CS ASSIGNMENT

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Q1 Find all the eigen values of the matrix using Jacobi's method. Iterate till off elements in magnitude are less than 0.0005

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Here, largest non-diagonal element $= a_{12} = a_{21} = a_{23} = a_{32} = 2$

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2 \times 2}{3-3} = \infty$$

$$2\theta = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Then, $B_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$B_1 = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta \quad B_1^{-1} = \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $D_1 = B_1^{-1} A B_1$

$$D_1 = \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 5 & 0 & -2.121 \\ 0 & 1 & 0.707 \\ -2.121 & 0.707 & 3 \end{bmatrix}$$

Now, the largest non-diagonal element is $a_{32} = a_{23} = 0.707$

$$\tan 2\theta = \frac{2 a_{32}}{a_{33} - a_{22}} = \frac{2 \times 0.707}{3 - 1}$$

$\theta = 0.307$

Then,

$$B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.953 & -0.302 \\ 0 & 0.302 & 0.953 \end{bmatrix}$$

& $B_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.953 & 0.302 \\ 0 & -0.302 & 0.953 \end{bmatrix}$

The 2nd Iteration gives:-

$D_2 = B_2^{-1} A B_2$

$$D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.953 & 0.302 \\ 0 & -0.302 & 0.953 \end{bmatrix} \begin{bmatrix} 5 & 0 & -2.121 \\ 0 & 1 & 0.707 \\ -2.121 & 0.707 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.953 & -0.302 \\ 0 & 0.302 & 0.953 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 6.345 & -0.379 & 0 \\ -0.379 & 1 & -0.597 \\ 0 & -0.597 & 1.655 \end{bmatrix}$$

Similarly, on 3rd iteration :-

$$D_3 = \begin{bmatrix} 6.345 & -0.193 & 0.326 \\ -0.193 & 2.008 & 0 \\ 0.326 & 0 & 0.646 \end{bmatrix}$$

On 4th iteration :-

$$D_4 = \begin{bmatrix} 6.364 & 0.193 & 0 \\ 0.193 & 2.008 & 0.011 \\ 0 & 0.011 & 0.628 \end{bmatrix}$$

On 5th iteration :-

$$D_5 = \begin{bmatrix} 6.372 & 0 & 0 \\ 0 & 2 & 0.011 \\ 0 & 0.011 & 0.628 \end{bmatrix}$$

On 6th iteration :-

$$D_6 = \begin{bmatrix} 6.372 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0.628 \end{bmatrix}$$

Hence, the eigen values of the given matrix is 6.372, 2 & 0.628.