STATS 202 | HW:1 | Sagar Ganapaneni | SUID# 06167633

Problem: 1

Background:

Error: The difference between actual or true value and predicted value derived from model.

Error term has two components:

1. Reducible error
   a. consists of variance and bias, need to strike a balance between these components to produce a high performance model
   b. Variance and bias depends on the Model flexibility (or) complexity and actual data nature whether it is more linear or non linear
2. Irreducible error
   a. due to unknown factors, uncaptured data, unpredictable factors
   b. nothing we can do much to reduce this component

Our main objective should be reducing the MSE of test data rather than train data as overfitting training data leads to include noise from train data which is not present in test data set.

a) Large Sample size *n* & Small number of Predictors *p*:

More Flexible model fits well with large sample size hence flexible model performance better than inflexible model

b) Small Sample size *n* & Large number of Predictors *p*:

With small sample size, flexible model over fits the training data which leads to poor performance with test data sets. Hence flexible model performs worse than inflexible model

c) Relationship between predictors and response is highly non-linear:

With highly non-linear data more flexible model will account for non-linearity in the data whereas inflexible model might lead to poor fit to the data hence Flexible model performance better than Inflexible model

d) $\sigma^2 = \mathrm{Var}(\boldsymbol{\varepsilon})$, is extremely high:

This means high noise in the train data, a flexible method could over fit the data and include this noise in the model which is not present in the test data. So Inflexible method performs better then inflexible method

Problem: 2

1. **Prediction:** predict an event or outcome value (Y) based on the data in hand (X) by computing $\hat{Y} = \hat{f}(X)$. $\hat{f}(X)$ could be a black box, we are most interested in the outcome variable than understanding f.
2. **Inference**: some times our goal may not be necessarily to make prediction instead we want to understand the relationship between X and Y or more specifically how Y varies with changes in X. with this we can find out the exact relationship between response and each predictor.

a)

i. Data set of 500 forms ➔ sample size n= 500
ii. Predictors: profit, number of employees, industry ➔ p = 3
iii. Output variable, CEO's Salary, a quantitative variable ➔ Scenario: Regression
iv. We are most interested the relationship between predictors and output ➔ Inference

b)

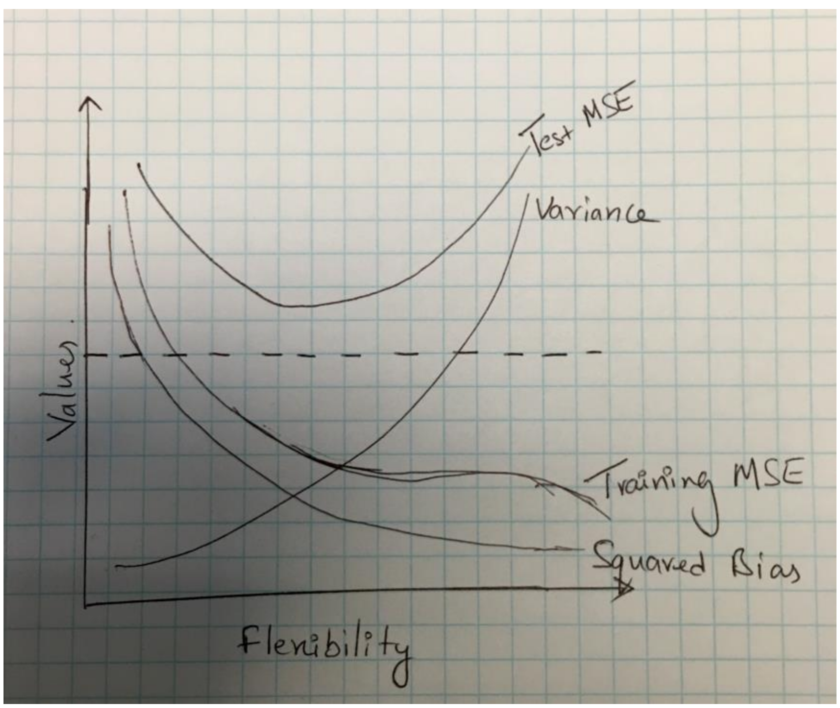
i. Data of 20 similar products launched previously ➔ sample size n= 20
ii. Predictors: price charged for the product, marketing budget, competition price, and ten other variables ➔ p = 13
iii. Output variable, whether new product will be a success or a failure, a categorical variable ➔ Scenario: Classification
iv. We are only interested in output ➔ Prediction

c)

i. Weekly data for all of 2012: 53 weeks➔ sample size n= 53
ii. Predictors: % change in the dollar, the % change in the US market, the % change in the British market, and the % change in the German market.➔ p = 3
iii. Response variable, % change in the US dollar, a quantitative variable ➔ Scenario: Regression
iv. We are only interested in predicting output ➔ Prediction

Problem: 3

a)



b)

Variance: measures how much $\hat{f}(x)$ changes as we change the training data set
Bias: measure how far the estimated value of $\hat{f}$(x) from the actual or true f(x)
Test MSE = Var $(\hat{f}(x_0))$ + [Bias$\hat{f}(x_0)]^2$ + Var(ϵ_0)

Variance: increases monotonically as flexibility increases. more flexible fit contains noise from the train data. As the training data changes $\hat{f}$ changes hence higher variance.

