5.2 Combinations

Consider the following situation:

From twenty members of a cricket team, three are chosen to receive awards. Are the total number of ways to receive an award the same if:

- There is a **1st**, **2nd**, and **3rd** place award?

- The three each receive an "**award of distinction**".

5.2 Combinations

Permutations - A selection from a group of items where the order of items matters. i.e., arrangements matter.

compared to ...

Combinations - A selection from a group of items where *order does not matter.*

The number of **combinations** of **r** objects chosen from a set of **n** distinct objects (order does not matter).

$$_{n}C_{r} = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

- A combination is a permutation divided by(the number sets with repeated items)!
- We can also refer to combinations of a certain number items from a collection of items as subsets of that set of items because order of subsets is not important

Combinations can now be used to redefine a permutation.

A combination is a subset of a collection of objects without regard to order

Given the set S = {a,b,c}

1. List all the subsets of set S that contain
a) no elements
b) one element
c) two elements
d) three elements

2. Evaluate the following ${}_3C_0$, ${}_3C_1$, ${}_3C_2$, ${}_3C_3$

3. How do your answers in 1 and 2 compare?

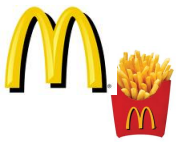
Ex: Compute the following, using the formula, then verify with your calculator:

(a) ${}_{10}C_5 =$ (b) $\binom{6}{4}$

(c) $\binom{7}{0}$ (d) $\binom{7}{1}$ (e) $\binom{7}{7}$

(f) ${}_8C_5$ (g) ${}_8C_3$

Ex:
Art, George, Fotini, Minnie, Lucy, and Stan all work at the local Mickey D's down the street.



(a) In how many ways could the restaurant manager fill the positions of chef, dining room attendant, cashier, and supervisor, assuming they are equally qualified?

(b) In how many ways could the restaurant manager choose two of them to do "trash" duty?

(c) How could we use combinations to answer part (a)?

Ex: From a class of 28 students, in how many ways can a six person committee be chosen to organize a party, if:

(a) There are no restrictions.

(b) Danielle absolutely insists that she must be on any committee chosen.

(c) The committee may consist of five members or six members?

Ex: The school's debate team has 8 females and 10 males. How many ways can the following be selected:

(a) any 7 students?

(b) 4 males and 3 females?

(c) 4 males and 3 females or 3 males and 4 females be chosen?

Ex. Suppose you are playing coed volleyball, with a team of 4 men and 5 women. The rules state that you must have at least 3 women on the floor at all times, 6 members in total how many combinations of team lineups are there?

Ex: How many three letter arrangements are there of the letters taken from the word WARRIORS? Hint: Consider all the cases with and without R's

Extend on the previous question:

How many different four letter words can be taken from the word MISSISSAUGA?

Using a standard 52 card deck of cards.

a) Determine the number of 5 card hands that can be dealt.

b) The number of 5 card hands with exactly 3 Kings

c) The number of 5 card hands with at least two spades.

5 Card Poker

a) How many 5 card poker hands are possible?

b) How many 4 of a kind hands are possible?

c) How many single pair hands are possible?

d) How many hands contain two pair?

e) How many hand have three of a kind?

f) How many full houses are there (three of one kind and two of another)?