DATE: ______________________________

KNOWLEDGE/UNDERSTANDING

MULTIPLE CHOICE - Circle the correct answer. $P(a) = r$

1. What is the remainder of the division $(7x^3 - 5x^2 - 4x + 12) \div (x+1)$?

   a. 10 b. 4 c. -4 d. 12

2. Which set of values for *x* should be tested as possible zeros of $2x^3 - 7x^2 + 3x - 4$? $\frac{b}{a}$

   a. $\frac{1}{2}, 1, 2, 4$ b. $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

   c. ±1, ±2, ±4 d. -1, -2, - 4

3. A polynomial function of degree 11 can have__________ turning points.

   a. 1 to 11 b. 0 to 11 c. 1, 3, 5, 7, 9 or 11 d. 0, 2, 4, 6, 8 or 10

4. A polynomial function of degree 11 can have__________ x-intercepts.

   a. 0 to 10 b. 1 to 10 c. 0 or 11 d. 1 or 11

5. The value of the leading coefficient of $f(x) = -3(x-4)(2x-1)^2(2x-3)$ is:

   a. -24 b. -12 c. -3 d. 12

6. The function that has the end behaviour, $x \to -\infty, y \to -\infty$ and $x \to \infty, y \to \infty$

   a. $y = (x-1)^3(x+2)$ b. $y = (x-1)^2(x+3)^5$

   c. $y = -2(x-3)^3(x+4)^3$ d. $y = -2x(x-1)(x-2)$

7. Factor $7x^4 - 448x$ $7x(x^3 - 64)$

   a. $7x(x+4)(x^2+4x+16)$ b. $7x(x-4)(x^2+4x+16)$

   c. $7x(x-4)(x^2-4x+16)$ d. Cannot be factored

   $7x($ $(x)^3 - (4)^3 = (x-4)(x^2+4x+16$

8. Let f(x) = $(x-a)^2(x+b)^3(x-c)^2$, where a, b, c are real numbers. Which of the following

   statements is <u>not true</u>?

   A. f has a root at x = -b B. f(c) = f(a)

   C. The graph of f crosses the x-axis at x = a D. x= c is a zero (order 2)

9. If a graph represents an Even-degree polynomial function, <u>what must be true</u>:

   A. As x→∞, y→ ∞ B. It must pass through the x-axis.

   As x→ - ∞, y→ - ∞

   C. It may not have any real roots. D. It must have at least one real root

[4, 4] $+ (3m+4)^3$

$= (5m+1)\left(4m^2 - 6m + 9 - (6m^2+8m-9m-12) + (9m^2-24m+16)\right)$

$= (5m+1)(7m^2 - 29m + 37)$

$(2m-3)^3 + (3m+4)^3 = (5m+1)(7m^2 - 24m + 37)$

-0.5

$(4x)^3 - \left(\frac{y}{4}\right)^3 = \left(4x - \frac{y}{4}\right)\left(16x^2 + \frac{4xy}{4} + \frac{y^2}{4^2}\right)$

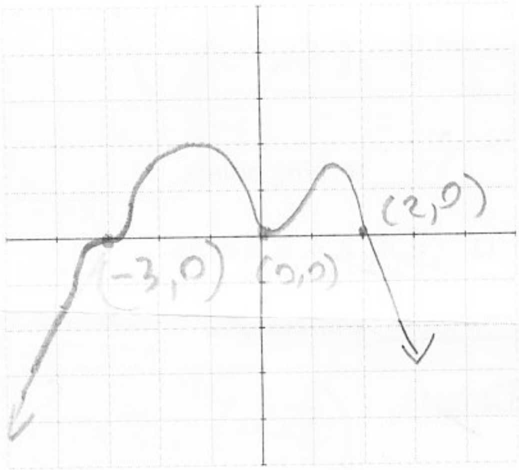
$= \left(\frac{16x}{4} - \frac{y}{4}\right)\left(16x^2 + xy + \frac{y^2}{16}\right)$

$= \left(\frac{16x-y}{4}\right)\left(16x^2 + xy + \frac{y^2}{16}\right)$

$64x^3 - \frac{y^3}{64} = \left(\frac{1}{4}(16x-y)\right)\left(16x^2+xy+\dots\right)$

* 11. Sketch the function $f(x) = -3x^2(x-2)(x+3)^3$

[5]



