Paramagnetic System.

$$dU = TdS + Bdm$$

Paramagnetism: M = Magnetization.

$$m = \frac{M}{V} = \text{magnetic moment}$$

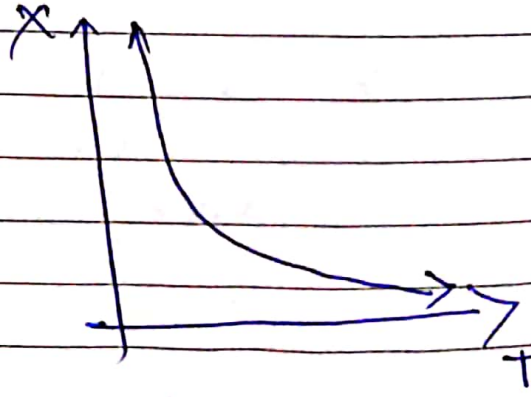
$$B = \mu_0 (H + m) \approx \mu_0 H$$

μ_0 = vacuum permeability.

$$\chi = \lim_{H \to 0} \frac{M}{H} = \frac{\mu_0 M}{B}$$

$$\boxed{\chi \propto \frac{1}{T}}$$

Curie's Law.



$$\left(\frac{\partial \chi}{\partial T}\right)_B < 0$$

Thermodynamics of magnetic system.

P V T

B M T

B = Magnetic Induction / Flux

M = Magnetization.

(1) $dU = TdS + BdM$

$U = U(S, M)$

(2) $H = U - MB$

$dH = TdS - MdB \Rightarrow H = H(S, B)$

(3) $F = U - TS$

$\Rightarrow dF = -SdT + BdM \Rightarrow F = F(T, M)$

4. $\Rightarrow G = U - TS - MB$

$\Rightarrow dG = -SdT - MdB \Rightarrow G = G(B, T)$

From (1)

$\Rightarrow T = \left(\frac{\partial U}{\partial S}\right)_M$ $\quad B = \left(\frac{\partial U}{\partial M}\right)_S$ Maxwell Relations

$\left(\frac{\partial T}{\partial M}\right)_S = \left(\frac{\partial B}{\partial S}\right)_M$

2021

Heat Capacity:

$$C_M = T\left(\frac{\partial S}{\partial T}\right)_M = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_M$$

$$C_B = T\left(\frac{\partial S}{\partial T}\right)_B = -T\left(\frac{\partial^2 G}{\partial T^2}\right)_B$$

Magnetic Susceptibility:

$$\chi_T = \left(\frac{\partial M}{\partial B}\right)_T = -\left(\frac{\partial^2 G}{\partial B^2}\right)_T$$

$$\chi_S = \left(\frac{\partial M}{\partial B}\right)_S = -\left(\frac{\partial^2 H}{\partial B^2}\right)_S$$

$$\alpha_B = \left(\frac{\partial M}{\partial T}\right)_B = \left(\frac{\partial^2 G}{\partial T \partial B}\right)$$

Show

$$\frac{C_B}{C_M} = \frac{\chi_T}{\chi_S}$$

$$\chi_T (C_B - C_M) = T\alpha_B^2$$

$$C_B (\chi_T - \chi_S) = T\alpha_B^2$$

Adiabatic Demagnetization:

Liquid Helium boiling ~~point~~

for Temp $< 4°K$ $\sim 0.7°K$

$$dQ = dU - B\,dM$$

$$\Rightarrow TdS = dU - B\,dM.$$

Maxwell's

$$\Rightarrow \left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial M}{\partial T}\right)_B \qquad \left(\frac{\partial T}{\partial B}\right)_S = -\left(\frac{\partial M}{\partial S}\right)_B$$

G H

$$\left(\frac{\partial T}{\partial B}\right)_S = -\left(\frac{\partial M}{\partial S}\right)_B = -\frac{\left(\partial M/\partial T\right)_B}{\left(\frac{\partial S}{\partial T}\right)_B} = -\frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B$$

Integrating we have

$$T_f - T_i = \Delta T = -\int_{B_i}^{B_f} \frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B dB.$$

$$\left(\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B .$$

For Paramagnetic $M \sim B$

$$\simeq \chi \, VB$$

$$\simeq \frac{C}{T} VB .$$

$\chi \sim \frac{1}{T}$ (Curie's Law)

$$\Rightarrow \left(\frac{\partial M}{\partial T}\right)_B = -\frac{C V B}{T^2}$$

$$\Rightarrow \left(\frac{\partial T}{\partial B}\right)_S = +\frac{T}{C_B}\frac{C V B}{T^2}$$

$$\Rightarrow \int_{T_i}^{T_f} T\,dT = \int_{B_0}^{0} \left(\frac{CV}{C_B}\right) B\,dB$$

$$\Rightarrow \left(T_f^2 - T_i^2\right) = -\left(\frac{CV}{C_B}\right) B_0^2$$

$$\Rightarrow \underbrace{\left(T_f - T_i\right)}_{\Delta T} = -\left(\frac{CV}{C_B}\right)\frac{B_0^2}{2\,T_{av}}$$

$$T_{av} = \frac{T_f + T_i}{2}$$