Squared bias: declines monotonically as flexibility increases. Inflexible models approximate the relationship between the variable to greater extent.

Var(ϵ), the irreducible error is constant unless we add new data or come up with new predictors

Test MSE declines at first, because as flexibility increases the bias decreases. However, increased flexibility leads
to increased variance, so at some point the benefits of decreasing bias are outweighed by the variance, which

Problem 4:

Bias Variance Decomposition Equation:
E$(y_0 - \hat{f}(x_0))^2$ = Var($\hat{f}(x_0)$) + $[Bias(\hat{f}(x_0))]^2$ +Var(ε_0)
Variance: measures how much $\hat{f}(x)$ changes as we change the training data set
Bias: measure how far the estimated value of $\hat{f}$(x) from the actual or true f(x)

b) We cannot estimate the *bias* component as we don't have the true f(x).
c) We can estimate the ***variance*** component by using already simulated models, find out corresponding estimated y_0 and measure the variance.
d) we cannot estimate the variance in ε_0 as $y_0 = f(x_0) + \varepsilon_0$ and we cannot simulate y_0
a) We cannot compute TEST MSE as well as we don't know true f

To summarize, with unknown f, we can only estimate the variance component.

Problem 5:

Read data

```
college <-read.csv('College.csv', header=TRUE)
```

view data

```
head (college)
```

```
##                               X Private Apps Accept Enroll Top10perc
```

```
## 1 Abilene Christian University     Yes 1660   1232    721         23
## 2           Adelphi University     Yes 2186   1924    512         16
## 3               Adrian College     Yes 1428   1097    336         22
## 4          Agnes Scott College     Yes  417    349    137         60
## 5    Alaska Pacific University     Yes  193    146     55         16
## 6            Albertson College     Yes  587    479    158         38
##   Top25perc F.Undergrad P.Undergrad Outstate Room.Board Books Personal PhD
## 1        52        2885         537     7440       3300   450     2200  70
## 2        29        2683        1227    12280       6450   750     1500  29
## 3        50        1036          99    11250       3750   400     1165  53
## 4        89         510          63    12960       5450   450      875  92
## 5        44         249         869     7560       4120   800     1500  76
## 6        62         678          41    13500       3335   500      675  67
##   Terminal S.F.Ratio perc.alumni Expend Grad.Rate
## 1       78      18.1          12   7041        60
## 2       30      12.2          16  10527        56
## 3       66      12.9          30   8735        54
## 4       97       7.7          37  19016        59
## 5       72      11.9           2  10922        15
## 6       73       9.4          11   9727        55
```

include college names in to the datafarame

```
rownames (college )<- college [,1]
college <- college [,-1]
```