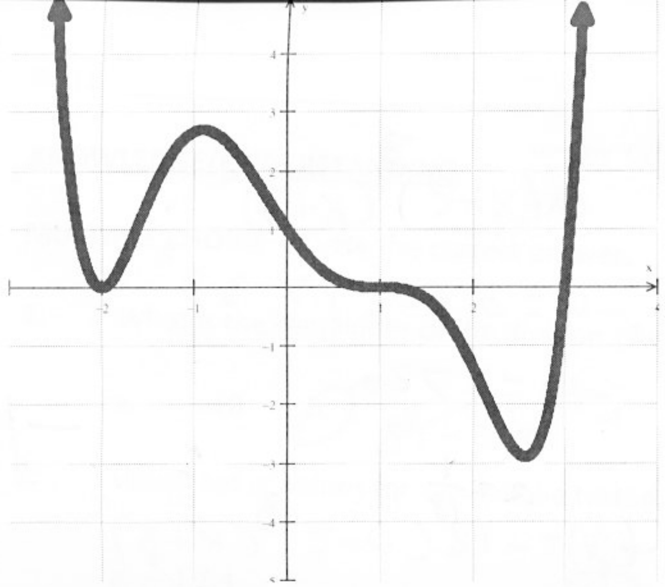
12. Divide $(6x^4 - 2x^3 + 3x - 1) \div (x^2 - 2x + 1)$ using long division. Express your answer in form P(x)=D(x)Q(x) + R(x).

$$
\begin{array}{r}
6x^2 + 10x + 14 \\
x^2 - 2x + 1 \,\big)\, \overline{6x^4 - 2x^3 + 0x^2 + 3x - 1} \\
\underline{6x^4 - 12x^3 + 6x^2} \\
0 + 10x^3 - 6x^2 + 3x \\
\underline{10x^3 - 20x^2 + 10x} \\
0 + 14x^2 - 7x - 1 \\
\underline{14x^2 - 28x + 14} \\
0 + 21x - 15
\end{array}
$$

[5]

$6x^4 - 2x^3 + 3x - 1 = (x^2 - 2x + 1)(6x^2 + 10x + 14) + 21x - 15$

$6x^4 - 2x^3 + 3x - 1 = 2(x^2 - 2x + 1)(3x^2 + 5x + 7) + 21x - 15$



$a(x+2)^2(x-1)^3(x-3)$

$1 = a(2)^2(-1)^3(-3)$

$1 = 12a$

$a = 1/12$

$f(x) = 1/12(x+2)^2(x-1)^3(x-3$

14. When $f(x) = 3x^3 + mx^2 + nx + 2$ is divided by (x - 3) the remainder is 32. A factor of f(x) is (x + 1). Determine the values of m and n.

[6]

$3(3)^3 + m(3)^2 + n(3) + 2 = 32$

$81 + 9m + 3n + 2 = 32 \quad -51 = 9m + 3n \quad -17 = 3m + n$

$3(-1)^3 + m(-1)^2 + n(-1) + 2 = 0$

$-3 + m - n + 2 = 0$

$m - n = 1$

$3m + n = -17$

$m - n = 1$

$-4 - n = 1$

$-n = 5 \quad n = -5$

$m - 1 = n$

$-17 - 3m = n$

$-17 - 3m = m - 1$

$-16 = 4m$

$m = -4$

$\boxed{n = -5 \quad m = -4}$

$f(x) = 3x^3 - 4x^2 - 5x$

$$\begin{array}{r|rrrr} -1 & 3 & -4 & -5 & 2 \\ & & -3 & 7 & -2 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

COMMUNICATION

15. **List** the two factors that determine the end behavior of a polynomial function. BE SPECIFIC!

[2]

The degree of the polynomial function, whether it is even or odd determines in which directions the polyn will point whether they will go opposite to each o or the same.
The sign of the leading coefficient determines the reflection so in a positive even function $x \to \infty$ $y \to \infty$ but in a negative even function $x \to \infty$ $y \to -\infty$

16. The equation $5x^3 + 7x^2 + 4 = 0$ cannot be factored. Does this mean there are no real roots (solutions) that satisfy this equation? Briefly explain.

