Good work!

KNOWLEDGE		APPLICATION		THINKING / INQUIRY		COMMUNICATION	
22	24	24	24	12	12	12	12

MCV4U TEST Limits, Rates of Change & Continuity

NAME: [redacted] **DATE:** Feb 16 2018

22

KNOWLEDGE / UNDERSTANDING

24

1. Evaluate the following limits, if they exist. Show all steps of the algebraic solutions. State the exact values **(NO DECIMALS!)**

-12
$(2x^2+4x)+(-3x-6)$
$2x(x+2)+-3(x+2)$
$(2x-3)(x+2)$

7 **8**

a) $\lim\limits_{x\to 0}\left[\dfrac{\frac{3}{x+2}-\frac{3}{2}}{x}\right]$

$= \lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)\left(\dfrac{3}{x+2}-\dfrac{3}{2}\right)$

$= \lim\limits_{x\to 0}\dfrac{1}{x}\left(\dfrac{3(2)-3(x+2)}{(x+2)(2)}\right)$

$= \lim\limits_{x\to 0}\dfrac{1}{x}\left(\dfrac{6-3x-6}{2x+4}\right)$

$= \lim\limits_{x\to 0}\dfrac{-3}{2x+4}$

$\boxed{= -\frac{3}{4}}$

b) $\lim\limits_{x\to -2}\left[\dfrac{2x^2+x-6}{2x^3+16}\right]$

$= \lim\limits_{x\to -2}\dfrac{(2x-3)(x+2)}{2(x^3+8)}$

$= \lim\limits_{x\to -2}\dfrac{(2x-3)(x+2)}{2(x+2)(x^2+2x+4)}$? X

$= \dfrac{2(-2)-3}{2[(-2)^2+(2)(-2)+4]}$

$2(12)=24$

? 8

8 **8**

c) $\lim\limits_{x\to 0}\left[\dfrac{\sqrt[3]{x+125}-5}{x}\right]$

as $x\to 0$, let $u=\sqrt[3]{x+125}$
$u\to 5$ $x=u^3-125$

$= \lim\limits_{u\to 5}\dfrac{u-5}{u^3-125}$

that's pretty good

$= \lim\limits_{u\to 5}\dfrac{(u-5)}{(u-5)(u^2+5u+25)}$

$= \dfrac{1}{5^2+5(5)+25}$

7 $\boxed{=\frac{1}{75}}$

56
$(4x^2-8x)+(7x-14)$
$4x(x-2)+7$

d) $\lim\limits_{x\to 2}\left[\dfrac{2x-\sqrt{x+14}}{x-2}\right]$

$= \lim\limits_{x\to 2}\dfrac{(2x-\sqrt{x+14})(2x+\sqrt{x+14})}{(x-2)(2x+\sqrt{x+14})}$

$= \lim\limits_{x\to 2}\dfrac{4x^2-x-14}{(x-2)(2x+\sqrt{x+14})}$

$= \lim\limits_{x\to 2}\dfrac{(4x+7)(x-2)}{(x-2)(2x+\sqrt{x+14})}$

$= \dfrac{4(2)+7}{2(2)+\sqrt{16}}$ $\boxed{=\frac{15}{8}}$

8

e) $\lim\limits_{x\to -\frac{1}{2}}\left[\dfrac{2x^3+x^2-8x-4}{8x^2+10x+3}\right]$

$-\frac{1}{2}\,|\,2 \quad 1 \quad -8 \quad -4$
$\quad\;\; -1 \quad 0 \quad 4$
$\quad\;\; 2 \quad 0 \quad -8 \,|\, 0$

$(2x+1)(x^2-4)$

$= \lim\limits_{x\to -\frac{1}{2}}\dfrac{(2x+1)(x^2-4)}{(4x+3)(2x+1)}$

$= \lim\limits_{x\to -\frac{1}{2}}\dfrac{x^2-4}{4x+3}$

$= \dfrac{\left(-\frac{1}{2}\right)^2-4}{4\left(-\frac{1}{2}\right)+3}$

$= \frac{1}{4}-4 = -\dfrac{15}{4}$!

? $\frac{4}{4}-\frac{16}{4}$ $\boxed{= -3}$ X

f) $\lim\limits_{x\to \frac{2}{3}}\dfrac{6x^2-5x-6}{|6x-9|}$

$= \dfrac{6\left(\frac{2}{3}\right)^2-5\left(\frac{2}{3}\right)-6}{|6\left(\frac{2}{3}\right)-9|}$

