Fermi Dirac Distribution & Semiconductor

BPT: 401: Electronics and Modern Physics

Tutorial - 2

Fermi-Dirac distribution and the Fermi-level

- The free electrons in the conduction band of metals are essentially in a field-free or equipotential space.

- Due to the thermal energy, these free electrons move in random directions like gas molecules move. Hence these electrons are said to form an electron gas.

- Due to very large number of free electrons in metal (i.e. ~10^{23} cm^{-3}), principle of statistical mechanics are employed to determine their average behaviour.

- Since the well known Maxwell-Boltzmann distribution function does not incorporate the Pauli's exclusion principle, Fermi-Dirac distribution function can be used to determine the energy distribution of free electrons in metals (ie. for finding the probability of occupancy of given states by electrons).

Fermi-Dirac distribution and the Fermi-level

The Fermi Fermi-Dirac distribution function $f(E)$ gives the number of the existing states at energy E that will be filled with electrons at temperature T.

In other words, $f(E)$ gives the probability of occupancy of the state with energy E.

$$\boldsymbol{f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}}$$

Where, E_F is the Fermi energy level and is a characteristic energy for a particular solid

T is the obsolete temperature in kelvin.

k_B is the Boltzmann constant (k_B = 1.38 × 10^{-23} J/K $= 8.6 \times 10^{-5}$ eV/K)

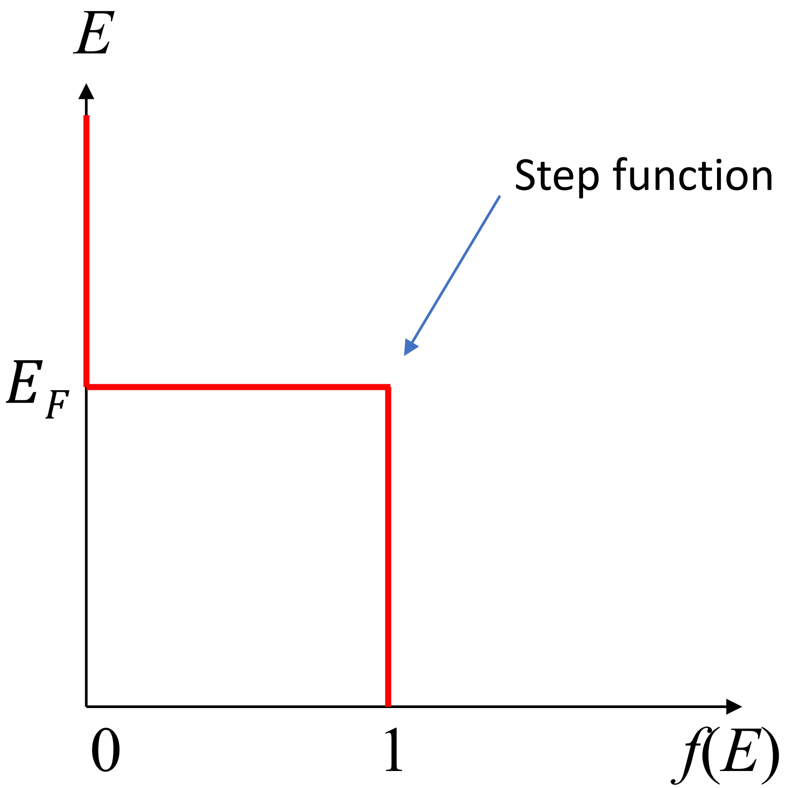
Fermi-Dirac distribution and the Fermi-level

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$$

At the absolute zero of temperature (T = 0 K)

For $E > E_F$ $$f(E > E_F) = \frac{1}{1+\exp(+\infty)} = 0$$

For $E < E_F$ $$f(E < E_F) = \frac{1}{1+\exp(-\infty)} = 1$$



Thus all the states below E_F are occupied by electrons (completely filled) and the all the states above E_F are empty)

The probability of finding electrons above the Fermi level E_F is zero and the probability of finding electrons below E_F is one.

Hence the Fermi Energy E_F denotes the maximum energy that can be occupied by electrons at 0 K.