[2]

Just because this equation cannot be factored does not mean there are no real roots. This function MUST have at least 1 root as the function has a degree of 3 which is odd and all odd functions have at least 1 root.
Some number inputted into x must result in 0 as there is no maximum or minimum in odd polynomials

- f(x) is increasing when $x \in (-\infty,-3) \cup (-1,\infty)$ and is decreasing when $x \in (-3,-1)$
- f(0) = - 4, f(2)=0
- two zeros
- degree 5
- End Behaviour $x \to \infty, y \to \infty,\ x \to -\infty, y \to -\infty$

[10]

$a(x-2)^3(x+3)^2$

$-4 = a(-2)^3(3)^2$

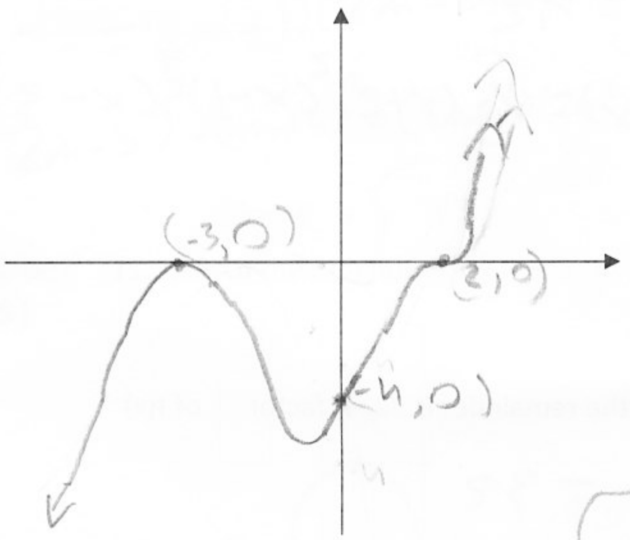
$-4 = -72a$

$a = -18$

$a = \frac{1}{18}$

$f(x) = -18(x-2)^3(x+3)^2$

-1



18. **Factor** $f(x) = -2x^4 + 6x^3 + 4x^2 - 24x + 16$. Use the factored form of the function along with its **y-intercept** to **sketch** a possible graph of f(x). **Label and use an appropriate scale.**

[11]

$a(x-1)(x-2)^2(x+2)$ ~~(x+4)~~

$$\begin{array}{r|rrrrr} 1 & -2 & 6 & 4 & -24 & 16 \\ & & -2 & 4 & 8 & -16 \\ \hline & -2 & 4 & 8 & -16 & 0 \end{array}$$

$-2x^3 + 4x^2 + 8x - 16$

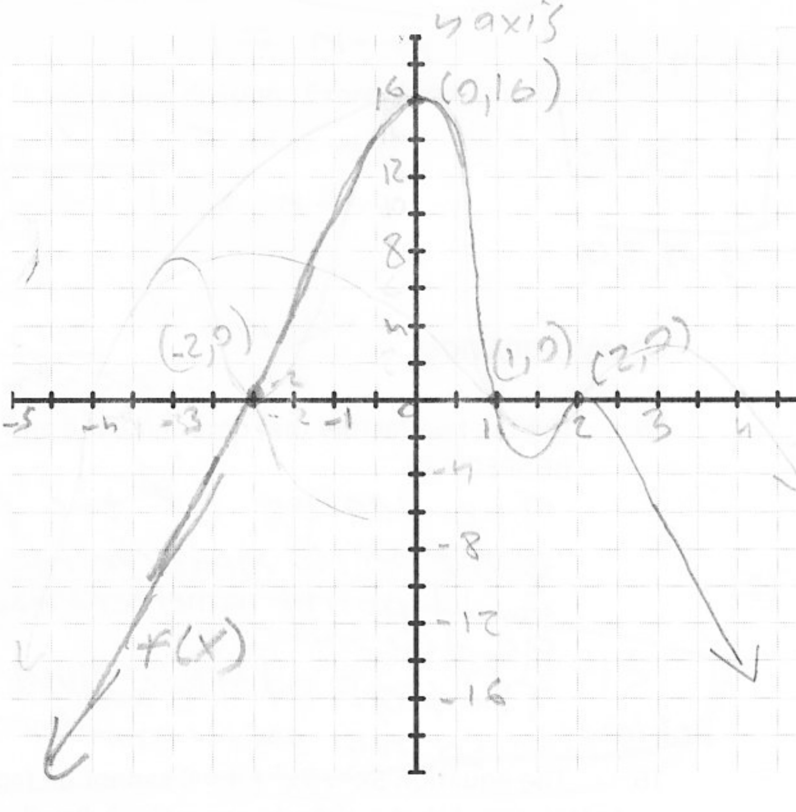
$$\begin{array}{r|rrrr} 2 & -2 & 4 & 8 & -16 \\ & & -4 & 0 & 16 \\ \hline & -2 & 0 & 8 & 0 \end{array}$$

$-2x^2 + 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$



$f(x) = -2(x-1)(x-2)^2(x+2)$

$16 = a(-1)(-2)^2(2)$

$= -8a$