

Problem ①:

a) no. of features $P = 1$

$X \rightarrow$ uniformly distributed on $[0, 1]$

10% range is used when predicting a test observation

i.e., for $X = 0.6$ we look at $[0.55, 0.65]$

①

$X = 0.5$ we consider $[0.45, 0.55]$

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but for $X = 0.4$ ($X < 0.5$ scenario)

②

we take only $[0, 0.9]$

③

Similarly for ($X > 0.95$) $X = 0.99$

we look at only $[0.99, 1]$

So on an average we consider, the fraction of available observations we consider for prediction is less than 10%

Exact value can be calculate using area method.

$$\text{for } [0.05, 0.95] \Rightarrow 10\%$$

$$\text{for } [0, 0.05] \Rightarrow (100x+5)\%$$

$$\text{for } [0.95, 1] \Rightarrow (105-100x)\%$$

So integral sum of these values gives total area (fraction we are interested in)

$$= \int_{0.05}^{0.95} 10 dx + \int_{0.0}^{0.05} (100x+5) dx + \int_{0.95}^{1} (105-100x) dx$$

$$= 9 + 0.375 + 0.375$$

$$= 9.750$$

$$= \boxed{9.75\%}$$

We expect to look at 9.75% observations on average.

(b) with $p=2$.

(x_1, x_2) uniformly distributed on $[0,1] \times [0,1]$

\Rightarrow Examples.

① $x_1 = 0.6; x_2 = 0.35$.

observation range would be.

$$x_1 \rightarrow [0.55, 0.65]$$

$$x_2 \rightarrow [0.3, 0.4]$$

② $x_1 = 0.04; x_2 = 0.5$

$$x_1 \rightarrow [0, 0.09] \quad x_2 \Rightarrow [0.45, 0.5]$$

It is similar to previous case.

in case $p=2$

the fraction would be $= (9.75) \times (9.75)$

$$= 95.0625 \approx 95\%$$

(c) $P = 100$

the fraction available observation for making prediction would be

$$(0.95)^{100} \approx 0\%$$

(d) as we increase P (number of testing) we left with fewer fraction of observation for prediction
as $P \rightarrow \infty$ this would be zero.

(e) 10% training observations are used for prediction

$$P=1 \rightarrow \text{line} \Rightarrow \text{length} = \left(\frac{1}{10}\right)$$

$$P=2 \rightarrow \text{Square} = \text{length} = \left(\frac{1}{10}\right)^2$$

$$P=100 \rightarrow \text{length} = \left(\frac{1}{10}\right)^{100}$$