Exp. 5. Young's Modulus of a Beam by Bending

Object

To determine the value of Young's modulus of the material from the flexure of a beam supported on two knife-edges and loaded at its middle point.

Apparatus

The experimental beam, two knife-edges fixed on rigid supports, a Lechlanche cell, a detecting device, vernier callipers, a screw gauge, a hanger and a set of 50 gm weights (fig. 5.1).

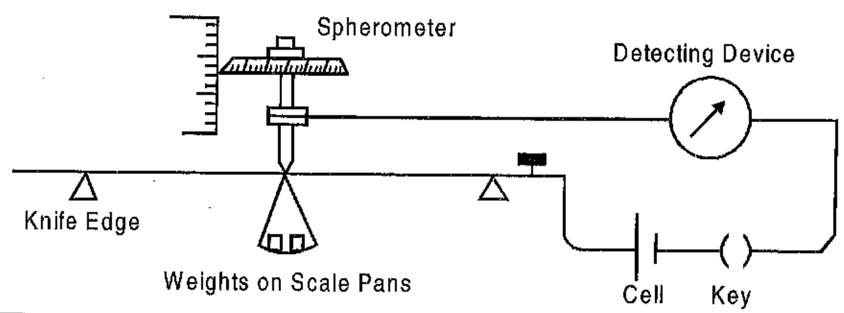

Fig. 5.1

Theory

A rod or a beam fixed vertically at one end and loaded at the other, is called a cantilever. Due to the load at the end, the beam bends with curvature changing along the length of the beam, being zero at the fixed end. For keeping any portion *PB* of the beam (fig. 5.2) in equilibrium, the force *W* acting vertically downwards at *B* is balanced by an equal vertical force acting upwards at *P*. These two forces constitute a couple whose moment is called the *bending moment.*

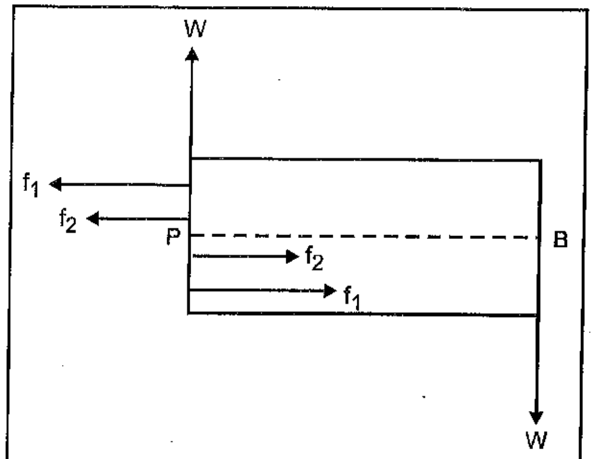

Fig. 5.2

There is a filament in the middle of the beam called *neutral axis,* unaltered in length due to bending, above which the filaments are lengthened and below which the filaments are shortened *longitudinally.* Extending and compressing forces f_1, f_2 etc. are thus set up which constitute a system of couples. The resultant of these couples balances the bending moment and keeps the portion *PB* in equilibrium.