Fermi-Dirac distribution and the Fermi-level

At temperatures greater than absolute zero (T > 0 K)

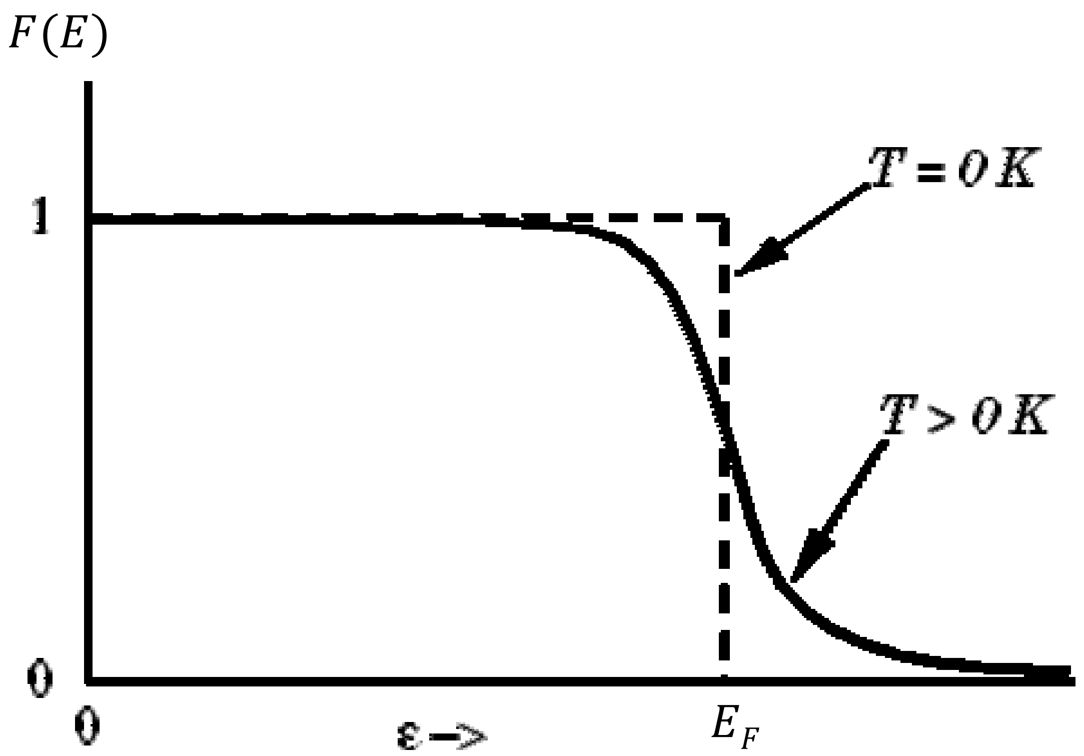
At finite temperature (T > 0 K), some of the electrons in the quantum states below E_F acquire thermal energy to move into states above E_F

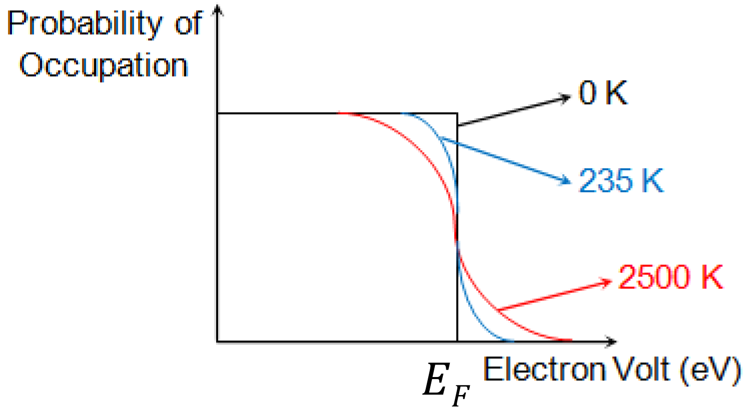
$$\color{red}{For\ E = E_F} \qquad f(E = E_F) = \frac{1}{1 + e^{0}} = \frac{1}{2}$$

The probability of finding electrons at the Fermi level E_F is ½.

For most metals, E_F is less than 10 eV.

