Problem 4

a) Prove $\frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2$

$$= 2 \sum_{i \in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2$$

given $\bar{x}_{kj} = \frac{1}{|C_k|} \sum_{i \in C_k} x_{ij}$

Expanding Squares on the Left hand Side

$$\frac{1}{C_k} \sum_{i \in C_k} \sum_{i' \in C_k} \sum_{j=1}^{p} \left(x_{ij}^2 - 2x_{ij} x_{i'j} + x_{i'j}^2 \right)$$

$$\sum_{i \in C_k} \sum_{j=1}^{p} \left(\frac{1}{|C_k|} \sum_{i' \in C_k} \left(x_{ij}^2 - 2x_{ij} x_{i'j} + x_{i'j}^2 \right) \right)$$

$$\sum_{i \in C_k} \sum_{j=1}^{p} \left[2 \times x_{ij}^2 - 4 x_{ij} \bar{x}_{kj} + 2 \bar{x}_{kj}^2 \right]$$

$\because$ as the Summation is over orderd pairs.

$$= 2 \sum_{i \in C_k} \sum_{j=1}^{p} (x_{ij} - \bar{x}_{kj})^2$$

hence proved