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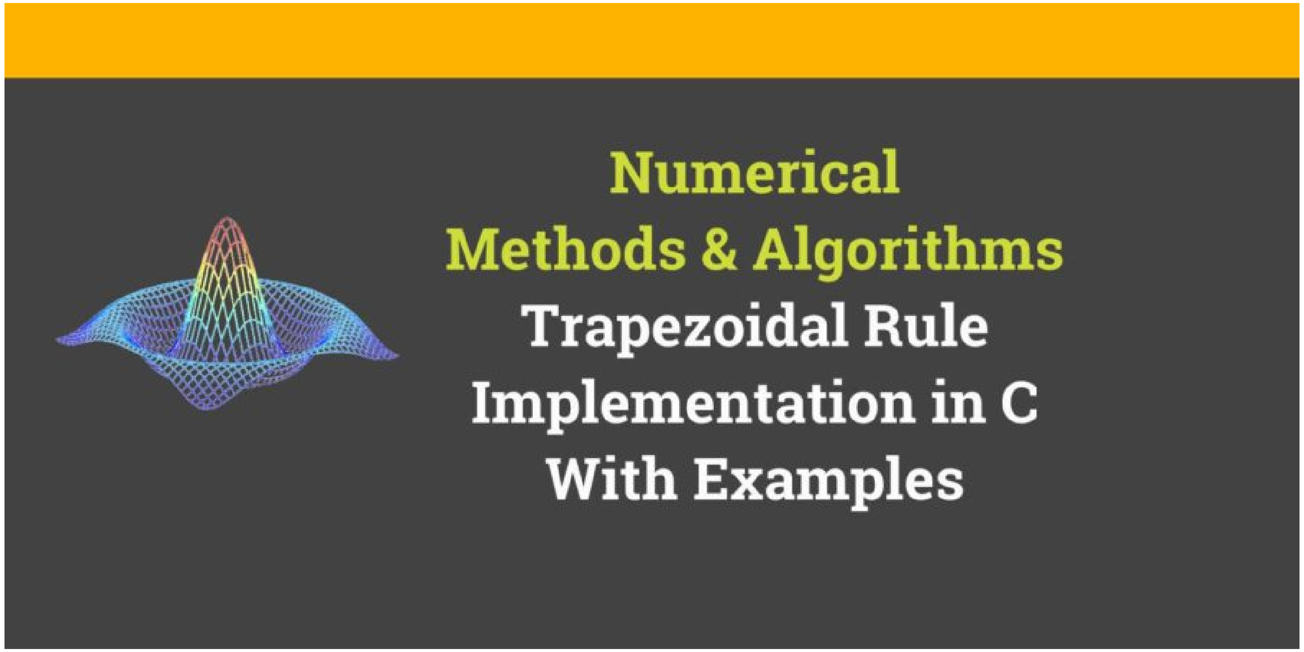

Trapezoidal Rule – Algorithm, Implementation in C With Solved Examples

NUMERICAL METHODS & ALGORITHMS / SUNDAY, OCTOBER 21ST, 2018

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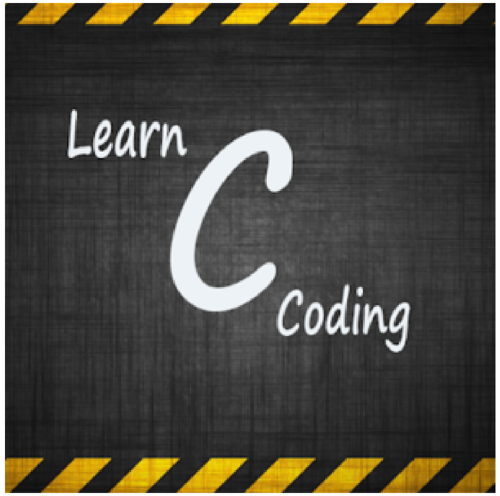

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At first we deduce the general integration formula based on Newton's forward interpolation formula and after that we will use it to formulate Trapezoidal Rule and Simpson's 1/3 rd rule.

The Newton's forward interpolation formula for the equi-spaced points **x_i, i =0, 1, ..., n, $x_i = x_0 + ih$** is

$$\phi(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots.$$

$$where\ \ u = \frac{x - x_0}{h},\ \ h\ \ is\ \ the\ \ spacing.$$

Let the interval [a, b] be divided into n equal subintervals such that **$a = x_0 < x_1 < x_2 < \ldots < x_n = b$**. Then

$$I = \int_a^b f(x)dx = \int_{x_0}^{x_n} \phi(x)dx$$

$$= \int_{x_0}^{x_n} \left[y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots.\right]dx$$

