Electrostatics

Date- 02/03/2021

q_3

q_2

q_1

Q

Test charge

source

Stationary charges $\Rightarrow$ electrostatics

The fundamental problem of the EM theory

$\Rightarrow$ find force due to source charges on test charge Q.

step by step : (i) superposition principle $\Longrightarrow$

(ii) Coulomb law

q_1

force

$\vec{F}_1$ on Q due to q_1

$\vec{F}_2$ . . Q . . q_2

Total force on Q due to all the source charges

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \cdots + \bar{F}_n$$

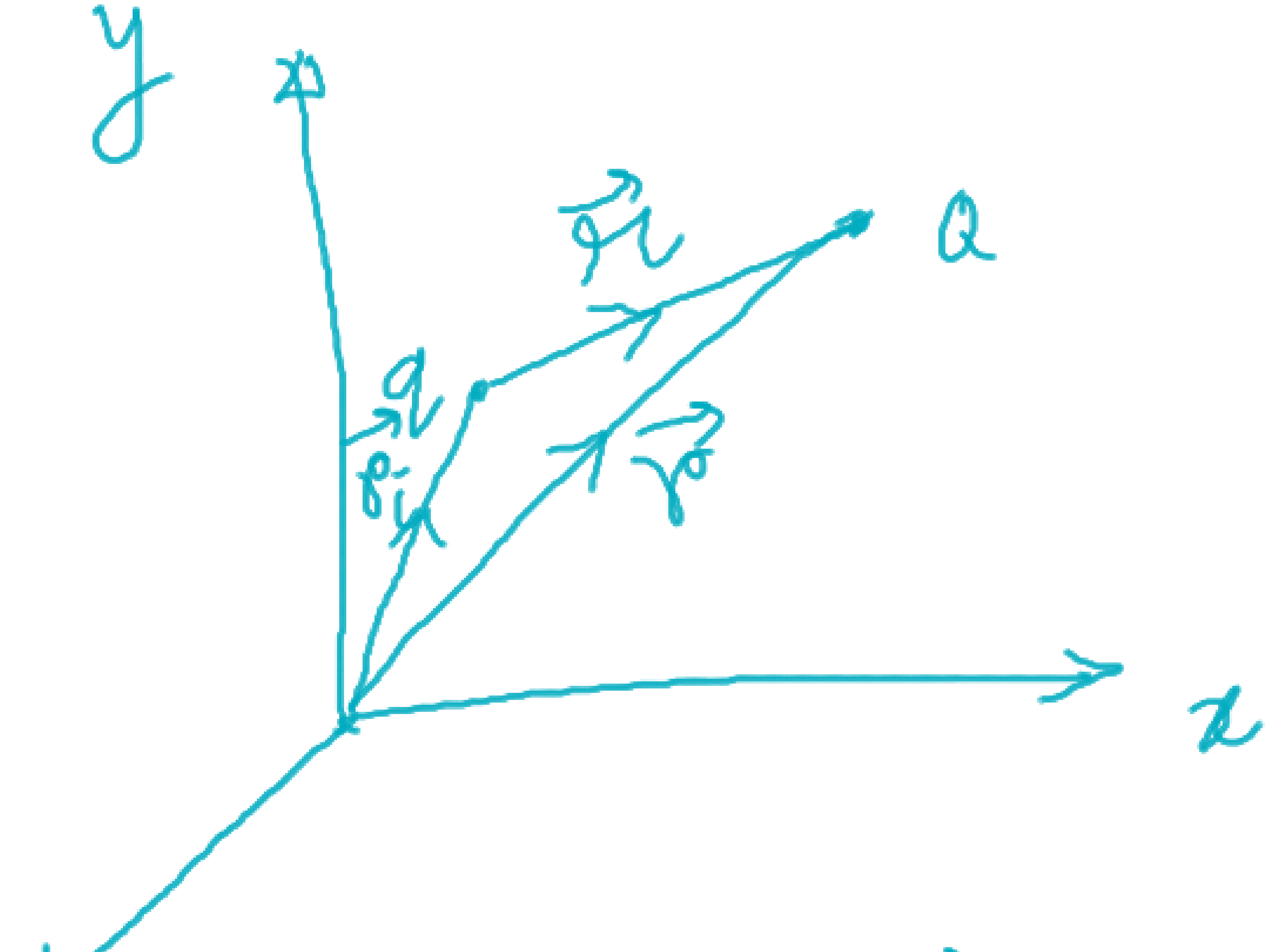
A B C

interaction ~~bed~~ between Q & q

Coulomb law :

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}}$$

ε_0 = free space permitivity = $8.85\times 10^{-12}\ \frac{C^2}{N\cdot m^2}$



$$\vec{\mathscr{R}} = \vec{r} - \vec{r}_i$$

The electric field :