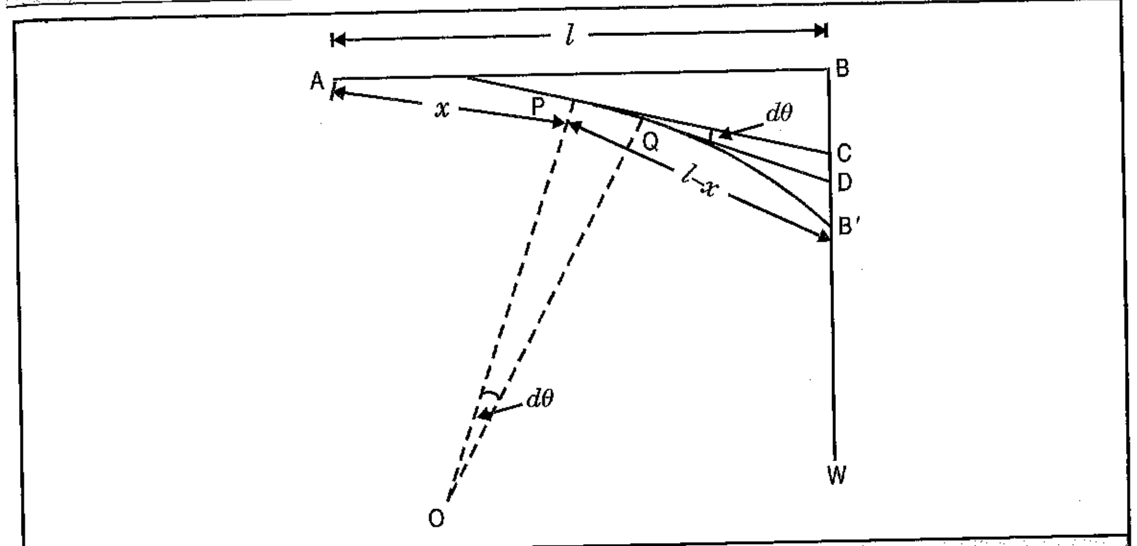

Fig. 5.3

Let *AB* represent the neutral axis of a cantilever of length l with the fixed end at *A* (fig. 5.3). Since the depression of *B* is small, *B'* (depressed portion of *B*) is practically in the same vertical line as *B*. For a section at *P* at a distance x from *A*, the moment of the external couple due to the load *W*

$$= W \times PB' = W(l - x).$$

Since the beam is in equilibrium, this must be equal to the bending moment $\frac{YI_g}{R}$ where I_g is the geometrical moment of inertia and *R* is the radius of curvature of the neutral axis at *P*.

Thus, $$\frac{YI_g}{R} = W(l-x) \qquad ...(1)$$

Since the movement of the applied load increases in going towards *A*, the radius of curvature is different at different points. However, for a point *Q* at a small distance dx from *P*, the radius of curvature is practically the same as at *P*.

$$\therefore \quad R\,d\theta = dx \qquad ...(2)$$

where $d\theta$ is the angle *POQ*.

Let the tangents to the neutral axis at *P* and *Q* meet the vertical line at *C* and *D* respectively.

The depression of *Q* below *P* will be given by,

$$dy = (l - x)\,d\theta$$

$$= (l-x)\frac{dx}{R} \qquad \text{(from eqn. 2)}$$

$$= (l-x)\,dx\,\frac{W(l-x)}{YI_g} \qquad \text{(from eqn. 1)}$$

Therefore, the total depression $y = BB'$ is obtained by integrating the expression for dy between the limits $x = 0$ and $x = l$.

Thus,
$$y = \int_0^l \frac{W(l-x)^2 dx}{YI_g} = \frac{Wl^3}{3YI_g}$$

$$= \frac{mgl^3}{3YI_g} \qquad ...(3)$$

In the actual experiment, the beam is supported on two knife-edges at a distance l apart and is loaded with a mass m at its middle point. The reaction at each knife-edge A and B (fig. 5.4) will clearly be $\frac{mg}{2}$ in the upward direction. The beam may now be considered as equivalent to two inverted cantilevers. Thus, the length of each cantilever is $l/2$ and the elevation of knife-edges A or B above C is given by

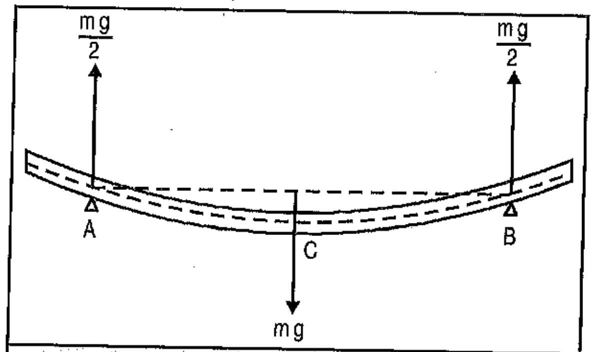

Fig. 5.4

$$y = \frac{\frac{mg}{2}\left(\frac{l}{2}\right)^3}{3YI_g} = \frac{mgl^3}{48YI_g} \qquad ...(4)$$

For a beam of rectangular cross-section of breadth b and thickness d, we have,

$$I_g = (bd) \times \frac{d^2}{12} = \frac{bd^3}{12}.$$

Hence, the depression, $$y = \frac{mgl^3}{4bd^3y} \qquad ...(5)$$

The Young's modulus for the material of the beam is determined from the relation

$$y = \frac{mgl^3}{4bd^3y} \qquad ...(6)$$

Procedure

(a) *Micrometer screw arrangement :*

(i) The thickness of the beam is measured by a screw gauge and the breadth by a vernier callipers at several places along the length of the beam.

