BPT-201 (semester II)
Topic: Blackbody Radiation-part 2
(Kirchhoff's Law)

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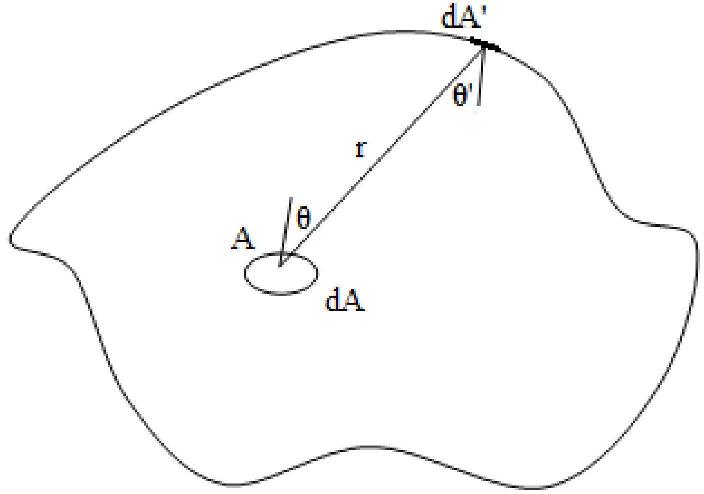
Prof of Kirchhoff's Law

- Let us imagine an enclosure which is thermally insulated from surrounding and opaque to all wavelength of radiation.
- Let the size and shape of enclosure is not well defined
- Let the temperature of the enclosure be T
- The enclosure is filled with thermal radiation emitted by its wall.
- Let a body 'A' of emissive power e_λ and absorptive power a_λ is kept inside the enclosure at large distance from walls.

- Now whatever was the initial temperature of body 'A' the final temperature (at equilibrium) will be T, because the equilibrium will be attained when the temperature difference vanishes.
- Even though a heterogeneous body (having different emissive and absorptive power) is kept inside the enclosure, since the final temperature of the body will be the temperature of enclosure
- Hence at equilibrium total energy absorbed by it will be total energy emitted by it irrespective to the position and orientation of the body with respect to enclosure's wall.



- The amount of energy emitted by the elemental area dA of body 'A' in the direction $\theta+d\theta$ and $\phi+d\phi$ will be $e_\lambda d\lambda dA \cos\theta \sin\theta \, d\theta \, d\phi$
- Hence the amount of energy emitted by dA in all direction of θ and ϕ will be $$dA\int_0^\infty e_\lambda \, d\lambda \int_0^{\pi/2} sin\theta \, cos\theta \, d\theta \int_0^{2\pi} d\phi = \pi dA \int_0^\infty e_\lambda \, d\lambda$$
- Hence for whole body the emission will be

$$\pi\Sigma(dA)\int_0^\infty e_\lambda \, d\lambda$$

- Now to calculate the total absorption by body 'A', the amount of energy emitted by elemental area of dA' of enclosure, in the direction of dA (the elemental area of body) is required
- This is given as $dQ_\lambda = E_\lambda \, d\lambda \, dA' cos\theta' \frac{dAcos\theta}{r^2}$
- Where r=distance between dA and dA'
- θ' is the angle between normal to dA' with the direction of emission
- E_λ is the emissive power of the enclosure
- Hence the amount of energy absorbed by the surface dA will be $a_\lambda dQ_\lambda = a_\lambda E_\lambda \, d\lambda \, dAcos\theta \frac{dA' cos\theta'}{r^2}$ or $a_\lambda dQ_\lambda = a_\lambda E_\lambda \, d\lambda \, dAcos\theta \, d\omega$
- or $a_\lambda dQ_\lambda = a_\lambda E_\lambda \, d\lambda \, dAcos\theta \, (sin\theta d\theta d\phi)$
- Where $d\omega = \frac{dA' cos\theta'}{r^2} = (sin\theta d\theta d\phi)$ is the solid angle subtended by dA' at dA

- Total energy absorbed by surface dA from the total surface of the enclosure will be given by

$$dA\int_0^\infty a_\lambda\, E_\lambda d\lambda \int_0^{\pi/2} sin\theta\, cos\theta\, d\theta \int_0^{2\pi} d\phi = \pi dA \int_0^\infty a_\lambda E_\lambda\, d\lambda$$

- Hence the absorption for the whole body will be given as

$$\pi\Sigma(dA)\int_0^\infty a_\lambda\, E_\lambda d\lambda$$

- In the equilibrium state the absorption must be equal to emission and so

$$\pi\Sigma(dA)\int_0^\infty e_\lambda\, d\lambda = \pi\Sigma(dA)\int_0^\infty a_\lambda\, E_\lambda d\lambda \quad \text{or} \quad \int_0^\infty e_\lambda\, d\lambda = \int_0^\infty a_\lambda\, E_\lambda d\lambda \quad \text{or} \quad e_\lambda = a_\lambda\, E_\lambda$$

- If the same body 'A' is kept in another enclosure with emissive power E'_λ having different shape and size then we will get $e_\lambda = a_\lambda E'_\lambda$
- Since the emissive and absorptive power of an body depends upon the body and its temperature and not on surrounding conditions and hence a_λ and e_λ of body 'A' will remain same and this give us $E_\lambda = E'_\lambda$
- let us consider body 'A' is a black body so $a_\lambda=1$
- i.e. $e_\lambda= E_\lambda$ that means the emissive power of enclosure is equal to emissive power of black body and is independent of shape and size

- From above mathematics we can conclude that the radiation in any hollow enclosure is independent of the nature and shape of the walls and is identical with black body radiation at the same temperature
- We also get that ratio of the emissive power to absorptive power for radiation of a given wavelength is the same/ constant for all bodies if they are at same temperature. This ratio is equal to the emissive power of a perfect black at same temperature. $E_\lambda = e_\lambda / a_\lambda$ Where E_λ is the emissive power of perfect black body. This is known as Kirchhoff's Law.