$q_1, q_2, q_3, \ldots, q_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ from a test charge Q

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \ldots$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

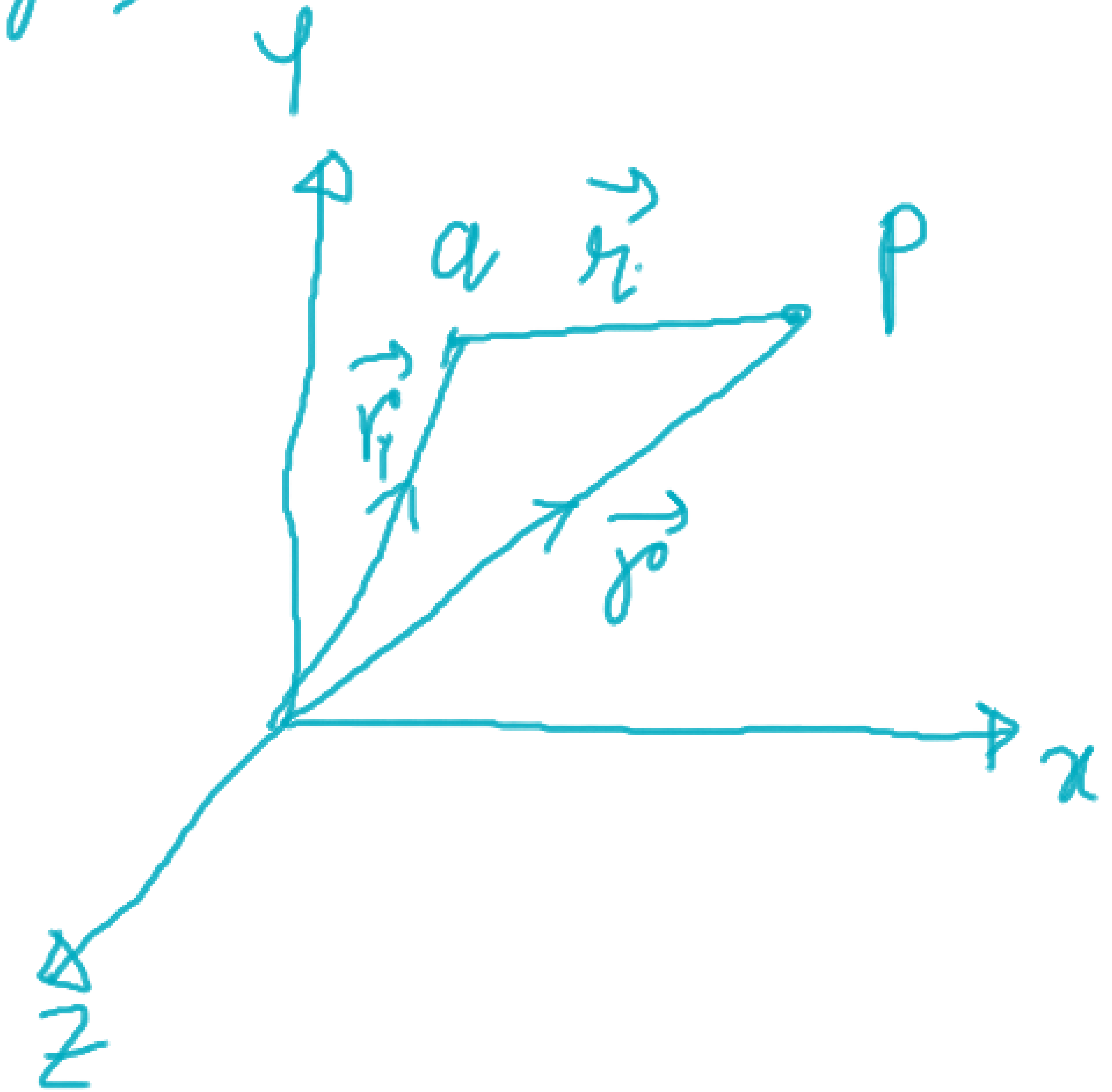
$$= Q\vec{E}$$

where, $\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$ Electric field

