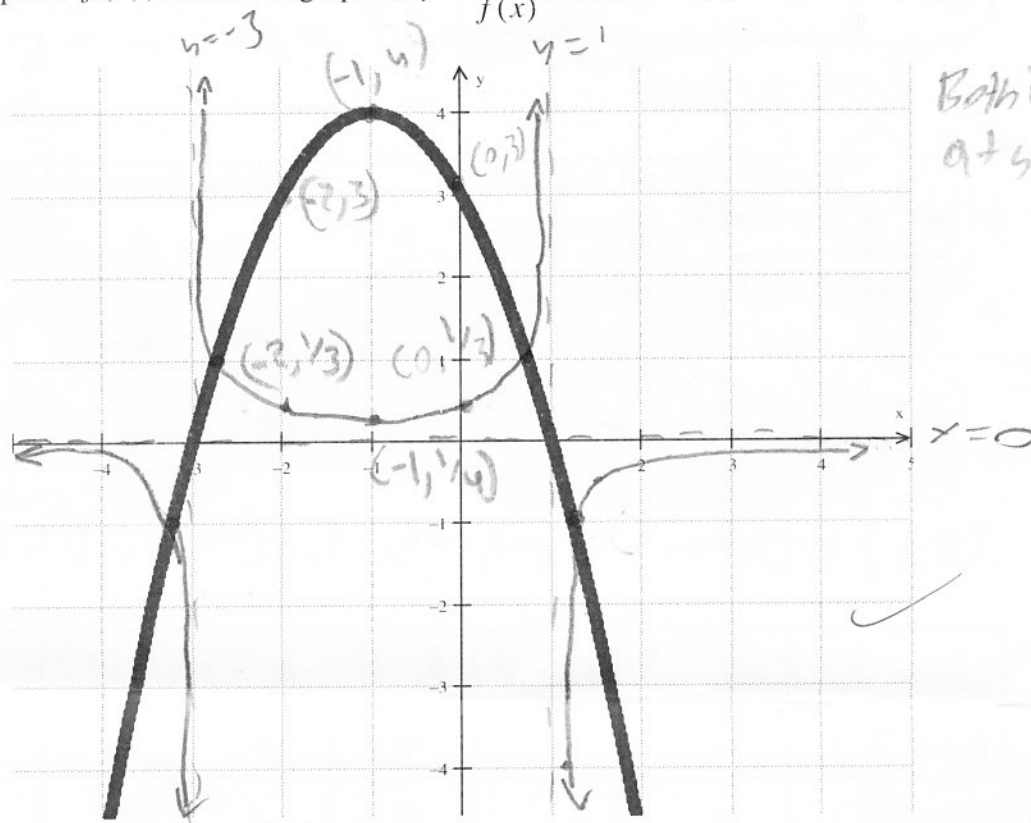


KNOWLEDGE/UNDERSTANDING

1. Given the graph of $f(x)$, sketch the graph of $y = \frac{1}{f(x)}$. Include key points and asymptotes

[4]



Both intersect
at $x+4=1$ and $y=1$

✓

2. Solve the following equation. State Restrictions and express your answer to two decimal places where necessary.

[8]

$$\frac{6}{x^2+x-6} - \frac{3}{x^2+4x+3} = \frac{12}{x^2+2x-3}$$

$$\frac{6}{(x+3)(x-2)} - \frac{3}{(x+3)(x+1)} = \frac{4}{(x+3)(x-1)}$$

$$\frac{6(x+1) - 3(x-2)}{(x+3)(x+1)(x-2)} = \frac{4(x-2)(x+1)}{(x+3)(x-1)(x-2)}$$

$$\frac{6x+6-3x+6}{(x+1)(x-2)} = \frac{4(x+1)(x-2)}{(x+1)(x-2)(x-1)}$$

$$\frac{3(-x+4)(x-1) - 4(x+1)(x-2)}{(x+1)(x-2)(x-1)} = 0$$

$$\frac{3x^2+9x-12-4x^2+4x+8}{(x+1)(x-2)(x-1)} = 0$$

$$x \neq -3, 2, -1, 1$$

$$-x^2+13x-4=0$$

$$-(x^2-13x+4)=0$$

$$x^2-13x+4=0$$

$$x = \frac{13 \pm \sqrt{13^2 - 4(4)}}{2}$$

$$= \frac{13 \pm \sqrt{153}}{2}$$

$$x = \frac{13 + \sqrt{153}}{2} = 12.68$$

$$x = \frac{13 - \sqrt{153}}{2} = 0.32$$

$$x = 12.68 \text{ or } x = 0.32$$

3. Solve the rational inequality using the interval chart method.

$$\frac{2x-1}{x+7} > \frac{x+1}{x+3}$$

Handwritten notes: $\frac{19}{12} > \frac{11}{13}$, $\frac{19}{12} > \frac{7}{11}$

[12]

$$\frac{(2x-1)(x+3)}{(x+7)(x+3)} - \frac{(x+1)(x+2)}{(x+3)(x+7)} > 0$$

$$\frac{(2x^2 + 5x - 3) - (x^2 + 3x + 2)}{(x+7)(x+3)} > 0$$

$$\frac{x^2 - 3x - 10}{(x+7)(x+3)} > 0$$

$$\frac{(x-5)(x+2)}{(x+7)(x+3)} > 0$$

-7, -3, -2, 5

	$(-\infty, -7)$	$(-7, -3)$	$(-3, -2)$	$(-2, 5)$	$(5, \infty)$	
$(x+7)$	-	+	+	+	+	
$(x+3)$	-	-	+	+	+	
$(x+2)$	-	-	-	+	+	
$(x-5)$	-	-	-	-	+	
TV	-10	-4	-2.5	0	10	
Answer	(+)	-	(+)	-	(+)	

