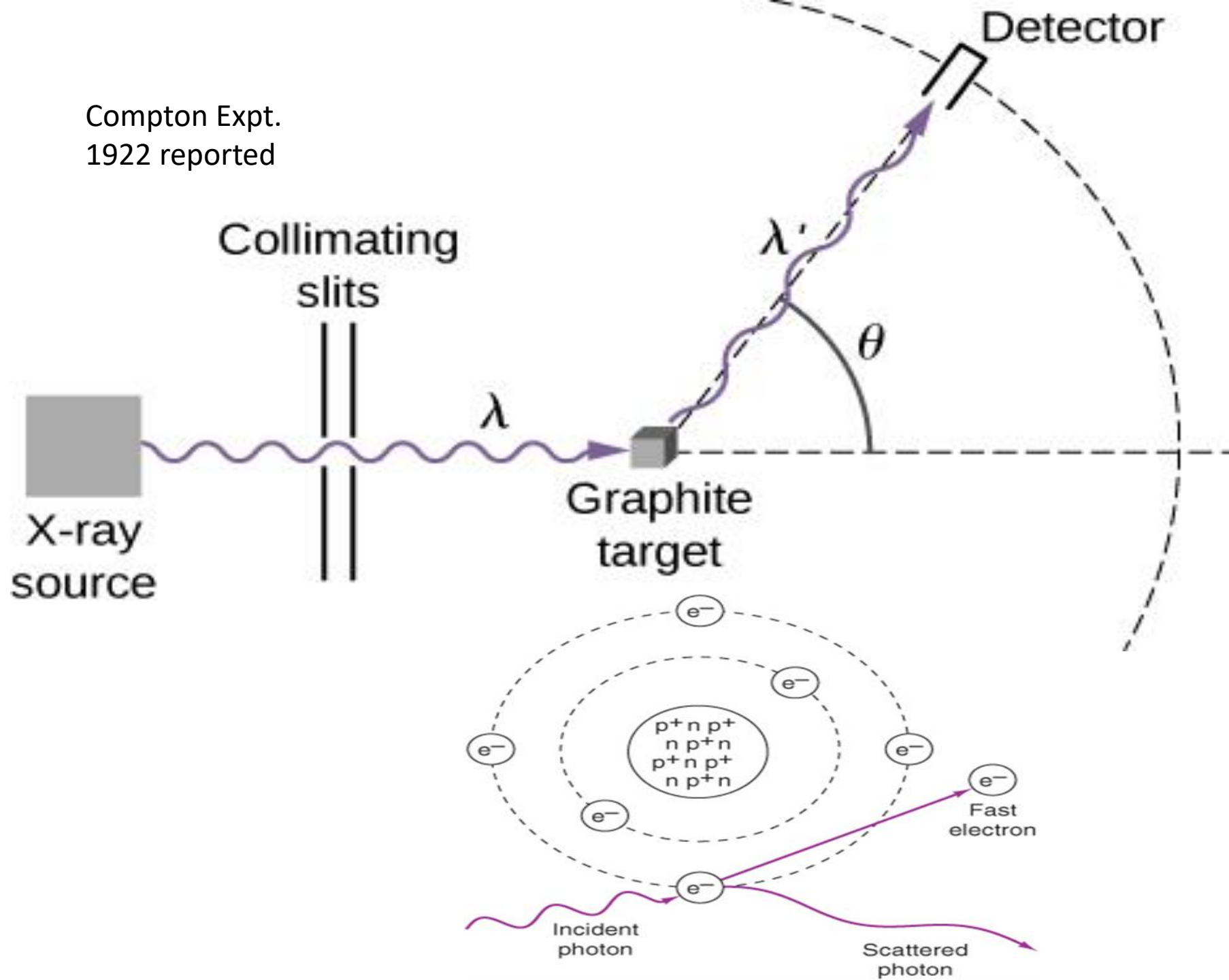