$$\boxed{\bar{E} = \frac{F}{Q}}$$

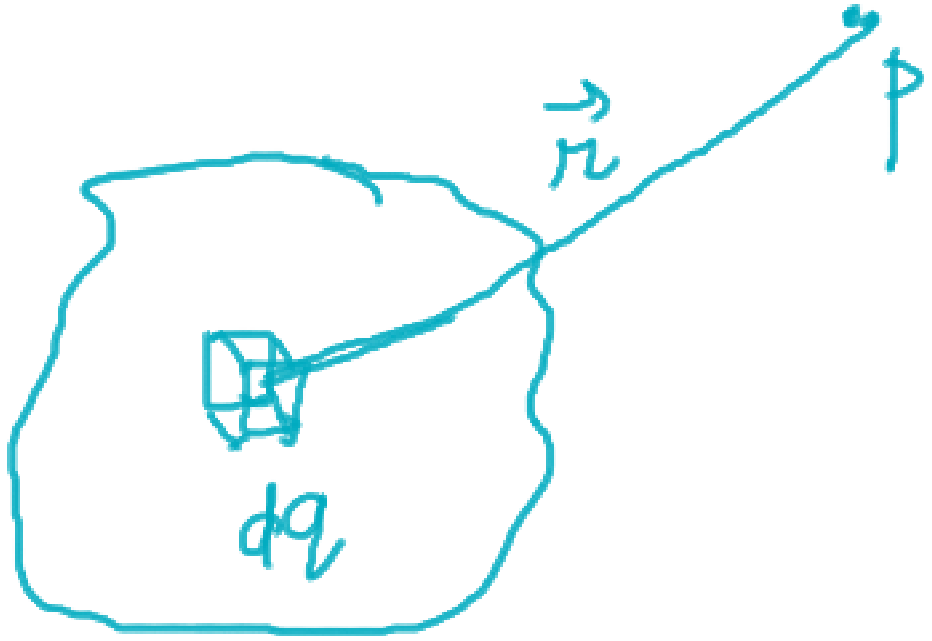
force per unit charge.

$\bar{E}(\bar{r})$



Continuous charge distribution :

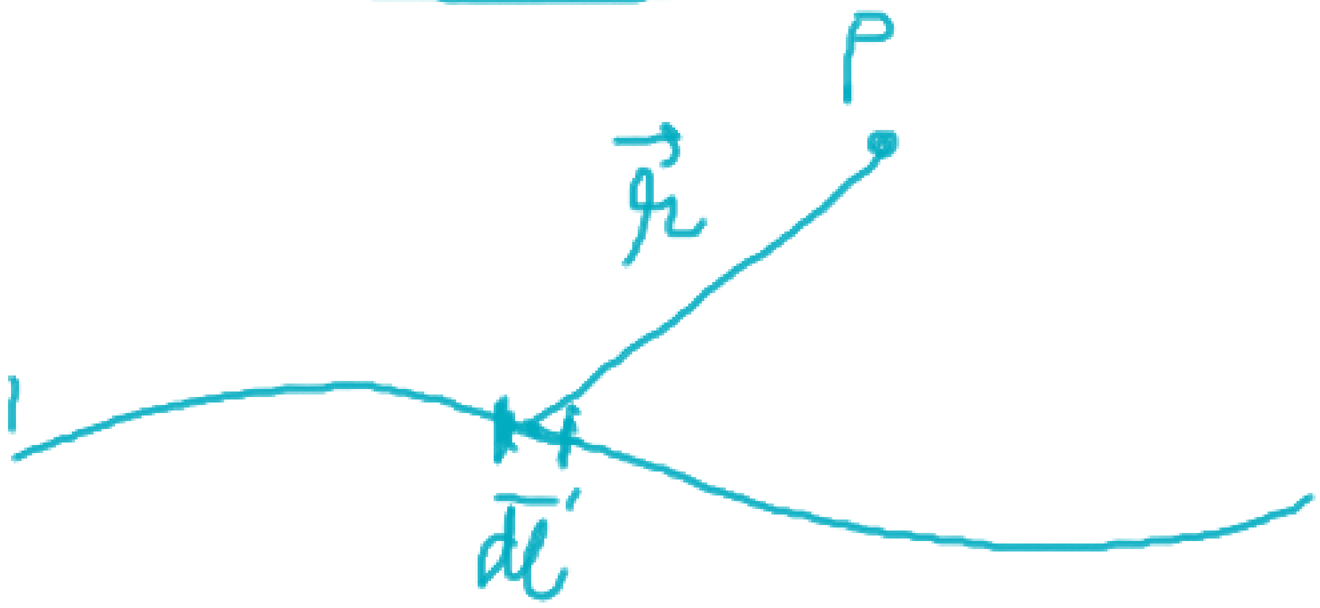
$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r^2}\,\hat{r}\,dq}$$