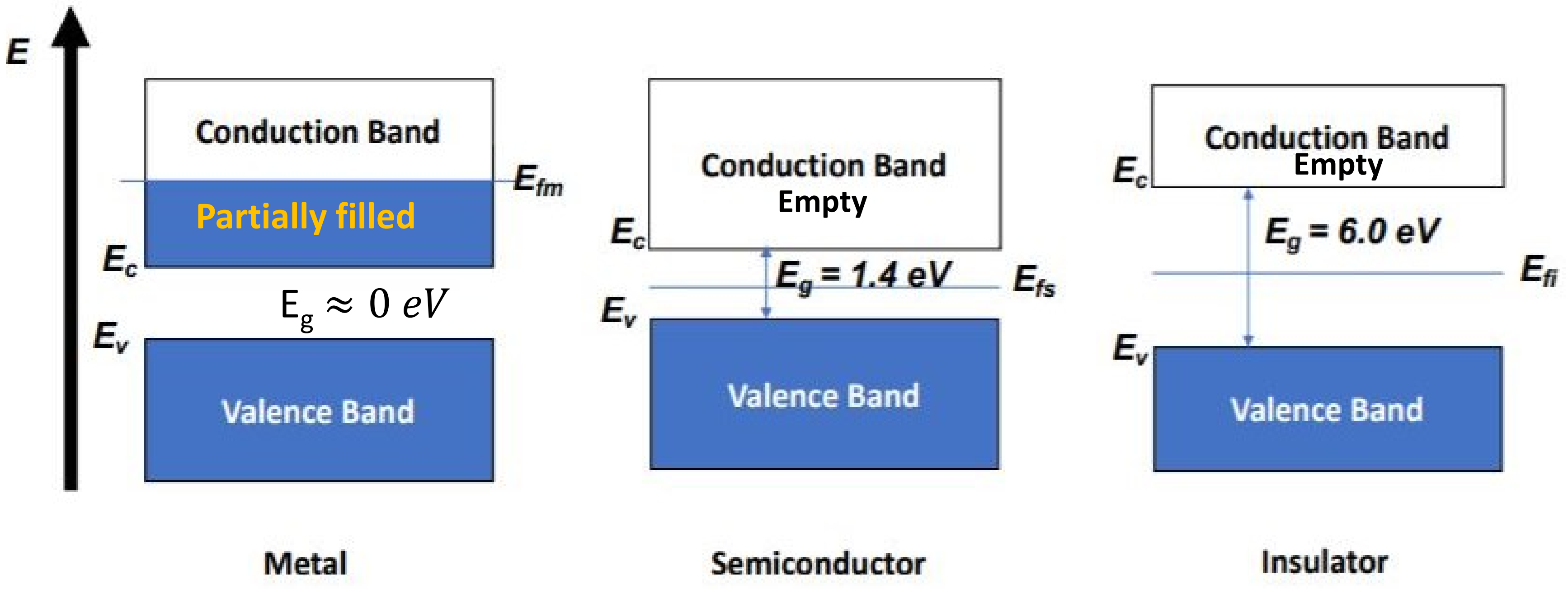
For pure semiconductor and insulator, E_F lies near the middle of band gap and for metals E_F lies in the conduction band.





Fermi-Dirac Distribution Function at Different Temperatures

Metal , Insulator and Semiconductor



This separation of the valence and conduction bands determines the electrical properties of the material. In terms of their electrical properties, materials can be classified into three groups: metals, semiconductors, and insulators.

Metal

- **Partially filled Conduction Band**
- **E_V and E_c overlap ($E_g \approx 0\ eV$)**
- **Fermi energy in Conduction Band**
- **Free electrons in Conduction Band**

Semiconductor

- **Empty Conduction Band**
- **Small energy gap between E_V and E_c (E_g between $0.3\ to\ 3\ eV$)**
- **Fermi energy in the middle of E_g**
- **At 0 K, no free electrons in Conduction Band**
- **Free electrons in Conduction Band increases with temperature**

