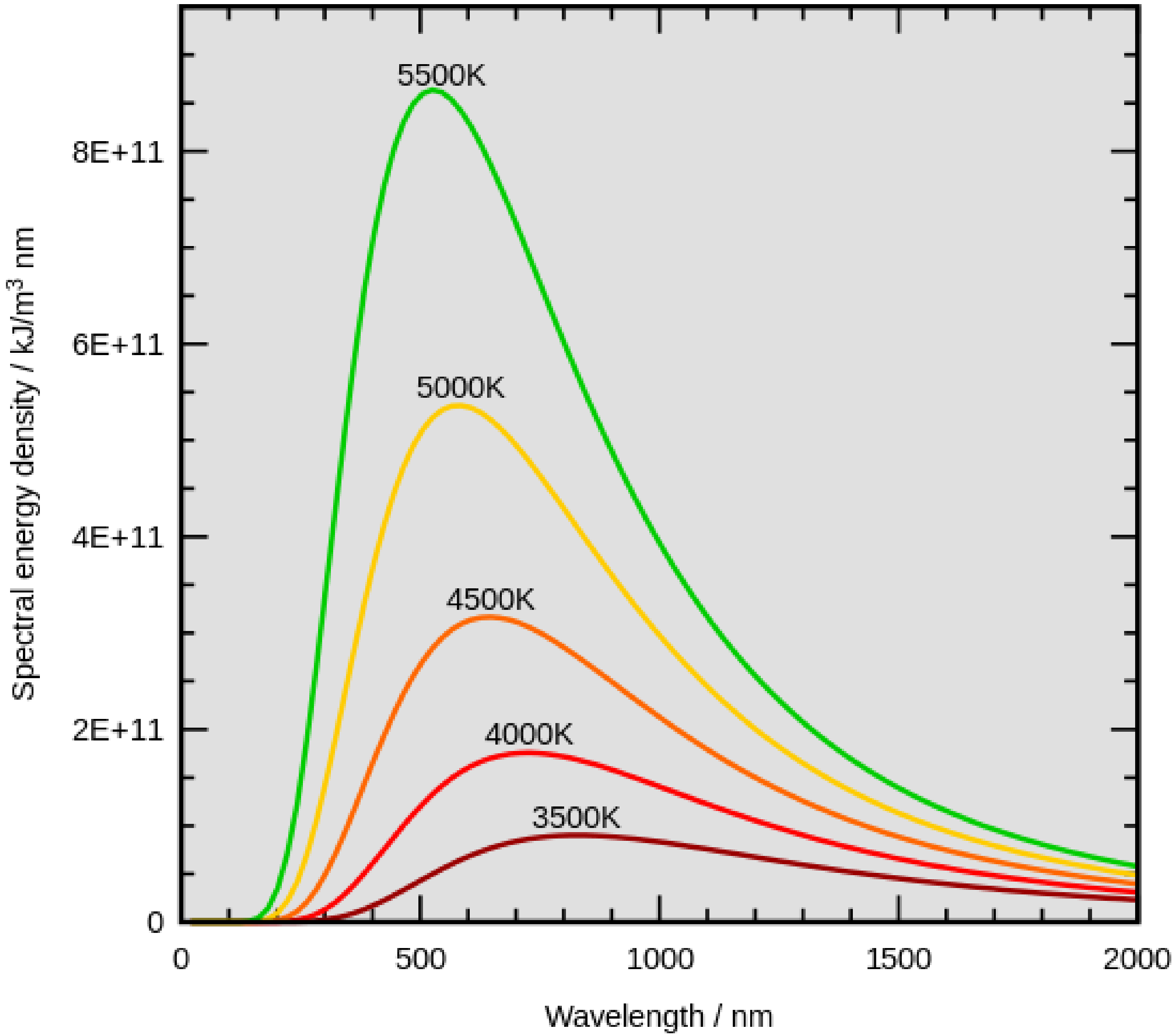
BPT-201 (semester II)
Topic: Blackbody Radiation-part 5
(Wien's Distribution Law)

Dr Neelam Srivastava
Department of Physics (MMV Section)
Banaras Hindu University
neelamsrivastava_bhu@yahoo.co.in
neel@bhu.ac.in

Experimental results of black body radiation energy distribution



What this result tells us

- Energy density is a function of temperature and wavelength both.
- For every temperature there is a particular wavelength which has maximum energy density.
- This maximum wavelength shows a shift with temperature. As the temperature increases λ_{max} shifts toward the lower value.
- In Wien's displacement law Wien has explained shifting relation between temperature and wavelength.
- In Wien's distribution law he has deduced a formula for estimating the energy density related with λ at given temperature.

Wien's Distribution Law

- In previous lecture (ppt) we studied regarding the Wien's displacement law.

- Which tells us how the wavelength of maximum intensity changes with change in temperature.

- Although long back Kirchhoff in 1860s, had shown that $u=u(\nu,T)$ can be presented by a universal function.

....................Wien's Distribution Law

- It could be formulated nearly after 40 years of his statement.
- Wien's in 1896, could find a relation for u in terms of ν and T,
- which is now known as Wien's distribution law.
- Here he tried to estimate the energy related to wavelength range at certain λ
- or better to say how energy is distributed in a radiation.

