Exp. 6. Modulus of rigidity by statical method

Object

To determine the modulus of rigidity for the material of a wire by statical method using Barton's apparatus.

Apparatus

Barton's apparatus, metre scale, vernier callipers, screw gauge and slotted weights of half-kilogram each.

The apparatus consists of a wire *AL* clamped at *A* with its lower end fixed to a heavy cylinder as shown in fig. 6.1. Two parallel flexible threads leave opposite sides of the cylinder tangentially at two diametrically opposite points and passing over two identical frictionless pulleys carry pans of equal weights. Pointers screwed to the experimental wire can move over graduated circular scales *B*, *C* and *D*.

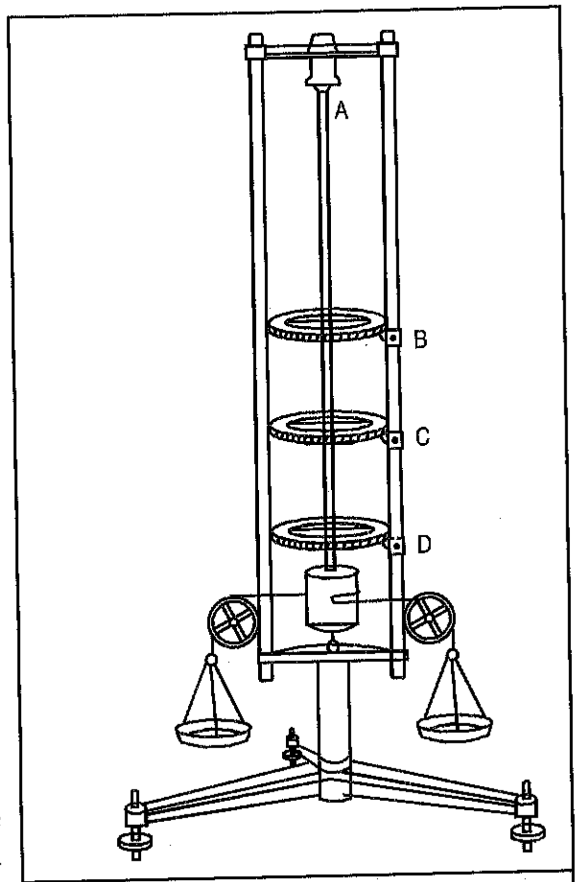

Fig. 6.1

Theory

Suppose, by applying a couple (placing equal weights on the pans), the lower end of the wire is twisted through an angle θ radian.

Let l, r, η be respectively the length, radius and co-efficient of rigidity of the material of the wire.

(*a*) The cylinder may be considered to consist of a large number of coaxial hollow cylindrical shells and in fig. 6.2, let us consider a cylindrical shell of radii x and $x + dx$ respectively. Let *AB* be a line (fig. 6.3) parallel to the axis *DC* before the cylinder is twisted. On twisting, B shifts to the position B′ and the hollow cylinder is sheared through an angle ϕ. If this hollow cylinder is cut and flattened out, it will form a rectangle *ABEF* of sides l and $2\pi x$ (fig. 6.4). After