$= \dfrac{6\left(\frac{4}{9}\right)-5\left(\frac{2}{3}\right)-6}{5}$

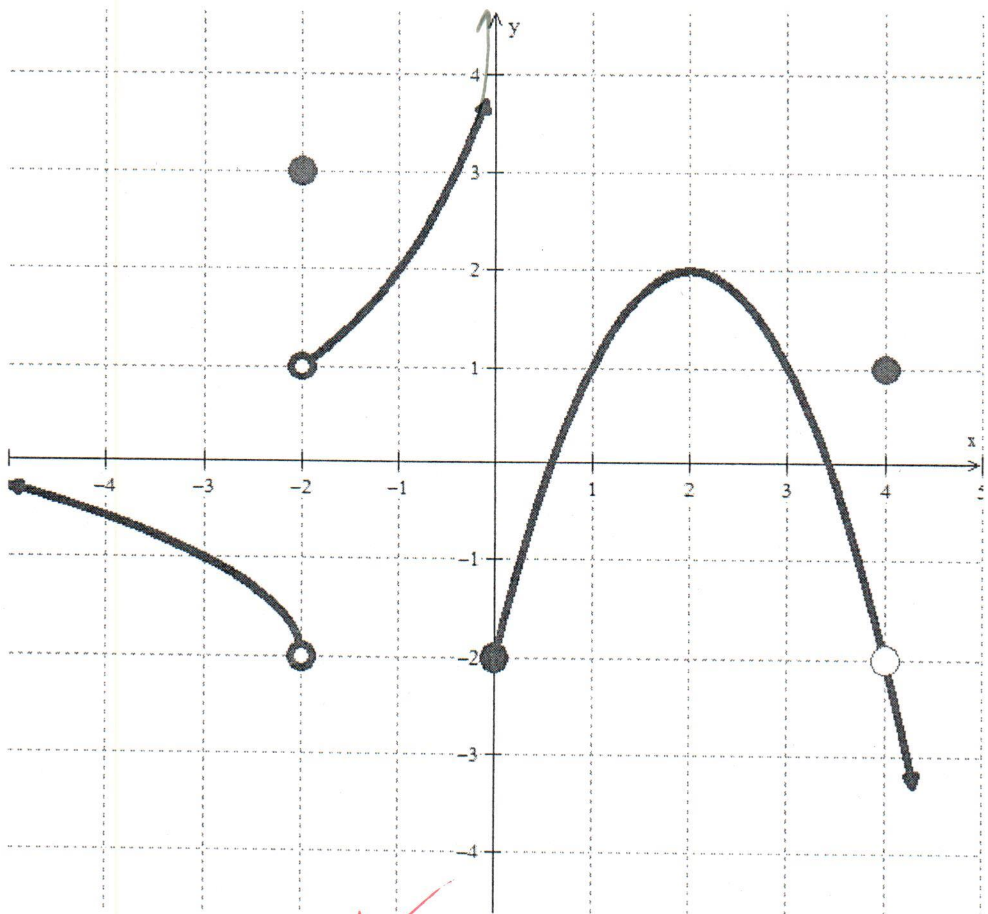
$= \dfrac{\frac{24}{9}-\frac{30}{9}-\frac{54}{9}}{5}$

$= \dfrac{-\frac{60}{9}}{5} = \dfrac{-\frac{20}{3}}{5} = \dfrac{-20}{15}$ $\boxed{= -\frac{4}{3}}$

APPLICATION

24 | 24

2. Use the graph of the piece-wise function $y = f(x)$ shown below to answer the following questions.



6 | 6

(a) $\lim_{x\to 0^+} f(x) =$ -2

(b) $\lim_{x\to 4} f(x) =$ -2

(c) $\lim_{x\to -2^-} f(x) =$ -2

(d) $\lim_{x\to 0^-} f(x) =$ ∞ (DNE)

(e) $\lim_{x\to -2^+} f(x) =$ 1

(f) $\lim_{x\to -2} f(x) =$ DNE

3. State the three conditions necessary for a function **f(x)** to be continuous at **x = a**

$f(a)$ must be defined

$\lim_{x\to a} f(x)$ must exist $\left(\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)\right)$

$\lim_{x\to a} f(x) = f(a)$

6 | 6

Referring to the graph in **#2**, and your answer above, explain what ***condition for continuity*** has been violated when:

a) **x = -2**

$\lim_{x\to a} f(x) = \text{DNE}$

$\lim_{x\to -2^+} f(x) \neq \lim_{x\to -2^-} f(x)$

since the left limit does not equal the right limit, the general limit does not exist. Therefore, the second condition is not met.

b) **x = 4**

$\lim_{x\to a} f(x) \neq f(a)$

$-2 \neq 1$

The general limit as x approaches a does not equal the value of the function when $x = a$. As such, the third rule is broken.

