Multiple Choice: Write the correct answer in the box to the left.

$\frac{2\pi}{3}-\frac{\pi}{4}=\frac{8\pi}{12}-\frac{3\pi}{12}$

$=\frac{5\pi}{12}$

c

1. A simpler expression for $\sin\frac{2\pi}{3}\cos\frac{\pi}{4}-\cos\frac{2\pi}{3}\sin\frac{\pi}{4}$ is

a) $\cos\frac{5\pi}{12}$ b) $\sin\frac{11\pi}{12}$ c) $\sin\frac{5\pi}{12}$ d) $\cos\frac{11\pi}{12}$ e) $\sin\frac{7\pi}{12}$

c

2. A simpler expression for $\cos^2 32° - \sin^2 32°$ is

a) $\cos 32°$ b) $\sin 64°$ c) $\cos 64°$ d) $\sin 32°$ e) $\tan 64°$

a

3. The value of $\cos(\pi+x)-\cos(\pi-x)$ is: $-\cos - -\cos$

a) 0 b) -1 c) $-2\cos\pi$ d) Dependent on the value of x e) $\cos 2x$

c

4. If $\tan A = 3$ and $\tan B = -2$, then the exact value of $\tan(A+B)$ is $\frac{3-2}{1-(3)(-2)}=\frac{1}{1+6}$

a) 1 b) -1 c) $\frac{1}{7}$ d) $-\frac{3}{2}$ e) -7

d

5. Which set of values is the solution of $(2\sin x+1)(\cos x-1)=0$, where $0\le x\le 2\pi$?

$\cos x = 1$

$\to 0, 2\pi$

$\sin x = -1/2$

a) $x=\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $x=\frac{\pi}{6}, \frac{5\pi}{6}, \pi$ c) $x=0, \frac{5\pi}{6}, \frac{7\pi}{6}, 2\pi$ d) $x=0, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

d

6. The expression $\cos\left(\frac{\pi}{4}+\theta\right)$ has the same value as $\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta$

$\frac{1}{\sqrt2}\cos\theta - \frac{1}{\sqrt2}\sin\theta$

a) $\frac{1+\sqrt2\sin\theta}{1-\sqrt2\sin\theta}$ b) $\frac{1}{\sqrt2}(\cos\theta+\sin\theta)$ c) $\frac{1}{\sqrt2}$ d) $\frac{1}{\sqrt2}(\cos\theta-\sin\theta)$ e) $\frac{\tan\theta}{\sqrt2}$

e

7. The value of $3\sin^2 A + 3\cos^2 A$ is $3(\sin^2 A+\cos^2 A)$

a) dependent on the value of A b) 18 c) 6 d) 1 e) 3

a

8. The solutions of $\sin x + \cos x = 0$ in the interval $0\le x\le 2\pi$ are

$x = 0, \pi, 2\pi$

a) $\frac{3\pi}{4}, \frac{7\pi}{4}$ b) $0, \pi, 2\pi$ c) $\frac{\pi}{2}, \frac{3\pi}{2}$ d) $\frac{\pi}{4}, \frac{5\pi}{4}$ e) No solutions

~~$\sin x - \cos x$~~ $\emptyset = -\tan x$

$\phi = \frac{-\sin x}{\cos x}$ $\tan^{-1}(\phi)=$ $*\cos x \ne 0$

9. Write an equivalent expression for the following using the identity indicated.

a) $\cos\frac{5\pi}{13}$ using a co-function (correlated) identity

$\cos\left(\frac{5\pi}{13}\right)=\sin\left(\frac{\pi}{2}-\frac{5\pi}{13}\right)=\sin\left(\frac{13\pi}{26}-\frac{10\pi}{26}\right)=\sin\left(\frac{3\pi}{26}\right)$

[2]

b) $\tan\frac{15\pi}{11}$ using a related identity

$\tan\frac{15\pi}{11}=\tan\left(\pi+\frac{4\pi}{11}\right)=\tan\left(\frac{4\pi}{11}\right)$

[2]

10. Use the compound angle formula to determine the exact value of $\tan\left(\frac{17\pi}{12}\right)$. Rationalize the denominator if necessary.

$\tan\left(\frac{17\pi}{12}\right)$ $\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-(\tan\frac{\pi}{6})(\tan\frac{\pi}{4})}$ $2+\sqrt3$

$=\tan\left(\pi+\frac{5\pi}{12}\right)$

$=\tan\left(\frac{5\pi}{12}\right)$

[5]

$=\frac{\left(\frac{1}{\sqrt3}\right)+1}{1-\left(\frac{1}{\sqrt3}\right)(1)}=\frac{\frac{1+\sqrt3}{\sqrt3}}{\frac{\sqrt3-1}{\sqrt3}}=\frac{1+\sqrt3}{\sqrt3-1}$

$(1+\sqrt3)(\sqrt3+1)$ $=\sqrt3+1+3+\sqrt3$ $=4+2\sqrt3$

a) $\csc 2x \cos x - \sqrt{2}\cos x = 0$ (Exact solutions only)

[6]

