BPT-201 (semester II)
Topic: Blackbody Radiation-part 7
(Planck's Law)

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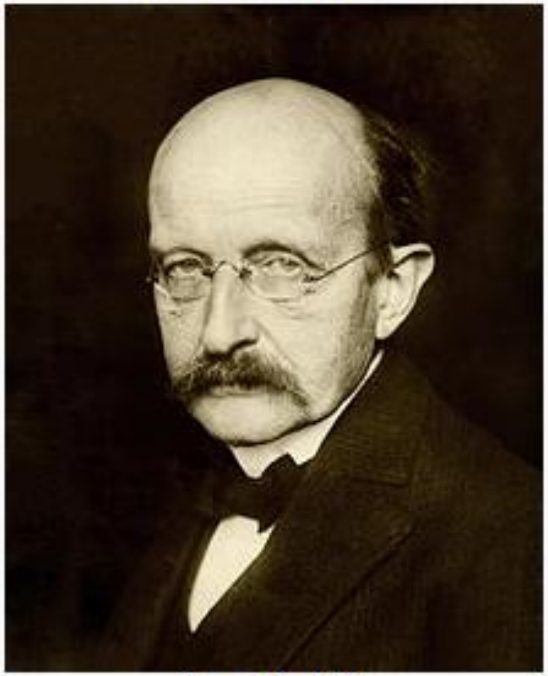
Max Karl Planck



Planck in 1933

Born	Max Karl Ernst Ludwig Planck 23 April 1858 Kiel, Duchy of Holstein
Died	4 October 1947 (aged 89) Göttingen, Lower Saxony, Germany
Nationality	German
Alma mater	Ludwig Maximilian University of Munich
Known for	Planck constant Planck postulate Planck's law of black body radiation Fokker–Planck equation Nernst–Planck equation Third law of thermodynamics
Spouse(s)	Marie Merck (m. 1887; died 1909) Marga von Hösslin (m. 1911)
Children	5
Awards	Nobel Prize in Physics (1918) Foreign Associate of the National Academy of Sciences (1926) Lorentz Medal (1927) Copley Medal (1929) Max Planck Medal (1929) Goethe Prize (1945)

Scientific career

Fields	Physics
Institutions	University of Kiel University of Göttingen Kaiser Wilhelm Society
Thesis	*On the Second Principles of Mechanical Heat Theory* (1879)
Doctoral advisor	Alexander von Brill Gustav Kirchhoff Hermann von Helmholtz
Doctoral students	Erich Kretschmann Gustav Ludwig Hertz Julius Edgar Lilienfeld Max Abraham Max von Laue Moritz Schlick Walter Schottky Walther Bothe Walther Meissner Richard Becker
Other notable students	Wolfgang Köhler Lise Meitner

Signature

Taken From :https://en.wikipedia.org/wiki/Max_Planck

Planck's Law

- In 1900, Max Planck was first to explain the complete blackbody radiation behavior.

- His theory came before the Rayleigh Jeans Law, but unfortunately it was not appreciated very much at that time because Rayleigh Jeans law was based on well accepted classical concept that energy has continuous value and Planck's came with a new concept (that means quantized value of energy) which was not discussed before.

- At that time there was no explanation for assuming energy, corresponding to particular wavelength, to be quantized rather than being continuous.

- Planck's law was accepted after Einstein accepted it and applied quantization of energy to explain specific heat of materials at low temperature in 1905.

- When Planck gave his radiation law, the quantum theory of radiation was not known. It was Planck's concept of quantization of radiation energy which led the path for quantum mechanism.

- At his time there were many challenges because many common understanding of the process through which atoms emit and accept the electromagnetic waves was not known.

- The model of Niels Bohr came in 1913. He reached to his concept by incorporating Planck's ideas of quantization and Einstein's finding that light consists of photons whose energy is proportional to their frequency, into the classical mechanics description of the atom.

- Hence Planck had to think of his own process for interaction of energy and atom. He came with the assumption of resonator of molecular size.

