TIPS: Combinatorics (PRACTICE)

Total
14 marks

	Level 4	Level 3	Level 2	Level 1
Thinking/Inquiry/ Problem Solving • applying the steps of an inquiry/problem-solving process	applies the steps of an inquiry/problem-solving process with a high degree of effectiveness (e.g.,selects the most effective strategies with an accurate solution or careless errors; justifies solution in a thorough and sophisticated manner)	applies the steps of an inquiry/problem-solving process with considerable effectiveness (e.g., selects effective strategies to solve the problem with minor errors; justifies solution with appropriate information)	applies the steps of an inquiry/problem-solving process with moderate effectiveness (e.g., selects fairly effective strategies to create a partially accurate solution; justifies solution with some appropriate information	applies the steps of an inquiry/problem-solving process with limited effectiveness (e.g., selects inappropriate strategies with many errors in the solution; justifies solution with limited information)

Read each question carefully. Show all your work, and state any assumptions you make (if necessary).

A palindrome is a word or phrase which reads the same frontwards and backwards. "I prefer pi" and "racecar" are examples of palindromes. Numbers which remain the same where the digits are reversed are also considered to be palindromes. For example 1287821, 4554 and 7 are possible palindromic numbers (single digits are considered palindromes). Determine the number of **positive integers less than 1 000 000** that are palindromes.

[8]

9 = 9

9 1 = 9

9 10 1 = 90

9 10 1 1 = 90

9 10 10 1 1 = 900

9 10 10 1 1 1 = 900

= 9+9+90+90+900+900

= 1998

Read each question carefully. Show all your work, and state any assumptions you make (if necessary).

Alex and Chris are working together on a science project. While working on her part at home, Alex needed to call Chris for some information. She had written his cell phone number down but could not find it. But when she originally wrote the number down, she had observed several things.

- Each one of the digits 0 to 9 appeared exactly once in the number.
- The area code (first three digits) was 517, the same as her area code.
- The tenth digit was a 0.
- The ninth digit was not a 9 and the eighth digit was not an 8.

If Alex were to try every possible 10-digit number with these properties, what would be the maximum number of calls she would have to make in order to contact Chris?

[8]

Total - undesired

5 1 7 _ _ _ _ _ _ 0

Total = 6! = 720

~~0~~ ~~1~~ 2 3 4 ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~

2 3 4 6 8 9 (left over)

Undesired case #1

_ _ _ _ 8 9

= 4!

= 24

Undesired case #2

4 3 2 1 8 4

4! · 4 = 96

↑ not a 9

Undesired case # 3

4 3 2 1 4 9

↑ not an 8.

= 4! · 4

= 96

Total = 720 - 24 - 96 - 96

= 504

Read each question carefully. Show all your work, and state any assumptions you make (if necessary).

A room contains 150 boxes. The boxes are identical except for colour. Each box contains a card which has one of three messages printed on it. In 78 of the boxes the card says "No Prize". In 66 of the boxes the card says "Winner $20". And in 6 of the boxes the card says "Winner $100".Contestants have been assigned a number corresponding to when they will make their selection. On a turn each contestant gets to randomly choose two boxes. Once a box is chosen it is removed from the room.You are the second contestant. In how many ways can you select a box containing $100.

[8]

Total - undesired (you do not win $100)

Total = $150P_4$

$= 38247000 48$

undesired Case #1 = 100 100 X X (don't win $100) $= 6P_2 \cdot 144P_2$

#2 = 100 X X X $= 6P_1 \cdot 144P_2$

#3 = X 100 X X $= 6P_1 \cdot 144P_3$

#4 = X X X X $= 144P_4$

How many different five letter words can be formed using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R? (Hint: Consider the cases with and without repeated letters)

[4]

MEDITERRANEAN

E ~~R~~ A N

~~E~~

E _ _ _ R

All different $\binom{8}{3} \cdot 3!$

$= 336$

Two the same $\binom{3}{1}\binom{7}{1} \cdot 3!$

$= 126$

Total = 462

Given the numbers -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, in how many ways could four different numbers be chosen so that their product is negative.

[4]

case #1 Choose one negative $\binom{6}{1} = 6$

choose three positive $\binom{5}{3} = 10$

$= 6(10)$

$= 60$

OR

choose 3 neg $\binom{6}{3} = 20$

choose 1 pos. $\binom{5}{1} = 5$

$= (20)(5)$

$= 100$

Total = 160

Twenty people are to travel in a bus from the airport to the hotel at a resort. The bus is designed for use in a tropical climate; it can carry twelve passengers outside and eight passengers inside. If four of the passengers refuse to travel outside and five will not travel inside, in how many ways can the passengers be seated if the order of the passengers is not important?

[4]

seat 4 inside $\binom{4}{4}$ × seat 5 outside $\binom{5}{5}$ × fill rest of inside $\binom{11}{4}$ × fill rest of outside $\binom{7}{7}$

$= 330$

A student must choose one course/spare for each of four periods for semester 1 and semester 2.

AMU 4M	MCV 4U
BAT 4M	MDM 4U
ENG 4U	SBI 4U
EBT 40	SCH 4U
PSE 4U	SPH 4U
IDC 4U	CGW 4U
FSF 4U	HZT 4U
MHF 4U	TGJ 4M
SPARE	ICS 4U

How many two semester timetables can be made if MCV 4U cannot be taken in first semester and no more than one spare can be taken per semester with no repetitions allowed (other than one spare per semester)?

[8]

case #1 - no spare either semester

$_{16}P_4 \cdot {}_{13}P_4 = 749548800$

case #2 - spare sem. 1

$S \; {}_{16}P_3 \cdot 4 \; {}_{14}P_4 = 322882560$

case #3 - spare sem 2.

$_{16}P_4 \cdot S \; {}_{13}P_3 \cdot 4 = 299819520$

case #4 - spare both sem

$S \; {}_{16}P_3 \cdot 4 \cdot S \; {}_{14}P_3 \cdot 4 = 117411840$

Total = 1489662720