a) Two pair $\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{2}\binom{44}{1}$ WRONG!

$nC_r = \frac{nP_r}{r!}$

b) Three of a kind $\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}$ $\binom{13}{2}\binom{4}{2}\binom{44}{1}$

c) Full house (3 of a kind, 2 of a kind) $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$

Sep 27-9:55 AM

5.3 Problem Solving with Combinations

Set Theory - Investigation:

Given sets A, B, C, D

A = {1} B = {1, 2} C = {1, 2, 3}

D = {1, 2, 3, 4}

List all the possible ***subsets*** of each set.

$\{\}$ $\{1\}$ $\{1,2\}$ $\{1,2,3\}$ $\{1,2,3,4\}$

$= {}_4C_0$ $\{2\}$ $\{1,3\}$ $\{2,3,4\}$ $= {}_4C_4$

$\{3\}$ $\{1,4\}$ $\{1,2,4\}$

$\{4\}$ $\{2,3\}$ $\{1,3,4\}$

$= {}_4C_1$ $= {}_4C_3$

$\{2,4\}$

$\{3,4\}$

$= {}_4C_2$

Total = $1+4+6+4+1$

$= 16$

2^n = Total number of subsets for a given set.

$2^4 = 16$

Sep 29-4:17 PM

A combination is simply a subset of a group of ***n*** distinct objects.

The total number of combinations containing at least one item chosen from a group of n distinct items is $2^n - 1$

Sep 29-4:26 PM

Ex: At the start of the semester, the following extracurricular activities are running: Multicultural Club, School Reach, SAC, Social Justice, Math Club, and Study Hall. Chris wants to get involved in at least one; how many different ways could he?

$2^n - 1$

$= 2^6 - 1$

$= 63$

$- {}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6$

$= 63$

Feb 22-6:25 PM

Combinations with some identical items:

If at least one item is chosen, the total # of selections that can be made from
- p items of one kind;
- q items of another kind;
- r items of another kind;

and so on ...

is $(p+1)(q+1)(r+1)\ldots - 1$

(+1: the option to choose nothing. −1: do not have the option to choose nothing.)

Sep 29-4:32 PM

Ex: Mrs Valliere, the librarian, is purchasing books. There have been requests for three copies of Twilight, six copies of Harry Potter and the Deathly Hallows, two copies of R&J, and one copy of the History of Mathematics.

Due to budget restrictions, she may not be able to buy all of these. How many different purchases can she make?

$(p+1)(q+1)(r+1) - 1$

$(3+1)(6+1)(2+1)(1+1) - 1$

$= 167$

$(p)(q)(r)(s)$

$(3)(6)\ldots$

Sep 29-4:30 PM

Ex: A DJ has 5 rock songs, 2 blues tunes, and 3 Jazz pieces in his playlist. In how many ways can he choose 3 pieces to play if the crowd wants to hear some Jazz?

Direct: 1J 2O $\binom{3}{1}\binom{7}{2}$ + 2J 1O $\binom{3}{2}\binom{7}{1}$ + 3J $\binom{3}{3}$

Indirect: Total $\binom{10}{3}$ − undesired (No jazz) $\binom{7}{3}$

$= 85$

Sep 29-4:35 PM

Ex: For a school play, 12 students, five boys and 7 girls, have auditioned for the roles of Peter Pan, TinkerBell, Captain Hook, Wendy, John and Michael.

(a) In how many ways can these roles be filled?
Assume that the guys only play male roles, while girls only play female roles.

$5P_4 \times 7P_2 = 5040$

5 CH, 4 PP, 3 J, 2 M $\quad 5C_1 \times 4! =$ _____

(b) What if the director does not care about gender specific roles?

$12P_6 = 665\,280$

Sep 30-11:49 PM

Ex: Josh has 15 different board games and wants to host a gaming party. In how many ways can he:

(a) Select *some* of his games to be played?

$2^{15}-1 = 32\,767$

(b) Set a schedule of playing 6 games starting with Settlers of Catan and finishing with Risk.

SC _ _ _ _ R

$_{13}P_4 = 17160$ $\quad {}_{13}C_4 \times 4! = 17160$

Feb 28-10:56 PM

Ex: From a standard deck of 52 playing cards:

(a) How many five card hands contain at most two black cards?

0B 5R 1B 4R 2B 3R

$\binom{26}{0}\binom{26}{5} + \binom{26}{1}\binom{26}{4} + \binom{26}{2}\binom{26}{3}$

$= 1\,299\,480$

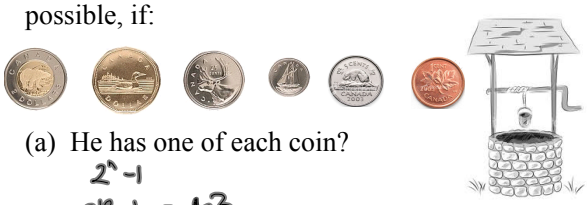
(b) How many five card hands contain at least one heart?

Total - undesired

$\binom{52}{5} - \binom{39}{5}$ No hearts OR 5D

Feb 28-9:10 PM

Ex: In his pocket, Patrick has some coins. He wishes to throw one or more of these coins into a wishing well. How many sums of money are possible, if:



(a) He has one of each coin?

$2^n - 1$

$2^6 - 1 = 63$

(b) He has 2 toonies, 3 pennies, 1 nickel, and 5 dimes, and 4 quarters?

$= (2+1)(3+1)(1+1)(5+1)(4+1) - 1$

$= (3)(4)(2)(6)(5) - 1$

$= 720$

c) Same as above but must throw in a penny

$= (2+1)(3)(1+1)(5+1)(4+1)$

$= (3)(3)(2)(6)(5)$

$= 540$

Sep 29-4:29 PM

Ex: A school DECA team has fifteen members: 4 grade 10s, 5 grade 11s, and 6 grade 12s. If a team of four is to be selected for a competition, how many ways are there to:

(a) Select a team captain, and an asst team captain, if they must be grade 11s or 12s?

$_{11}P_2$ Grade 11/12 — Two specific positions

(b) Select a team with exactly two senior students, and two junior students?

$\binom{4}{2}\binom{11}{2}$ 2 Junior 2 Senior

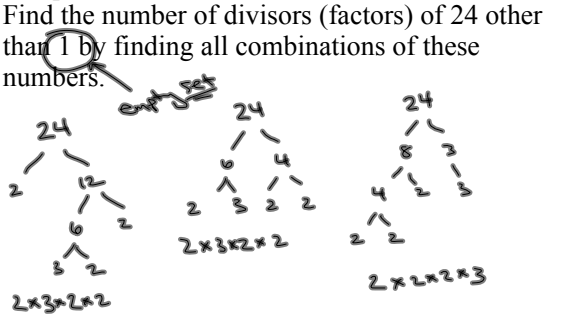
(c) Select a team with at least two senior students?

Total - undesired

$\binom{15}{4} - \binom{11}{0}\binom{4}{4} - \binom{11}{1}\binom{4}{3}$ No seniors, 4 others; 1S 3O

Feb 22-5:47 PM

The prime factorization of 24 is 2 x 2 x 2 x 3. Find the number of divisors (factors) of 24 other than 1 by finding all combinations of these numbers.



Any combination of prime factors will give a factor of 24!

$(3+1)(1+1)-1$

$= (4)(2)-1$

$= 7$

Sep 27-10:09 PM