$\frac{1}{\sin 2x}\cos x - \sqrt{2}\cos x = 0$

$\frac{\cos x}{2\sin x \cancel{\cos x}} - \sqrt{2}\cos x = 0$

$\frac{1}{2\sin x} - \sqrt{2}\cos x = 0$

$\frac{1}{2\sin x} = \sqrt{2}\cos x$

$1 = \sqrt{2}\cos x \, 2\sin x$

$\frac{1}{\sqrt{2}} = 2\sin x \cos x$

$\frac{1}{\sqrt{2}} = \sin 2x$

$2x = \sin^{-1}(\frac{1}{\sqrt{2}})$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4} \quad (0 \le x \le 4\pi)$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

b) $4\sin x + 2\cos 2x + 1 = 0$ (Exact solutions only)

[6]

$4\sin x + 2(1 - 2\sin^2 x) + 1 = 0$

$4\sin x + 2 - 4\sin^2 x + 1 = 0$

$4\sin^2 x - 4\sin x - 3 = 0$

$(2\sin x - 3)(2\sin x + 1) = 0$

$2x \;\; -3$
$2x \;\; 1$

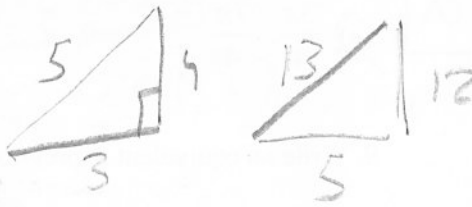
① $2\sin x - 3 = 0$ $\sin x > 1$
$\sin x = 3/2$ ← so No soluti
$x = N.A$

② $2\sin x + 1 = 0$
$\sin x = -1/2$
$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

Q3

12. Given $\cos A = -\frac{3}{5}$ where $\pi \le A \le \frac{3\pi}{2}$ and $\sin B = -\frac{12}{13}$ where $\frac{3\pi}{2} \le B \le 2\pi$, determine EXACT VALUES (no decimals) of the following

Q4



a) sinA and cosB

[4]

$\sin A = \frac{opp}{hyp} = -\frac{4}{5}$ ← negative

$\cos B = \frac{5}{13}$ ← positive

b) cos(A+B)

[2]

$\cos A \cos B - \sin A \sin B$

$= \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right)$

$= \frac{-15}{65} - \frac{48}{65} = \frac{-63}{65}$

c) csc2B

$1 \qquad 1 \qquad -164$

Communication

13. Given the trig equation $\sec\theta(2\sec\theta+1)=0$, over the interval $0\le x\le 2\pi$, how many solutions does it contain? Explain how you know without solving the equation.

[3]

There are no solutions, we do not even need to solve to discover this. $\sec\theta=\frac{1}{\cos\theta}$ therefore it is impossible for $\sec\theta$ to be zero as the numerator is one. Thus since $\sec\theta\ne0$ then the first part of the equation can be rejected. The second half $2\sec\theta+1=0$ can never be equal to zero either as for $\sec x$ to have a solution equal to 0 it must be translated up or down greater then or equal to the amplitude.

14. Identify the error(s) in the solution to the following equation and explain how to correct the error(s). **You do not have to solve the equation!**

$$(\cos x)\cot x=\cos x$$
$$\cot x=1$$
$$x=\frac{\pi}{4}\quad\text{OR}\quad x=\frac{5\pi}{4}$$

There is no range for example $x\in[0,2\pi]$ thus the solution of $\cot x=1$ is not $x=\frac{\pi}{4}$ or $x=\frac{5\pi}{4}$ it is $k\pi+\frac{\pi}{4}$. $\cot x=1$ can be simplified to $\tan x=1$. There should also technically be a restriction that $\cos x\ne0$ and $\tan x\ne0$ but that does not effect solution.

[3]

Not enough steps are shown it should be

$\cos x\cot x=\cos x$

$\cot x=\frac{\cos x}{\cos x}$

$\frac{1}{\tan x}=1$

$\tan x=1$

2

Thinking

15. Prove the following identities (6 marks each)

Let $2y=x$

a) $\sin^2(\frac{x}{2})=\frac{\csc(\pi-x)-\cot(\pi+x)}{2\csc(\pi-x)}$

$=\sin^2 y$

$=\frac{\frac{1}{\sin2y}-\frac{\cos2y}{\sin2y}}{\frac{2}{\sin2y}}$

$\frac{\frac{1-\cos2y}{\cancel{\sin2y}}}{\frac{2}{\cancel{\sin2y}}}$

$=\frac{1-(1-2\sin^2y)}{2}$

$+2\sin^2y$

b) $\frac{1-\cos2x+\sin2x}{1+\cos2x+\sin2x}=\tan x$

R.S

$\frac{\sin x}{\cos x}$

$\frac{1-(1-2\sin^2x)+2\sin x\cos x}{1+(2\cos^2x-1)+2\sin x\cos x}$

$\frac{1-1-2\sin^2x+2\sin x\cos x}{1-1+2\cos^2x+2\sin x\cos x}$

$\frac{-2\sin^2x+2\sin x\cos x}{-2\cos^2x+2\sin x\cos x}$

$\frac{2\sin x(\cancel{\sin x+\cos x})}{2\cos x(\cancel{\sin x+\cos x})}$

$=\frac{2\sin x}{2\cos x}$

$=\frac{\sin x}{\cos x}$

no hole