

A. Motion Terminology

1. a) $\Delta d = 700 \text{ m} + 1200 \text{ m}$
 $= 1900 \text{ m}$

b) $\vec{\Delta d} = 700 \text{ m}[\text{E}] + 1200 \text{ m}[\text{W}]$
 $= -700 \text{ m}[\text{W}] + 1200 \text{ m}[\text{W}]$
 $= 500 \text{ m}[\text{W}]$

c) $35 \text{ min} = 2100 \text{ s}$

$v = \frac{1900 \text{ m}}{2100 \text{ s}}$
 $= 0.90 \text{ m/s}$

d) $\vec{v} = \frac{500 \text{ m}[\text{W}]}{2100 \text{ s}}$
 $= 0.24 \text{ m/s} [\text{W}]$

2. a) $\Delta d = 1000 \text{ m} + 1000 \text{ m}$
 $= 2000 \text{ m}$

b) $\vec{\Delta d} = 1000 \text{ m}[\text{W}] + 1000 \text{ m}[\text{E}]$
 $= -1000 \text{ m}[\text{E}] + 1000 \text{ m}[\text{E}]$
 $= 0 \text{ m}[\text{E}]$

3. $\Delta t = \frac{\Delta d}{v}$
 $= \frac{128 \text{ cm}}{4.0 \text{ cm/min}}$
 $= 32 \text{ min}$

A2)

$$4. \vec{v} = \frac{862 \text{ m [E]}}{240 \text{ s}} \\ = 3.6 \text{ m/s [E]}$$

$$4.0 \text{ min} = 240 \text{ s}$$

$$5. a) \Delta \vec{d} = 70.0 \text{ m [W]} + 30.0 \text{ m [E]} + 30.0 \text{ m [W]} \\ = 70.0 \text{ m [W]} - 30.0 \text{ m [W]} + 30.0 \text{ m [W]} \\ = 70.0 \text{ m [W]}$$

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} \\ = \frac{70.0 \text{ m [W]}}{480 \text{ s}} \\ = 0.146 \text{ m/s [W]}$$

$$8.0 \text{ min} = 480 \text{ s}$$

$$b) \Delta d = 70.0 \text{ m} + 30.0 \text{ m} + 30.0 \text{ m} \\ = 130.0 \text{ m}$$

$$v = \frac{\Delta d}{\Delta t} \\ = \frac{130.0 \text{ m}}{480 \text{ s}}$$

$$v = 0.271 \text{ m/s}$$

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$$v = 0.271 \text{ m/s}$$

(B)

B. 1D Motion and Motion Graphs

1. a) bc and de (Hint: an object will have an acceleration if it is speeding up or slowing down)

b) ab and ef

c) fg

$$\begin{aligned} \text{d) } \Delta \vec{d} &= \vec{d}_{25} - \vec{d}_{10} \\ &= 40\text{m}[\text{N}] - 10\text{m}[\text{N}] \\ &= 30\text{m}[\text{N}] \end{aligned}$$

$$\text{e) (i) } 0\text{m/s}[\text{N}]$$

$$\begin{aligned} \text{(ii) } \vec{v}_{av} &= \frac{40-10}{25-10} \\ &= 2\text{m/s}[\text{N}] \end{aligned}$$

$$\begin{aligned} \text{(iii) } \vec{v}_{av} &= \frac{50-40}{40-25} \\ &= 0.67\text{m/s}[\text{N}] \end{aligned}$$

$$\begin{aligned} \text{(iv) } \vec{v}_{av} &= \frac{60-50}{55-40} \\ &= 0.67\text{m/s}[\text{N}] \end{aligned}$$

$$\text{(v) } 0\text{m/s}[\text{N}]$$

$$\begin{aligned} \text{(vi) } \vec{v}_{av} &= \frac{30-60}{75-65} \\ &= -3\text{m/s}[\text{N}] \text{ or } 3\text{m/s}[\text{S}] \end{aligned}$$

$$\begin{aligned} \text{f) } \vec{v}_{inst} &= \frac{25-10}{25-15} \\ &= 1.5\text{m/s}[\text{N}] \end{aligned}$$

tangent should be drawn on graph

Answers may vary

2. a) a (section a is the only segment that starts at a point with a y-coordinate of $0 \text{ m} [\text{E}]$)

