ROOT FINDING METHODS (1)

Bisection Method

(1) Find the real root of the eqn $x^3 - 2x - 5 = 0$, correct to two decimal places.

(2) Find the real root of the function $f(x) = xe^x - 1$ correct to 3 decimal places, which lies b/w 0 & 1.

(1) Given $f(x) = x^3 - 2x - 5 = 0$

Let $a = 2$ & $b = 3$.

$f(a) = -1$ (-ve) & $f(b) = 16$

As $f(a) \times f(b) < 0$

So, first approximation, $x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

Now, $f(2.5) = 5.625 > 0$

So, root lies b/w 2 & 2.5.

$x_1 = \frac{2+2.5}{2} = 2.25$

Now, $f(2.25) = 1.890625 > 0$

So, root lies b/w 2 & 2.25

repeating this procedure till the desired result is obtained.

$x_2 = 2.125$
$x_3 = 2.0625$
$x_4 = 0.093750$
$x_5 = 2.109375$
$x_6 = 2.101562$

As, $|x_6 - x_5| = 0.0007813$

The computed result is correct to 2 decimal place.

final root is (upto 2 decimal accuracy)
is 2.101562

(1) Given function is $f(x) = xe^x - 1$

Let $a = 0$ & $b = 1$
$f(a) = -1$ (-ve) & $f(b) = e - 1$ (+ve)

As $f(a) \times f(b) < 0$
So, first approximation $x_0 = \frac{a+b}{2} = 0.5$

Now, $f(0.5) = -0.175639 < 0$

So, root lies b/w 0.5 & 1

$$x_1 = \frac{0.5+1}{2} = 0.75 > 0$$

So, root lies b/w 0.5 & 0.75.

⇒ repeating the procedure till the desired result is obtained.

$x_2 = 0.625$
$x_3 = 0.5625$
$x_4 = 0.59375$
$x_5 = 0.578125$
$x_6 = 0.570312$
$x_7 = 0.566406$
$x_8 = 0.568359$
$x_9 = 0.567383$

$x_{10} = 0.566825$
$x_{11} = 0.567139$
$x_{12} = 0.567261$

As, $|x_{12} - x_{11}| = 0.000122$

The computed result is correct to 3 decimal places.

final root is (upto 3 decimal accuracy) ≈ 0.567261

Regula-Falsi Method

Q1) final a real root of eqn $f(x) = x^3 - 4x - 9$ using Regula falsi Method correct to 3 decimal places.

Let $a = 2$ & $b = 3$
$f(a) = -9$ (-ve) & $f(b) = 6$ (+ve).

Let x_1 be the 1st approximation,

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2 \times 6 - 3 \times (-9)}{6 - (-9)} = \frac{39}{15} = \frac{13}{5} = 2.6$$

$$f(x_1) = f(2.6) = -1.824002 < 0$$

So, root lies b/w 2.6 & 3 :-

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{(2.6)(6) - 3(-1.824002)}{6 - (-1.824002)}$$

$$x_2 = 2.699252$$

Repeating this process the successive approximations are.

$x_3 = 2.704918$ $\quad x_4 = 2.706333$

$x_5 = 2.706504$

So, one of its root correct to 3 decimal places is 2.706504

(2) Find the real root of the eqn $3x - \cos x - 1 = 0$ using false posn method, correct to 2 decimal places.

(3) Find the real root of the function $f(x) = xe^x - 1$ using false posn method correct to three decimal places, which lies b/w 0 & 1.

(2) Let $a = 0.5$ & $b = \frac{\pi}{4}$

$f(a) = -0.377583$, $f(b) = 0.649088$

Let x_1 be the first approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = 0.604962$$

$f(x_1) = -0.007638 < 0$

So, root lies b/w $a = 0.604962$ & $b = \pi/4$

$x_2 = 0.607060$

Similarly, $x_3 = 0.607101$

So, one of its root correct to 3 decimal places is 0.607101

(3) Let $a = 0$, $b = \ln 2 = 0.693147$

$f(0) = -1 < 0$, $f(b) = 0.386294 > 0$

Let x_1 be the 1st approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(0.386294) - \ln 2(-1)}{0.386294 - (-1)}$$

$$x_1 = 0.5$$

$\Rightarrow f(x_1) = -0.175639$

So, root lies b/w $a = 0.5$ & $b = \ln 2$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.5)(0.386294) - \ln 2(-0.175639)}{0.386294 - (-0.175639)}$$

$$x_2 = 0.560371$$

Repeating this process, the successive approximations are:-

$x_3 = 0.566473$
$x_4 = 0.567077$
$x_5 = 0.567137$

So, one of its root correct to 3 decimal places is 0.567137.