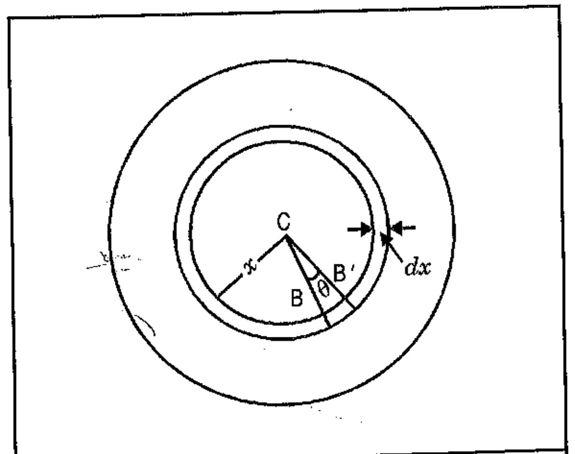

Fig. 6.2

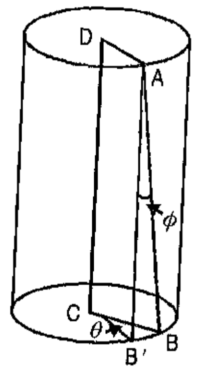

Fig. 6.3

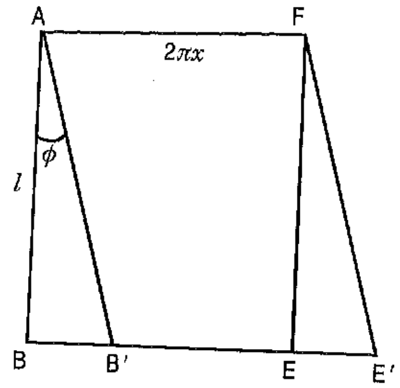

Fig. 6.4

twisting, a parallelogram $AB'E'F$ is obtained. The angle through which this hollow cylinder is sheared is clearly ϕ.

(*b*) Now, $\quad \text{arc } BB' = x\theta$

$$= l\phi. \qquad \text{(fig. 6.3)}$$

$\therefore$

$$\phi = \frac{x\theta}{l}.$$

Since,

$$\eta = \frac{\text{shearing stress}}{\text{angle of shear}} = \frac{\text{shearing stress}}{\phi}$$

$\therefore$ we have, shearing stress $= \eta \times \phi = Y_1 \dfrac{x\theta}{l}$.

Now face area of this hollow cylinder $= 2\pi x dx$.

Therefore, total shearing force on this area

$$= 2\pi x dx \times \frac{\eta x\theta}{l}$$

$$= 2\pi\eta \frac{\theta}{l} x^2 dx$$

Moment of the force about the axis CD

$$= 2\pi\eta \frac{\theta}{l} x^2 dx \times x$$

The total twisting couple on the cylinder

$$= \int_0^r 2\pi\eta \ \frac{\theta}{l} x^3 dx$$

$$= \frac{\pi\eta\theta r^4}{2l}$$

(*c*) In the Barton's apparatus, if d be the diameter of the cylinder, θ radian be the angle of twist and m be the mass placed on each of the pans, the twisting couple $= mgd$.

$\therefore$

$$mgd = \frac{\eta\theta\pi r^4}{2l}$$

and hence

$$\eta = \frac{2mgld}{\pi\theta r^4}$$

When the angle of twist θ is measured in degrees, the expression for η is modified to

$$\eta = \frac{360\, mgld}{\pi^2 r^4 \theta} \qquad \text{...(1)}$$

If θ_1 and θ_2 are the angles of twist at lengths l_1 and l_2 measured from the fixed end, then

$$\eta = \frac{360\, mgd(l_1 - l_2)}{\pi^2 r^4 (\theta_1 - \theta_2)} \qquad \text{...(2)}$$

Procedure

(*i*) After levelling the apparatus, the pointers are clamped at three suitable distances from the fixed end of the rod adjusting their positions near about zero mark on the circular scales. The initial readings of the pointers at their two ends are taken with no load on the pans.

(*ii*) The pulleys are adjusted at the same height. Next, a load of 500 gm (say)* is placed on each pan. Again the readings of the pointers on both the ends for different lengths are recorded.**

(*iii*) The load is gradually increased on each pan on steps of 500 gm up to 3500 gm taking down the readings of the pointers at each step. The procedure is repeated in the same stages with decreasing loads till all the loads are removed.

(*iv*) The three lengths of the wire upto the pointers from its fixed end are noted.

(*v*) The diameter of the wire is measured with a screw gauge and that of the cylinder with a vernier callipers.