4. In a particle physics experiment, an electrically charged particle is moving along a straight line in a controlled electromagnetic field. Its position, relative to the frame of reference, during a **10 second interval**, can be modelled by the function:

$$\mathbf{S(t) = t^3 - 4t^2 + 5t} \quad , \quad t \in [0,10]$$

where **S** is measured in **millimetres** and **t** in **seconds**.

a) Find the average velocity of the particle in the interval: $t \in [1,5]$

12

12

b) Use the limit of the Difference Quotient when $h \to 0$ to obtain the equation of the velocity function: $\mathbf{V}(t) = \lim_{h \to 0} \frac{S(t+h) - S(t)}{h}$

c) Use $\mathbf{V(t)}$ to calculate the velocity when $\mathbf{t = 6}$ seconds.

d) Are there any moments within the first 10 seconds, when the particle stops: $\mathbf{V(t) = 0}$?

a) $aV = \frac{s(5) - s(1)}{5-1}$

$aV = \frac{5^3 - 4(5)^2 + 5(5) - (1^3 - 4(1)^2 + 5(1))}{4}$

$aV = \frac{125 - 100 + 25 - 1 + 4 - 5}{4}$

$aV = \frac{48}{4}$

$\boxed{aV = 12 mm/s}$

$$\begin{array}{ccc} 1 & 0 & \\ 1\ 1 & 1 & \\ 1\ 2\ 1 & 2 & \\ 1\ 3\ 3\ 1 & 3 & \end{array}$$

$-t^3 + 4t^2 - 5t$

b) $V(t) = \lim_{h \to 0} \frac{S(t+h) - S(t)}{h}$

$\lim_{h \to 0} \frac{(t+h)^3 - 4(t+h)^2 + 5(t+h) - (t^3 - 4t^2 + 5t)}{h}$

$= \lim_{h \to 0} \frac{t^3 + 3t^2h + 3th^2 + h^3 - 4(t^2 + 2th + h^2) + 5t + 5h - t^3 + 4t^2 - 5t}{h}$

$= \lim_{h \to 0} \frac{\cancel{t^3} + 3t^2h + 3th^2 + h^3 - \cancel{4t^2} - 8th - 4h^2 - \cancel{t^3} + \cancel{4t^2} - \cancel{5t} + \cancel{5t} + 5h}{h}$

$= \lim_{h \to 0} \frac{3t^2h + 3th^2 + h^3 - 8th - 4h^2 + 5h}{h}$

$= \lim_{h \to 0} 3t^2 + 3\cancel{th} + \cancel{h^2} - 8t - 4\cancel{h} + 5$

$\boxed{V(t) = 3t^2 - 8t + 5}$

c) $V(6) = 3(6)^2 - 8(6) + 5$

$\boxed{V(6) = 65 mm/s}$

d) $V(t) = 0$

$0 = 3t^2 - 8t + 5$

$0 = (t-1)(3t-5)$

$t - 1 = 0$ or $0 = 3t - 5$

$\boxed{t = 1}$ $\boxed{t = \frac{5}{3}}$

$\therefore$ the particle comes to rest at $t = 1s$ and $t = \frac{5}{3}s$

THINKING / INQUIRY

12

12

5. Find the values for **a** and **b**, so that **f(x)** is continuous for all real values of **x**

6

6

$$f(x)=\begin{cases} ax-b, & \text{if } x<-2 \\ 2x^2+bx-14, & \text{if } -2\le x\le 3 \\ ax+b, & \text{if } x>3 \end{cases}$$

when $x=2$, $ax-b=2x^2+bx-14$
$-2a-b=2(4)-2b-14$
$-2a+b=-6$
$b=2a-6$

when $x=3$, $2x^2+bx-14=ax+b$
$18+3b-14=3a+b$
$2b+4=3a$
$2(2a-6)+4=3a$
$4a-12+4=3a$
$a-12+4=0$
$a-8=0$
$a=8$

$b=2(8)-6$
$b=10$

$\boxed{\therefore\ a=8,\ b=10}$

6. Find an equation of the tangent line to the curve $f(x)=\sqrt{4x+1}$ at **x = 6**
Write the equation in **standard form**.

6

6

$f(6)=5$

$\lim\limits_{h\to 0}\dfrac{\sqrt{4(6+h)+1}-\sqrt{4(6)+1}}{h}$

$=\lim\limits_{h\to 0}\dfrac{\sqrt{25+4h}-5}{h}$

$=\lim\limits_{h\to 0}\dfrac{(\sqrt{25+4h}-5)(\sqrt{25+4h}+5)}{h(\sqrt{25+4h}+5)}$

$=\lim\limits_{h\to 0}\dfrac{\cancel{25}+4\cancel{h}-\cancel{25}}{\cancel{h}(\sqrt{25+4h}+5)}$

$=\dfrac{4}{\sqrt{25+4(0)}+5}$

$=\dfrac{4}{5+5}$

$=\dfrac{4}{10}=\dfrac{2}{5}$ slope

$y=\frac{2}{5}x+b$
$5=\frac{2}{5}(6)+b$
$5=\frac{12}{5}+b$
$b=\frac{25}{5}-\frac{12}{5}$
$b=\frac{13}{5}$

$y=\frac{2}{5}x+\frac{13}{5}$ $y=mx+b$ form

$0=\frac{2}{5}x+\frac{13}{5}-y$

$\boxed{2x-5y+13=0}$ standard form

12

C12

Note: The 12 Communication marks are allocated for using proper mathematical notation, clearly communicating your reasoning, showing all your work and writing full solutions.