Numerical Computing

Root Finding Methods

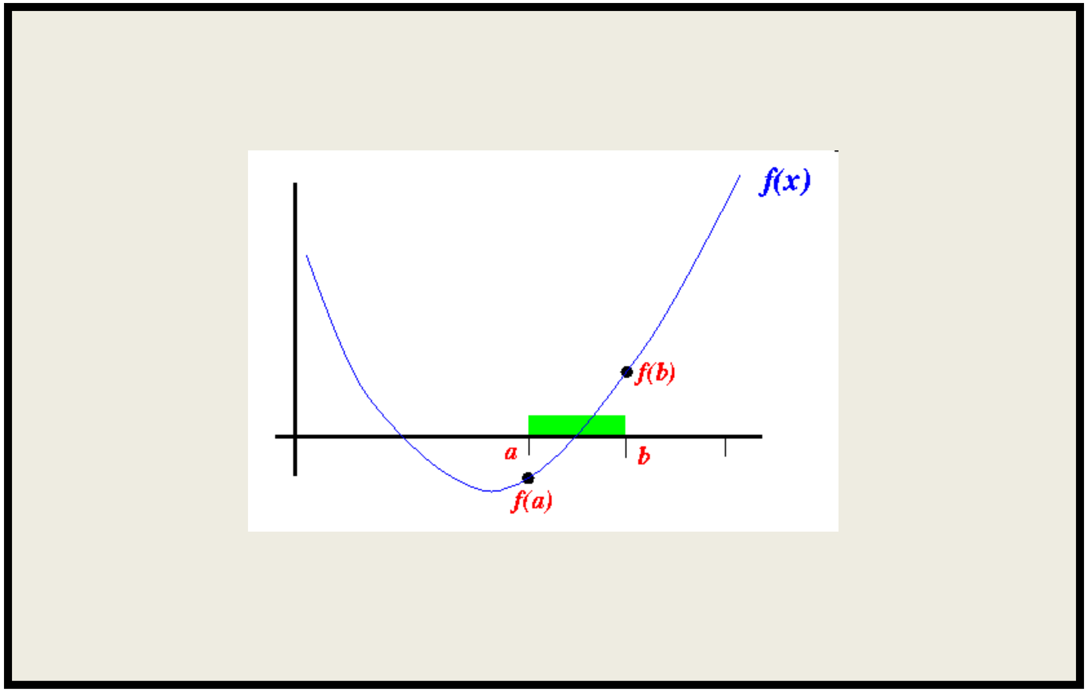
1. *Bisection* Method

2. *Regula Falsi* method

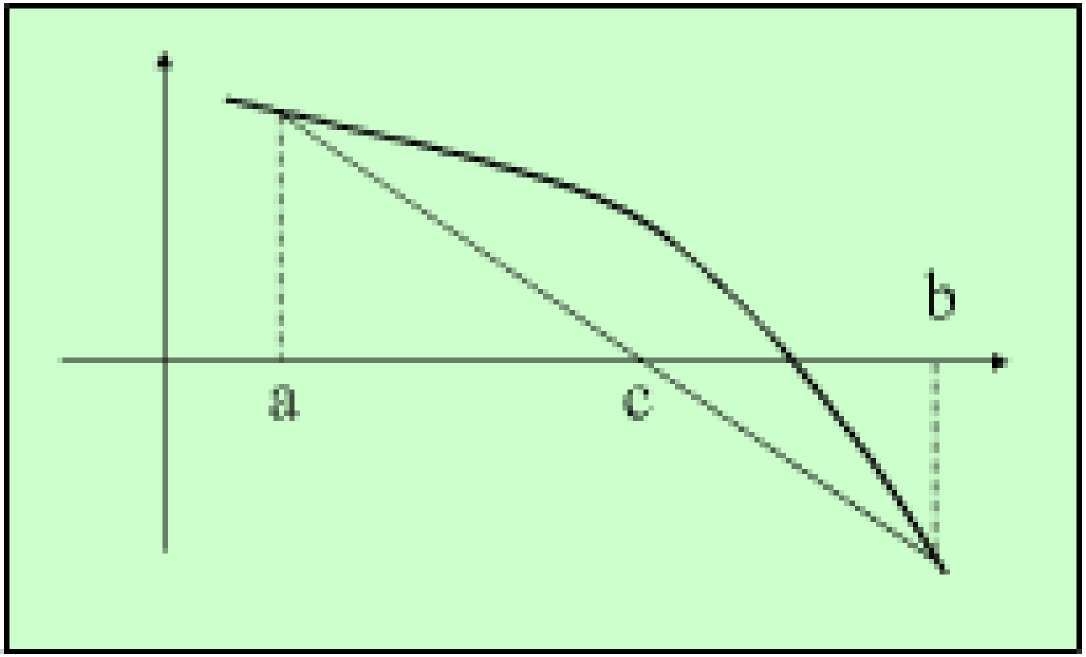
3. *Secant* Method

4. *Newton Raphson* method

Bisection Method



The False-Position Method (Regula-Falsi) Cont..

