

this one

THINKING- CHOOSE ONE OF THE FOLLOWING QUESTIONS TO ANSWER

- 10a. Seven-year old Miles has just come home from a birthday party in which he collected candy from a pinata. He collected 5 tootsie rolls; 4 red lollipops, 3 Hershey's Kisses, 2 boxes of Nerds and 1 Airhead. Miles' mom tells him that he can eat four pieces of candy right now. How many ways can Miles line up the four pieces of candy he chooses to eat?

all different  ${}^5P_4 = 120$  ✓  
 one pair  $\binom{4}{1} \cdot {}^4P_2 \cdot \frac{n!}{2!} = 576$   
 [6] two pairs  $\binom{4}{1} \binom{2}{1} \cdot \frac{n!}{2!2!} = 72$   
 three  $\binom{3}{1} \binom{4}{1} \cdot \frac{n!}{3!} = 48$  ✓  
 four in a row  $\binom{2}{1} \left( \frac{n!}{4!} \right) = 2$  ✓

$$120 + 576 + 72 + 48 = 816$$

816 ways to line up candy

(4)

- 10b. Twelve people, including the siblings Mickey, Dicky, Ricky and Vicky, are going on a canoe trip using six canoes. Each canoe seats two people. The twelve people are to be randomly assigned to the six canoes in pairs. How many ways can the 12 people be seated in the six canoes if no two of Mickey, Dicky, Ricky and Vicky can be seated together?

M D R V 3 2

[6]

$$6^C_4 (8!) = 604800$$

604800 ways to sit in 6 canoes

6 canoes put

upside

M, D, R, V

plug in the rest

all  
12  
people  
or  
12  
people  
or  
12  
people

8. In Hannah's first-year University program, she must take 10 courses (5 per semester). There are six different science courses, five different mathematics courses, four different language courses and five different business courses to choose from. Assume she does not repeat any courses.

20 courses 6S 5M 4L 5B 5 per 2 semester

- a) How many ways can she choose her course load if she must take at least two mathematics courses?

[3] 
$$20C_{10} - \binom{5}{1}\binom{15}{9} - \binom{5}{2}\binom{15}{8} - \binom{5}{3}\binom{15}{7}$$
  

$$= 92378$$

(2)

- b) How many ways can she choose her course load if she takes three science courses, 3 math courses, 2 business courses and 2 language courses?

[2] 
$$\binom{6}{3}\binom{5}{3}\binom{5}{2}\binom{4}{2} = 12000$$

(2)

- c) How many **schedules** for the entire year could she make if she takes two science courses and she must take one language course and she wants the language course to be the first course of the morning?

[4] 
$$\binom{4}{1} \cdot \binom{6}{2} \cdot \binom{13}{7} \cdot 1! \cdot 4$$
  

$$= 3.736 \times 10^{10}$$
 possible arrangements

or she

## COMMUNICATION

9. Explain the key difference between permutations of three out of a group of seven objects and combinations of three out of a group of seven objects. In your response discuss, both logically and mathematically, why the number of combinations in this scenario will be less than the number of permutations.

$\frac{7!}{(7-3)!} > \frac{7!}{(7-3)! \cdot 3!}$

${}^7P_3$  vs  ${}^7C_3$

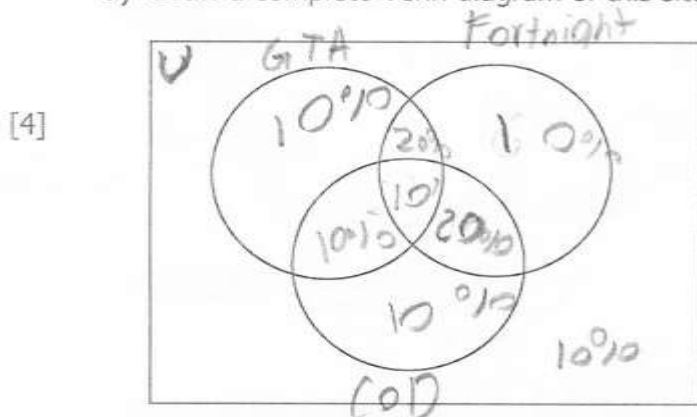
Permutations are where order matters while combinations it does not. If I have 7 objects and I pick three of them up and lay them down. There is only one combination of objects with that set because we ignore order but if it's a permutation there are 3! permutations for that set because it can be ABC or BAC or CAB... Mathematically combinations are always less than permutations.  $nPr = \frac{n!}{(n-r)!}$  then  $nCr = \frac{n!}{(n-r)! \cdot r!}$ .  $nCr$  is  $\frac{nPr}{r!}$  and since  $r!$  is always  $\geq 1$

