BPT-401

Date - 23/03/2021

Divergence theorem,

$$\int_V (\bar{\nabla}\cdot\bar{v})\, d\tau = \oint_S \bar{v}\cdot d\bar{a}$$

dielectric material

✓ The electric field $\vec{E}_1$ due to $\vec{P}$ is same as the field produced by surface bound charges $\sigma_b = \vec{P}\cdot\hat{n}$ and volume bound charges $\rho_b = -\vec{\nabla}\cdot\vec{P}$

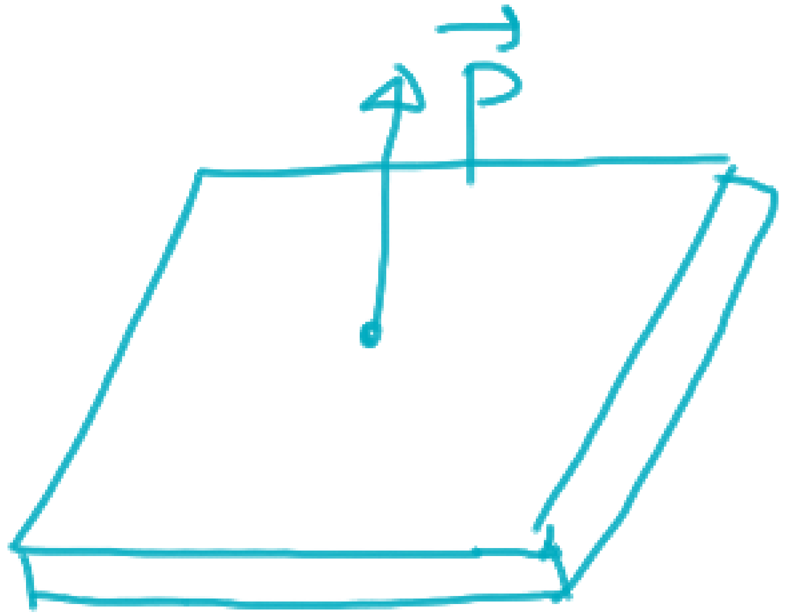
✓ check

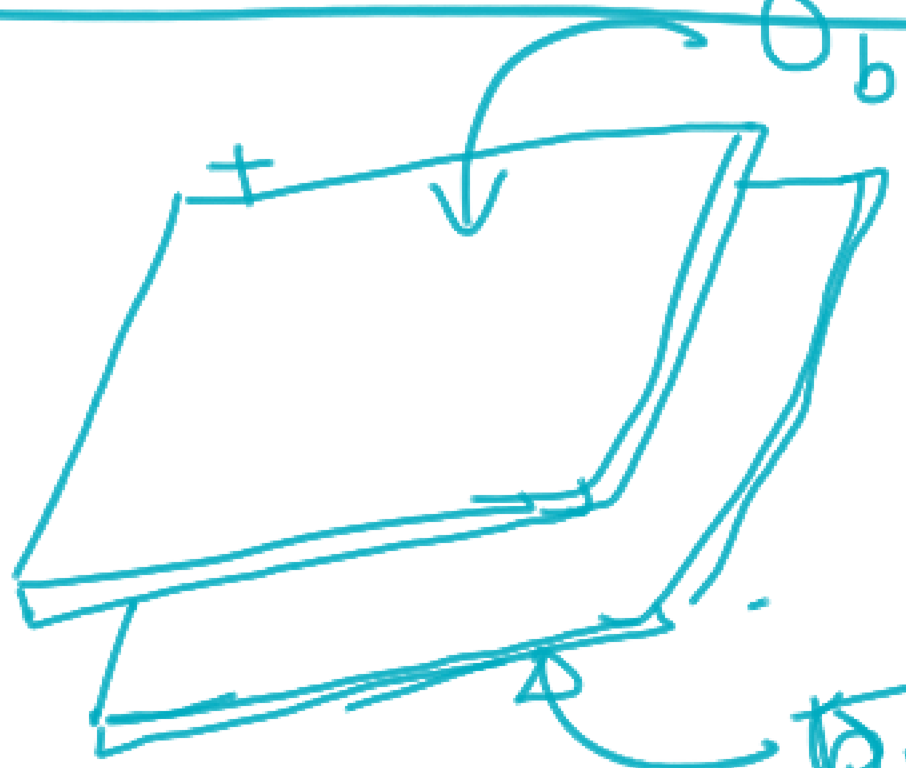
$$\vec{E} = \vec{E}_0 + \vec{E}_1$$

$\vec{E}_1 \rightarrow$ Field due to $\vec{P}$

$\vec{E}_0 \rightarrow$ Field by which you created $\vec{P}$

$\vec{E} \rightarrow$ total field