Summary

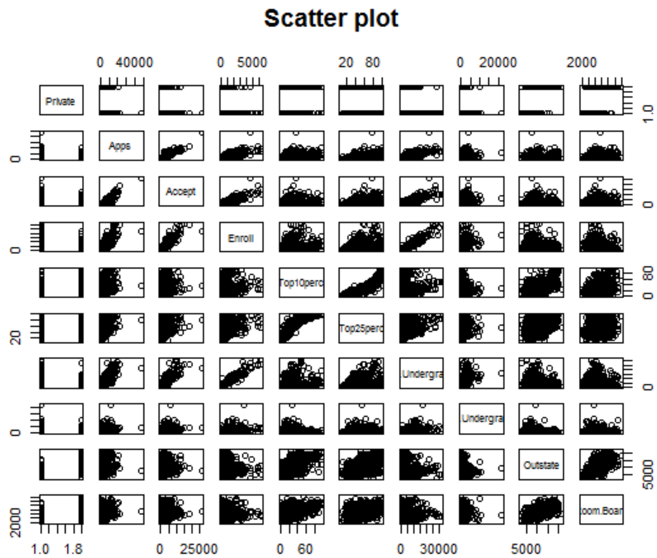
```
summary(college)
```

```
##                              X        Private        Apps
##  Abilene Christian University:  1   No :212   Min.   :   81
##  Adelphi University          :  1   Yes:565   1st Qu.:  776
##  Adrian College              :  1             Median : 1558
##  Agnes Scott College         :  1             Mean   : 3002
##  Alaska Pacific University   :  1             3rd Qu.: 3624
##  Albertson College           :  1             Max.   :48094
##  (Other)                     :771
```

```
##      Accept          Enroll        Top10perc       Top25perc
##  Min.    :   72   Min.    :  35   Min.    : 1.00   Min.    :  9.0
##  1st Qu.:  604   1st Qu.: 242   1st Qu.:15.00   1st Qu.: 41.0
##  Median : 1110   Median : 434   Median :23.00   Median : 54.0
##  Mean   : 2019   Mean   : 780   Mean   :27.56   Mean   : 55.8
##  3rd Qu.: 2424   3rd Qu.: 902   3rd Qu.:35.00   3rd Qu.: 69.0
##  Max.   :26330   Max.   :6392   Max.   :96.00   Max.   :100.0
##
##   F.Undergrad     P.Undergrad         Outstate        Room.Board
##  Min.   :  139   Min.   :    1.0   Min.   : 2340   Min.   :1780
##  1st Qu.:  992   1st Qu.:   95.0   1st Qu.: 7320   1st Qu.:3597
##  Median : 1707   Median :  353.0   Median : 9990   Median :4200
##  Mean   : 3700   Mean   :  855.3   Mean   :10441   Mean   :4358
##  3rd Qu.: 4005   3rd Qu.:  967.0   3rd Qu.:12925   3rd Qu.:5050
##  Max.   :31643   Max.   :21836.0   Max.   :21700   Max.   :8124
##
##      Books           Personal          PhD            Terminal
##  Min.   :  96.0   Min.   : 250   Min.   :  8.00   Min.   : 24.0
##  1st Qu.: 470.0   1st Qu.: 850   1st Qu.: 62.00   1st Qu.: 71.0
##  Median : 500.0   Median :1200   Median : 75.00   Median : 82.0
##  Mean   : 549.4   Mean   :1341   Mean   : 72.66   Mean   : 79.7
##  3rd Qu.: 600.0   3rd Qu.:1700   3rd Qu.: 85.00   3rd Qu.: 92.0
##  Max.   :2340.0   Max.   :6800   Max.   :103.00   Max.   :100.0
##
##    S.F.Ratio      perc.alumni        Expend          Grad.Rate
##  Min.   : 2.50   Min.   : 0.00   Min.   : 3186   Min.   : 10.00
##  1st Qu.:11.50   1st Qu.:13.00   1st Qu.: 6751   1st Qu.: 53.00
##  Median :13.60   Median :21.00   Median : 8377   Median : 65.00
##  Mean   :14.09   Mean   :22.74   Mean   : 9660   Mean   : 65.46
##  3rd Qu.:16.50   3rd Qu.:31.00   3rd Qu.:10830   3rd Qu.: 78.00
##  Max.   :39.80   Max.   :64.00   Max.   :56233   Max.   :118.00
##
```

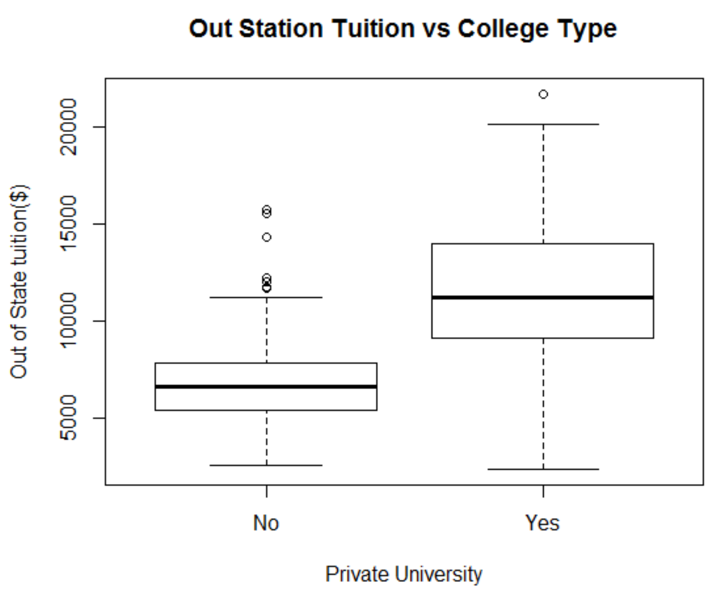
Use the pairs () function to produce a scatterplot matrix of the first ten columns or variables of the data.

```
pairs(college[, 1:10], main='Scatter plot')
```



Use the plot() function to produce side-by-side boxplots of 'Outstate' vs. 'Private'

```
plot(college$Private, college$Outstate, xlab = "Private University", ylab ="Out of Sta
te tuition($)", main = "Out Station Tuition vs College Type")
```



- *Median Out of station Tuition fee is higher for Private colleges compared to Public colleges*

Create a new qualitative variable, called Elite, by binning the Top10perc variable. We are going to divide universities into two groups based on whether or not the proportion of students coming from the top 10% of their high school classes exceed 50%.