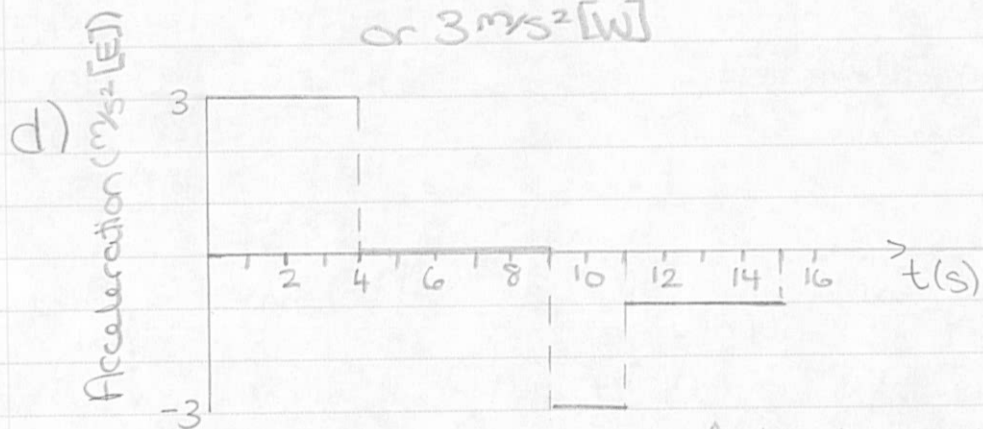
b) c

$$\text{c) (i) } \vec{a}_{aw} = \frac{12-0}{4-0} = 3 \text{ m/s}^2 [\text{E}]$$

$$\text{(ii) } \vec{a}_{aw} = 0 \text{ m/s}^2 [\text{E}]$$

$$\text{(iii) } \vec{a}_{aw} = \frac{6-12}{11-9} = -3 \text{ m/s}^2 [\text{E}] \\ \text{or } 3 \text{ m/s}^2 [\text{W}]$$

$$\text{(iv) } \vec{a}_{aw} = \frac{10-6}{15-11} = 1 \text{ m/s}^2 [\text{E}]$$



$$\text{e) (i) } \Delta d_A = \frac{1}{2} (4)(12) = 24 \text{ m} [\text{E}]$$

Actual position req'd for f.

$$d_A = 0 + \Delta d_A = 24 \text{ m} [\text{E}]$$

$$\text{(ii) } \Delta d_B = (5)(12) = 60 \text{ m} [\text{E}]$$

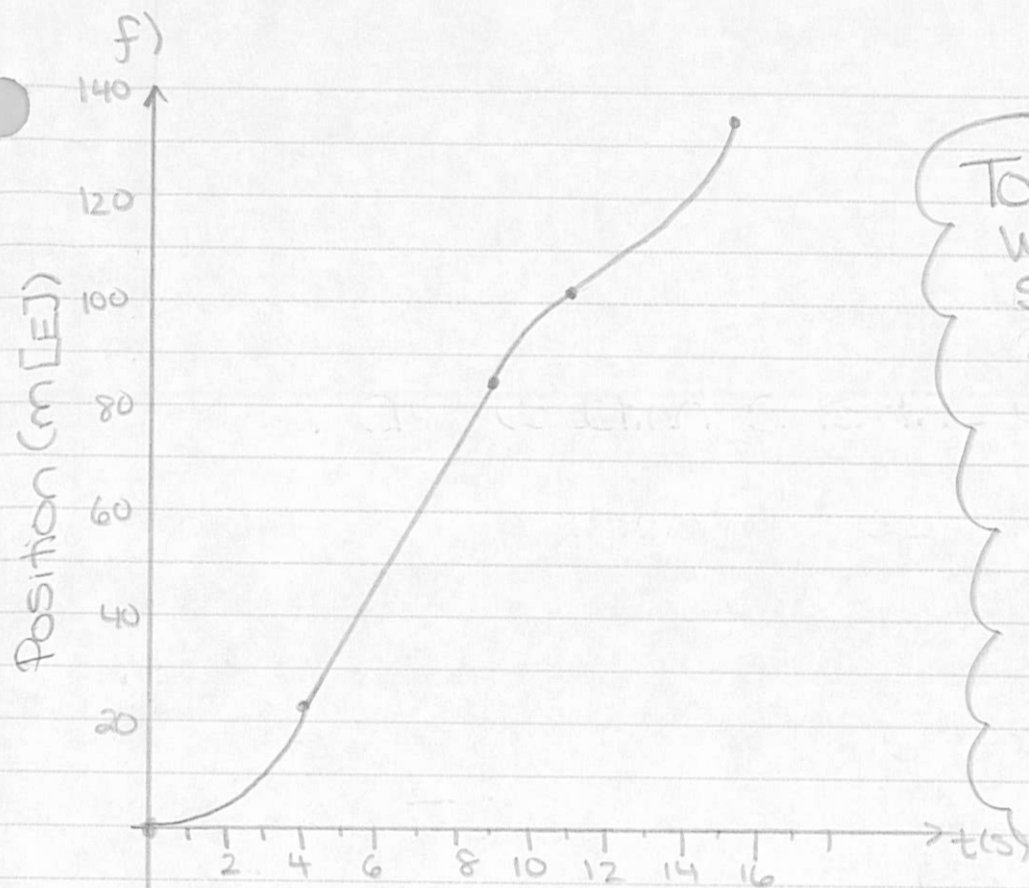
$$d_B = 24 \text{ m} [\text{E}] + \Delta d_B = 84 \text{ m} [\text{E}]$$