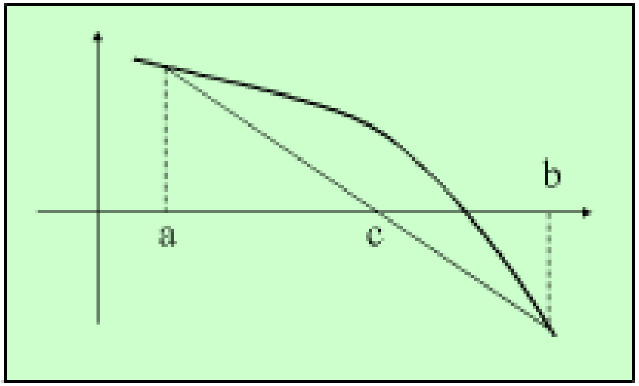


The False-Position Method (Regula-Falsi)

- To refine the bisection method, we can choose a ‘false-position’ instead of the midpoint.

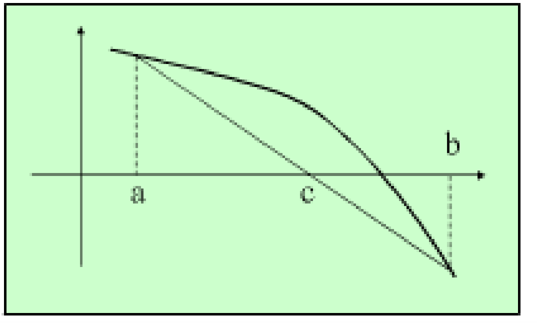
- The false-position is defined as the x position where a line connecting the two boundary points crosses the axis

The False-Position Method (Regula-Falsi) Cont..

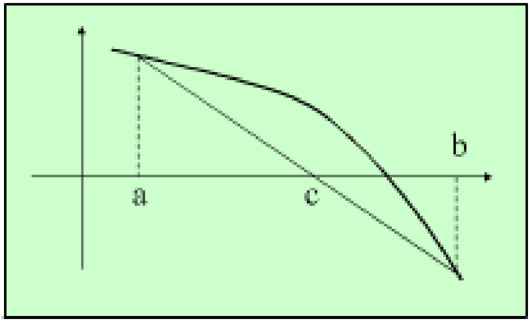


- Choose two points a and b such that f(a) and f(b) are of opposite sign i.e $f(a) \times f(b) < 0$

- So root must lie between these two points.



- The equation of chord joining two points [a, f(a)] and [b, f(b)] is given by:



$$y-y1 = \frac{(y2-y1)}{(x2-x1)} (x-x1) \;........(1)$$

$$y-f(a) = \frac{f(b) - f(a)}{b-a} (x- a) \qquad ....(2)$$

Note: This method consists in replacing the part of the curve between the points [a, f(a)] and [b, f(b)] by means of the chord joining these points.

take the point of intersection of the chord with x axis as an approximation to the root.

→ y=0

$$0-f(a) = \frac{f(b)-f(a)}{b-a}(x-a) \qquad \text{...............(3)}$$

$$x = a - \frac{f(a)}{f(b)-f(a)}(b-a) \qquad \text{..................(4)}$$

Let x1 be the first approximation

$$x1 = \frac{a\,f(b) - bf(a)}{f(b)-f(a)} \qquad \text{...................(5)}$$

- If $f(x1)=0$, then it is the required root, else if $f(x1)$ and $f(a)$ are of opposite signs, then the root must lie between a and x1 and we replace b by x1. Otherwise replace a by x1.

- Note: the procedure is repeated till the root is obtained to the desired accuracy.

Find the real root of the equation x^3-2x-5=0 using Regula falsi method, correct to 3 decimal places.

Let $f(x)=x^3-2x-5$,

Now, f(2)=-1 and f(3)= 16 i.e. a root lies between 2 and 3.

