Recall Pascal's Formula

$t_{n,r} = t_{n-1, r-1} + t_{n-1, r}$

Pascal's Formula - Combination version

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

The visual "proof "is within Pascal's triangle

Looking at the terms in Pascal's Triangle again:

$$\binom{0}{0}\ 1$$

$$\binom{1}{0}\ 1 \quad \binom{1}{1}\ 1$$

$$\binom{2}{0}\ 1 \quad \binom{2}{1}\ 2 \quad \binom{2}{2}\ 1$$

$$\binom{3}{0}\ 1 \quad \binom{3}{1}\ 3 \quad \binom{3}{2}\ 3 \quad \binom{3}{3}\ 1$$

$$\binom{4}{0}\ 1 \quad \binom{4}{1}\ 4 \quad \binom{4}{2}\ 6 \quad \binom{4}{3}\ 4 \quad \binom{4}{4}\ 1$$

The terms correspond to Pascal's triangle!
Wow! ^_^

Using combinations we can quickly find ANY term in Pascal's Triangle.

Ex: Re-write each as an equivalent expression using Pascal's Formula. Then verify the result.

(a) $\binom{17}{2}$

(b) $\binom{13}{5} + \binom{13}{6}$

(c) $\binom{20}{6} - \binom{19}{6}$

The Binomial Theorem:

Provides a nice connection between expanding binomials, combinations, and Pascal's Triangle

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}a^0 b^n$$

- For the **n^{th} power** of the binomial, the coefficients represent the **n^{th} row** of Pascal's triangle.
- This provides an easy method to expand higher power binomials.

Ex: State the value the exponent k in the binomial expansion of $(a + b)^9$

a) a^3b^k b) a^kb^2 c) $a^{k+1}b^k$

Ex: State the coefficient for the following terms in the expansion of $(a + b)^{11}$

a) a^9b^2 b) a^3b^8 c) $a^{11-r}b^r$

Alternatively, we can write the BT as:

$$\boxed{(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r}$$

$\sum$ represents the "sum of" all the terms in the expansion

r is a value ranging between 0 to n **($0 \le r \le n$)**

Ex: Expand $(5x + y)^4$ using the BT.

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}a^0 b^n$$

Ex: Expand the 4th to 7th terms of $(3x^2 - 2y)^9$ using the BT.

Factor the expression

$1+10x^{2}+40x^{4}+80x^{6}+80x^{8}+32x^{10}$