$$\text{(iii) } \Delta d_C = \frac{1}{2} (12+6)(2) = 18 \text{ m} [\text{E}]$$

$$d_C = 84 \text{ m} [\text{E}] + \Delta d_C = 102 \text{ m} [\text{E}]$$

$$\text{(iv) } \Delta d_D = \frac{1}{2} (6+10)(4) = 32 \text{ m} [\text{E}]$$

$$d_D = 102 \text{ m} [\text{E}] + \Delta d_D = 134 \text{ m} [\text{E}]$$



To determine whether segments should be straight or curved, consider acceleration.

no acceleration
→ straight

constant acceleration
(nonzero)
→ curved

Handwritten notes in the top left corner, possibly including the word "Hypothesis".

C. Equations of Uniform Acceleration

1. $\vec{v}_i = 0 \text{ m/s [f]}$
 $\vec{v}_f = 150 \text{ m/s [f]}$
 $\Delta t = 0.040 \text{ s}$
 $\vec{a} = ?$

$$\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$$
$$150 = 0 + a(0.040)$$
$$\vec{a} = 3750 \text{ m/s}^2 \text{ [f]}$$

2. $\vec{v}_i = 25.0 \text{ m/s [S]}$
 $\vec{a} = 1.20 \text{ m/s}^2 \text{ [S]}$
 $\Delta t = 3.00 \text{ s}$
 $\vec{v}_f = ?$

$$v_f = v_i + a \Delta t$$
$$= 25.0 + 1.20(3.00)$$
$$= 28.6 \text{ m/s [S]}$$

3. $v_i = 25.0 \text{ m/s [S]}$
 $a = 3.50 \text{ m/s}^2 \text{ [N]}$
 $= -3.50 \text{ m/s}^2 \text{ [S]}$
 $\Delta t = 3.00 \text{ s}$
 $v_f = ?$

$$v_f = v_i + a \Delta t$$
$$= 25.0 + (-3.50)(3.00)$$
$$= 14.5 \text{ m/s [S]}$$

4. $\Delta d = 33 \text{ cm [f]}$
 $= 0.33 \text{ m [f]}$
 $v_f = 0 \text{ m/s [f]}$
 $\Delta t = 0.0020 \text{ s}$
 $v_i = ?$

$$\Delta d = \left(\frac{v_i + v_f}{2} \right) \Delta t$$
$$0.33 = \left(\frac{v_i + 0}{2} \right) (0.0020)$$
$$v_i = 330 \text{ m/s [f]}$$

