BPT-201 (semester II)
Topic: Blackbody Radiation-part 6
(Rayleigh Jeans Law)

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The Right Honourable
The Lord Rayleigh
OM PC PRS

Born	12 November 1842 Langford Grove, Maldon, Essex, England
Died	30 June 1919 (aged 76) Terling Place, Witham, Essex, England
Nationality	English

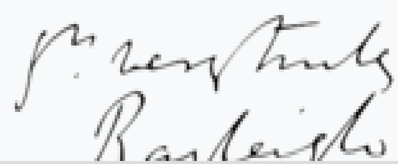
Discovery of argon
Rayleigh waves
Rayleigh scattering
Rayleigh criterion
Rayleigh–Bénard convection
Rayleigh's criterion
Rayleigh's method of dimensional analysis
Rayleigh–Ritz method
Rayleigh–Ritz inequality
Rayleigh quotient
Rayleigh–Lorentz pendulum
Rayleigh–Gans approximation
Duplex theory
Sound theory
Rayleigh flow
Rayleigh problem
Rayleigh–Plesset equation
Rayleigh–Schrödinger perturbation theory
Rayleigh–Taylor instability
Rayleigh–Jeans law
Rayleigh's equation

Awards	1865 Smith's Prize 1882 Royal Medal 1890 De Morgan Medal 1894 Matteucci Medal 1895 Faraday Lectureship Prize 1899 Copley Medal 1904 Nobel Prize in Physics 1905 Albert Medal 1913 Elliott Cresson Medal 1914 Rumford Medal

Scientific career

Fields	Physics, optics, acoustics
Institutions	Trinity College, Cambridge
Academic advisors	Edward John Routh Sir George Stokes[1]
Notable students	J. J. Thomson Jagadish Chandra Bose

Signature



Taken from: https://en.wikipedia.org/wiki/John_William_Strutt,_3rd_Baron_Rayleigh

Sir James Hopwood Jeans
OM FRS

Born	11 September 1877 Southport (Ormskirk Registration District), Lancashire, England
Died	16 September 1946 (aged 69) Dorking, Surrey, England
Nationality	British
Alma mater	Merchant Taylors' School; Cambridge University
Known for	Jeans instability Rayleigh–Jeans law Jeans mass Jeans length Method of image charges
Awards	Smith's Prize (1901) Adams Prize (1917) Royal Medal (1919)
	Scientific career
Fields	Astronomy, mathematics, physics
Institutions	Trinity College, Cambridge; Princeton University
Notable students	Ronald Fisher

Taken from :https://en.wikipedia.org/wiki/James_Jeans

Rayleigh Jeans Law

- Wien's law failed to explain the complete wavelength behavior of blackbody spectrum

- Then Rayleigh came with a different approach in 1900 where he showed that the energy density is a function of λ^{-4}.

- The present form of Rayleigh –Jeans law came in 1905.

- Rayleigh approach was based on following two points
  – theorem of standing waves in hollow space
  – theorem of equipartition of energy

Proof of Rayleigh Jeans Law

- Let the black body radiation be filled with diffused radiation of frequencies 0 to ∞

- Radiation is composed of electromagnetic waves in space

- And due to multiple reflection from walls they form standing wave in the space of enclosure

- Rayleigh through his theory predicted that number of possible independent vibrations between the frequencies ν to $\nu+d\nu$ per unit volume will be proportional to $\nu^2 d\nu$

- To prove it Rayleigh applied the analogy of stretched string
  - Which can vibrate in one, two or more segments resulting in fundamental or overtones.
  - Here each is considered to be independent considering that energy of each is independent of each other
- Rayleigh applied this analogy to radiation in following way
- Let there are two walls separated by a distance ‘a’
- A wave (λ) is moving between them at 90°

- The waves are reflected from boundaries and hence produces stationary waves characterized by 'N' nodes and 'A' antinodes

- let us considered that incident nad reflected waves are represented as

$$Y_1 = A \ \sin\frac{2\pi}{\lambda}(ct - x) \quad \text{or} \quad Y_2 = A \ \sin\frac{2\pi}{\lambda}(ct + x)$$

- Where 'c' is the velocity of wave.

- The resultant displacement of the particle at time 't' due to two wave trains will be

$$Y = Y_1 + Y_2 = A \ sin\frac{2\pi}{\lambda}(ct - x) + A \ sin\frac{2\pi}{\lambda}(ct + x)$$

- or $$Y = 2A \cos\frac{2\pi x}{\lambda} sin\frac{2\pi ct}{\lambda} = K\, sin\frac{2\pi ct}{\lambda}$$

Where $K = 2A \cos\frac{2\pi x}{\lambda}$

- k is resultant amplitude of the which is not a function of time but only the distance

- So for any time t, from the above equation Y=0 if

$$\cos\frac{2\pi x}{\lambda} = 0 \Rightarrow \frac{2\pi x}{\lambda} = \frac{(2r+1)\pi}{2} \quad or\, x = \frac{(2r+1)\lambda}{4} \quad where\, r = 0,1,2 \ldots.$$