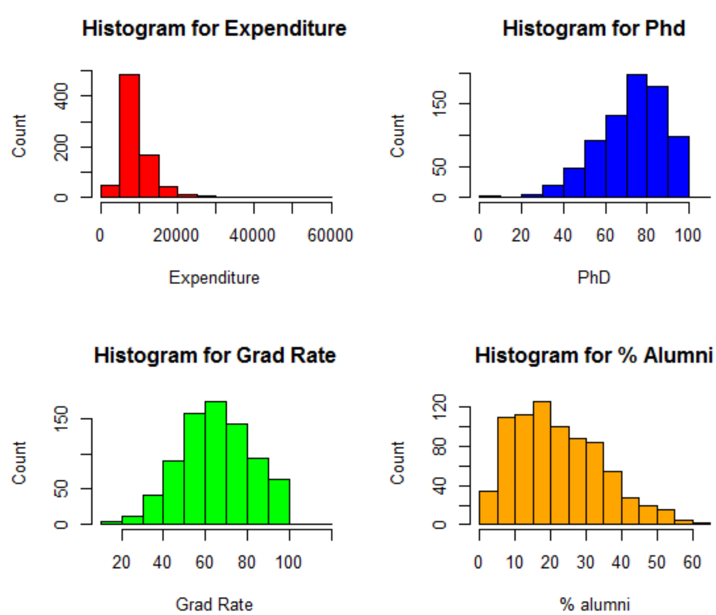
```
college$Elite<-'No'
```

```
college[college$Top10perc > 50,]$Elite<-'Yes'

college$Elite<- as.factor(college$Elite)

summary(college$Elite)
```

```
##   No Yes
## 699  78
```

##Use the hist() function to produce some histograms with differing numbers of bins for a few of the quantitative variables

```
par(mfrow = c(2,2))

hist(college$Expend, col = 'red', xlab = "Expenditure", ylab = "Count",

main='Histogram for Expenditure')

hist(college$PhD, col = 'blue', xlab = "PhD", ylab = "Count",

main='Histogram for Phd')

hist(college$Grad.Rate, col = 'green', xlab = "Grad Rate", ylab = "Count",

main='Histogram for Grad Rate')

hist(college$perc.alumni, col = 'orange', xlab = "% alumni", ylab = "Count",

main='Histogram for % Alumni')
```

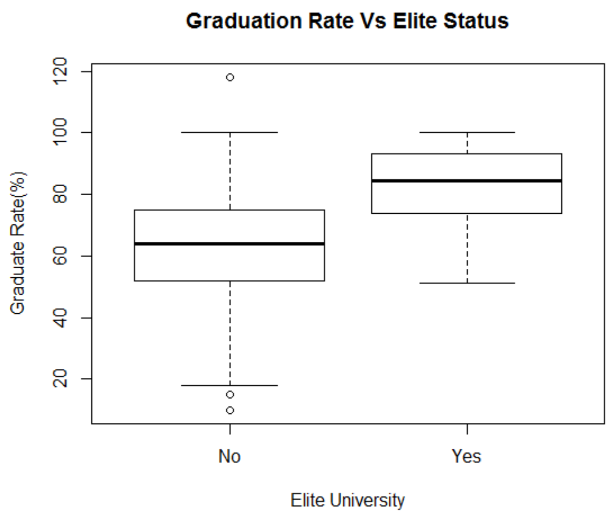


- % Phd Professors metric is negatively skewed
- % Alumni metric is positively skewed

further data exploration

i. If we look at summary of Grad. Rate & PhD columns the max value is greater than 100, this could be an issue with data entry
ii. Median Graduation rate is higher in Elite Colleges compared to non- Elite colleges and there is an outlier in the non-Elite college data

```
plot(college$Elite, college$Grad.Rate, xlab = "Elite University", ylab ="Graduate R
ate(%)", main = "Graduation Rate Vs Elite Status")
```



.

iii. Median Acceptance rate is lower Elite Colleges compared to non- Elite colleges

```
college$acceptance_rate <- (college$Accept/college$Apps)*100

plot(college$Elite, college$acceptance_rate, xlab = "Elite University", ylab ="Accepta
nce Rate(%)", main = "Acceptance Rate Vs Elite Status")
```