(ii) The beam is placed symmetrically on the two knife-edges and the distance between them is measured by means of a metre scale.

(iii) The hanger is suspended at the middle of the two knife-edges and the spherometer reading is taken when the central leg of the spherometer just touches the beam. This is the initial reading when the spherometer is just in contact with the experimental beam. This can be ascertained by the deflection of the galvanometer needle or the glowing of the small electric bulb due to the completion of the circuit as a detecting device.

(iv) Weights are then gradually added in steps of 50 gm (say) on the pan and the corresponding readings for each load are recorded.

(v) The observations are repeated by gradually removing the load till all the weights are taken out of the pan. The mean depression for 200 gm is determined as shown in the observtion table [B].

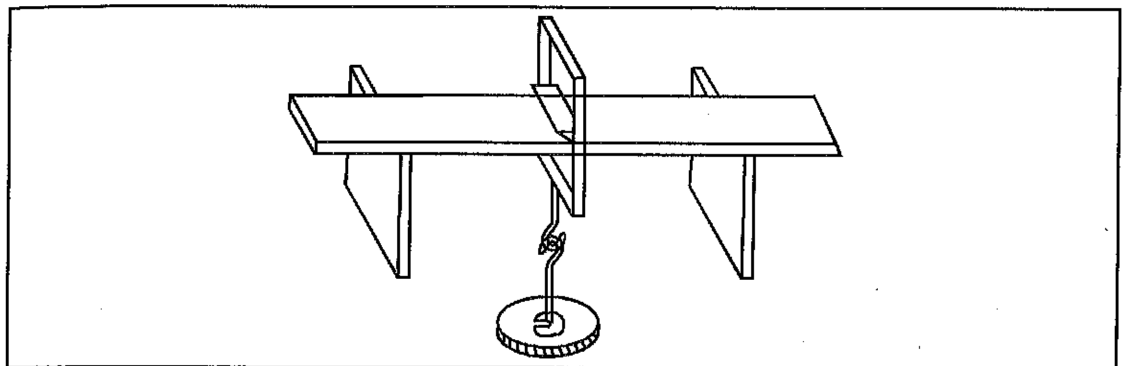

Fig. 5.5

(b) *Travelling microscope arrangement* (Fig. 5.5) :

A rectangular frame carries a knife-edge whose sharp edge is placed on the bar. There is a sharp pointer at the top of the frame and a hanger is attached at its bottom.

Due to increase in load placed on the hanger, the bar depresses alongwith the pointer. At each step, the horizontal cross-wire of a travelling microscope is made tangential to the tip of the pointer and the readings are noted from which y can be calculated.

(c) *Optical lever arrangement :* (fig. 5.6)

It consists of a plane mirror on a tripod stand whose two legs are parallel to the plane of the mirror and lie on a support fixed behind the bar. The third leg rests on the middle point of the bar. A vertical scale is set up in front of the mirror at a distance of about one metre. The cross-wire of a telescope (very close to the scale) is focussed on the image of the scale reading.

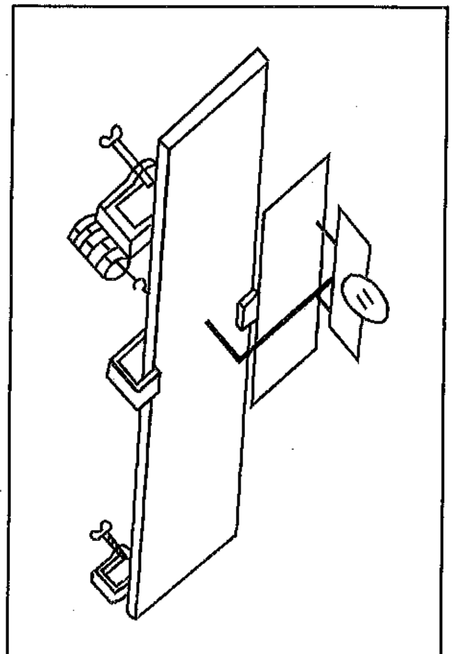

Fig. 5.6

As the beam is loaded, it depresses along the third leg of the lever arrangement thereby tilting the mirror slightly. At each step of increasing and decreasing loads, the scale reading coinciding with that horizontal cross-wire is noted.

As the mirror tilts through an angle θ, the direction of the incident ray must change by an angle 2θ to get the reflected ray in the same direction (fig. 5.7). Since the angle θ is small, $2\theta = x/D$, where 'x' is the shift in the scale image seen through the telescope for a known load and D is the distance of the mirror from the scale.

