Qualitative questions and answers in STR

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1. You are in a spaceship sailing along in outer space. Is there any way you can measure your speed without looking outside?

   Answer: There are two points to be made here. First, the question is meaningless, because absolute speed doesn't exist. The spaceship doesn't have a speed; it only has a speed relative to something else. Second, even if the question asked for the speed with respect to, say, a given piece of stellar dust, the answer would be "no." Uniform speed is not measurable from within the spaceship. Acceleration, on the other hand, is measurable (assuming there is no gravity around to confuse it with).

2. Two people, A and B, are moving with respect to each other in one dimension. Is the speed of B as viewed by A equal to the speed of A as viewed by B?

   Answer: Yes. The first postulate of relativity states that all inertial frames are equivalent, which implies that there is no preferred location or direction in space. If the relative speed measured by the left person were larger than the relative speed measured by the right person, then there would be a preferred direction in space. Apparently people on the left always measure a larger speed. This violates the first postulate. Likewise if the left person measures a smaller speed. The two speeds must therefore be equal.

3. Is the second postulate of relativity (that the speed of light is the same in all inertial frames) really necessary? Or is it already implied by the first postulate (that the laws of physics are the same in all inertial frames)?

   Answer: It is necessary. The speed-of-light postulate is not implied by the laws-of physics postulate. The latter doesn't imply that base balls have the same speed in all inertial frames, so it likewise doesn't imply that light has the same speed. It turns out that nearly all the results in special relativity can be deduced by using only the laws-of-physics postulate. What you can find (with some work) is that there is some limiting speed, which may be finite or infinite. The speedof-light postulate fills in the last bit of information by telling us what the limiting speed is.

4. Is the speed of light equal to c, under all circumstances?

Answer: No. In the postulate we stated as, “The speed of light in vacuum has the same value *c* in any inertial frame.” There are two key words here: “vacuum” and “inertial.” If we are dealing with a medium (such as glass or water) instead of vacuum, then the speed of light is smaller than *c*. And in an accelerating (noninertial) reference frame, the speed of light can be larger or smaller than *c*.

5. Can information travel faster than the speed of light?

   Answer: No. If it could, we would be able to generate not only causality-violating setups, but also genuine contradictions.

6. Can an object (with nonzero mass) move at the speed of light?

   Answer: The answer is no due to the following reason.

   (1) Energy: $v = c$ implies $\gamma = 1$, which implies that $E = \gamma mc^2$ is infinite. The object must therefore have an infinite amount of energy (unless $m = 0$, as for a photon).
   (2) Momentum: again, $v = c$ implies $\gamma = 1$, which implies that $p = \gamma m v$ is infinite. The object must therefore have an infinite amount of momentum (unless $m = 0$, as for a photon).
   (3) Velocity-addition formula: no matter what speed you give an object with respect to the frame it was just in (that is, no matter how you accelerate it), the velocity-addition formula always yields a speed that is less than *c*. The only way the resulting speed can equal *c* is if one of the two speeds in the formula is *c*.

7. Person *A* chases person *B*. As measured in the ground frame, they have speeds v_A and v_B. If they start a distance *L* apart (as measured in the ground frame), how much time will it take (as measured in the ground frame) for *A* to catch *B*?

   Answer: In the ground frame, the relative speed is $v_A - v_B$. Person *A* must close the initial gap of *L*, so the time it takes is $L=(v_A - v_B)$. There is no need to use any fancy velocity-addition or length-contraction formulas, because all quantities in this problem are measured with respect to the *same* frame. So it quickly reduces to a simple “(rate)(time) = (distance)” problem. Alternatively, the two positions in the ground frame are given by $x_A = v_A t$ and $x_B = L + v_B t$. Setting these positions equal to each other gives $t = L=(v_A - v_B)$. Note that no object in this setup moves with speed $v_A - v_B$. This is simply the rate at which the gap between *A* and *B* closes, and a gap isn’t an actual thing.

8. Two clocks at the ends of a train are synchronized with respect to the train. If the train moves past you, which clock shows a higher time?

Answer: The rear clock shows a higher time. It shows $L\mathrm{v}=c^2$ more than the front clock, where L is the proper length of the train.

9. Does the rear-clock-ahead effect imply that the rear clock runs faster than the front clock?

Answer: No. Both clocks run at the same rate in the ground frame. It's just that the rear clock is always a fixed time of $L\mathrm{v}=c^2$ ahead of the front clock.

10. Moving clocks run slow. Does this result have anything to do with the time it takes light to travel from the clock to your eye?

Answer: No. When we talk about how fast a clock is running in a given frame, we are referring to what the clock actually reads in that frame. It will certainly take time for the light from the clock to reach your eye, but it is understood that you subtract off this transit time in order to calculate the time (in your frame) at which the clock actually shows a particular reading. Likewise, other relativistic effects, such as length contraction and the loss of simultaneity, have nothing to do with the time it takes light to reach your eye. They deal only with what really *is*, in your frame.

11. Someone says, "A stick that is length-contracted isn't *really* shorter, it just *looks* shorter." How do you respond?

Answer: The stick really *is* shorter in your frame. Length contraction has nothing to do with how the stick looks, because light takes time to travel to your eye. It has to do with where the ends of the stick are at simultaneous times in your frame. This is, after all, how you measure the length of something. At a given instant in your frame, the distance between the ends of the stick is genuinely less than the proper length of the stick.

12. If you move at the speed of light, what shape does the universe take in your frame?

Answer: The question is meaningless, because it's impossible for you to move at the speed of light. A meaningful question to ask is: What shape does the universe take if you move at a speed very close to c (with respect to, say, the average velocity of all the stars)? The answer is that in your frame everything will be squashed along the direction of your motion, due to length contraction. Any given region of the universe will be squashed down to a pancake.

13. How can you prove that $E = \gamma mc^2$ and $p = \gamma mv$ are conserved?

    Answer: You can't. Although there are strong theoretical reasons why the E and p given by these expressions should be conserved, in the end it comes down to experiment. And every experiment that has been done so far is consistent with these E and p being conserved. But this is no proof, of course. As is invariably the case, these expressions are undoubtedly just the limiting expressions of a more correct theory.

14. The energy of an object with mass m and speed v is $E = \gamma mc^2$. Is the statement, "A photon has zero mass, so it must have zero energy," correct or incorrect?

    Answer: It is incorrect. Although m is zero, the γ factor is infinite because $v = c$ for a photon. And infinity times zero is undefined. A photon does indeed have energy, and it happens to equal hf, where h is Planck's constant and f is the frequency of the light.

15. How does the relativistic energy γmc^2 reduce to the nonrelativistic kinetic energy $\frac{1}{2}mv^2$?

    Answer: The Taylor approximation $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$ turns γmc^2 into $mc^2 + \frac{1}{2}mv^2$. The first term is the rest energy. If we assume that a collision is elastic, which means that the masses don't change, then conservation of γmc^2 reduces to conservation of $\frac{1}{2}mv^2$.

16. In a nutshell, why isn't F equal to ma (or even γma) in relativity?

    Answer: F equals dp/dt in relativity, as it does in Newtonian physics. But the relativistic momentum is $p = \gamma mv$, and γ changes with time. So $dp/dt = m(\gamma\dot{v} + \dot{\gamma}\mathrm{v}) = m\gamma\dot{v} + m\dot{\gamma}v$). The second term here isn't present in the Newtonian case.