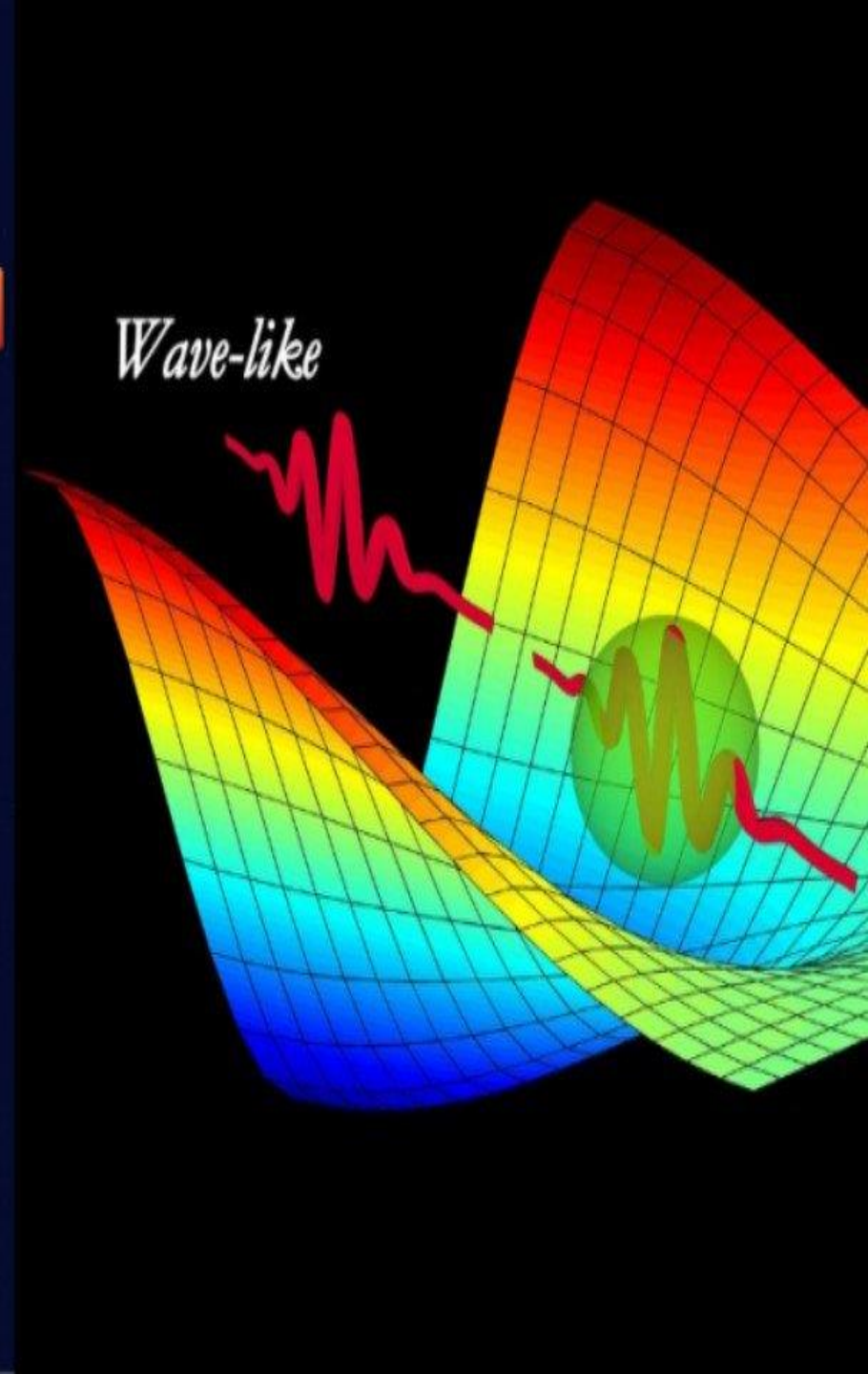