5. $v_{iy} = 0 \text{ m/s [d]}$
 $a_y = 9.8 \text{ m/s}^2 \text{ [d]}$
 $\Delta d_y = 20.0 \text{ m [d]}$

a) $\Delta t = ?$ $\Delta d_y = v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$
 $20.0 = 0 \Delta t + \frac{1}{2} (9.8) \Delta t^2$
 $\Delta t = 2.0203 \text{ s}$

b) If $t = 0.50 \text{ s}$, $v_f = ?$

$v_f = v_i + a \Delta t$
 $= 0 + (9.8)(0.50)$
 $= 4.9 \text{ m/s [d]}$

6. Car $v_i = 0 \text{ m/s [E]}$
 $a = 3.1 \text{ m/s}^2 \text{ [E]}$ uniform acceleration

Truck $v = 15 \text{ m/s [E]}$ uniform motion
 $(d = v \Delta t)$

a) At $t = 2.0 \text{ s}$

Car: $\Delta d_c = 0(2.0) + \frac{1}{2} (3.1)(2.0)^2$
 $= 6.2 \text{ m}$

Truck: $\Delta d_T = (15)(2.0)$
 $= 30. \text{ m}$

At $t = 8.0 \text{ s}$

Car: $\Delta d_c = 0(8.0) + \frac{1}{2} (3.1)(8.0)^2$
 $= 99.2 \text{ m}$

$\Delta d_T = (15)(8)$
 $= 120 \text{ m}$

C3
b)

$$\Delta d_c = \Delta d_t$$

$$0\Delta t + \frac{1}{2}(3.1)\Delta t^2 = 15\Delta t$$

$$1.55\Delta t^2 = 15\Delta t$$

$$1.55\Delta t^2 - 15\Delta t = 0$$

$$\Delta t(1.55\Delta t - 15) = 0$$

$$\begin{array}{ccc} \swarrow & & \searrow \\ \Delta t = 0 & \text{or} & 1.55\Delta t - 15 = 0 \\ & & \Delta t = 9.6774 \end{array}$$

\therefore the vehicles will pass each other after ~ 9.7 s.

c) Can use car or truck because the displacement will be the same.

Car:

$$\begin{aligned} \Delta d &= 0(9.6774) + \frac{1}{2}(3.1)(9.6774)^2 \\ &= 145.1607 \text{ m} \end{aligned}$$

Truck

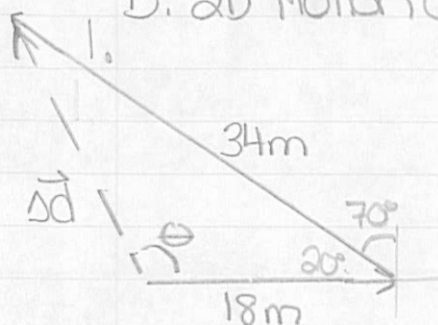
$$\begin{aligned} \Delta d &= (15)(9.6774) \\ &= 145.1610 \text{ m} \end{aligned}$$

\therefore they will pass each other $\sim 1.5 \times 10^2$ m in front of the traffic light.



D1

D. 2D Motion and Methods for Vector Algebra



$$\Delta d^2 = 18^2 + 34^2 - 2(18)(34)\cos 20^\circ$$

$$= 329.8162$$

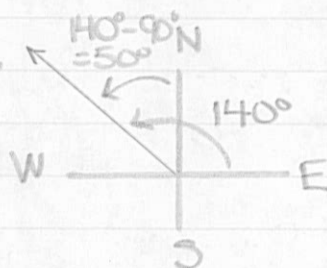
$$\Delta d = 18.1808 \text{ m}$$

$$34^2 = 18^2 + 18.1808^2 - 2(18)(18.1808)\cos \theta$$

$$1156 = 654.5415 - 654.5088 \cos \theta$$

$$\theta = 140^\circ$$

Let's look at our angle:

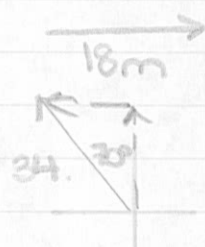


will give related acute angle if using Sine law (ambiguous)

∴ the displacement is $\sim 18 \text{ m [N} 50^\circ \text{W]}$

OR

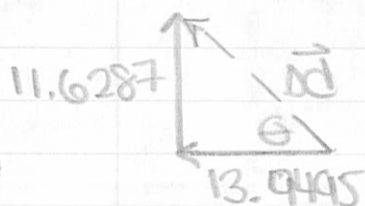
Sketch x comp y comp



$$18 \text{ m [E]} \quad 0 \text{ m [N]}$$

$$34 \sin 70^\circ = 31.9495 \text{ m [W]} \quad 34 \cos 70^\circ = 11.6287 \text{ m [N]}$$