Let 'p' be the perpendicular distance of the front leg of the lever from the line joining the hindlegs. The actual depression 'y' of the bar is given by

$$y = \theta \times p = px/2D$$

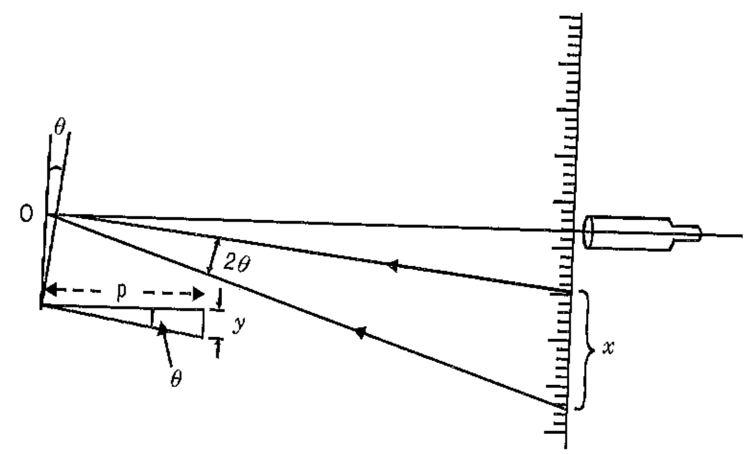

Fig. 5.7

Observations

[A] *Measurement of breadth and thickness of the beam :*

Pitch of the screw gauge	= ... cm.	One main Sc. div. of vernier	= ... cm.
No. of div. in circular scale	=	No. of vernier div.	= ...
Least count	= ... cm.	Least count of vernier	= ... cm.
Zero-error	= ... cm.	Zero-error	= ... cm.

S. No.	Thickness by screw gauge			Breadth by vernier		
	Main Sc. reading	Circular Sc. reading	Total in cm	Main Sc. reading	Vernier reading	Total in cm
		Mean			Mean	

[B] *Measurement of depression of the beam :*

(*a*) *by micrometer or*

(*b*) *by microscope*

Pitch of the micrometer	= ... cm.	Value of one main Sc. div.	= ... cm.
No. of div. in circular Sc.	=	No. of div. in vernier	=
Least count	= ... cm.	Least count	= ... cm.
Zero-error	= ... cm	Zero-error	= ... cm.

or (c) by optical lever arrangement :

Distance 'p' of the front leg from the line joining the hind ones (found from foot impression on a paper) = ... cm.

Distance 'D' of the mirror from the scale = ... cm.

Calculations

S. No.	Distance between knife-edges (l) in cm	Load on the pan in gm	Micrometer or microscope reading *or* position of cross-wire on the image-scale: Load increase – Main	Vernier	Total	Load decreasing – Main	Vernier	Total	Mean	Depression or shift of the image-scale for ... gm
1.		...							...	
2.		...							...	
3.		...							...	...
4.		...							...	...
5.		...							...	...
6.		...							...	...
7.		...							...	
8.		...							...	
								Mean	...	

$$Y = \frac{mgl^3}{4bd^3y}*$$

In optical lever arrangement,

y (mean shift in image-scale) = $p/2D$.

% *error* :

Result

The value of Young's modulus for the material of the beam (correct to significant figures) = ... dyne/cm^2.

Precautions

(*i*) Care must be taken in measuring the length and depth of the beam as they occur in the third power.

(*ii*) The beam should be placed symmetrically on the two knife-edges.

(*iii*) Back-lash error should be avoided.

(*iv*) The loads should be placed or removed from the pan very gently and readings should be noted only after waiting for sometime.

QUESTIONS

(*i*) Define : Bending moment, neutral surface, geometrical moment of Inertia.

(*ii*) How are longitudinal stress and strain produced by bending ?

(*iii*) How does the bending moment act on the cantilever ?

(*iv*) Is there any other kind of stress produced in addition to longitudinal one ?

(*v*) Why do you not use a single cantilever ?

(*vi*) Why do you load and unload the bar in small steps ?

(*vii*) State some practical applications of your knowledge of Y.

(*viii*) Which of the two methods will you prefer to determine Y, by stretching or by bending moment ? Give reasons.

(*ix*) What will happen if the two knife-edges are not equidistant from the c.g. of the bar ?

