Boolean Algebra

Logic Gates

B.Sc. 2nd Semester

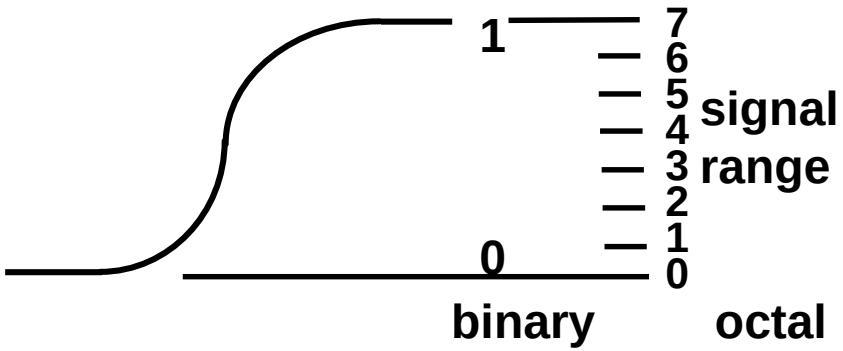
Introduction

- Developed by English Mathematician George Boole in between 1815 - 1864.
- It is described as an algebra of logic or an algebra of two values i.e True or False.
- The term logic means a statement having binary decisions i.e True/Yes or False/No.

Application of Boolean algebra

- It is used to perform the logical operations in digital computer.
- In digital computer True represent by ‘1’ (high volt) and False represent by ‘0’ (low volt).
  - **Why *BINARY* ? instead of Decimal or other number system ?**

* Consider electronic signal



***** Consider the calculation cost - Add

	0	1
0	0	1
1	1	10

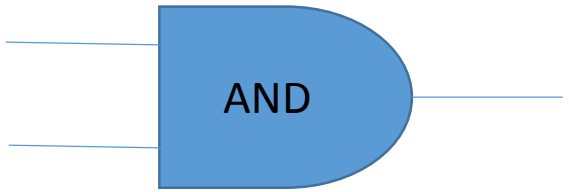
	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- Logical operations are performed by logical operators. The fundamental logical operators are:
  1. AND (conjunction)
  2. OR (disjunction)
  3. NOT (negation/complement)

AND operator

- It performs logical multiplication and denoted by (.) dot.

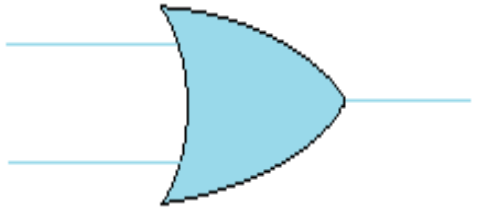
X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1



OR operator

- It performs logical addition and denoted by (+) plus.

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

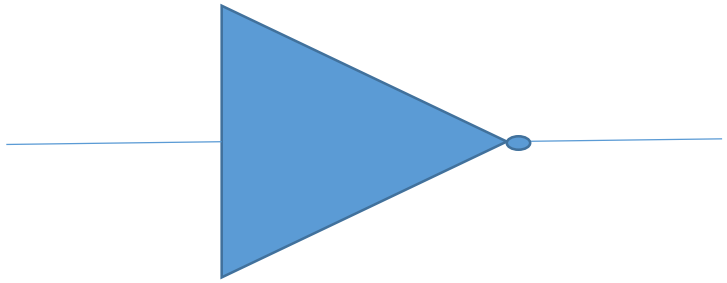


NOT operator

- It performs logical negation and denoted by (-) bar. It operates on single variable.