Taking a=2 and b=3

f(a)=-1 and f(b)=16

First approximation,

Let x1 is the first approximation

$$x_1 = \frac{a\,f(b) - b f(a)}{f(b)-f(a)} = \frac{2\times16-3\times-1}{16-(-1)} = \frac{35}{17} = 2.0588$$

$$f(x_1)=f(2.0588)=2.0588^3-2\times2.0588-5= -0.3908$$

Now, $f(x1)=f(2.0588)= -0.3908$, i.e. the root lies between 2.0588 and 3 so, taking $a=2.0588$ and $b= 3$, $f(a)= - 0.3908$, $f(b)= 16$

Second approximation,

$$x2 = \frac{a\,f(b) - bf(a)}{f(b)-f(a)} = \frac{2.0588\times16-3\times -0.3908}{16-(-0.3908)} = 2.0813$$

Repeating this process the successive approximations are:

$x3=2.0862$,

$x4=2.0915$,

$x5=2.0934$,

$x6= 2.0941$,

$x7=2.0943$, hence 2.0943 is the root correct to 3 decimal places

Practice Problem

1. Find the real root of the equation $3x-\cos x-1=0$ using false position method, correct to 2 decimal places.

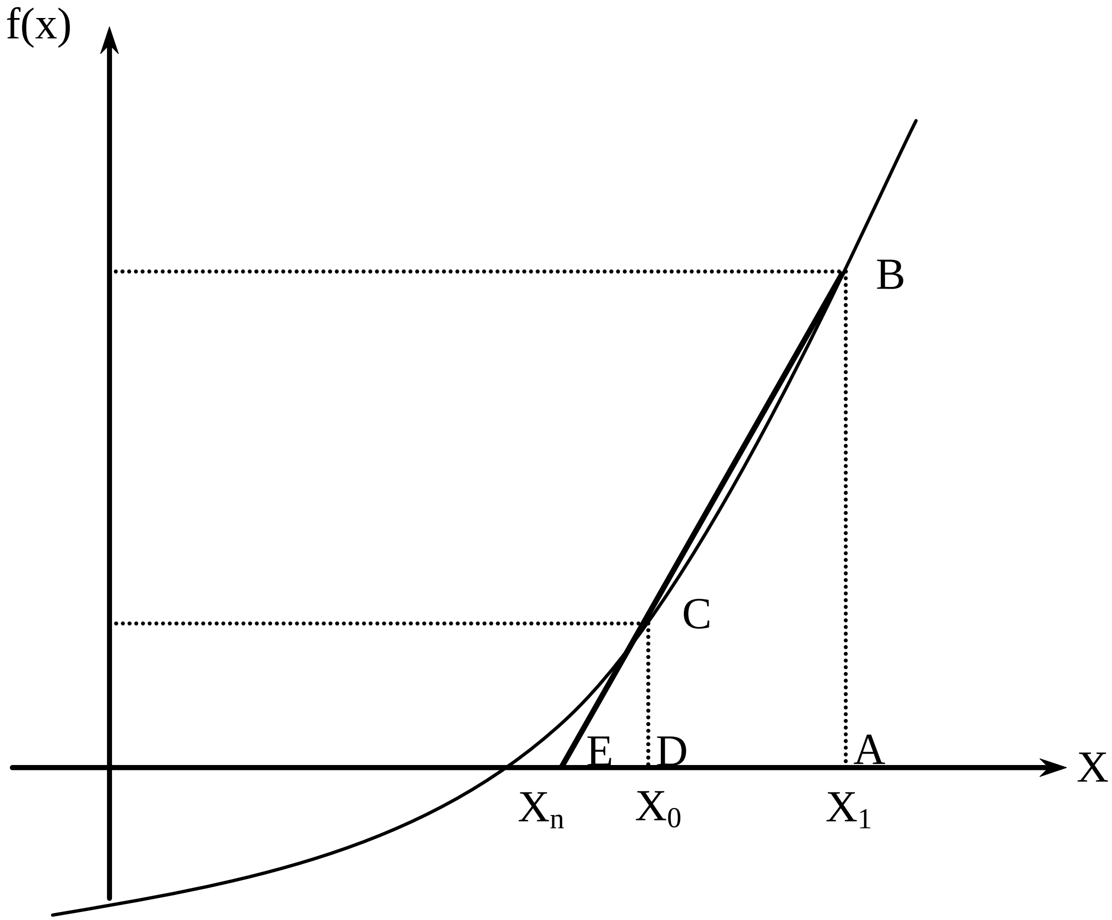
2. Find the real root of the function $f(x)=xe^x-1$ using false position method correct to three decimal places, which lies between 0 and 1.