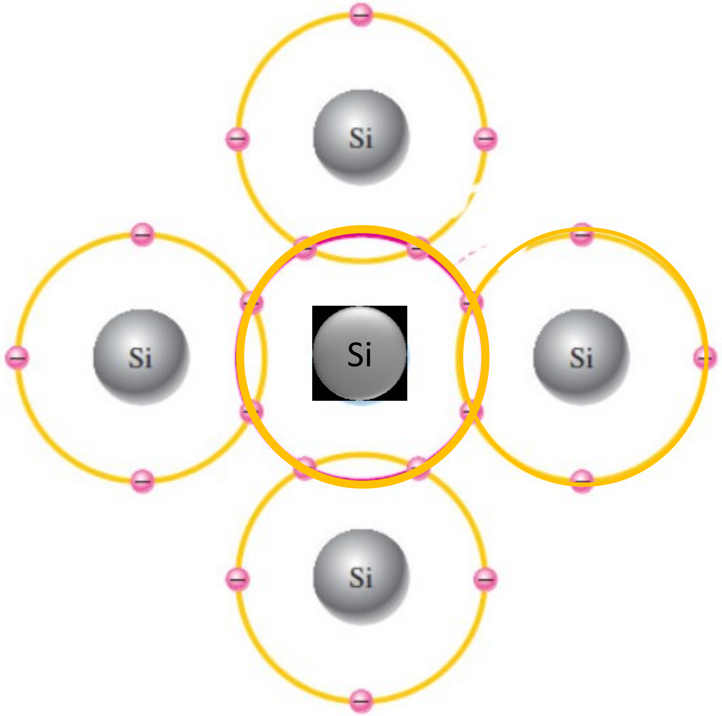
Insulator

- **Empty Conduction Band**
- **Large energy gap between E_V and E_c ($E_g \gg 3\ eV$)**
- **Fermi energy in the middle of E_g**

Intrinsic (pure), N-type and P-type semiconductors

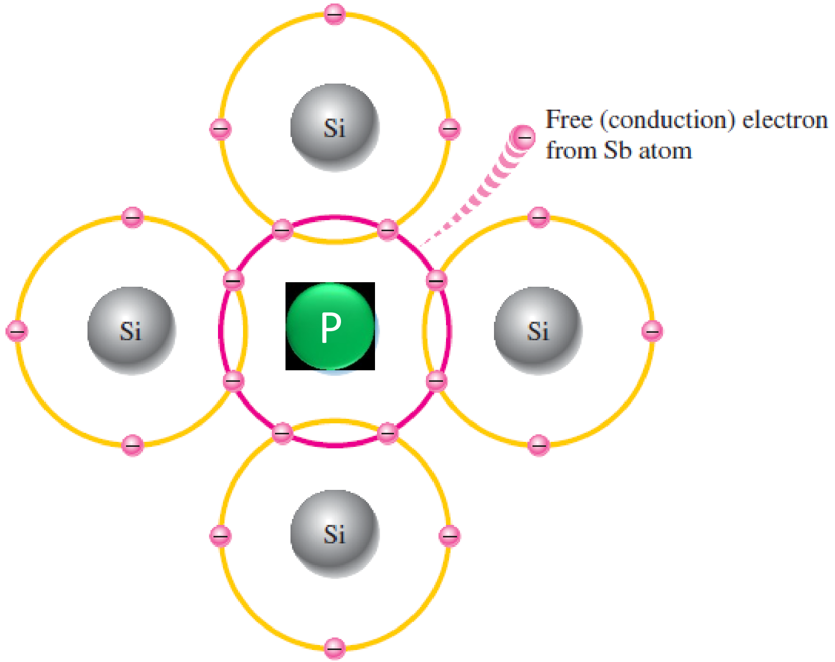
Since intrinsic state (pure) of semiconductors are generally poor conductors (due to limited number of free electrons in conduction band and holes in valance band), their conductivity can be drastically increased by the controlled addition of impurities to the intrinsic semiconductive material. This process, called **doping**, increases the number of current carriers (electrons or holes). The two categories of impurities are **n-type** and **p-type**.

