$P \neq NP$

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1 Abstract

The question, whether NP- and P-problems are logically equal, is fiercely debated. Using *Strict logic* (by Walther Brüning), this question can be answered. It is resulted, that NP-problems have an *Applied Logic*-correspondent in *Pure Logic*.

2 Introduction

As an introduction, it is briefly explained what *Strict Logic* is, and the use of *Modus ponens* and the transcription of *Barbara* is illustrated.

2.1 General information about Strict Logic

Strict Logic has been established in Walther Brüning's 1996 book *Grundlagen der Strengen Logik* ('Foundations of strict logic'). The preface describes how it is to be understood. Its constitution relies solely on *the principle of identity* and *the principle of limitation*, as well as *affirmation* (A) and *negation* (N).

The *principle of identity* he states (positively) as follows: 'Everything stated in logic is identical with itself and only with itself.'

The *principle of limitation*, he states (positively) as follows: 'Everything stated is limitatively different

1. from other things
2. but only from other things.'

For a more detailed explanation of the principles, see the book (page 58ff).

'If a logic violates the principles, it is called *transgressive*.' (see the book, page 50).

'Returning to the start of logical thinking' Brüning goes from the *dyadic level* (concerning two cases) to the *henadic level* (concerning one case). Here, firstly, the *henadic level* is explained (page 52ff):

Henadic means exactly one case is considered. Through the logical principles one base domain is split up into two *partitions* (for a more detailed explanation, see the book). A domain relating to a case (domain-name: B) and a complement of B (domain-name: $\sim B$). Because of the logical principles, these partitions can combine to four possibilities:

	B	$\sim B$
1	A	A
2	N	N
3	A	N
4	N	A

Table 1: Henadic level

These are *synthetic states*. It is to be noted that in formal logic, affirmation does not take precedent over negation.

'The four complete henadic formulas contradict each other. At least at one position two formulas A and N meet:

If incomplete formulas are permitted (Au, uA, Nu, uN), it is already possible to demonstrate a precursor of logical deduction. Indeed, logical deduction in a strict sense means detaching a part from a given piece of information.

E. g., given AN, Au follows logically:

$AN \rightarrow Au$

[...].'

Likewise, it is possible to proceed with further formulas. These are examples of *direct conclusions*.

Further:

'From the assumption

No not-humans exist (uN)

It doesn't follow that:

Humans exist (Au)

[...].'

Now it is possible to expand the stipulation of the first split of the base domain (which led to the henadic level).

'At the first expansion, four affirmations and/or negations are used at the same time. Thus, the base domain once again is split up and accordingly a new case including complement (C, $\sim C$) is introduced. From that, a dyadic level results.

Between the first case's level and the second case's level there exist four possibilities for overlap (which they embody in corresponding variables) [...].'

As with the henadic level, 16 possible complete *case connections* result.

'Any analogous expansions can be joined[.]

[...]

Considering all that has been said, it is to be noted that in the area of Pure Logic cases are of a general nature. Individual cases are only considered in Applied Logic.'

It can be concluded between levels or within levels. In order to make conclusions (here and in the following: on the same level) the following is to be said (see the book, page 21ff): The transition to indirect conclusions requires the introduction of a second term. In addition: 'The value-formulas are to be extended accordingly[.]' Further (see the book, page 23): 'What applies to direct conclusions, applies also to indirect conclusions:

A logical inference in the strict sense is detaching a part (conclusion) from a given piece of information (premise). What thereto is needed, is only the information and the principle of identity.'

The inference should detach the part from a given piece of information, which is relevant for the conclusion. Further: 'Thereby the par-positions of the expected conclusion [...] are reviewed in regard to possible information from the premises, and that way the formula of the conclusion is built up step by step[.]'

'The [...] inference process can be traced back to the two following rules:

1. If one of two of the a-par-positions of premises is blocked (through an n of the other premise), the a of the other par-position must go into the conclusion (the par-positions of the conclusion are thus a).

2. If two par-positions of the conclusion are blocked by the premises through at least one n, they both must be n in the conclusion.

The main question of this process is easy: Which information of the premises is relevant for the conclusion.' (see the book, page 26f)

If both rules were to be applied simultaneously (at certain par-positions of the conclusion), the premises would contradict each other (in addition see the book, page 88).