State your answer in:

Set Notation: $\{x \in \mathbb{R} \mid -\infty < x < -7 \text{ or } -3 < x < -2 \text{ or } 5 < x < \infty\}$

Interval Notation: $x \in (-\infty, -7) \cup (-3, -2) \cup (5, \infty)$

APPLICATION

4. Given $f(x) = \frac{x^2 - 5x + 4}{x - 2}$, state the following:

$$\begin{array}{c} (-\infty, 0) \quad (1, 2) \quad (2, 4) \quad (4, \infty) \\ \begin{array}{cccc} x-1 & - & + & + \\ x-2 & - & - & + \\ x-4 & - & - & + \end{array} \\ \hline \begin{array}{cccc} - & + & - & + \end{array} \end{array}$$

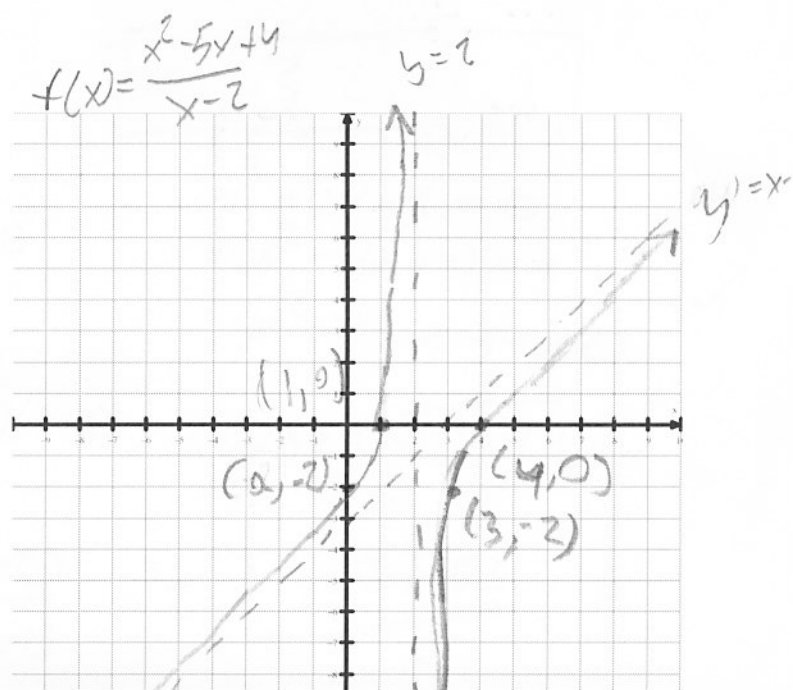
$$= \frac{(x-4)(x-1)}{(x-2)}$$

$$\begin{array}{r} x-3 \\ x-2 \overline{) x^2 - 5x + 4} \\ \underline{x^2 - 2x} \\ -3x + 4 \\ \underline{-3x + 6} \\ -2 \end{array}$$

<p>State the domain</p> <p>[1] $\{x \in \mathbb{R} \mid x \neq 2\}$ ✓</p>	<p>x and y intercepts</p> <p>x-int $x=4, 1$ ✓</p> <p>y-int $y=-2$ ✓</p> <p>[2]</p>
<p>State the equation of any vertical asymptote(s)</p> <p>[1] V.A at $x=2$</p>	<p>State the equation of any horizontal/oblique asymptote</p> <p>[1] Oblique asymptote $y = x - 3$</p>
<p>Determine the behaviour near the vertical asymptote(s)</p> <p>$x \rightarrow 2^- \quad y \rightarrow \infty$</p> <p>$x \rightarrow 2^+ \quad y \rightarrow -\infty$</p> <p>[2]</p>	<p>Determine the end behaviour</p> <p>$x \rightarrow -\infty \quad y \rightarrow -\infty$</p> <p>$x \rightarrow \infty \quad y \rightarrow \infty$</p> <p>$f(x) \rightarrow y = x - 3$</p> <p>[2]</p>
<p>Intervals where $f(x) > 0$</p> <p>$x \in (1, 2) \cup (4, \infty)$</p> <p>[1] ✓</p>	<p>Intervals where $f(x) < 0$</p> <p>$x \in (-\infty, 1) \cup (2, 4)$</p> <p>[1] ✓</p>

Graph the function. Label all asymptotes and key points.

[4]



5. In the event of a power failure, a computer model estimates the temperature, T , in degrees Celsius, in a food-processing plant's freezer to be $T = \frac{2t^2}{t+1} - 15$, where t is the time, in hours, after the power failure. How long would it take for the temperature to reach 0°C ?

[5]

$$T = 0^\circ\text{C} \quad T = \frac{2t^2}{t+1} - 15$$

$$\begin{aligned} 0 &= \frac{2t^2}{t+1} - 15 \\ &= \frac{2t^2}{t+1} - \frac{15(t+1)}{(t+1)} \\ &= \frac{2t^2 - 15t - 15}{t+1} \end{aligned}$$

$$t = \frac{15 \pm \sqrt{15^2 - 4(2)(-15)}}{2(2)}$$

$$t = \frac{15 \pm \sqrt{345}}{4}$$

$$t = 8.3935$$

$$t = -0.8935$$

$$0 = 2t^2 - 15t - 15$$

$$t = 8.3935$$

$t = -0.8935$ $\because t < 0$ \therefore The second solution is not acceptable only positive time is acceptable

$$t = 8.39 \leftarrow \text{rounded to 2 decimals}$$

\therefore The refrigerator will reach 0°C after around 8.39 hours or approximately 8 hours and 23 minutes.

$$0.39 \times 60 = 23.4$$