Intrinsic state of Silicon



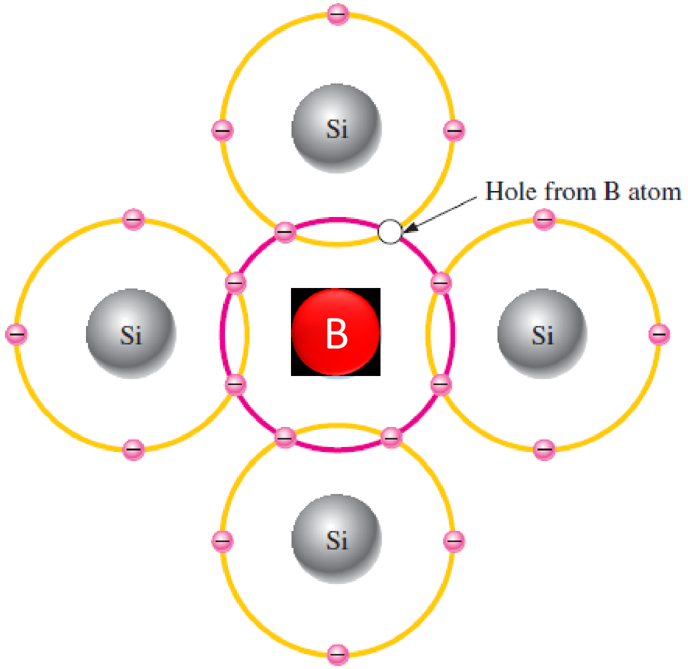
Free carriers are generated only by thermal excitation

N type Silicon due to pentavalent P doping



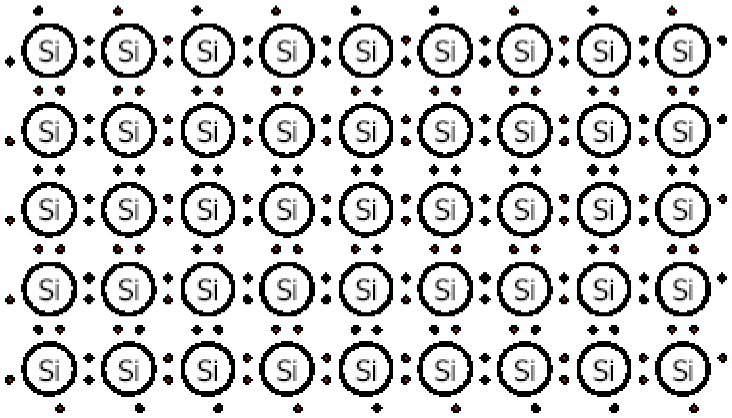
Majority carriers are electrons (Donor atom doping)

P type Silicon due to trivalent B doping

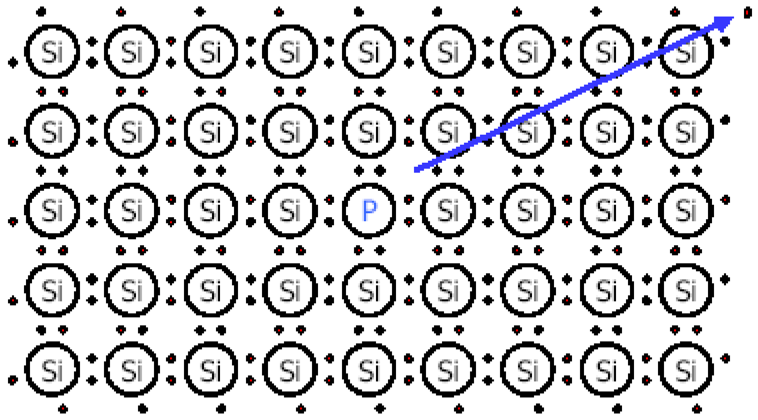


Majority carriers are holes (Acceptor atom doping)