2.2 Conditional inference and Modus ponens as example thereof in Strict Logic

General. The given inference process can be applied to Modus ponens:

'In the different sections of strict Specific Logic, different aspects are highlighted, which prove especially beneficial in the corresponding context.

That is the case, for example, for the conditional conclusion in predicate logic [...].

It is however important to see, that all these aspects have fundamental validity in all of Pure Logic [...]' (see the book, page 100)

Conditional inference. 'The conditions, of which the conclusions follow here, are formulated in case-connections whose value-formulas, beside unknown-signs, only contain N.' (see the book, page 104).

Modus ponens. The first premise is a condition of dyadic (i.e. two cases are connected with each other) level (therefore capital letter). It is a conditional proposition (*There is no F without G.*). The second premise is a dyadically extended henadic (i.e. one case in question) formula (therefor lower case letters and par-positions); as well as the conclusion. (The underlined a goes into the conclusion because the other a of the par-position is blocked by an N). The superscript c means *condicio* (see mainly page 106):

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
$F \supset^c G$	u	u	N	u
F	$\underline{a}$	u	a	u
G	a	a	u	u

Table 2: Modus ponens in Strict Logic

The columns are separated by different possible combinations . The extension of the value-formulas of the premise F and accordingly the conclusion G result from introduction of the second term G and/or F. Therefore their value remains constant (par-positions are formed). a – short for affirmation (corresponds to true); N – short for negation (corresponds to false); u – short for unknown; $\sim X$ – short for complement of X.

Thus, this would be an example of conditional inference.

2.3 Strict syllogistic by Brüning

Judges. Universal judges draw a negative judge over a partition of a dyadic formula (page 8ff):

	S P	$\sim S$ P	S $\sim P$	$\sim S$ $\sim P$
SaP	u	u	N	u
SeP	N	u	u	u

Table 3: Universal judges in Strict Syllogistic

Namely, that S <u>without P</u> are not (All S are P, SaP) and that S <u>without $\sim P$</u> are not (No S are P, SeP).

Particular judges draw a affirmative judge over a partition of a dyadic formula:

	S P	$\sim S$ P	S $\sim P$	$\sim S$ $\sim P$
SiP	A	u	u	u
SoP	u	u	A	u

Table 4: Particular judges in Strict Syllogisitc

Namely, that some S are P (SiP) and that some S are $\sim P$ (SoP).

It is known, that traditional syllogistic makes some preassumptions about the openness along existences of the terms. Therefore universal judges specify themselves. Openness, in Strict Logic, is only meant to be for now, that A-positions must be able. And, in fact, for every term and complementary term which shows up in the premises. Therefore, following specifications result:

	S P	$\sim S$ P	S $\sim P$	$\sim S$ $\sim P$
SaP	A	u	N	A
SeP	N	A	A	u

Table 5: Universal judges with preassumptions in Strict Syllogistic

Namely, that for example the first partition of SaP (SP) must become affirmative, because the other partition of S ($S \sim P$) is not open certainly (N).

Barbara in Strict Syllogistic. For syllogistic reasoning we just only have to extend the formulas of the judges about a mid-term. That is happening, by doubling the positions of the formula accordingly, each, to the, so to speak, uninvolved term; thus doubling the values of the correspondent columns:

	S M P	$\sim S$ M P	S $\sim M$ P	$\sim S$ $\sim M$ P	S M $\sim P$	$\sim S$ M $\sim P$	S $\sim M$ $\sim P$	$\sim S$ $\sim M$ $\sim P$
MaP	a	a	u	u	n	n	a	a
SaM	a	u	n	a	a	u	n	a

Table 6: Examples for extended judges (here: MaP and SaM) in Strict Syllogistic

Know it is possible to conclude stepwise for the deducible relationship ($S{\bullet}P$) with the help of the two deduction rules. Step-by-step therefore, because one must be aware their self about the par-positions of the conclusion. It must be pointed out, that all in traditional syllogistic made inferences, the pair of premises is never in contradiction to each other (This would be the case, if both rules would be made applicable for a par-position-pair of the conclusion at once.):

	S M P	$\sim S$ M P	S $\sim M$ P	$\sim S$ $\sim M$ P	S M $\sim P$	$\sim S$ M $\sim P$	S $\sim M$ $\sim P$	$\sim S$ $\sim M$ $\sim P$
MaP	a	a	u	u	$\underline{n}$	n	a	$\underline{a}$
SaM	$\underline{a}$	u	n	a	a	u	$\underline{n}$	a
SaP	a	u	n	a	a	u	n	a

Table 7: Example for an inference of traditional syllogistic in Strict Syllogistic (here: *Barbara*)

The underlined letters go into the conclusion.