$\vec{P}$ is the polarisation in the slab

upper surface, $\sigma_b = \hat{n}\cdot\vec{P} = P$, $\hat{n}$ upwards

lower surface, $\sigma_b = \hat{n}\cdot\vec{P} = -P$, $\hat{n}$ downwards

Using Gauss's theorem, electric field

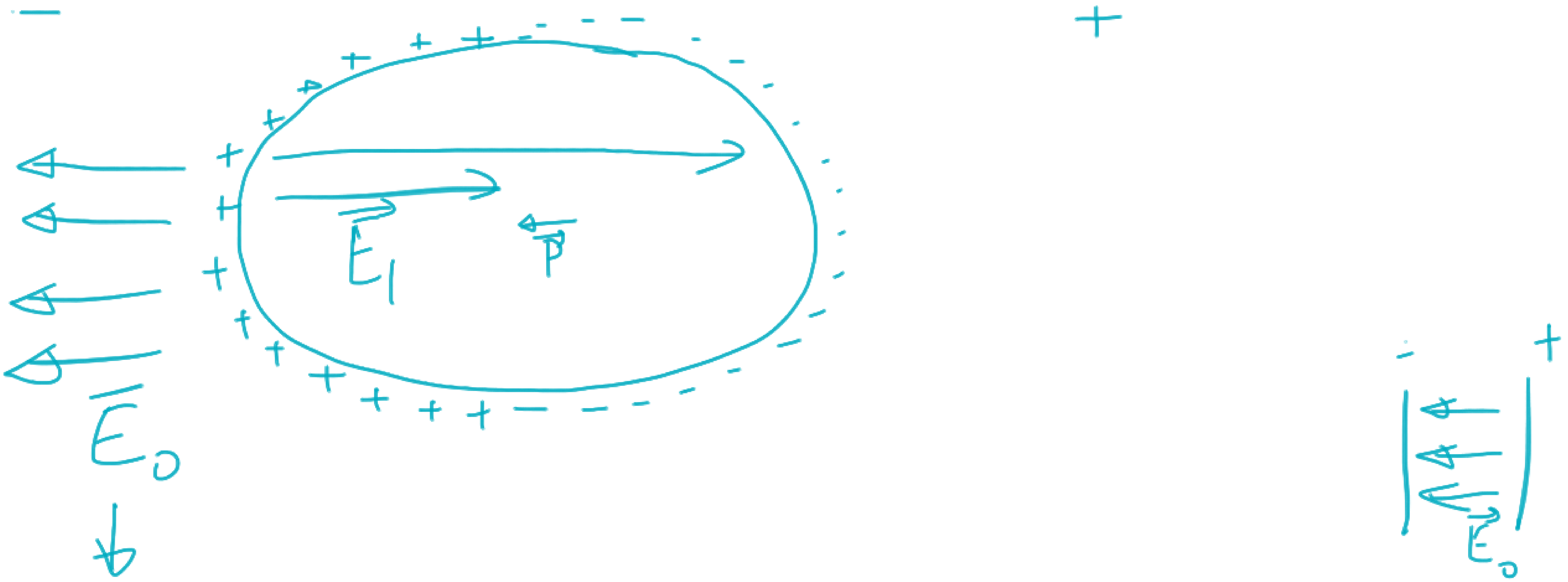
$$\bar{E}_1 = -\frac{|\sigma|}{\varepsilon_0} = -\frac{P}{\varepsilon_0}$$ ✓

total field,

$$\bar{E} = \bar{E}_0 + \bar{E}_1 = \bar{E}_0 - \frac{P}{\varepsilon_0}\hat{z}$$

Applied field

✓ Polarisation occurs due to $\bar{E}_0$

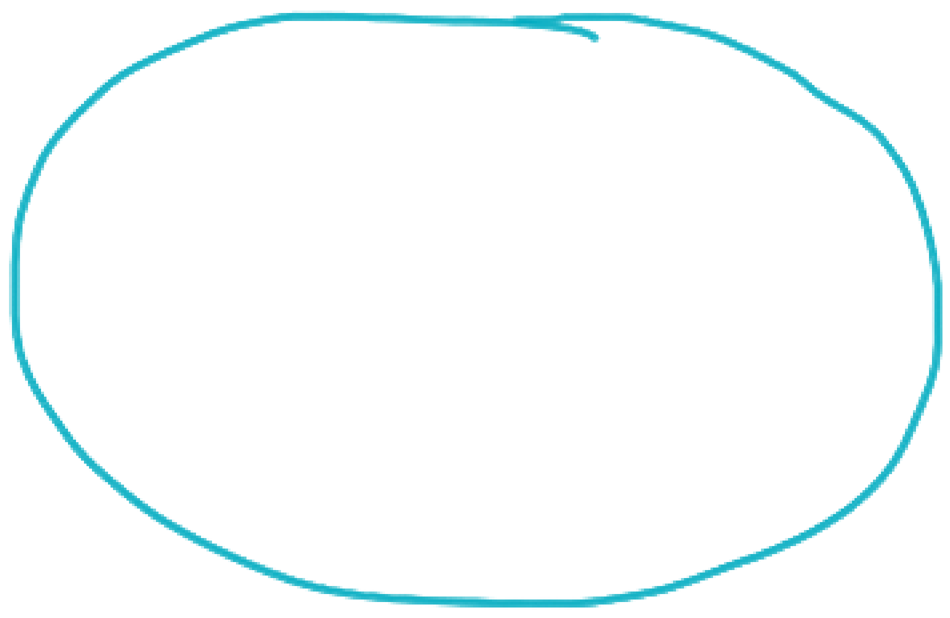


Depolarisation field $\underline{\underline{E_1}}$

Gauss's law in dielectrics :