X	$\overline{X}$
0	1
1	0

(means complement of x)



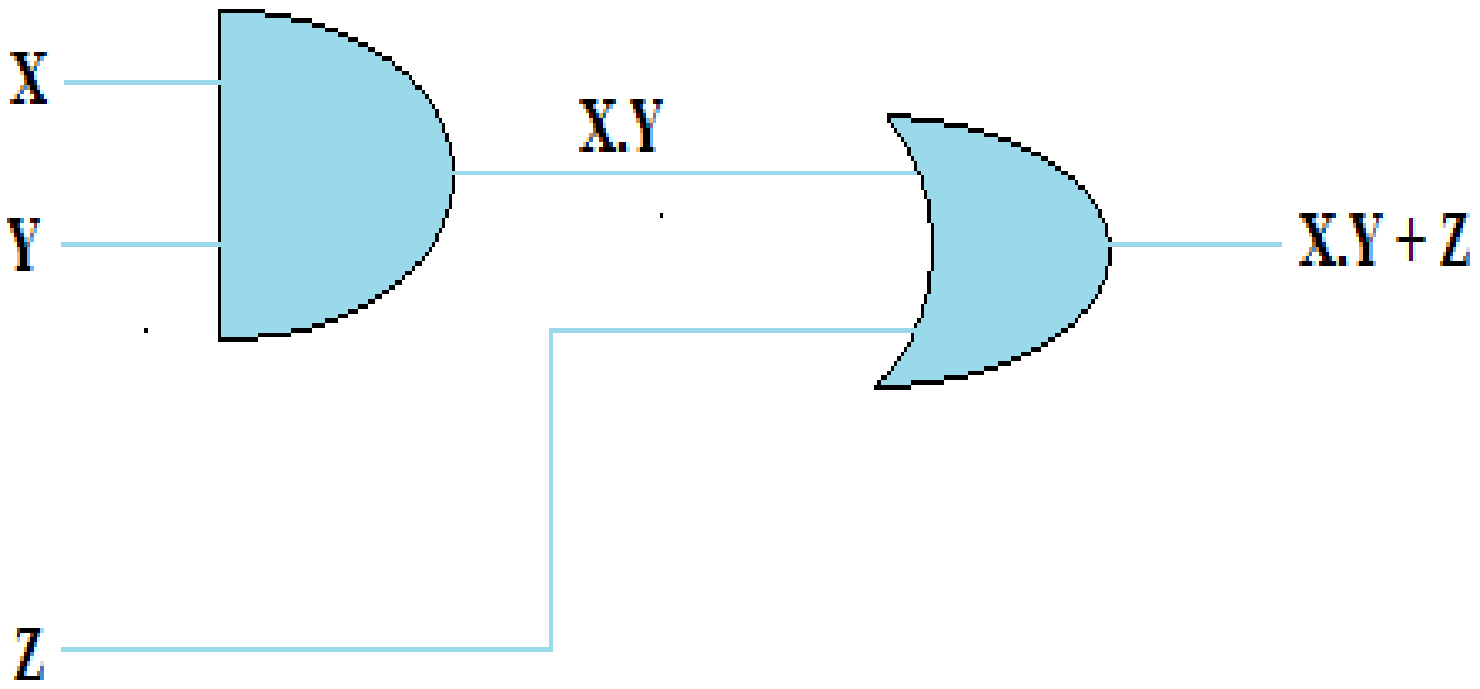
Truth Table

- Truth table is a table that contains all possible values of logical variables/statements in a Boolean expression.
- No. of possible combination = 2^n, where n=number of variables used in a Boolean expression.

Example

- The truth table for X.Y + Z is as follows:

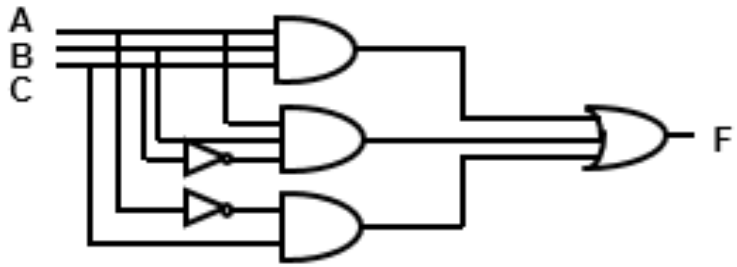
Dec	X	Y	Z	X.Y	X.Y + Z
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	1	1
7	1	1	1	1	1


