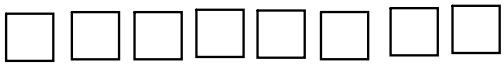
4.2 Factorials and Permuations

A family of eight people want a portrait. How many different ways could the photographer arrange the family
assuming they pose for the photo in a single row.

How many people could we put in position 1, then position 2, and so on....



Many counting and probability calculations involve multiplying consecutive integers. We often use factorial notation to write such expressions.

For a **natural number** n ($n = 1, 2, 3, \ldots$)

$n! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$

Read as: "*n factorial*"

Note: You CANNOT multiply factorials together i.e. 15!2! DOES NOT EQAUL 30!

Example:

(a) **4!** (b) 6!

(c) 1!

(d) $\frac{8!}{6!}$ (e) $\frac{75!}{71!}$

(g) $\frac{17!}{15!\,2!}$ (h) 0!

On your (scientific) calculator, there should be a button "**n!**" that will calculate factorials.

Why is 0!= 1?

If n! is defined as the product of all positive integers from 1 to n,
then:
4! = 4*3*2*1 = 24
3! = 3*2*1 = 6
2! = 2*1 = 2
1! = 1
...
n! = n*(n-1)*(n-2)*(n-3)*..... 3*2*1
and so on.
Logically, n! can also be expressed n*(n-1)! .

Therefore, at n=1, using n! = n*(n-1)!
1! = 1*0!, we know that 1!= 1 (not 0) so therefore 0! must =1

Simplify the following. Represent as a single factorial.

a) 56x6!

b) $\frac{n!}{(n-2)!}$

c) (n+6)(n+5)(n+4)!

Solve for n in the following equations

a) n!= 42(n-2)!

b) $\frac{(n+1)!}{(n-1)!} = 2$

How many ways are there to arrange a deck of cards?

Now look up how many atoms there are in the universe or how many seconds have existed since time began! Which number is bigger?

Ex.
How many ways can a 12 volume encyclopedia set be arranged incorrectly?

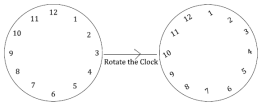
Ex.
In how many ways can a 5 member Data Management group sit in desks arranged in a:

a) row

THINKING QUESTION

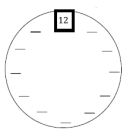
b) circle

Why is a circle different than a row?
What is rotational symmetry and how does that effect this problem?



Are these the same arrangement?

Does the problem with rotational symmetry exist if we fix one person in a position?



Ex.
a) How many six-digit numbers are there?

b) How many *even* six-digit numbers are there?

c) How many even six-digit numbers have all their digits different?
(Consider different cases)

A **permutation** of ***n*** *distinct* items is an arrangement of all the items in a definite order.

The total number of such permutations is denoted by **P(n, n)** or $_nP_n$

P(n, n) = n x (n-1) x (n-2) x ... x 3 x 2 x 1 = **n!**

On your calculator, there should be a button **nPr**

A permutation of ***n*** distinct items *taken r at a time*, is an arrangement of *r of the n items* in a definite order.

"***r*** *arrangements of* ***n*** *items*"

The total # of possible arrangements of **r** items out of a set of **n** items is **P(n, r)** or $_nP_r$

$$_nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

Ex. Suppose we have a group of 10 people and we want to choose a president, vice-president and treasurer. How many ways can this be done?

You Purchase 215 songs online and download them to listen to on your computer

a) By using the shuffle feature of your music software, how many different playlists can your computer make if it uses all the songs?

b) Suppose the software will show only the next ten songs in the playlist. How many different arrangements of songs are possible to be shown?

C) Many software programs that play music and will play those songs more often. How many different ten-song playlists can the software create if each one must start and end with one of the ten songs by your favorite artists?

(a) Find the number of permutations of the letters of the word: **DIPLOMA**

(b) Find the number of arrangements if "**L**" must remain in the middle.

(c) How many arrangements are there if "**O**" and "**I**" must be together?

(d) How many arrangements are there if "O" and "**I**" are at least one letter apart?

(e) How many arrangements are there if "O" and "I" are one letter apart?

Stack Jack

http://www.yummymath.com/2012/stacking-jack/