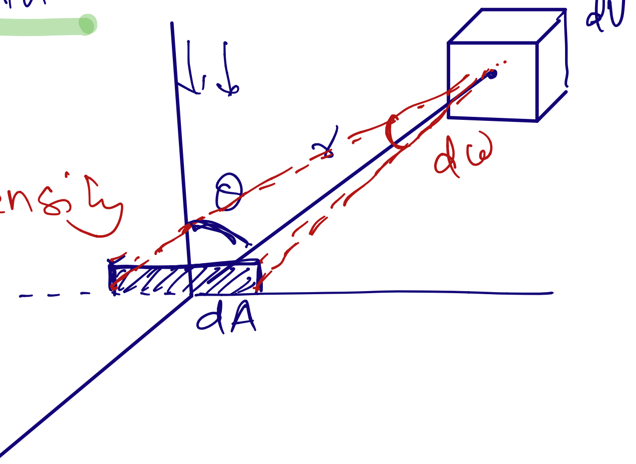
Pressure equation from Mean Free Path



n = number density

- Number of molecules in dV volume
$$dN = n\,dV$$

- If the collision frequency $= f_c$

- Then the total number of collision within dV in time dt.
$$= \frac{1}{2} f_c \, n\,dV \, dt$$
[Because in a collision two molecules interact at a time.]

- So the number of free path.
$$= 2 \times \frac{1}{2} f_c \, n\,dV \, dt$$

[Because each collision will result in two free paths]

$$= f_c \, n \, dV \, dt$$

- These free paths travel in each direction.
- So number of free paths in solid angle $d\omega$.

$$N_0 = \frac{d\omega}{4\pi} f_c n \, dV \, dt$$

$$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$d\omega = dA \cos\theta / r^2$$

$$N_0 = f_c \frac{n}{4\pi} \cdot \frac{dA \cos\theta}{\cancel{r^2}} \cancel{r^2} \sin\theta \, d\theta \, d\phi \, dr \, dt \cdot$$

$$= \frac{1}{4\pi} \cdot f_c \cdot n \cdot dt \, dA \cdot \sin\theta \cos\theta \, d\theta . d\phi . dr .$$

But we know the Survival Equation.

$$N = N_0 e^{-r/\lambda}$$

So the number of molecules reaching dA

$$N = \frac{1}{4\pi\sqrt{c}}\, n\, dt\, dA\, \sin\theta \cos\theta\, d\theta\, d\phi\, e^{-r/\lambda} dr$$

Now the total momentum change due to collision of one molecules at the surface.

$$= m\bar{c}\cos\theta - (-m\bar{c}\cos\theta)$$

$$= 2m\bar{c}\cos\theta.$$

for N molecules the momentum change

$$= 2m\bar{c}\cos\theta \;\; N$$

The total change of momentum by all molecules (coming from all the directions and distance) stricking the area dA in the time interval dt.

$$= 2m\bar{c}\cos\theta \; \frac{1}{4\pi\sqrt{c}}\, n\, dt\, dA\, \sin\theta \cos\theta\, d\theta\, d\phi\, e^{-r/\lambda} dr$$

$$= \frac{1}{4\pi} f_c n \cdot 2m\bar{c} \cdot dA\,dt \int_0^{\pi/2} \sin\theta \cos^2\theta \, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} e^{-r/\lambda} \, dr$$

$$= \frac{1}{\cancel{4\pi}} f_c n \, \cancel{2} m\bar{c} \; dA\,dt \; \frac{1}{3} \cdot \lambda \cdot \cancel{2\pi}$$

$$dF\,dt = \frac{1}{3} f_c \, m n \bar{c} \, \lambda \; dA\,dt$$

$$\frac{dF\,\cancel{dt}}{\cancel{dt}} = \frac{1}{3} f_c \, m n \bar{c} \, \lambda \, dA$$

$$\frac{dF}{dA} = \frac{1}{3} f_c \, m n \bar{c} \, \lambda$$

$$P = \frac{1}{3} m n \bar{c} \, \cancel{\lambda} \, \frac{\bar{c}}{\cancel{\lambda}}$$

$$\boxed{P = \frac{1}{3} m n \bar{c}^2}$$

TRANPORT PHENOMENA :

- In undisturbed state gas remains in equilibrium state.

- If this equilibrium state is disturbed either by thermal energy, mass motion or by adding a small amount of molecule at one end.

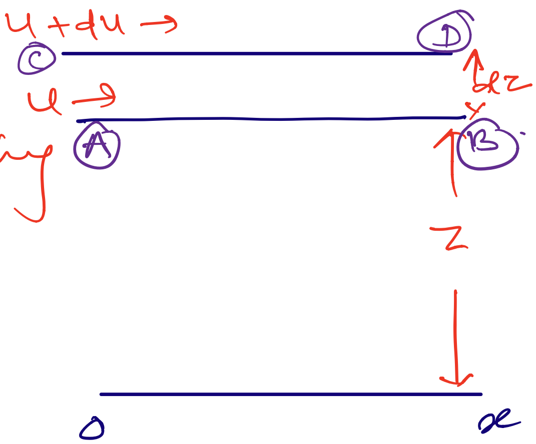
- Gas molecules will start moving so that they will have equilibrium state.

- and these moment actually result in transport of thermal energy / momentum / mass giving rise to:

Thermal conduction
Viscosity
Diffusion.

] Transport Phenomena.

Viscosity:



Two layers are having relative motion

- A force of friction works between two layer.
- This will increase the velocity of slow layer and decrease the velocity of fast layer.

This force of friction is called **VISCOUS FORCE** and the phenomenon is known as **Viscosity**

—x—

If the motion is not turbulent it is found that experimentally viscous force is proportional to

$$\left.\begin{aligned} F &\propto A \\ &\propto \frac{dU}{dZ} \end{aligned}\right] \quad F = \eta A \cdot \frac{dU}{dZ}$$

If the contact area $A = 1$

and velocity gradient $\frac{dU}{dZ} = 1$

then $\boxed{F = \eta}$ $\Rightarrow$ η = coefficient of viscosity.