Within this procedure all in traditional syllogistic valid inferences can be gained.

3 Transcriptions

3.1 P-problems

P-problems can be transcribed with Pure Logic as follows:

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
F and $\sim F$ not	$\underline{a}$	n	a	n
$F \cap^c G$	u	N	N	u
G and $\sim G$ not	a	a	n	n
$Total - formula$	A	N	N	N

Table 8: P-problems

Thus, they are bidirectional (Elsegood, 'F and $\sim F$ not' follows from 'G and $\sim G$ not'.). *Total-formulas* summarize (relevant) information on a higher level.

Transcribing P-problems in Applied Logic For this Brüning writes: 'In Applied Logic it is possible to introduce the worth of individual cases in the first instance in the meaning of bare 'existence'-stipulations. Is a A-position called by an e-index additionally (A^e), so it is meant to be, that the position is not only 'open', but that here also individuals 'exist' (of unkown number). For this 'existence' is meant to be a logical stipulation. It is not meant to be about real in being. In certain clear relationships e-indexing can also be replaced easily by recognising all A-positions as stipulated existentially (page 162).

3.2 NP-problems

Transcribing of NP-problems. At NP-problems (*decision problems*), on the other side, it is known that the solution is an individual case (e. g. SAT-problem: There is at least one assignment (satisfiability)). Thus, it is about

Applied Logic. Here it is called g^i. The formula therefore is A^iA^i. I. e. for individual circumstances only one A^i is able to become A and the remaining A^is then become Ns. (See page 164ff and page 109ff. An individual circumstance cannot be divided.). The case that both A^iA^i become N, is no contradiction (NN, ibid.). At reasoning, A^is always stay together (ibid.).

As a premis of NP-problems is valid, that additionally to an conditional sentence $F \supset^c G$ ($uuNu$), also a *verifier* $F \subset^c G$ ($uNuu$) must be present. Therefore results as an conditional sentence: $F \cap^c G$ ($uNNu$).

In the first instance it is equal, whether $F \supset^c G$ ($uuNu$) or $F \subset^c G$ ($uNuu$) is interpretated as *verifier*, because both are transitive, but not symmetric. For this Brüning writes, that at stipulation of certain meaning of the terms it is necessary to be aware of a difference. (Another inference-context is produced, page 43).

Firstly it is concluded with the conditional sentence $F \supset^c G$ ($uuNu$). As mentioned A^i-positions must stay together at the inference process. Therefore follows either g^i (A^iA^i) or f^i (A^iA^i):

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
f^i	(a^i)	a^i	a^i	(a^i)
$F \supset^c G$	u	u	N	u
Either:				
g^i	a^i	a^i	a^i	a^i
Or:				
f^i	a^i	a^i	a^i	a^i
$Total - formula$	(A^i)	u	N	(A^i)

Table 9: First step of NP-problems

Now, the problem could be solved already. Just, it could not be proven. Thus, it is concluded with the *verifier* $F \subset^c G$ ($uNuu$) again.

Per synthesis it is only possible to conclude once from enlonged individual variables.

Thus, it follows anew synthesis of the premisis. As the case may be, whether the individual variable is acknowledged the selfsame (identical), two possibilities result:

The first possibility now shall stipulate, that the individual from the premis does not change (identical):

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
f^i	(a^i)	a^i	a^i	(a^i)
$Total - formula$	(A^i)	u	N	(A^i)
$F \subset^c G$	u	N	u	u
$Total - formula$	A^i	N	N	A^i

Table 10: Second step of NP-problems with an identical individual

It follows $P \neq NP$. (entanglement: Trust cannot be proven.)