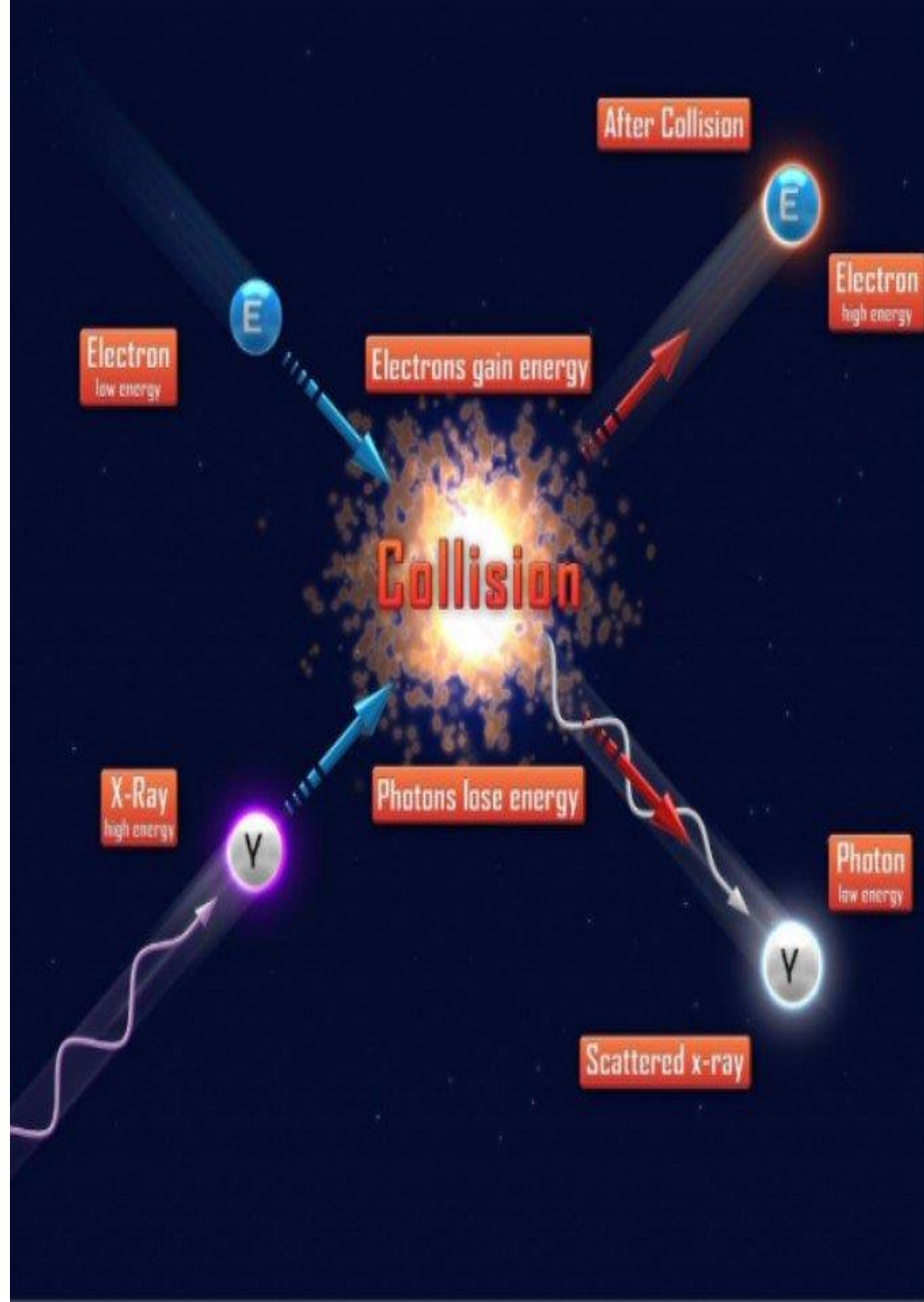