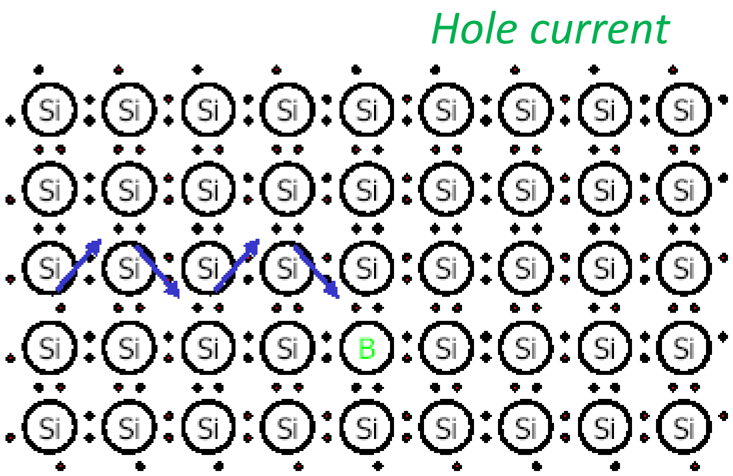
Intrinsic (pure), N-type and P-type Silicon crystal



Intrinsic (Pure) Silicon
(4 valance electron)



N-type (doped) Silicon
(5 valance electron from dopant)

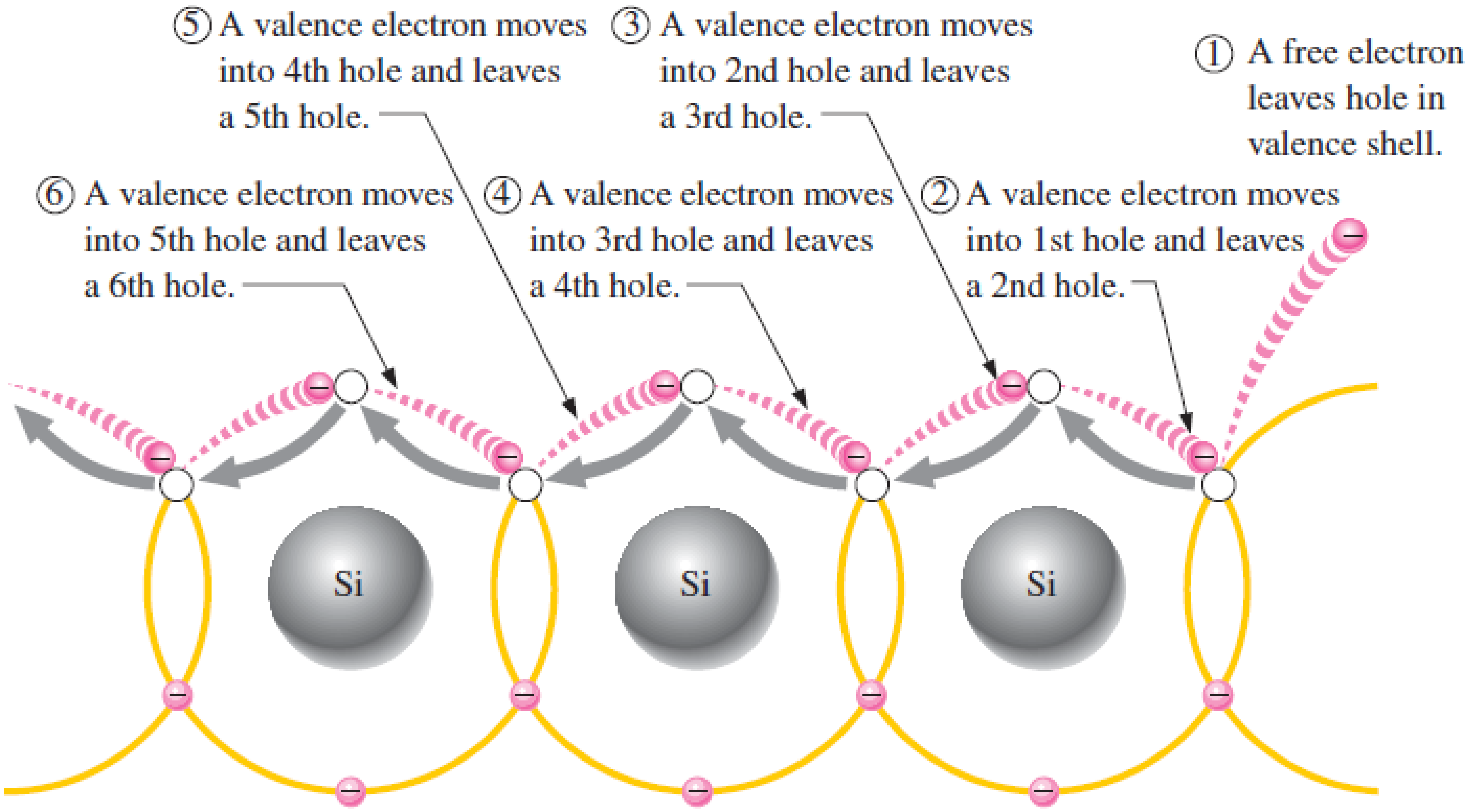


P-type (doped) Silicon
(3 valance electron from dopant)

