

Handwritten notes for Example 1:

Case 1:  $\frac{6!}{7! \cdot 6!}$

Case 2:  $\frac{6!}{6! \cdot 6! \cdot 7 \cdot 5}$

Case 3:  $\frac{6!}{7! \cdot 6}$

Total - undesired

$9! - 8!2! - 8!2! + 7!2!$

$= 211,680$

Handwritten calculations for Example 2:

$\frac{6!}{2!2!} = 180$

$\frac{4!}{2!} = 12$

Feb 10-1:36 PM

## 4.3 Permutations with Identical Elements

Sometimes permutations will occur where some items are **identical**. In other words, you cannot tell them apart.

The number of permutations of a set of  $n$  objects containing:

- $a$  identical objects of one kind;
- $b$  identical objects of another kind;
- $c$  identical objects of a third kind.

$$\frac{n!}{a! b! c!}$$

and so on, if there are more identical objects.

Sep 3-12:18 AM

Example 1: How many distinct permutations of the letters of the word OTTAWA?

$$\frac{6!}{2!2!} = 180$$

Example 2: How many distinct permutations of the letters of the word OTTAWA are there that begin and end with the letter T?

$$\frac{4!}{2!} = 12$$

Sep 11-6:40 AM

Example 3:

- a) A stuffed animal game machine has 15 identical monkeys, 10 identical bears, and 6 identical elephants. If you are guaranteed a prize for every game and you play the game until the machine is empty, how many arrangements of prizes could you win?

$$\frac{31!}{15! 10! 6!} = 2,406,725,882,000$$

- b) If the question is reworded to become "a stuffed animal game machine has 15 distinct monkeys, 10 distinct bears, and 6 distinct elephants and if you are guaranteed a prize for every game and you play the game until the machine is empty, how many arrangements of prizes could you win?"

$$31!$$

- c) If the question is reworded to become "a stuffed animal game machine has 15 identical monkeys, 10 distinct bears, and 6 distinct elephants and if you are guaranteed a prize for every game and you play the game until the machine is empty, how many arrangements of prizes could you win?"

$$\frac{31!}{15!} = 6.288 \times 10^{14}$$

Sep 3-12:19 AM

Ex:

- (a) How many distinct seven digit numbers can be constructed from the digits 1, 2, 2, 3, 3, 4, 5?

$$\frac{7!}{2!2!} = 1260$$

- (b) How many of these would be odd numbers?

Case 1: ends in 1 or 5

Case 2: ends in 3

Total =  $\frac{6! \cdot 2}{2!2!} + \frac{6!}{2!} = 720$

- (c) How many of these would be less than 3,000,000?

Starts with a 1

Starts with a 2

$\frac{6!}{2!2!} + \frac{6!}{2!} = 540$

Sep 2-11:12 PM