

CS Assignment

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Q1 Given the values

x	$f(x)$
0	2
1	3
2	12
5	147

find the polynomial of the lowest possible degree using Newton's divided difference interpolation.

$$x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 5$$
$$f(x_0) = 2, f(x_1) = 3, f(x_2) = 12, f(x_3) = 147$$

Divided difference table :-

x	$f(x)$	Δ	Δ^2	Δ^3
0	2	$\frac{3-2}{1-0} = 1$	$\frac{9-4}{2-0} = 4$	$\frac{9-4}{5-0} = 1$
1	3	$\frac{12-3}{2-1} = 9$	$\frac{45-9}{5-1} = 9$	
2	12	$\frac{147-12}{5-2} = 45$		
5	147			

So, the Newton divided difference ~~for~~ interpolating polynomial becomes :-

$$P_3(x) = f(x_0) + (x-x_0)f[x_0, x_1] + (x-x_0)(x-x_1)f[x_0, x_1, x_2] + (x-x_0)(x-x_1)(x-x_2)f[x_0, x_1, x_2, x_3]$$
$$= 2 + (x-0) \cdot 1 + (x-0)(x-1) \cdot 4 + (x-0)(x-1)(x-2) \cdot 1$$
$$= 2 + x + 4x(x-1) + x(x-1)(x-2)$$

$$\Rightarrow 2 + x + 4x^2 - 4x + x^3 - 3x^2 + 2x$$

$$\Rightarrow x^3 + x^2 - x + 2$$

$$\text{So, } p_3(x) = x^3 + x^2 - x + 2$$