

Recall Pascal's Formula

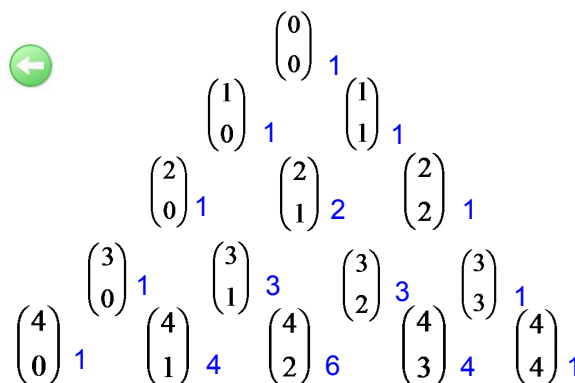
$$t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$$

Pascal's Formula - Combination version

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

The visual "proof" is within Pascal's triangle

Looking at the terms in Pascal's Triangle again:



The terms correspond to Pascal's triangle!

Wow! ^ _ ^

Using combinations we can quickly find ANY term in Pascal's Triangle.

Ex: Re-write each as an equivalent expression using Pascal's Formula. Then verify the result.

(a) $\binom{17}{2}$

(b) $\binom{13}{5} + \binom{13}{6}$

(c) $\binom{20}{6} - \binom{19}{6}$

The Binomial Theorem:

Provides a nice connection between expanding binomials, combinations, and Pascal's Triangle

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}a^0 b^n$$

- For the **nth power** of the binomial, the coefficients represent the **nth row** of Pascal's triangle.
- This provides an easy method to expand higher power binomials.

Ex: State the value the exponent k in the binomial expansion of $(a + b)^9$

a) a^3b^k

b) a^kb^2

c) $a^{k+1}b^k$

Ex: State the coefficient for the following terms in the expansion of $(a + b)^{11}$

a) a^9b^2

b) a^3b^8

c) $a^{11-r}b^r$

Alternatively, we can write the BT as:

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

\sum represents the "sum of" all the terms in the expansion

r is a value ranging between 0 to n ($0 \leq r \leq n$)

Ex: Expand $(5x + y)^4$ using the BT.

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + \binom{n}{n} a^0 b^n$$

Ex: Expand the 4th to 7th terms of $(3x^2 - 2y)^9$ using the BT.

Factor the expression

$$1+10x^2+40x^4+80x^6+80x^8+32x^{10}$$