source charge
continuous distribution

l

(i) Line charge

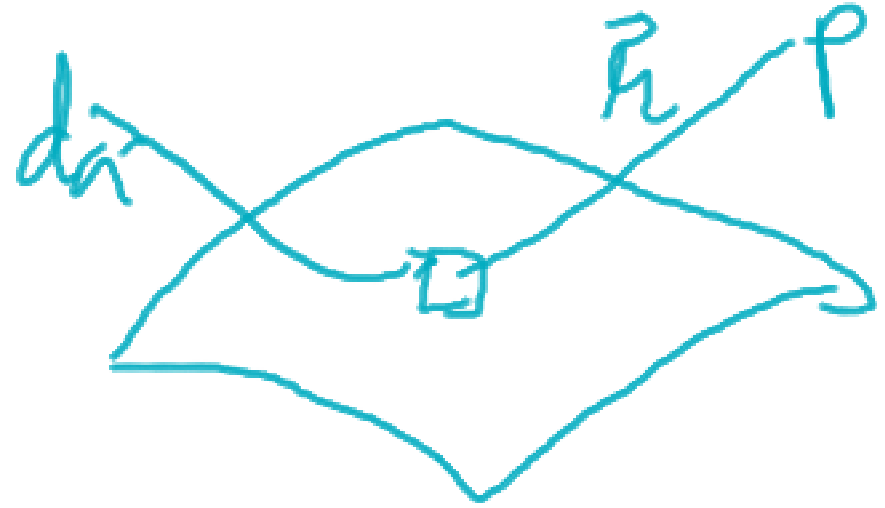


$$\lambda = \frac{Q}{l}$$

$$dq = \lambda\, dl'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r^2}\lambda(\vec{r}')\,\hat{r}\,dl'$$

(ii) Surface charge



$\sigma = \dfrac{q}{A}$

$dq = \sigma \, da'$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{r}\,')}{\mathscr{r}^2} \hat{\mathscr{r}} \, da'$$

(iii) Volume charge :

$\rho =$

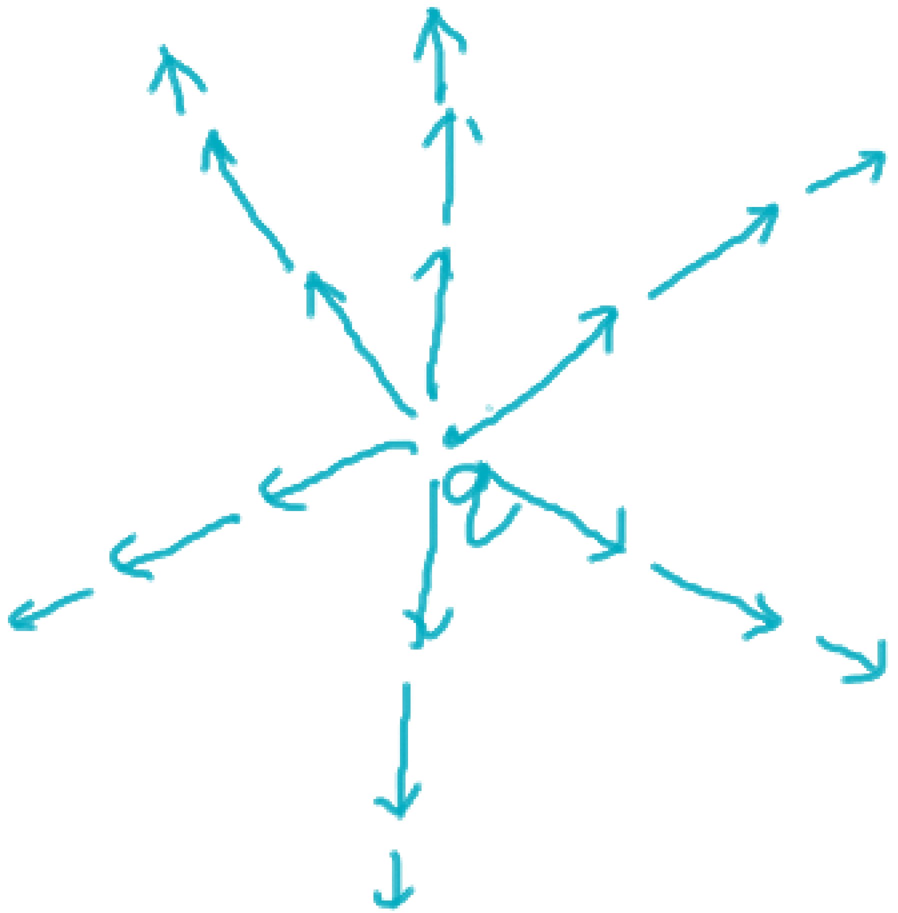
$dq = \rho \, d\tau'$,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}^2} \hat{\mathscr{r}} \, d\tau'$$

Field lines, flux and Gauss law:

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

$$\boxed{E \propto \frac{1}{r^2}}$$



Field strength is given by the density of lines

2D deceptive

$\bar{E} = \frac{n}{2\pi r}$ $E \propto \frac{1}{r}$ actually, $\bar{E} \propto \frac{1}{r^2}$

3D Spherically outward

$E \propto \frac{n}{4\pi r^2}$

$\bar{E} \propto \frac{1}{r^2}$ captures the concept of field lines