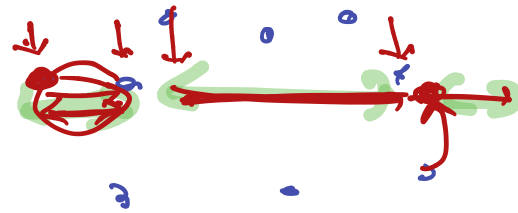
Mean Free Path

Free Path :



Path traversed by a molecule between two successive collision ⟹ Free Path

It is continously changing and depends upon the collision frequency.

That means we have to define an average free path — Mean Free Path.

The mean free path (λ) of a molecule is the average distance traversed by the molecule in between two successive collisions.

The Distribution of Free Path:

Let us find how many molecules have a free path between $x - x + dx$

Let us start

N_0 = Number of molecules in any system at a given instant and follow a group as the molecules collide

After every collison a molecules is knocked out.

N = the number of molecules remaining in the group after a distance x has been travelled by the group.

N = it is the number of molecules which did not incounter any collision.

dN = number of molecules which have been knocked out after distance dx.

$dN \propto N$

$\propto dx$.

$$dN = -P_c N dx \quad \text{———} \quad (A)$$

P_c = Probability of collision which depends upon physical condition of gas BUT not upon N and x

$$\frac{dN}{N} = -P_c dx$$

$$\ln N = -P_c x + \text{constant}.$$

$x = 0 \qquad N = N_0$

$$\ln N = -P_c x + \ln N_0$$

$$N = N_0 e^{-P_c x}. \quad \text{———} \quad (B)$$

So the number falls exponentially from eq (A) and (B)

$$dN = -P_c N_0 e^{-P_c x} dx \quad \text{(C)}$$

This is the number of molecules which have free path between x and $x + dx$.

Mean Free Path / Average free path:

$$\lambda = \frac{x_1 dN_1 + x_2 dN_2 + x_2 dN_3 + \ldots}{dN_1 + dN_2 + dN_3 + \ldots}$$

$$\lambda = \frac{1}{N_0} \int x \, dN$$

$$= \frac{1}{N_0} \int (-P_c) N_0 e^{-P_c x} x \, dx$$

$$= -\frac{P_c \cancel{N_0}}{\cancel{N_0}} \int_0^\infty e^{-P_c x} x \, dx = (-P_c)\left(-\frac{1}{P_c^2}\right)$$

$$= \frac{1}{P_c}$$

$$\lambda = \frac{1}{P_c} \Rightarrow P_c = \lambda^{-1}$$

From equation (B)

SURVIVAL EQUATION

$$N = N_0 e^{-P_c x} = \boxed{N_0 e^{-x/\lambda} = N}$$

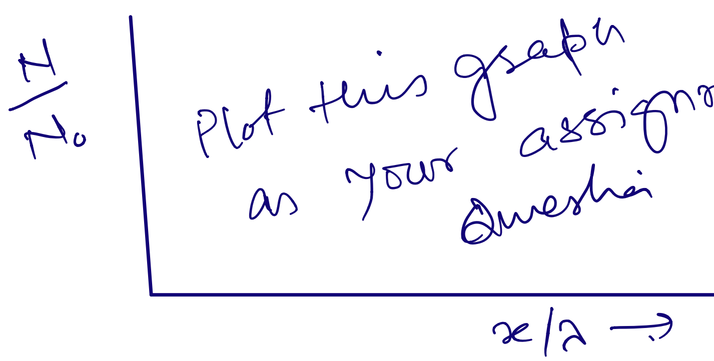
$N \Rightarrow$ represents the number of molecules after traversing the x distance

From equation (C)

$$dN = -\frac{N_0}{\lambda} e^{-x/\lambda}\, dx$$

$$\boxed{|dN| = \frac{N_0}{\lambda} e^{-x/\lambda}\, dx.}$$

From Survival Equation

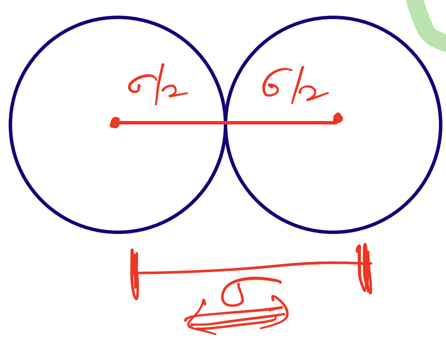


Expression for Mean Free Path:

Assumption:

- All molecules are at rest except one
- It is having velocity $\bar{c}$
- It moves through the frozen molecules
- Perfect elastic molecule having diameter $= \sigma$

$n =$ number density = or number of molecules in unit volume.



$X = \bar{c}t$

$\pi\sigma^2 = e$

$v = e\,\bar{c}t$

So effective cross sectional area of molecule $= \rho = \pi\sigma^2$

Let in 't' interval molecules sweeps out a cylindrical volume $= \rho \bar{c} t$

number of collisions in 't' $= \rho \bar{c} t n = A$

Collision frequency $= \boxed{Z = \rho \bar{c} n}$
$\Rightarrow$ number of collision per sec.

Mean Free Path $= \dfrac{X}{A}$

$$= \frac{\bar{c} t}{\rho \bar{c} t n} = \frac{1}{\rho n} = \frac{1}{\pi\sigma^2 n}$$

$$\boxed{\lambda = 1/\pi\sigma^2 n}$$

Mean free path estimated through <u>ELEMENTARY</u> <u>ANALYSIS</u>

Other Formulas

Clasius Method $\lambda = \frac{3}{4} \frac{1}{\pi \sigma^2 n}$

Maxwell Method $\lambda = \frac{1}{\sqrt{2}} \frac{1}{\pi \sigma^2 n}$

Elementary Analysis $\lambda = ? \frac{1}{\pi \sigma^2 n}$