## APPLICATION

5. A survey of teenagers who play video games was taken at a local high school and produces the following data:

60% play Fortnite, 50% play Grand Theft Auto and 50% play Call of Duty. Additionally, 30% play both Fortnite and Grand Theft Auto while 20% play both Grand Theft Auto and Call of Duty. 30% of students play both Fortnite and Call of Duty. Of the students who play both Grand Theft Auto and Call of Duty, 10% do not play Fortnite.

- a) Draw a complete Venn diagram of this situation and label each section.



$$GTA \cap COD = 10\%$$

$$GTA \cap COD \cap \text{Fortnite} = 20\% - 10\% = 10\%$$

- b) Of the students surveyed, what percentage of students play at least one of these games?

[1]

$$100\% - 10\% = 90\%$$

90% play these games

- c) Of the students surveyed what percentage of students played Grand Theft Auto and/or Fortnite but do not play Call of Duty?

[1]

$$10\% + 20\% + 10\% = 40\%$$

40% do not play COD but play at least 1 of the other games

7. An 8 card hand is dealt from a standard deck of 52 cards.

- a) How many hands contain at most two spades?

[3]

$$\binom{1}{0} \binom{13}{2} \binom{39}{6} = 2541851$$

$$+ \binom{1}{1} \binom{13}{1} \binom{39}{7} + \binom{1}{2} \binom{13}{0} \binom{39}{8} = 515469523$$

- b) How many hands contain 3 aces, another pair of non-face cards, and a triple of face cards?

[4]

$$\binom{1}{1} \binom{4}{3} \binom{9}{1} \binom{4}{2} \binom{3}{1} \binom{4}{3} = 2592$$

ace suit nonface suit

## Test 2: Combinations

|          |           |       |       |
|----------|-----------|-------|-------|
| 12/14 KU | 21/21 App | 4/6 T | 4/4 C |
|----------|-----------|-------|-------|

## Knowledge/Understanding

Give solutions to three significant digits where necessary

1. Let  $S$  be the set of letters in the word KEYBOARD  
 Let  $A$  be the set of vowels in the word KEYBOARD  
 Let  $B$  be the set of letters in the word BOARD

Determine the following:

a/  $A \cup B$   
 $\{E, B, O, A, R, D\}$

b/  $A \cap B$   
 $\{A, O\}$

c/  $(A \cup B)'$   
 $\{K, Y\}$

d/  $B'$   
 $\{K, E, Y\}$

e/  $n(S \cap B)$   
 $\{B, O, A, R, D\}$   
 $= 5 \text{ elements}$

f/ the number of subsets of set  $B$   
 $2^5 = 32$

[6]

2. Mrs. Back is grocery shopping with her children and when she turns her back she has found that they have put one box of Lucky Charms, one box of Cinnamon Toast Crunch, one box of Cookie Crisp, one box of Cap'n Crunch and one box of Ore-Os into the cart. After much begging Mrs. Back agrees to purchase at least one box of cereal. How many ways can Mrs. Back purchase cereal?

5 boxes

(2)  $2^5 - 1 = 31$  or  $(5C_5 + 5C_4 + 5C_3 + 5C_2 + 5C_1)$

[2]

She has 31 possible ways to purchase cereal

3.

- At the University Fair you are collecting information for you and a few of your friends who couldn't be there. The University of Waterloo has of a limited number of pamphlets about 6 different programs that they offer. The university representative tells you to take any that you want but not more than two of any one kind. Assuming that you take at least one pamphlet, how many choices could you make?

6 programs at least 1 all different

[3]

$2^6 - 1 = 63$  (1)

4. Of the 1341 students at Port Credit Secondary School, 802 attended the first dance of the year but the music and food were not very good so only 498 decided to attend the next dance. If 357 did not attend either dance, how many attended both?

1341 total

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$   
 $= 802 + 498 - (1341 - 357)$   
 $= 316$

[3]

3316 people  
 attended both  
 dances