- So the particles at distances $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$

are permanently at rest positions i.e. they are node positions

- If we consider three intersecting edges of the cubes at x, y, z

- $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are direction cosines of the direction of propagation of particular wave

- The projection of the edge of the cube on the direction of propagation will be ($a\cos\alpha$), ($a\cos\beta$) and ($a\cos\gamma$)

- Here only those waves are allowed for which the faces of cubes are nodal plane

- Hence $l = \cos\alpha$, $m = \cos\beta$, and $n = \cos\gamma$ which will give following where n_1, n_2 and n_3 are numbers $la = \frac{n_1\lambda}{2}, ma = \frac{n_2\lambda}{2}, and\ na = \frac{n_3\lambda}{2}$

- So we will get

$$n_1^2 + n_2^2 + n_3^2 = \frac{4}{\lambda^2}(l^2 + m^2 + n^2)a^2 = \frac{4a^2}{\lambda^2} \; becuase\; direction\; cosine\; (l^2 + m^2 + n^2) = 1$$

- Or

$$(n_1^2 + n_2^2 + n_3^2)^{\frac{1}{2}} = \frac{2a}{\lambda}$$

..............................(A)

- This gives the permissible value of wavelength

- Each choice of n_1, n_2 and n_3 corresponds to a frequency (or mode of vibration)

- The total number of modes of vibration are the total number of possible set of n_1, n_2 and n_3

- The number of modes of vibration between ν to $\nu+d\nu$ can be found from equation 'A'

- It can be proved that number of modes of vibration (proof given as annexure) per unit volume with frequency range

  ν to $\nu+d\nu$ will be $4\pi\nu^2 d\nu/c^3$

- The black body radiation travels with the velocity of light and are transverse in nature

- In case of transverse waves there are two possible polarization of each wave

- Considering this factor the number of possible modes of what we have previously estimated i.e. $2 \times 4\pi v^2 dv/c^3 = 8\pi v^2 dv/c^3$

- Now using the principle of equipartition of energy of each vibration will be given as $$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

- This is known as Rayleigh Jeans Law

Limitation of Rayleigh Jeans Law

- Rayleigh-Jeans law explains the energy distribution only for longer wavelengths region and it is not applicable at shorter wavelength region,

- Another important shortcoming of the law is that the energy density increases enormously as wavelength decreases

- This is a clear deviation from the experimental observations.

- The failure of the Rayleigh-Jeans law towards the lower wavelength side of the spectrum is particularly referred as 'Ultra-violet Catastrophe'.

- Although the concept of "ultraviolet catastrophe" originated in 1900 with the derivation of the Rayleigh-Jeans law.

- But Paul Ehrenfest was the first to use the term "ultraviolet catastrophe" in 1911.

- The phrase refers to the observation that below 105 GHz, Rayleigh jeans Law diverges with empirical observations specially when the frequencies reach the ultraviolet region of the electromagnetic spectrum.

- The ultraviolet catastrophe or the Rayleigh–Jeans catastrophe, is related with the prediction of classical physics.

- Which says that an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases.

- Hence when we calculate the total amount of radiated energy by summing the emissions in all frequency ranges, it can be shown that a blackbody is likely to release an arbitrarily high amount of energy.

- This would cause all matter to instantaneously radiate all of its energy until it is near absolute zero - indicating that a new model for the behavior of blackbodies was needed.

Annexure- calculation of number of modes

- We have already seen that $\lambda = 2a/n$ where $n = 1, 2, 3, 4 \ldots\ldots$

- And hence the corresponding frequencies are $\nu = c/\lambda = nc/2a$

- We have already seen that $(n_1^2 + n_2^2 + n_3^2)^{\frac{1}{2}} = \dfrac{2a}{\lambda}$