Total 13.9495 m [W] 11.6287 m [N]



$$\Delta d = \sqrt{13.9495^2 + 11.6287^2}$$

$$= 18.1608$$

$$\theta = \tan^{-1}\left(\frac{11.6287}{13.9495}\right)$$

$$= 39.8155^\circ$$

slightly different due to rounding

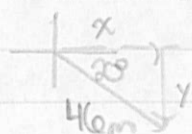
∴ displacement is $\sim 18 \text{ m [W} 40^\circ \text{N]}$ (or [N 50° W])

2)

2. Sketch

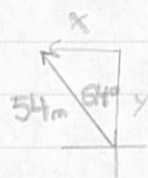
x comp

y comp



$$46 \cos 20^\circ \\ = 43.2259 \text{ m [E]}$$

$$46 \sin 20^\circ \\ = 15.7329 \text{ m [S]}$$



$$54 \sin 24^\circ \\ = 21.9638 \text{ m [W]}$$

$$54 \cos 24^\circ \\ = 49.3315 \text{ m [N]}$$



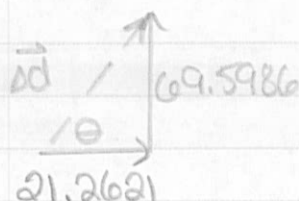
$$0 \text{ m [E]}$$

$$36 \text{ m [N]}$$

Total

$$21.2621 \text{ m [E]}$$

$$69.5986 \text{ m [N]}$$



$$\Delta d = \sqrt{69.5986^2 + 21.2621^2} \\ = 72.7739$$

$$\theta = \tan^{-1} \left(\frac{69.5986}{21.2621} \right) \\ = 73.0123^\circ$$

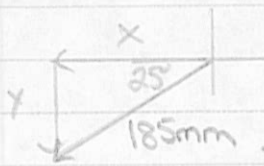
\therefore the displacement is $73 \text{ m [E } 73^\circ \text{ N]}$

D3

3. a) Sketch

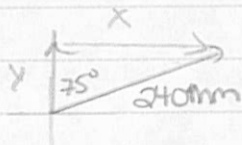
x comp

y comp



$$185 \cos 25^\circ = 167.6669 \text{ mm [W]}$$

$$185 \sin 25^\circ = 78.1844 \text{ mm [S]}$$



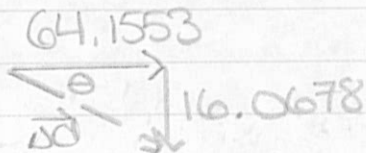
$$240 \sin 75^\circ = 231.8222 \text{ mm [E]}$$

$$240 \cos 75^\circ = 62.1166 \text{ mm [N]}$$

Total

$$64.1553 \text{ mm [E]}$$

$$16.0678 \text{ mm [S]}$$



$$\Delta d = \sqrt{64.1553^2 + 16.0678^2} = 66.1368 \text{ mm}$$

$$\theta = \tan^{-1}\left(\frac{16.0678}{64.1553}\right) = 14.0606^\circ$$

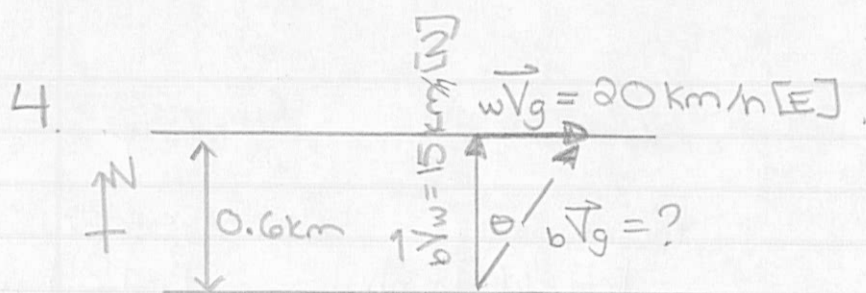
∴ the displacement of the snail is
~66 mm [E14°S]

$$b) \vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{66.1368 \text{ mm [E14°S]}}{14 \text{ s}}$$

$$= 4.7241 \text{ mm/s [E14°S]}$$

∴ the snail's average velocity is
~4.7 mm/s [E14°S]



a) Using vertical values:

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} \rightarrow \Delta t = \frac{\Delta \vec{d}}{\vec{v}}$$

$$\begin{aligned} \Delta t &= \frac{0.6 \text{ km [N]}}{15 \text{ km/h [N]}} \\ &= 0.04 \text{ h [N]} \end{aligned}$$

b) Using horizontal values:

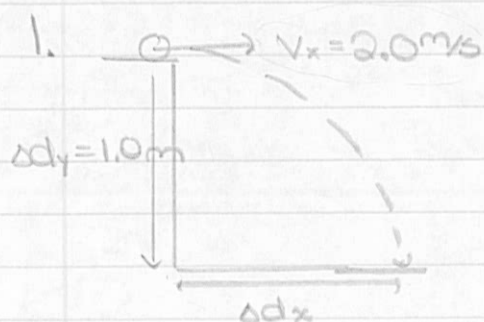
$$\begin{aligned} \Delta d &= v \Delta t \\ &= (20 \text{ km/h [E]})(0.04 \text{ h}) \\ &= 0.8 \text{ km [E]} \end{aligned}$$

$$\begin{aligned} \text{c) } |b\vec{V}_g| &= \sqrt{15^2 + 20^2} \\ &= 25 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{20}{15}\right) \\ &= 53^\circ \end{aligned}$$

\therefore the boat's resultant velocity is $25 \text{ km/h [N } 53^\circ \text{ E]}$

E. Projectile Motion



x comp
$v_x = 2.0 \text{ m/s [f]}$
$\Delta x = ?$
$\Delta t = ?$

y comp
$v_{iy} = 0 \text{ m/s [d]}$
$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$
$\Delta y = 1.0 \text{ m [d]}$
$\Delta t = ?$
$v_{fy} = ?$

$$\Delta y = v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$$

$$1.0 = 0 \Delta t + \frac{1}{2} (9.8) \Delta t^2$$

$$\Delta t = 0.4518 \text{ s}$$

$$\Delta x = (2.0)(0.4518)$$

$$= 0.9035 \text{ m}$$

\therefore it will land 0.90 m in front of the table.



x comp
N/A

y comp
$v_{iy} = 325 \text{ m/s [up]}$
$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$

a) $v_{fy} = 0 \text{ m/s [u]}$

$\Delta y = ?$

$$v_{fy}^2 = v_{iy}^2 + 2 a_y \Delta y$$

$$0^2 = 325^2 + 2(-9.8) \Delta y$$

$$\Delta y = 5389.0306 \text{ m}$$

\therefore it will rise 5389 m above the 2.4 m from which it was launched.

b) $a_y = 9.8 \text{ m/s}^2 \text{ [down]}$

c) $-2.4 = 325 \Delta t + \frac{1}{2} (-9.8) \Delta t^2$

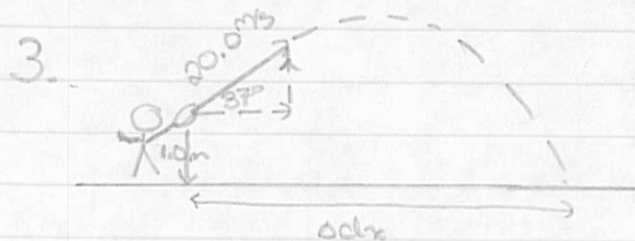
$$\Delta t = -0.007384 \text{ or } 66.3339 \text{ s}$$

d) $v_{fy} = v_f$ since there is no x-component

$$v_f^2 = 325^2 + 2(-9.8)(-2.4)$$

$$v_f = 325.0633$$

\therefore final velocity is $\sim 3.3 \times 10^2 \text{ m/s [down]}$



x comp

$$v_x = 20 \cos 37^\circ$$

$$= 15.9727 \text{ m/s [f]}$$

$$\Delta dx = ?$$

$$\Delta t = ?$$

y comp

$$v_{iy} = 20 \sin 37^\circ$$

$$= 12.0363 \text{ m/s [u]}$$

$$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$$

$$\Delta dy = 1.0 \text{ m [d]}$$

$$v_{fy} = ?$$

$$\Delta t = ?$$

$$a) \Delta dy = v_{iy} \Delta t + \frac{1}{2} a \Delta t^2$$

$$1.0 = -12.0363 \Delta t + \frac{1}{2} (9.8) \Delta t^2$$

$$\Delta t = 2.5368 \text{ s or } -0.0804 \text{ s}$$

\therefore it will be in the air for 2.5 s

$$b) \Delta dx = (15.9727)(2.5368)$$

$$= 40.5195$$

\therefore it will travel $\sim 41 \text{ m}$ [forward]