Secant Method

- This method is an improvement over the method of false position. As it does not require the condition $f(a) \times f(b) < 0$ of that method.

- The graph of the function $y=f(x)$ is approximated by a line but at each iteration two most recent approximations to the root is used to find the next approximation.

- It is not necessary that the interval must contain the root .



f(x)
B
C
E
D
A
X
X_n
X_0
X_1

Secant Method

In the neighborhood of an exact root, we approximate the curve by a straight line.

i.e. $f(X)= a0\ X+a1 =0$ ……………(1)

$X= -a1 /a0$ ……………(2)

Now these constant can be found as follows:

Let X_{n-1} and X_n be any two approximation to the root.

Now represent $f(X_{n-1})= f_{n-1}$ and $f(X_n)=f_n$

Since it satisfies equation 1

$f(X_{n-1}) =a0\ X_{n-1} +a1$

$f_{n-1} = a0\ X_{n-1} +a1$ …………(3)

& $f_n = a0\ X_n +a1$ …………(4)

$f(X_{n-1}) = a0\, X_{n-1} + a1$

$$f_{n-1} = a0\, X_{n-1} + a1 \quad \ldots\ldots\ldots\ldots(3)$$

$$\& \qquad f_n = a0\, X_n + a1 \quad \ldots\ldots\ldots\ldots(4)$$

On solving eq (3) and (4) for a0 and a1

$$f_n - f_{n-1} = a0\,(X_n - X_{n-1})$$

$$a0 = \frac{(f_n - f_{n-1})}{(X_n - X_{n-1})}$$

$a1 = f_n - a0\, X_n$

$$a1 = f_n - \frac{(f_n - f_{n-1})}{(X_n - X_{n-1})} X_n = \frac{f_{n-1} X_n - f_n x_{n-1}}{(X_n - x_{n-1})}$$

So the required approximated root X_{n+1},

$X_{n+1} = -a1 / a0$ ………………. (5)

From equation 4, $a1 = (f_n - a0 X_n)$, place it in eq. (5)

$X_{n+1} = -(f_n - a0 X_n) / a0 = X_n - f_n / a0$

$$a0 = \frac{(f_n - f_{n-1})}{(X_n - X_{n-1})}$$

$$X_{n+1} = X_n - \frac{(X_n - X_{n-1})}{(f_n - f_{n-1})} f_n$$

$$X_{n+1} = \frac{X_{n-1} f_n - X_n f_{n-1}}{f_n - f_{n-1}}, \quad n = 1, 2, 3 \ldots\ldots\ldots$$

A real root of the equation $x^3-5x+1=0$ lies in the interval (0, 1). Perform four iterations of the secant method.

We have, $x0=0$, $x1= 1$

$f(x0)=1$ and $f(x1)=-3$

- By Secant method

$$Xn+1= \frac{Xn-1\ fn - Xn\ fn-1}{fn - fn-1}, \quad n=1,2,3\ldots\ldots\ldots$$

$$x2= \frac{x0f1-x1f0}{f1-f0} = \frac{0\times-3-1\times1}{-3-1} = \frac{-1}{-4} = 0.25$$

$f(x2)= -0.234375$

$$X_{n+1} = \frac{X_{n-1} f_n - X_n f_{n-1}}{f_n - f_{n-1}}, \quad n=1,2,3\ldots\ldots\ldots$$

As, x2= 0.25 and f(x2)= -0.234375

$$x_3 = \frac{x_1 f_2 - x_2 f_1}{f_2 - f_1} = 0.186441$$

f(x3)= 0.074276

$$x_4 = \frac{x_2 f_3 - x_3 f_2}{f_3 - f_2} = 0.201736$$

f(x4)= -0.000470

$$x_5 = \frac{x_3 f_4 - x_4 f_3}{f_4 - f_3} = 0.201640$$

x5=0.2016 is the required approximated root correct to 3 decimal places

Practice problems

1. Find the real root of the equation $x^3-2x-5=0$ using secant method, correct to 2 decimal places.

2. Find the real root of the equation $3x-\cos x-1=0$ using secant method, correct to 2 decimal places.

Suggested books

1. Numerical Methods by **S.R.K Lyenger & R.K. Jain.**

2. Introductory methods of Numerical analysis by **S.S. Sastry**.

Thank you