The brackets shall indicate a case-differentiation. Thus, in the premisis there are $2^3 = 8$ possibilities differentiated. It is settled close, therefore to research the fourth level for that. There is following result for the possibility of a random solution on the tetradic level (from: *Logische Grundlagen der Quantenphysik 2*), if it is inferenced within the level (2^3 variable-positions) :

Possibility, that a resulting triadic enlonged
tetradic Part-formula contains no
uncertain places in the Total-formula

$$\varpi_\pi \approx \frac{16\,777\,215}{66\,577} + \frac{65\,535}{8\,649} = 259.574... \rightarrow \varpi = \frac{\varpi_\pi}{\pi^4_{\text{App.}}} = \underline{2.6}426...$$

Possibility, that a resulting triadic enlonged
tetradic Part-formula is displayable
as a tetradic Total-formula:

$$2_e \approx \frac{16\,777\,215}{88\,412{,}438\,09} + \frac{65\,535}{13\,568{,}061\,9} + \frac{255}{178} = 196.0234... \rightarrow 2 = \frac{2_e}{\pi^4_{\text{App.}}} = 1.9956...$$

Thus, chance, that it is an actually realised NP-problem and not a P-problem, is about $\frac{\pi}{\varpi}$ (≈ 1.20). Whereas chance for a realised P-problem in opposition to an NP-problem is $\frac{\varpi}{\pi}$ ($\approx 0,83$).

The second possibility now shall stipulate, that the individual from the premis changes (different):

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
f^i_{two}	(a^i)	a^i	a^i	(a^i)
$F \cap^c G$	u	N	N	u
$Total - formula$	(A^i)	N	N	(A^i)

Table 11: Second step of NP-problems with a different individual

Because, again, it can be concluded only once, follows $P = NP$. (sovereignty: Trust must be proven.)

Excursion. Since conventional computers do not rule superstition, the individual is changed. Therefore, they only can solve P-problems fast (stepwise calculation). Thereby the conditional sentence $F \sqcap^c G$ $(uNNN)$ is enlonged and a dummy-variable 'H and $\sim H$ not' (AN) is introduced. It contains all information, which constitute individuality (f. e. a pseudo-random-information) and separates it from other individuals.

	F G H	$\sim F$ G H	F $\sim G$ H	$\sim F$ $\sim G$ H	F G $\sim H$	$\sim F$ G $\sim H$	F $\sim G$ $\sim H$	$\sim F$ $\sim G$ $\sim H$
$F \sqcap^c G$	u	N	N	N	u	N	N	N
H and $\sim H$ not	$\underline{a}$	a	a	a	n	n	n	n
F and $\sim F$ not	a	n	a	n	a	n	a	n
G and $\sim G$ not	a	a	n	n	a	a	n	n

Table 12: First step of conventional computers

So, 'F and $\sim F$ not' and 'G and $\sim G$ not' is deducable at the same time.

Thus, within a second step there is to be synthesized a causalisation between 'F and $\sim F$ not' and 'G and $\sim G$ not': $F \supset^c G$ $(uuNu)$. - Reality is created:

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
$F \supset^c G$	u	u	N	u
F and $\sim F$ not	$\underline{a}$	n	a	n
G and $\sim G$ not	a	a	n	n

Table 13: Modus Ponens in computer science

Thus, all in all it is inferenced between 'F and $\sim F$ not' and 'G and $\sim G$ not' with three ciscumstances, thus a doubalisation.

Transformation of NP-problems to P-problems. Known solutions are easy to prove. The total-formula denotes as: 'F only with G and $\sim F$ only with $\sim G$' $(A^i NNA^i)$. If from the solved NP-problem G (Au) result, follows further (s. p. 113):

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
$F \cap^i G$	A^i	N	N	A^i
G	a	a	u	u
F	a	u	a	u

Table 14: At resulting G it it possible to take F for transformation to P-problems.

If Not G (G', Nu) results of a solved NP-problem, alternatively follows:

	F G	$\sim F$ G	F $\sim G$	$\sim F$ $\sim G$
$F \cap^i G$	A^i	N	N	A^i
Not G (G')	n	n	u	u
$\sim F$	u	a	u	a

Table 15: At resulting Not G (G') it is possible to take $\sim F$ for transformation to P-problems.

Transcribing NP-problems in Pure Logic. A^is correspont in Pure Logic to A^1s.(A^ks stand for collective circumstances in Applied Logic.)

4 Conclusion

When NP-problems are postulated (synthesized), an on avarage (significantly) longer run-time compared to (synthesized) P-problems follows. Decision problems are individual cases. The longer run-time is compensated for clearness over a decision.

So far, areas of logic had a transgressive character. With the help of Strict Logic it is provable:

Purely logically, NP- and P-problems are different.

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