$

$\cap C)$

16 = n(H

$8+16+14+12=50$ ✓

EXAMPLE

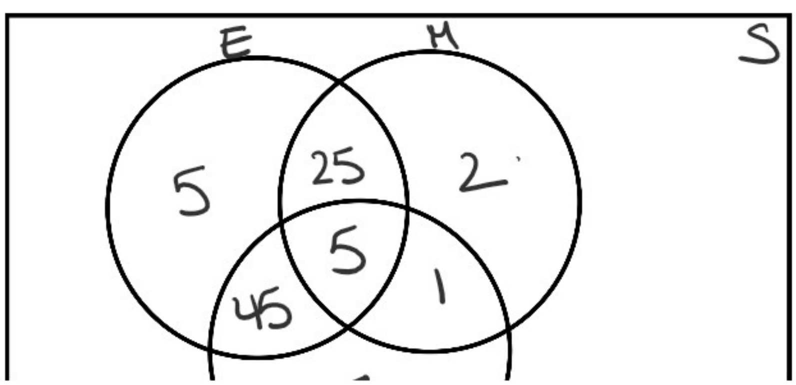
- A survey of 110 students

Course Taken	No. of students
English	80
Mathematics	33
French	68
English and Mathematics	30
French and Mathematics	6
English and French	50
All three courses	5

a) How many students take English only?

b) How many students take Math or French?

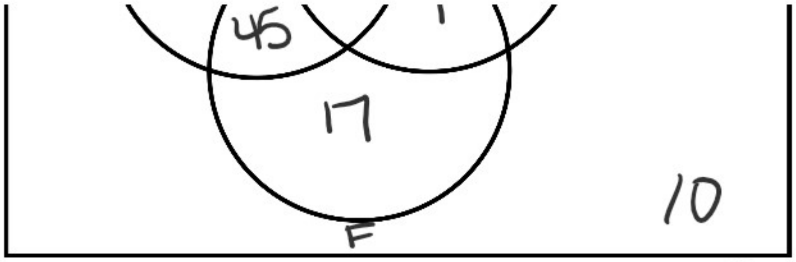
c) How many students do not take French?



Work from

nc)

inside out!



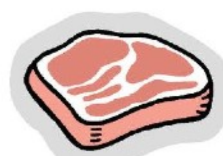
$5+25+2+45+5+1+17=100$

$\therefore$ 10 are in the universe.

Ex: Working with 3 sets

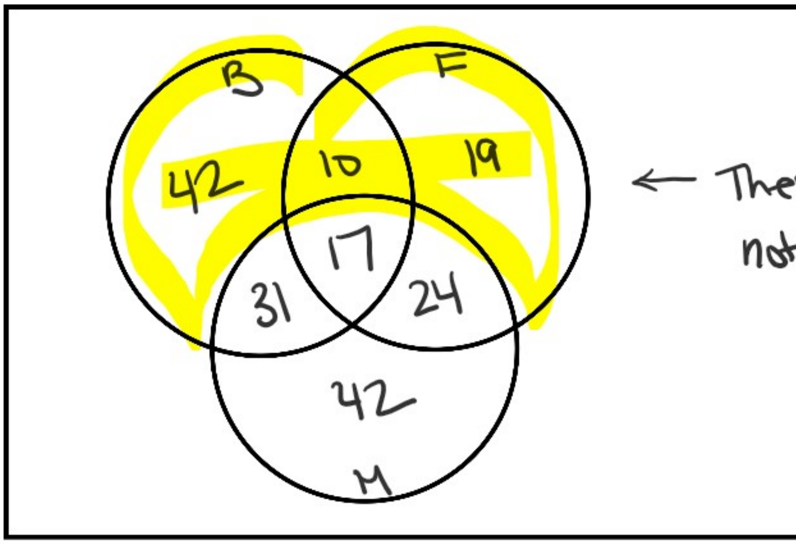
Data from a recent survey of PCSS Students reveals that

- 114 eat meat
- 100 eat bread
- 70 eat fruit
- 48 eat meat and bread
- 41 eat meat and fruit
- 27 eat bread and fruit
- 17 eat all 3 foods



(a) Draw a Venn Diagram. How many students were surveyed, **assuming all students eat at least one of the 3 foods?**

(b) How many students did not eat meat?



← These students
not eat meat

42+10+1

71 student
not eat m

Ex: Working with three sets

In an office with 28 employees, the following information was obtained about the staff:

- *6 are managers*
- *8 are over 50 years old*
- *17 are female*
- *There are 3 managers are over 50*
- *There are 3 female managers*

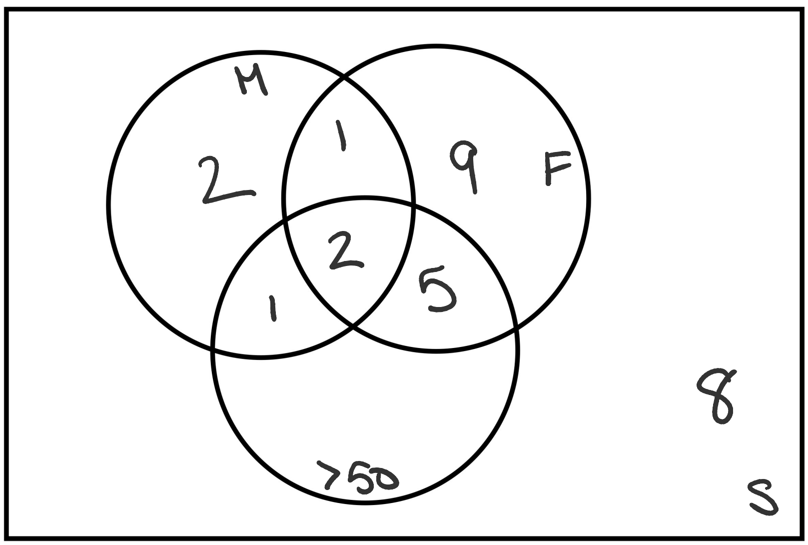
do

$1 = 71$

s do
eat.

- *There are 3 managers are over 50*
- *There are 3 female managers*
- *There are 7 females over 50*
- *There are 2 female managers over 50*

Draw a Venn Diagram and determine how many of the staff are male, under 50 and not a manager.



2+1+9+1+

28-20=8

8 people

male, un

and not

mana

+2+5=20

are
der 50
a
ger.

Principle of Inclusion and Exclusion: 3 sets

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$
$$- n(A \cap B) - n(A \cap C) - n(B \cap C)$$
$$+ n(A \cap B \cap C)$$

If we do not know the intersection of the

sets we need to solve using the principle

of inclusion and exclusion.

Be careful of the signs!!

Think: Why is the intersection of all three s

ADDED back in???

e

ets