Pascal's Triangle Pathways.notebook

MDM 4U0
Unit 1: Permutations
Lesson 6: Pascal's Triangle Part 1 (Textbook 4.4)

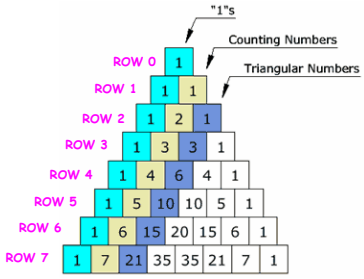
Pascal's Triangle

One of the most interesting Number Patterns is Pascal's Triangle (named after *Blaise Pascal*, a famous French Mathematician and Philosopher).

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
```

Each term, except for the 1's on the outer diagonals, is equal to the sum of terms immediately above it.

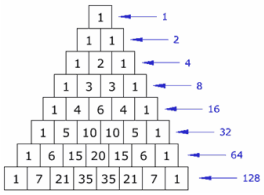
Patterns Within the Triangle



This sequence is generated from a pattern of dots which form a triangle.

1 dot 3 dots 6 dots 10 dots

Horizontal Sums



All terms can be represented in the form $t_{n,r}$
where
n is the row number
r is the column number
The first row is referred to as ROW 0.
The first column is referred to as COLUMN 0.

$t_{n,r}$ = term in row n, position r.

In general $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

Pascal's Formula

The terms in the next row always begin and end with a 1.
Each successive term is the sum of the two terms above it
Even rows have an odd number of terms.
Odd rows have an even number of terms.

Example 1: State the value of

a) $t_{2,1}$ b) $t_{4,2}$ c) $t_{5,4}$

Example 2: Express each of the following terms as a combination of the terms in the previous row.

a) $t_{2,1}$ b) $t_{4,2}$ c) $t_{9,5}$

Example 3: Express as a single term in Pascal's Triangle.

a) $t_{7,2} + t_{7,3}$ b) $t_{4,5} + t_{4,4}$ c) $t_{9,4} - t_{8,4}$

Example 4: Determine the row in Pascal's Triangle whose terms have a sum of 4096.

Example 5:
The first six terms in row 25 of Pascal's Triangle are given below. Determine the first six terms in row 26.

1, 25, 300, 2300, 12650, 53130.

Example 6: Fill in the missing numbers using Pascal's Triangle

_____ _____ 495 330

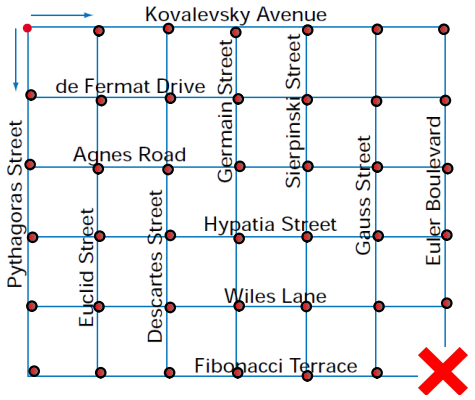
_____ _____ 825

3003 2112

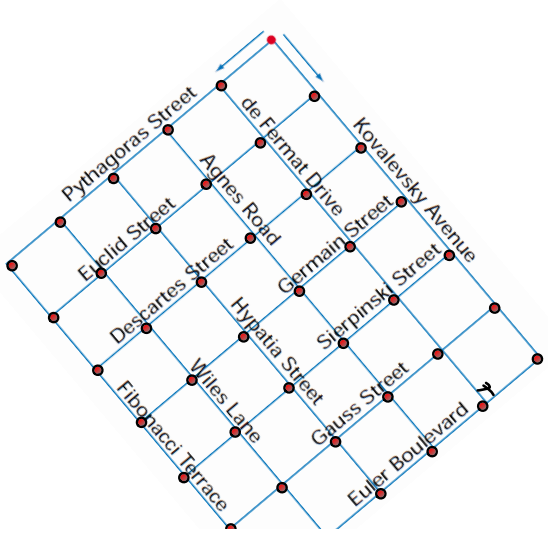
_____

Applying Pascal's Triangle

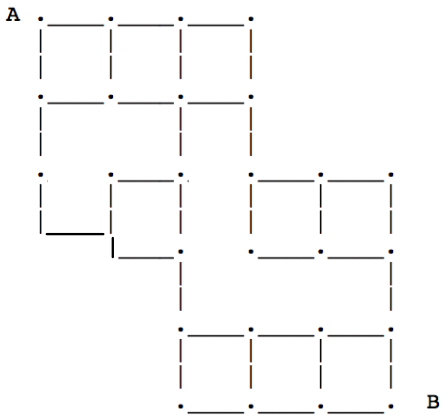
How many ways are there to get to the corner of Fibonacci Terrace and Euler Boulevard?



Use an iterative process: for each intersection as you move away from the starting point, count the number of routes to that particular intersection by adding together the number of routes from each possible preceding intersection (there will be one or two possible predecessors.)



How many paths are there from A to B?



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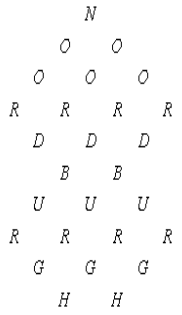
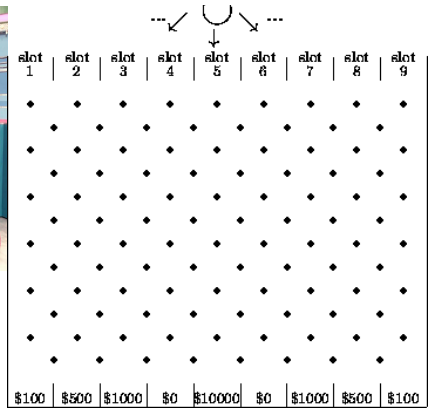
1. Find the total number of paths which spell the word NOORDBURGH.

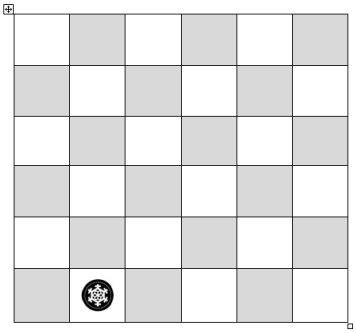


Paths = _____

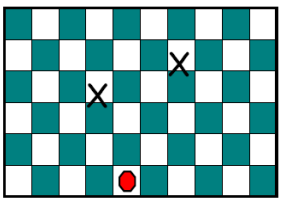


https://www.theatlantic.com/entertainment/archive/2013/09/how-to-game-plinko/280088/

In the checkerboard below, you are allowed to move the checker diagonally up left or right at one time. How many different ways are there to get to the opposite side of the board?



Example 2: On a the checkerboard shown, the checker can travel only diagonally upward. It cannot move through a square containing an X. Determine the number of paths from the checker's current position to the top of the board.



How many ways can the checker reach the other side if it may jump over any X. The checker can only move upward but into any colour of square (ie. It can move vertically or diagonally upward)?