$$c) \text{ @ max height } v_{fy} = 0 \text{ m/s [up]}$$

$$v_{iy} = 12.0363 \text{ m/s [u]}$$

$$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$$

$$\Delta dy_{\text{max}} = ?$$

$$v_{fy}^2 = v_{iy}^2 + 2 a_y \Delta dy_{\text{max}}$$

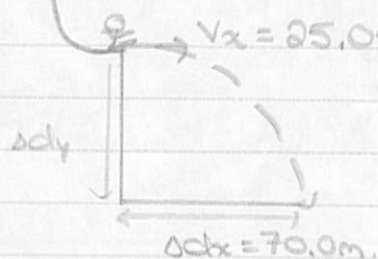
$$0^2 = 12.0363^2 + 2(-9.8) \Delta dy_{\text{max}}$$

$$\Delta dy_{\text{max}} = 7.3915$$

\therefore the max height is 7.4 m

E3

4.



x comp	y comp
$v_x = 25.0 \text{ m/s [E]}$	$v_{iy} = 0 \text{ m/s [d]}$
$\Delta x = 70.0 \text{ m [E]}$	$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$
$\Delta t = ?$	$\Delta y = ?$
	$\Delta t = ?$
	$v_{fy} = ?$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$= \frac{70.0}{25.0}$$

$$= 2.80 \text{ s}$$

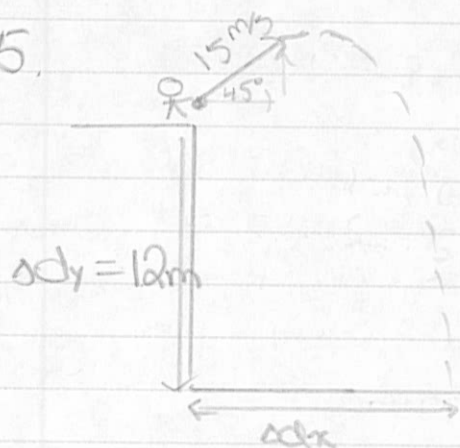
$$\Delta y = v_{iy} \Delta t + \frac{1}{2} a \Delta t^2$$

$$= 0(2.80) + \frac{1}{2} (9.8) (2.80)^2$$

$$= 38.416$$

∴ the ramp was 38.4 m above the ground

5.



x comp	y comp
$v_x = 15 \cos 45^\circ$	$v_{iy} = 15 \sin 45^\circ$
$= 10.6066 \text{ m/s [E]}$	$= 10.6066 \text{ m/s [up]}$
$\Delta x = ?$	$a_y = 9.8 \text{ m/s}^2 \text{ [d]}$
$\Delta t = ?$	$\Delta y = 12 \text{ m [d]}$
	$v_{fy} = ?$
	$\Delta t = ?$

$$a) \Delta y = v_{iy} \Delta t + \frac{1}{2} a \Delta t^2$$

$$12 = (-10.6066) \Delta t + \frac{1}{2} (9.8) \Delta t^2$$

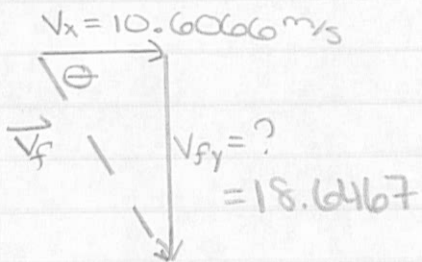
$$\Delta t = 2.9850 \text{ s or } -0.8204 \text{ s}$$

∴ it will be in the air for $\sim 3.0 \text{ s}$

$$b) \Delta x = (10.6066)(2.9850) \\ = 31.6607$$

\therefore it will land $\sim 32\text{m}$ in front of the cliff.

$$c) \vec{v}_f = \vec{v}_x + \vec{v}_{fy}$$



$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \\ = (-10.6066)^2 + 2(9.8)(12) \\ = 347.6999 \\ v_{fy} = 18.6467 \text{ m/s [d]}$$

$$v_f = \sqrt{10.6066^2 + 18.6467^2} \\ = 21.4522 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{18.6467}{10.6066}\right) \\ = 60.3680^\circ$$

\therefore the final velocity is $\sim 21 \text{ m/s}$ [60° below horizontal]