- Planck used the similar concept as Rayleigh Jeans used that means concept of standing waves and calculation of possible number of modes of vibration within certain frequency range.

- The difference between two theories was that Planck did not use the concept of equipartition of energy.

- He assumed that in the enclosure along with radiation and ideal gas molecules there exists resonators which facilitate the energy transfer between radiation and molecules.

- These resonators on collision with gas molecules can exchange their energy totally or partially with gas molecules to establish the equilibrium.

- These resonators are electric dipoles having motion along a fixed axis and having a stationary center of mass.

- He assumed these resonators can emit energy only when the energy absorbed has a certain minimum value ε

- Thus the radiation of energy ε can only be obtained from the resonators having energy content ε, 2ε, 3ε, 4ε, 5ε ....

Computation of mean energy of these resonators

- The mean energy of these resonators can be calculated using the Maxwell's classical formula.
- Which says that the probability of a resonator having energy E is $e^{-E/kT}$
- So the mean energy of resonator will be

$$\bar{\epsilon} = \frac{\sum_0^\infty n \in e^{-n\epsilon/kT}}{\sum_0^\infty e^{-n\epsilon/kT}} = \frac{\sum_0^\infty n \in e^{-n\beta\epsilon}}{\sum_0^\infty e^{-n\beta\epsilon}} = \frac{d}{d\beta} ln\left(\sum_0^\infty e^{-n\beta\epsilon}\right) = \frac{d}{d\beta} ln \frac{1}{1 - e^{-\beta\epsilon}}$$

$$= \frac{\in e^{-\beta\epsilon}}{1 - e^{-\beta\epsilon}} = \frac{\in}{e^{\beta\epsilon} - 1} = \frac{\in}{e^{\epsilon/kT} - 1}$$

- So the mean energy is very much different from kT as considered in Rayleigh-Jean Law on the basis of equipartition energy

- Planck considered that $\varepsilon = h\nu$ where 'h' is Planck's constant and hence the mean energy of resonator can be given as

$$\bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

- For estimating the number of modes of vibration in the frequency range ν to $\nu + d\nu$ he considered the concept of standing wave. It is similar as discussed in case of Rayleigh Jeans law

- So considering that number of modes of vibration between ν to $\nu + d\nu$ is $8\pi\nu^2 d\nu/c^3$ we get Planck's Law as

$$u_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

- or in terms of λ it is given as

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

- This expression gives the exact form of experimental results.

- The limiting cases of this expression gives the Wien's and Rayleigh-Jean's expression. Let us get these two laws from Planck's Law

- i) Wien's Law: for shorter wavelength and low temperature λT is small so we get so

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}}-1} = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}}}$$

when $e^{hc/\lambda kT} \gg 1$ at small λ and low T

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}}} = C_1\lambda^{-5}e^{\frac{-C_2}{\lambda T}}$$

where $C_1 = 8\pi hc$ and $C_2 = \frac{hc}{k}$ are constants

- This is Wien's expression

- ii) Rayleigh-Jeans Law: for long wavelength and high temperature we will get $u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}}-1} = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\left(1+\frac{hc}{\lambda kT}+\cdots\right)-1}$

$$when\ e^{hc/\lambda kT} \ll 1\ at\ long\ \lambda\ and\ high\ T$$

- or $u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\frac{hc}{\lambda kT}} = \frac{8\pi kT}{\lambda^4}d\lambda$ This is Rayleigh-Jeans Law.

- iii) Wien's Displacement Law: To get the wavelength with maximum energy, method of maxima-minima will be adopted, so by differentiating the equation we will get

$$\frac{5\times 8\pi hc}{\lambda_m^6}\frac{1}{e^{\frac{hc}{\lambda_m kT}}-1} + \frac{8\pi hc}{\lambda_m^5}\frac{e^{\frac{hc}{\lambda_m kT}}}{e^{\frac{hc}{\lambda_m kT}}-1}\frac{hc}{\lambda_m^2 kT} = 0$$

- The solution of this equation gives us $\lambda_m T$=constant