(*vi*) Graphs are plotted between the load and the twist for each value of length of the wire. The formula is then applied to determine the modulus of rigidity of the given rod.

*The exact magnitude of loads is so selected that the depression of the beam caused by it is appreciable and conveniently measurable.

**The percentage error will be small if the values of $(l_1 - l_2)$ and $(\theta_1 - \theta_2)$ are large. This is possible if the entire length of the wire is considered and reading only on one of the circular scales is taken by keeping it at the bottom. In this case, formula (1) is to be used.

Observations

[A] *Measurement of angle of twist*

	S. No.	Load on each pan in gm	Twist in degrees (θ)						Mean twist θ in degrees	Twist for 2000 gm in degrees
			Reading with load increasing			Reading with load decreasing				
			left end	right end	average twist	left end	right end	average twist		
Length of the rod under twist = l_1 cm	1.	0	...	...	...	...	...	...	...	
	2.	500	...	...	...	...	...	...	...	
	3.	1000	...	...	...	...	...	...	...	
	4.	1500	...	...	...	...	...	...	...	
	5.	2000	...	...	...	...	...	...	...	
	6.	2500	...	...	...	...	...	...	...	
	7.	3000	...	...	...	...	...	...	...	
	8.	3500	...	...	...	...	...	...	...	
Length = l_2 cm	1.	0	...	...	...	...	...	...	...	
	2.	500	...	...	...	...	...	...	...	
	3.	1000	...	...	...	...	...	...	...	
	4.	1500	...	...	...	...	...	...	...	
	5.	2000	...	...	...	...	...	...	...	
	6.	2500	...	...	...	...	...	...	...	
	7.	3000	...	...	...	...	...	...	...	
	8.	3500	...	...	...	...	...	...	...	
Length = l_3 cm	1.	0	...	...	...	...	...	...	...	
	2.	500	...	...	...	...	...	...	...	
	3.	1000	...	...	...	...	...	...	...	
	4.	1500	...	...	...	...	...	...	...	
	5.	2000	...	...	...	...	...	...	...	
	6.	2500	...	...	...	...	...	...	...	
	7.	3000	...	...	...	...	...	...	...	
	8.	3500	...	...	...	...	...	...	...	

Mean twist for 2000 gm for
(*a*) length, $l_1 = ...$
(*b*) length, $l_2 = ...$
(*c*) length, $l_3 = ...$

[B] Diameter of the cylinder = ... cm

[C] *Measurement of diameter of the wire* :

Pitch of the screw gauge = ... cm
No. of divisions in the circular scale = ... div
∴ Least count of the screw gauge = ... cm
Zero-error = ... cm.

S. No.	Diameter (in cm) along						Average diameter in cm
	any direction			perp. direction			
	Linear scale reading	Circular scale reading	Total (x)	Linear scale reading	Circular scale reading	Total (y)	$\frac{x+y}{2}$
1.	...	...		...	...	...	
2.	...	...		...	...	...	
3.	...	...		...	...	...	
						Mean	

Calculations

From the formula, the modulus of rigidity for the material of the rod is given by

$$\eta = 360\ mgdl/\pi^2 r^4 \theta \text{ dyne/cm}^2.$$

or

$$\eta = \frac{360\, mgd(l_1 - l_2)}{\pi^2 r^4 (\theta_1 - \theta_2)} \text{ dyne/cm}^2$$

% *error* :

Result

The modulus of rigidity for the material of the wire (correct to significant figures) = ... dyne/cm^2.

Precautions

(*i*) The base should be levelled by levelling screws so that the pillars are vertical.

(*ii*) The threads leaving the cylinder in opposite directions should be parallel and tangential to the cylinder.

(*iii*) The loads should be placed or removed as gently as possible.

(*iv*) The loads should never exceed the maximum permissible value.

(*v*) The rod should be clamped firmly at a top.

(*vi*) The pointers should be screwed firmly to the rod close to the respective circular scales.