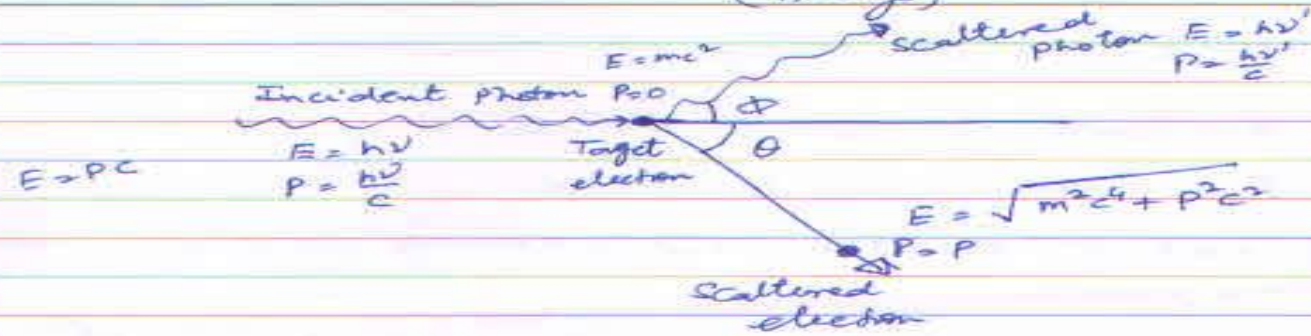
Compton Expt.
1922 reported



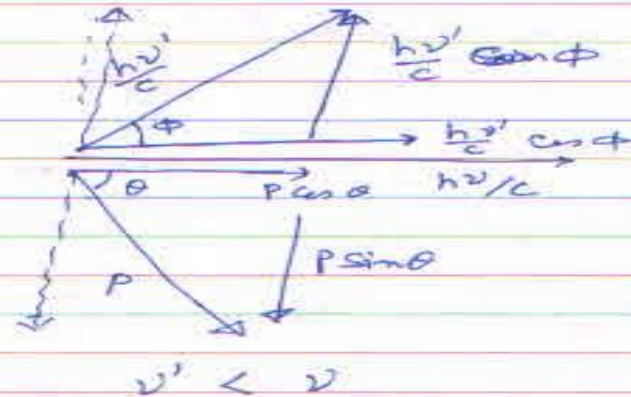


Compton Effect : (i) x-rays were emitted by electrons. ①

(ii) Photon - electron collision (x-rays)



Vector diagram.



$$\nu' < \nu$$

Loss in photon energy = gain in electron energy

$$h\nu - h\nu' = K.E. \quad - (1)$$

Energy - Momentum ^{relation} of photon

$$E = PC \quad - (2)$$

$$\text{Photon momentum } P = \frac{E}{c} = \frac{h\nu}{c} \quad - (3)$$

Law of momentum conservation at the time of collision.

$$\text{in the direction } \frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + P \cos \theta \quad - (4)$$

Perpendicular to the direction.

Initial momentum = Final momentum

$$0 = \frac{h\nu'}{c} \sin \phi - P \sin \theta \quad \text{--- (5)}$$

multiply by 'c' in eqn. (4) & (5), hence

$$P c \cos \theta = h\nu - h\nu' \cos \phi$$

$$P c \sin \theta = h\nu' \sin \phi$$

By squaring each of equation and adding them to eliminate 'θ'.

$$P^2 c^2 = (h\nu)^2 - 2(h\nu)(h\nu') \cos \phi + (h\nu')^2 \quad \text{--- (6)}$$

We know total energy of particle

$$E = K.E + mc^2$$

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$

$$E^2 = (m^2 c^4 + p^2 c^2)$$

$$(K.E + mc^2)^2 = m^2 c^4 + p^2 c^2$$

$$p^2 c^2 = (KE)^2 + 2mc^2(KE) + \cancel{m^2 c^4} - \cancel{m^2 c^4}$$

$$p^2 c^2 = (KE)^2 + 2mc^2(KE) \quad \text{--- (7)}$$

We know from eqn. (1)

$$K.E = h\nu - h\nu'$$

hence

$$p^2 c^2 = (h\nu - h\nu')^2 + 2mc^2(h\nu - h\nu')$$

$$= (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu')$$

--- (7)