- Silicon has 4 valence electrons, it covalently bonds with four adjacent atoms in the crystal.
- There are two types of doping (N-type and P-type) to change the electronic properties of intrinsic silicon.
- The N in N-type stands for negative. The extra valence electron from N-type dopant (Eg. Phosphorous from group V elements of periodic table) is free to move about the silicon crystal lattice.
- The P in P-type stands for positive. Electrons from the surrounding Silicon move to fill the “hole” (missing electron due to doping of P-type dopant, e.g. boron from group III elements of periodic table)

- Many extra electrons → “donor” or N-type material.
- Extra electrons needed for crystal bonding → “acceptor” or P-type material.
- Holes (missing electrons) can move to other areas of semiconductor if electrons continually fill holes. This moment of holes constitute ***hole current***

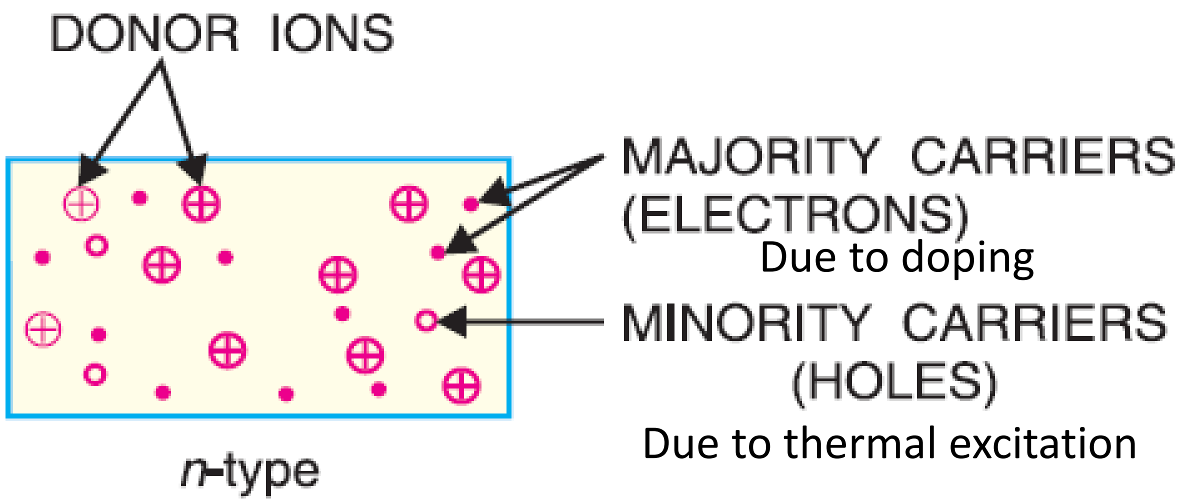
Hole Current in semiconductor



When a valence electron moves left to right to fill a hole while leaving another hole behind, the hole has effectively moved from right to left. Gray arrows indicate effective movement of a hole.