Proof of Wien's Distribution Law

- For estimating the energy content in particular wavelength let us start from our previous imaginary experiment (used for Wien's displacement law).

- Let us isolate wavelength between λ and $\lambda+d\lambda$ in the spherical enclosure subjected to adiabatic expansion.

- The work done = $p\ \Delta V$

- $P = u_\lambda\, d\lambda/3$ (as we have define energy density and pressure relation $p=u/3$ and here we are considering only wavelength $d\lambda$ range hence the if the energy density for λ is u_λ).

- So the work done = $u_\lambda\, d\lambda\, \Delta V/3$.

- The total energy content for particular wavelength = $u_\lambda d\lambda V$

- The work done must be equal to change in total energy available in volume V as calculated above hence we have

$$\frac{1}{3} u_\lambda d\lambda \Delta V = -\Delta(u_\lambda d\lambda V)$$

- or $$\frac{1}{3} u_\lambda d\lambda \Delta V = -\Delta(u_\lambda) d\lambda V - u_\lambda \Delta(d\lambda V)$$

or $$\frac{1}{3} u_\lambda d\lambda \Delta V = -\Delta u_\lambda \, d\lambda V - u_\lambda d\lambda \Delta V - u_\lambda V \Delta(d\lambda)$$

......................(A)

- Dividing the equation (A) by $u_\lambda d\lambda V$ we get

$$\frac{1}{3}\frac{\Delta V}{V} = -\frac{\Delta u_\lambda}{u_\lambda} - \frac{\Delta V}{V} - \frac{\Delta(d\lambda)}{d\lambda}$$

- or by rearranging and using $\dfrac{\Delta(d\lambda)}{d\lambda} = \dfrac{\Delta\lambda}{\lambda}$

- we get $\dfrac{4}{3}\dfrac{\Delta V}{V} = -\dfrac{\Delta u_\lambda}{u_\lambda} - \dfrac{\Delta\lambda}{\lambda}$ ....(B)

- Taking $V = \dfrac{4\pi}{3}r^3$ we get $\Delta V = \dfrac{4\pi}{3}3\,r^2\,\Delta \mathrm{r} = 4\pi r^2\,\Delta \mathrm{r}$

- So we get $$\frac{\Delta V}{V} = \frac{4\pi r^2\, \Delta r}{\frac{4\pi}{3} r^3} = 3\frac{\Delta r}{r}$$

- Hence $$\frac{\Delta r}{r} = \frac{1}{3}\frac{\Delta V}{V} = \frac{\Delta \lambda}{\lambda}$$

- Putting this in equation (B) we get $$5\frac{\Delta \lambda}{\lambda} + \frac{\Delta u_\lambda}{u_\lambda} = 0$$

- or $$u_\lambda \lambda^5 = constant = u_{\lambda'}\, \lambda'^5 \quad \text{........................(C)}$$

- Where $u_{\lambda'}$ is the energy density of wavelength λ' to which λ has transformed due to expansion

- Or we can say that u_λ is function of temperature

- Wien's displacement law says λT=constant so we can have

$$u_\lambda \lambda^5 = c\, f(\lambda T) \qquad \text{.....................(D)}$$

- Where c is also a constant.
- Hence from equations (C)and (D)

$$u_\lambda d\lambda = \frac{c}{\lambda^5}\, f(\lambda T)\, d\lambda$$

- This is Wien's distribution Law

- or we can write it as $u_\lambda d\lambda = \frac{cT^5}{\lambda^5 T^5} f(\lambda T)\, d\lambda$
- or $u_\lambda d\lambda = \frac{cT^5}{(\lambda T)^5} f(\lambda T)\, d\lambda = cT^5 F(\lambda T)\, d\lambda \; where \; F(\lambda T) = \frac{f(\lambda T)}{(\lambda T)^5}$
- so we get $\frac{u_\lambda}{T^5} = cF(\lambda T) = constant$
- Which results in $\frac{u_\lambda}{u_\lambda'} = \frac{T^5}{T'^5} = \left(\frac{T}{T'}\right)^5$
- Indicating that energy density directly increases with fifth power of absolute temperature.

Limitations of Wien's Distribution Law

- Although law successfully explains the energy distribution in shorter wavelengths

- But it fails in explaining the energy distribution in longer wavelength region.

- Another important limitation of the law is that according to Wien's distribution law, energy density has finite value even at absolute zero temperature.

- According to Stefan's Law it should be zero hence his law is a contradiction to Stefan's law at absolute zero.

Nice set of Lecture ppts for understanding the physics in better way

- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_relativity.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_chaos.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_dimension.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_galileo.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_maxwell.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_quanta1.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_quanta2.pdf

Study Material

- http://www1.itp.tu-berlin.de/brandes/public_html/qm/umist_qm/node3.html
- http://www.mrao.cam.ac.uk/~mph/concepts/
- http://galileo.phys.virginia.edu/classes/252/black_body_radiation.html#A%20Note%20on%20Wien%E2%80%99s%20Displacement%20Law
- http://hyperphysics.phy-astr.gsu.edu/hbase/wien.html