Substituting P_C^2 from eq. (6)

(3)

$$(h\nu)^2 - 2(h\nu)(h\nu') \cos\phi + (h\nu')^2$$

$$= (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2$$

$$+ 2mc^2(h\nu - h\nu')$$

$$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')$$

$$- 2(h\nu)(h\nu') \cos\phi$$

$$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\phi)$$

$$mc^2 h(\nu - \nu') = h^2 \nu \nu' (1 - \cos\phi) \quad \text{--- (8)}$$

$$\frac{mc}{h} \left(\frac{\nu}{c} - \frac{\nu'}{c} \right) = \frac{\nu}{c} \cdot \frac{\nu'}{c} (1 - \cos\phi)$$

$$\frac{mc}{h} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = \frac{1 - \cos\phi}{\lambda \lambda'}$$

$$\boxed{\lambda' - \lambda = \frac{h}{mc} (1 - \cos\phi)} \quad \text{--- (9)}$$

Eqn. (9) Derived by A. H. Compton in early 1920. and effects

$\lambda' > \lambda$ or $\nu' < \nu$ is called Compton effect

change in wavelength, called Compton wavelength

$$\lambda_c = \frac{h}{mc}$$

$$\text{at } \cos\phi = 0 \\ \phi = 90^\circ$$

λ_c is independent on λ of incident wave

hence

$$\boxed{\lambda' - \lambda = \lambda_c (1 - \cos \phi)} \quad \text{--- (10)}$$

(4)

$\phi = 180^\circ$ then $\lambda' - \lambda = \text{Maximum}$
or twice

$$2\lambda_c$$

$$\lambda_c = 2.426 \times 10^{-12} \text{ meter}$$

$$= 2.426 \text{ pm}$$

$$\lambda' - \lambda = 2\lambda_c = 2 \times 2.426 = 4.852 \text{ pm}$$

for visible light = 0.01 ~~pm~~
x-rays = .1

Meaning :

x-rays have energy when pass through matter

De - broglie wave : Matter wave

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$E = h\nu$$
$$pc = h\nu$$

$$\lambda = \frac{h}{p}$$

The momentum of particle $p = \gamma m v$

$$\lambda = \frac{h}{\gamma m v}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{relativistic factor}$$

Wave & matter properties can not measure at the same time.

If $v \ll c$, $\gamma \approx 1$

$$\Rightarrow \boxed{\lambda = \frac{h}{mv}}$$

$$p = \sqrt{2mE_k}$$

$$\lambda = \frac{h}{\sqrt{2mE_k}} \quad \text{or} \quad \frac{h}{\sqrt{2mqV}}$$

- Q. Find the de Broglie's wavelength of a
- 46 g mass with a velocity 30 m/sec
 - an electron of $v \sim 10^7$ m/sec.

(a)
$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J.s}}{(0.046 \text{ kg})(30 \text{ m/s})}$$

$$\lambda = 4.8 \times 10^{-34} \text{ meter}$$

\Rightarrow Size of particle \gg Wavelength.

No wave phenomenon will be observed.

(b)
$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J.s}}{(9.1 \times 10^{-31} \text{ kg})(10^7 \text{ m/s})} = 7.3 \times 10^{-11} \text{ m}$$

Radius of hydrogen atom is $5.3 \times 10^{-11} \text{ m}$

eq. to $7.3 \times 10^{-11} \text{ m}$

It will show wave nature

- Q. Find the de Broglie wavelength of (a) an electron whose speed is 1.0×10^8 m/s and (b) $v = 2.0 \times 10^8$ m/s

- Q. The equivalent wavelength of a moving electron is 0.24×10^{-10} meter. What voltage applied between two grids will bring it to rest.