


Program for Picard's iterative method | Computational Mathematics

Difficulty Level : Medium • Last Updated : 28 Jun, 2019

The **Picard's** method is an iterative method and is primarily used for approximating solutions to differential equations.

This method of solving a differential equation approximately is one of successive approximation; that is, it is an iterative method in which the numerical results become more and more accurate, the more times it is used.

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The Picard's iterative method gives a sequence of approximations $Y_1(x)$, $Y_2(x)$, ... $Y_k(x)$ to the solution of differential equations such that the n th approximation is obtained from one or more previous approximations.

The Picard's iterative series is relatively easy to implement and the solutions obtained

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Picard's iteration method formula:

$$y(t) = y_0 + \int_{t_0}^t f(x, y(x)) dx$$

Picard's iteration formula.

Steps involved:

- Step 1: An approximate value of y (taken, at first, to be a constant) is substituted into the right hand side of the differential equation:
 $dy/dx = f(x, y)$.
- Step 2: The equation is then integrated with respect to x giving y in terms of x as a second approximation, into which given numerical values are substituted and the result rounded off to an assigned number of decimal places or significant figures.
- Step 3: The iterative process is continued until two consecutive numerical solutions are the same when rounded off to the required number of decimal places.

Picard's iteration example:

Given that:

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$$\frac{dy}{dx} = x + y^2,$$

and that $y = 0$ when $x = 0$, determine the value of y when $x = 0.3$, correct to four places of decimals.

Solution:

We may proceed as follows:

$$\int_{x_0}^x \frac{dy}{dx} dx = \int_{x_0}^x (x + y^2) dx,$$

where $x_0 = 0$. Hence:

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where $y_0 = 0$. which becomes:

$$y = \int_0^x (x + y^2) dx.$$

- **First Iteration:**

We do not know y in terms of x yet, so we replace y by the constant value y_0 in the function to be integrated.

The result of the first iteration is thus given, at $x = 0.3$, by:

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$$y_1 = \int_0^x x dx = \frac{x^2}{2} \simeq 0.0450$$

- **Second Iteration:**

Now, we use:

$$\frac{dy}{dx} = x + y_1^2 = x + \frac{x^4}{4}.$$

Therefore,

$$\int_0^x \frac{dy}{dx} dx = \int_0^x \left(x + \frac{x^4}{4} \right) dx,$$

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which gives:

$$y - 0 = \frac{x^2}{2} + \frac{x^5}{20}.$$

The result of the second iteration is thus given by:

$$y_2 = \frac{x^2}{2} + \frac{x^5}{20} \simeq 0.0451$$

at $x=0.3$.

- **Third Iteration:**

Now we use:

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Therefore,

$$\int_0^x \frac{dy}{dx} dx = \int_0^x \left(x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400} \right) dx,$$

which gives:

$$y = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400} + \dots$$

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The result of the third iteration is thus given by:

$$y_3 = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400} \approx 0.0451 \text{ at } x = 0.3$$

at $x = 0.3$.

- Hence, $y = 0.0451$, correct upto four decimal places, at $x = 0.3$.

Program for Picard's iterative method:

```
// C program for Picard's iterative method

#include <math.h>
#include <stdio.h>

// required macros defined below:
#define Y1(x) (1 + (x) + pow(x, 2) / 2)
#define Y2(x) (1 + (x) + pow(x, 2) / 2 + pow(x, 3) / 3 + pow(x, 4) / 8)
#define Y3(x) (1 + (x) + pow(x, 2) / 2 + pow(x, 3) / 3 + pow(x, 4) / 8 + pow(x, 5) / 40)

int main()
{
    double start_value = 0, end_value = 3,
           allowed_error = 0.4, temp;
    double y1[30], y2[30], y3[30];
    int count;
```

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```
        y2[count] = Y2(temp);
        y3[count] = Y3(temp);
    }

    printf("\nX\n");
    for (temp = start_value;
        temp <= end_value;
        temp = temp + allowed_error) {

        // considering all values
        // upto 4 decimal places.
        printf("%.4lf ", temp);
    }

    printf("\n\nY(1)\n");
    for (temp = start_value, count = 0;
        temp <= end_value;
        temp = temp + allowed_error, count++) {

        printf("%.4lf ", y1[count]);
    }

    printf("\n\nY(2)\n");
    for (temp = start_value, count = 0;
        temp <= end_value;
        temp = temp + allowed_error, count++) {

        printf("%.4lf ", y2[count]);
    }

    printf("\n\nY(3)\n");
    for (temp = start_value, count = 0;
        temp <= end_value;
        temp = temp + allowed_error, count++) {

        printf("%.4lf ", y3[count]);
    }
    return 0;
}
```

Output:

X

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$Y(2)$

1.0000 1.5045 2.3419 3.7552 6.0645 9.6667 15.0352 22.7205

 $Y(3)$

1.0000 1.5053 2.3692 3.9833 7.1131 13.1333 24.3249 44.2335

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