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Since $x = x_0 + uh$, $dx = hdu$, when $x = x_0$ then $u = 0$ and $x = x_n$ then $u = n$. Thus,

$$I = \int_0^n \left[y_0 + u\Delta y_0 + \frac{u^2 - u}{2!}\Delta^2 y_0 + \frac{u^3 - 3u^2 + 2u}{3!}\Delta^3 y_0 + \ldots.\right] hdu$$

$$= h\left[y_0[u]_0^n + \Delta y_0\left[\frac{u^2}{2}\right]_0^n + \frac{\Delta^2 y_0}{2!}\left[\frac{u^3}{3} - \frac{u^2}{2}\right]_0^n + \frac{\Delta^3 y_0}{3!}\left[\frac{u^4}{4} - u^3 + u^2\right]_0^n + \ldots.\right]$$

$$= nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{2n^2 - 3n}{12}\Delta^2 y_0 + \frac{n^3 - 4n^2 + 4n}{24}\Delta^3 y_0 + \ldots.\right] \ldots\ldots\ldots(1)$$

From this formula, one can generate different integration formulae by substituting n = 1, 2, 3, ...

Trapezoidal Rule

Substituting n = 1 in the equation (1). In this case all differences higher than the first difference become zero. Then

$$\int_{x_0}^{x_n} f(x)dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right]$$

$$\Rightarrow \int_{x_0}^{x_n} f(x)dx = h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right] = \frac{h}{2}(y_0 + y_1)\ldots\ldots\ldots.(2)$$

The formula (2) is known as the Trapezoidal Rule.

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In this formula, the interval [a, b] is considered as a single interval, and it gives a very rough answer. But, if the interval [a, b] is divided into several subintervals and this formula is applied to each of these subintervals then a better approximate result may be obtained.

This formula is known as composite formula, deduced below.

Composite Trapezoidal Rule

Let the interval [a, b] be divided into n equal subintervals by the points **$a = x_0 < x_1 < x_2 < \ldots < x_n = b$, where $x_i = x_0 + ih$, i = 1, 2, 3, …,n.**

Applying the trapezoidal rule to each of the subintervals, one can find the composite formula as

$$\int_a^b f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \ldots . + \int_{x_{n-1}}^{x_n} f(x)dx$$

$$= \frac{h}{2}[y_0 + y_1] + \frac{h}{2}[y_1 + y_2] + \ldots . + \frac{h}{2}[y_{n-1} + y_n]$$

$$\therefore \int_a^b f(x)dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \ldots . + y_{n-1}) + y_n]$$

Algorithm of Trapezoidal Rule

```
Step 1. Input f(x);
Step 2. Rread a,b,n; //the lower and upper limits and number of subintervals
Step 3. Compute h=(b-a)/n;
Step 4. Set sum =[f(a)+f(a+nh)]/2;
Step 5. for i=1 to n-1 do
 Compute sum = sum + f(a+ih);
 endfor;
Step 6. Compute result = sum * h;
Step 7. Print result;
```

Trapezoidal Rule Implementation in C

```
/* This program finds the value of integration of a function
   by Trapezoidal rule. Here we assume that f(x) = x^3. */

#include<stdio.h>

void main()
{
 float a,b,h,sum;
 int n,i;
 float f(float);
 printf("Enter the values of a, b: ");
 scanf("%f%f",&a,&b);
 printf("Enter the value of n: ");
 scanf("%d",&n);
 h=(b-a)/n;
 sum=(f(a)+f(a+n*h))/2;
 for(i=1;i<n;i++)
 sum+=f(a+i*h);
 sum=sum*h;
```

```c
 printf("The value of the integration is %8.5f: ",sum);
}

float f(float x)
{
 return(x*x*x);
}
```

Output

Enter the values of a, b: 0 1

Enter the value of n: 100

The value of the integration is 0.25002

Example 01

Find the value of

$$\int_{1.2}^{1.6} \left(x + \frac{1}{x}\right) dx$$

Taking 4 subintervals, correct up to four significant figures.

Solution:

$$Let\ \ f\left(x\right) = \left(x + \frac{1}{x}\right)$$

$$Here\ \ x_0 = 1.2,\ \ x_n = 1.6,\ \ n = 4$$

$$\therefore h = \frac{1.6 - 1.2}{4} = 0.1$$

The tabulated values of f(x) for different values of x are given below:

x	1.2	1.3	1.4	1.5	1.6
f(x)	2.033333	2.069231	2.114286	2.166667	2.225

By Trapezoidal Rule, we have

$$\int_a^b f(x)dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4]$$

$$\int_{1.2}^{1.6} \left(x + \frac{1}{x}\right) dx = \frac{0.1}{2}[2.033333 + 2(2.069231 + 2.114286 + 2.166667) + 2.225]$$

$$\therefore \int_{1.2}^{1.6} \left(x + \frac{1}{x}\right) dx = 0.8477$$

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