X
Y
X.Y
Z
X.Y + Z

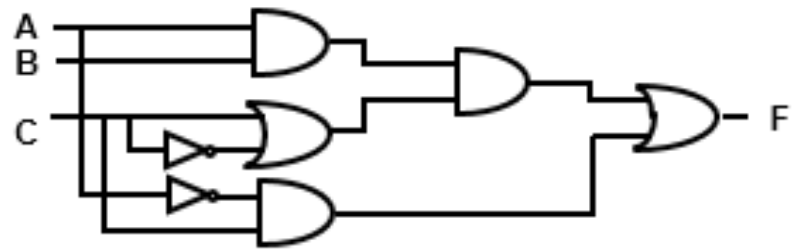
Many different logic diagrams are possible for a given Function

$$F = ABC + ABC' + A'C$$
$$= AB(C + C') + A'C$$
$$= AB \cdot 1 + A'C$$
$$= AB + A'C$$

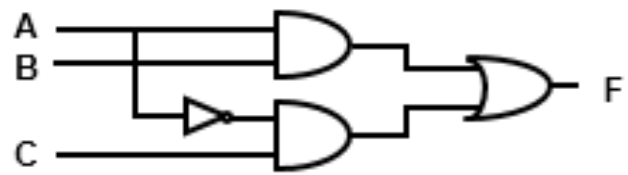
(1)



(2)



(3)



Tautology & Fallacy

- If the output of Boolean expression is always True or 1 is called Tautology.
- If the output of Boolean expression is always False or 0 is called Fallacy.

P	P’	$P \cap P'$	$P \cup P'$
0	1	0	1
1	0	0	1

$$P \cup P' \rightarrow Tautology$$

$$P \cap P' \rightarrow Fallacy$$

Exercise

- Evaluate the following Boolean expression using Truth Table.

  (a) X’Y’+X’Y

  (b) X’YZ’+XY’

  (c) XY’(Z+YZ’)+Z’

- Verify that P+(PQ)’ is a Tautology.
- Verify that (X+Y)’=X’Y’

Function of Boolean variables

X_1	X_2	Y
0	0	0
0	1	0
1	0	0
1	1	1

Y = 1 when X_1 = 1 and X_2 = 1

Y = $X_1 . X_2$

Function of Boolean variables

X_1	X_2	Y
0	0	0
0	1	0
1	0	0
1	1	1

Y = 1 when $\boldsymbol{X_1}$ = 1 and $\boldsymbol{X_2}$ = 1

Y = $\boldsymbol{X_1 . X_2}$

Y = $(\overline{\boldsymbol{X_1}} + \overline{\boldsymbol{X_2}}).(\overline{\boldsymbol{X_1}} + \boldsymbol{X_2}).(\boldsymbol{X_1} + \overline{\boldsymbol{X_2}})$

X_1	X_2	Y
0	0	0
0	1	1
1	0	1
1	1	0

X_1	X_2	Y
0	0	0
0	1	1
1	0	1
1	1	0

$$Y = (X_1 + X_2).(\overline{X_1} + \overline{X_2})$$
$$Y = \overline{X_1}.X_2 + X_1.\overline{X_2}$$

X_1	X_2	X_3	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

X_1	X_2	X_3	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$Y = (X_1 + X_2 + X_3).(X_1 + \overline{X_2} + X_3).(\overline{X_1} + X_2 + X_3).(X_1 + \overline{X_2} + \overline{X_3}) \quad \text{(POS)}$$

$$Y = (\overline{X_1}.\overline{X_2}.X_3) + (\overline{X_1}.X_2.X_3) + (X_1.\overline{X_2}.X_3) + (X_1.X_2.X_3) \quad \text{(SOP)}$$