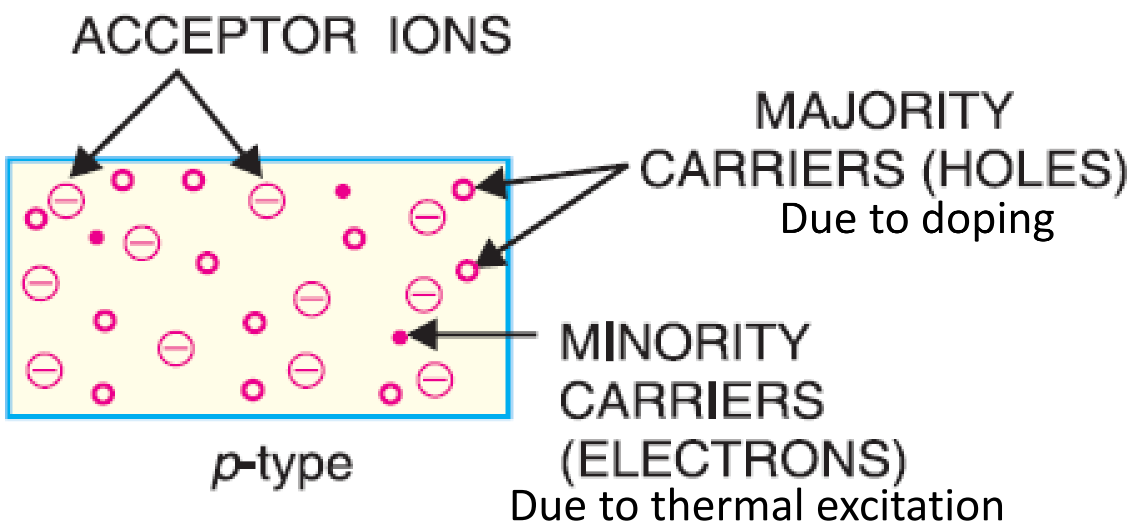
Majority and Minority Charge Carriers

N-type Semiconductor



In *n*-type semiconductive materials, the free electrons are considered ***majority carriers*** (majority portion of current is by the flow of free electrons) and the holes are the ***minority carriers***.

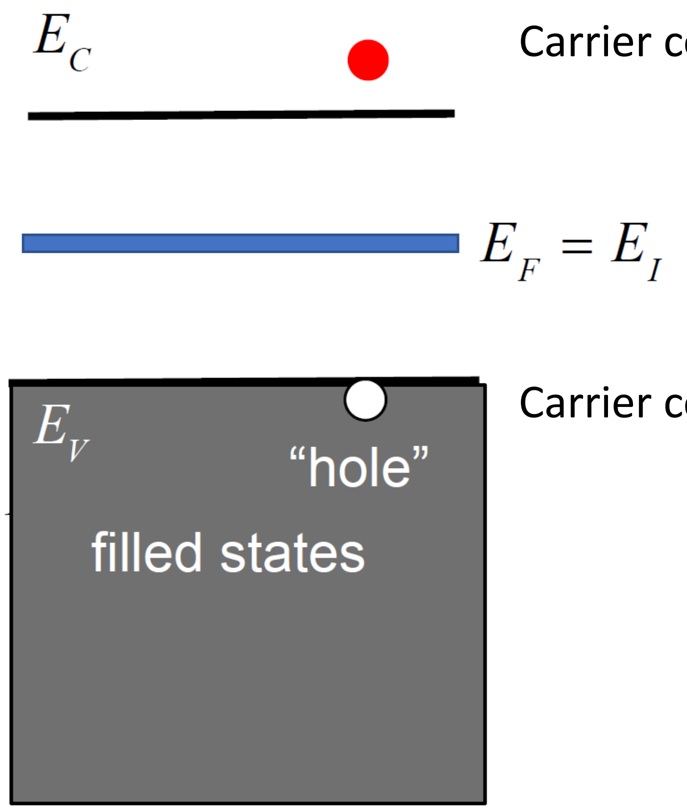
P-type Semiconductor



In a *p*-type semiconductive material, holes are the ***majority carriers*** (majority portion of current is by the movement of holes) and free electrons are the ***minority carriers***.

intrinsic semiconductor

Carrier generation by **thermal excitation**; INTRINSIC CONDUCTIVITY



Carrier concentration in CB, $\boldsymbol{n = n_i}$

$\boldsymbol{E_F}$ – Fermi level

$\boldsymbol{E_i}$ – Fermi level of intrinsic semiconductor

Carrier concentration in VB, $\boldsymbol{p = n_i}$

Mass Action Law

$$np = n_i^2$$

$$n_i(300\,K) \approx 10^{10} \text{ cm}^{-3}$$

n → electron concentration, p →holes concentration , n_i →intrinsic charge carrier concentration intrinsic | n-type | p-type