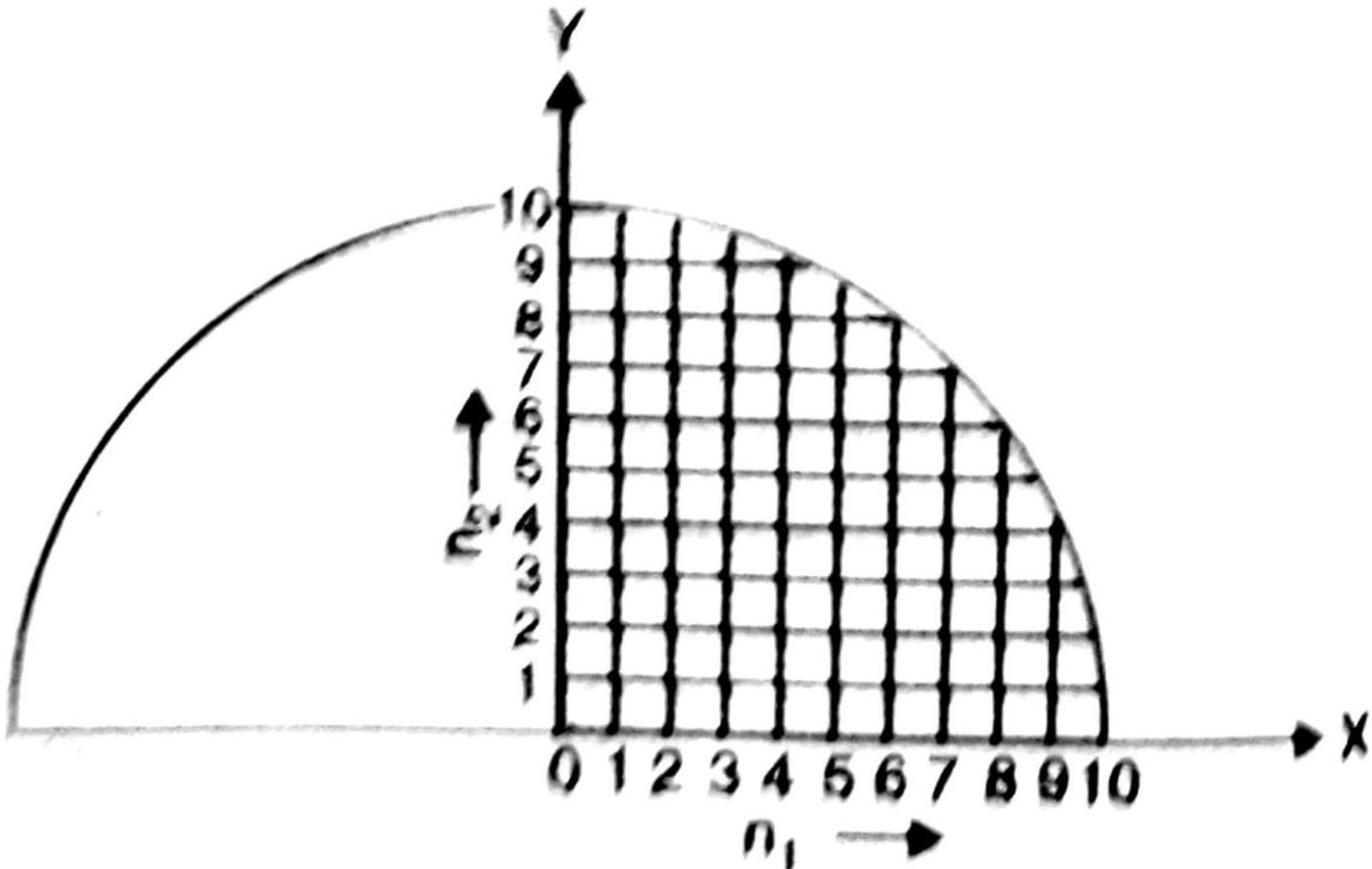
- Which can be written in terms of frequency as $(n_1^2 + n_2^2 + n_3^2)^{\frac{1}{2}} = \dfrac{2a\nu}{c}$

- Which gives the allowed frequencies (or mode of vibrations) which depends upon the possible combination of n_1, n_2 and n_3

- Just for simplicity let us start from two dimensional analogy

$$n_1^2 + n_2^2 = \left(\frac{2av}{c}\right)^2$$

- If we consider n_1 and n_2 at x and y-axis then it is an equation of circle, whose radius is $2a\nu/c$



- So each point on the circle is related with a possible value of ν

- While those points which are inside the circle correspond to frequency less than ν

- Intersection of lines, in figure, which are drawn at unit distance represent all possible combinations of n_1 and n_2

- The so formed squares have unit area and number of squares represent the are of quadrant of circle.

- Here we have to note that ‘n’ can have only positive values hence only positive quadrant is considered here

- Hence the number of modes of vibration with in frequency range ν to $\nu+d\nu$ will be equal to area of circle in positive quadrant lining between $2a\nu/c$ and $2a(\nu+d\nu)/c$

$$\frac{\pi}{4}\left[\left\{\frac{2a(\nu+d\nu)}{c}\right\}^2-\left(\frac{2a\nu}{c}\right)^2\right]=\frac{2\pi\nu a^2}{c^2}d\nu$$

- Now let us visualize it in 3-dimension -the positive quadrant of a sphere

- The volume of positive quadrant of sphere-volume of the sphere/8

- So the number of modes of vibration in 3-dimension in the range ν to $\nu+d\nu$ will be = volume of the spherical shell having radius from $2a\nu/c$ and $2a(\nu+d\nu)/c$

- Which will give us $$\frac{1}{8}\times\frac{4\pi}{3}\left[\left\{\frac{2a(\nu+d\nu)}{c}\right\}^3-\left(\frac{2a\nu}{c}\right)^3\right]=\frac{1}{8}\times\frac{4\pi}{3}\frac{8a^3}{c^2}3\nu^2 d\nu$$

$$=\frac{4\pi a^3\nu^2}{c^3}d\nu=\frac{4\pi V\nu^2}{c^3}d\nu \text{ where } V=\text{volume of cube}$$

- So the number of modes per unit volume will be $$=\frac{4\pi\nu^2}{c^3}\,d\nu$$

Nice set of Lecture ppts for understanding the physics in better way

- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_relativity.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_chaos.pdf
- http://www.mrao.cam.ac.uk/~mph/concepts/concepts_dimension.pdf
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Study Material

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