On the surface, $\sigma_b = \bar{P} \cdot \hat{n}$,

$\Rightarrow$ Volume bound charges $\rho_b = -\bar{\nabla} \cdot \bar{P}$



Total charge density

$$\underbrace{\rho_b}_{\text{bound charge due to polarisation}} \quad \rho = \rho_b + \rho_f \leftarrow \text{free charge}$$

Gauss's law in differential form

$$\bar{E} = \bar{E}_0 + \bar{E}_1$$

$$\varepsilon_0 \vec{\nabla} \cdot \bar{E} = \rho = \rho_b + \rho_f$$

$$\Rightarrow \varepsilon_0 \bar{\nabla} \cdot \bar{E} = \rho_b + \rho_f$$, we know, $\rho_b = -\bar{\nabla} \cdot \vec{P}$

$$\Rightarrow \varepsilon_0 \bar{\nabla} \cdot \bar{E} = -\bar{\nabla} \cdot \vec{P} + \rho_f$$

$$\Rightarrow \bar{\nabla} \cdot (\varepsilon_0 \bar{E} + \vec{P}) = \rho_f$$

Gauss's law in Dielectrics $\boxed{\bar{\nabla} \cdot \vec{D} = \rho_f}$ where, $\vec{D} = \varepsilon_0 \bar{E} + \vec{P}$

↳ electric displacement

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho_f}$$

$\vec{P}$

integral form

$$\oint \vec{D}\cdot d\bar{a} = Q_{enc}$$

$\Downarrow$

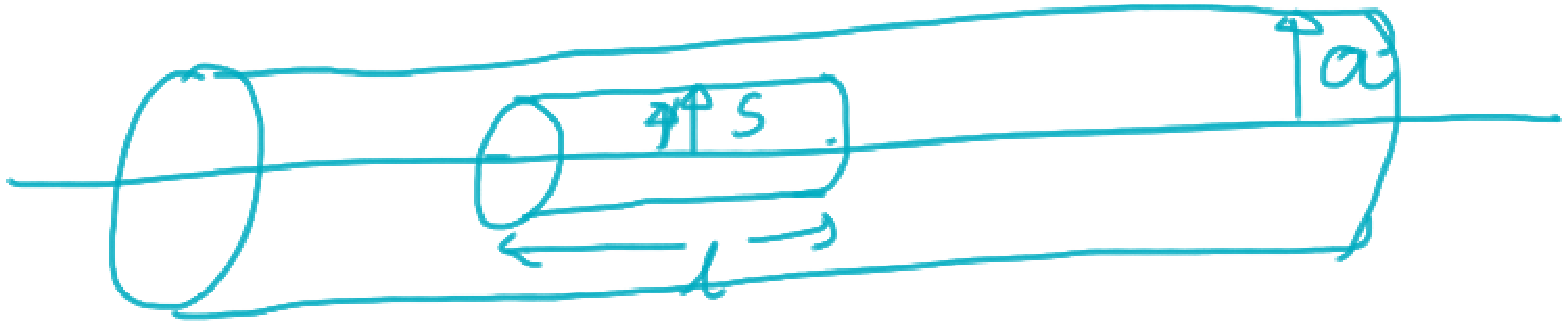
Represents total free charge enclosed in the volume

Example : A long straight wire carrying uniform line charge λ, is surrounded by rubber insulation out to a radius 'a'. Find $\vec{D}$.

Dielectric

λ = charge per unit length

$= \frac{q}{L}$,



Gauss's law,

$$\oint \vec{D} \cdot d\bar{a} = Q_{free}^{enc}.$$

$$Q_{free}^{enc} = \lambda l$$

$$D(2\pi s l) = \lambda l$$

$$\vec{D} = \frac{\lambda}{2\pi s}\vec{s}$$

→ electri displacement

$\vec{P}$ = value we do not know

Inside the rubber

outside

$$\vec{D} = \varepsilon_0 \bar{E} + \bar{P} = \frac{\lambda}{2\pi s}\hat{s}$$

$$\bar{P} = 0$$

$$\vec{D} = \varepsilon_0 \bar{E} = \frac{\lambda}{2\pi s}\hat{s}$$