(*x*) How will the depression change for the same load if :

(*a*) the length of the bar is made half,

(*b*) the breadth of the bar is made half,

(*c*) the thickness of the bar is made half ?

*The exact magnitude of loads is so selected that the depression of the beam caused by it is appreciable and conveniently measurable.

*For greater accuracy in the result, a number of sets of observations (atleast three) should be recorded for the *depression* with different distances between the knife-edges of the beam. A graph is then plotted between loads in gm and corresponding depressions. All these will be straight lines passing through the origin. Any vertical line is drawn cutting the three lines giving three values of ml^3/y for the different values of 'l'. The mean of ml^3/y is then found out which is substituted in the formula for determining the value of Y.

Exp. 6. Modulus of rigidity by statical method

Object

To determine the modulus of rigidity for the material of a wire by statical method using Barton's apparatus.

Apparatus

Barton's apparatus, metre scale, vernier callipers, screw gauge and slotted weights of half-kilogram each.

The apparatus consists of a wire AL clamped at A with its lower end fixed to a heavy cylinder as shown in fig. 6.1. Two parallel flexible threads leave opposite sides of the cylinder tangentially at two diametrically opposite points and passing over two identical frictionless pulleys carry pans of equal weights. Pointers screwed to the experimental wire can move over graduated circular scales B, C and D.

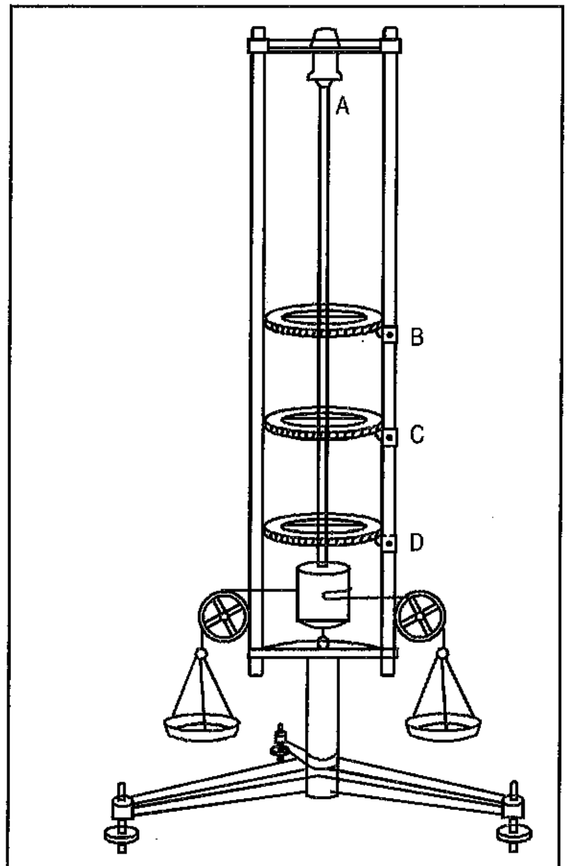

Fig. 6.1

Theory

Suppose, by applying a couple (placing equal weights on the pans), the lower end of the wire is twisted through an angle θ radian.

Let l, r, η be respectively the length, radius and co-efficient of rigidity of the material of the wire.

(*a*) The cylinder may be considered to consist of a large number of coaxial hollow cylindrical shells and in fig. 6.2, let us consider a cylindrical shell of radii x and $x + dx$ respectively. Let AB be a line (fig. 6.3) parallel to the axis DC before the cylinder is twisted. On twisting, B shifts to the position B$'$ and the hollow cylinder is sheared through an angle ϕ. If this hollow cylinder is cut and flattened out, it will form a rectangle $ABEF$ of sides l and $2\pi x$ (fig. 6.4). After

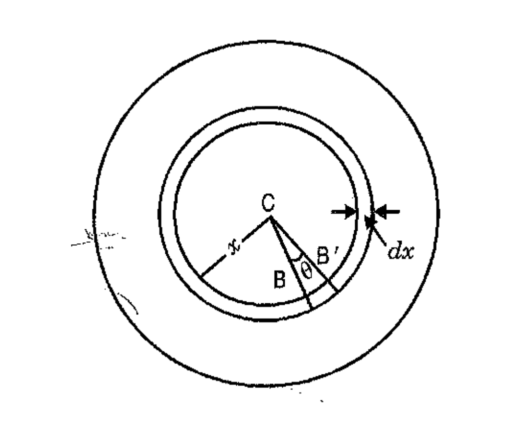

Fig. 6.2