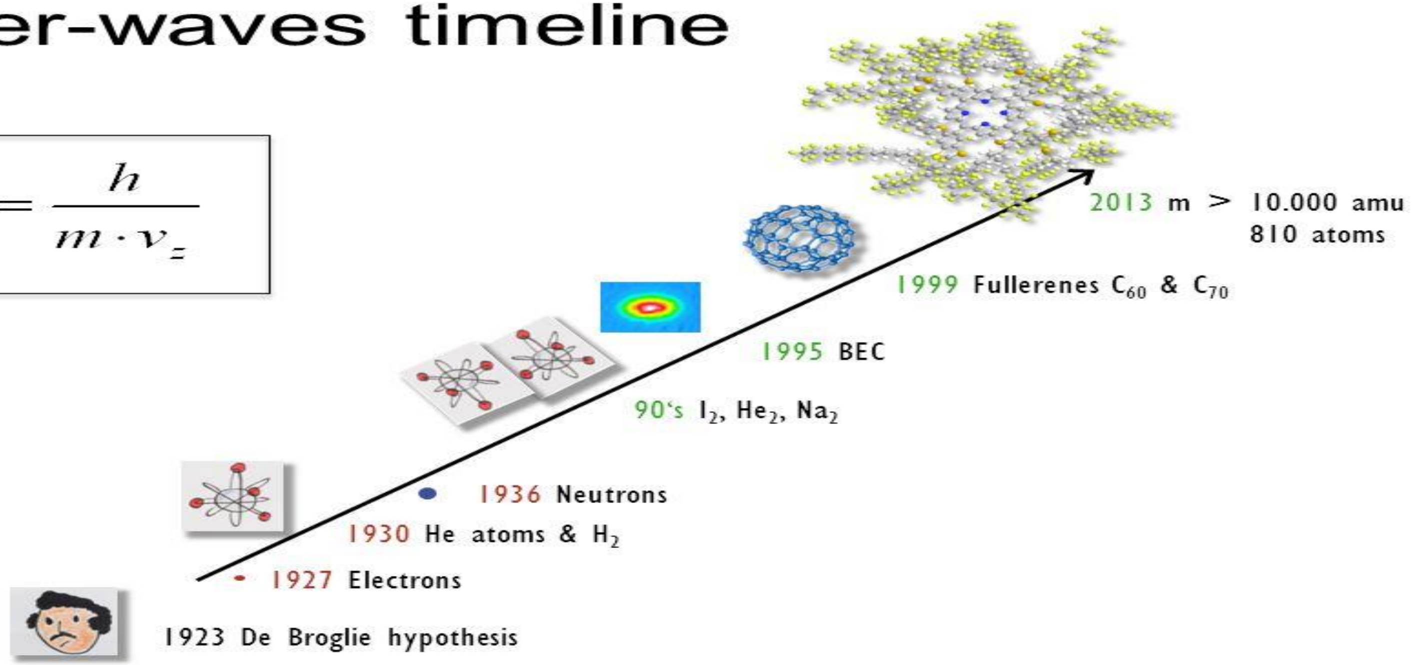
Matter-waves timeline

$$\lambda_{dB} = \frac{h}{m \cdot v_z}$$



Matter waves: water waves → Hight of water surface
Sound wave → Pressure.
E.M. waves → E & B.

representation $\psi(r, t)$

Probability density $|\psi|^2$ Max Born in 1926

de De-broglie wave velocity

$$V_p = \nu \lambda$$

$$\lambda = \frac{h}{\gamma m v}$$

$$E = \gamma m c^2 = h\nu$$

$$\nu = \frac{\gamma m c^2}{h}$$

De-broglie's phase velocity $V_p = \nu\lambda = \left(\frac{\gamma m c^2}{h}\right)\left(\frac{h}{\gamma m v}\right) = \frac{c^2}{v}$

means $V_p > c$ unexpected result

General wave formula. $y = A \cos 2\pi \nu t$

$$y = A \cos 2\pi\nu \left(t - \frac{x}{V_p}\right)$$

or $y = A \cos 2\pi \left(\nu t - \frac{\nu x}{V_p}\right)$

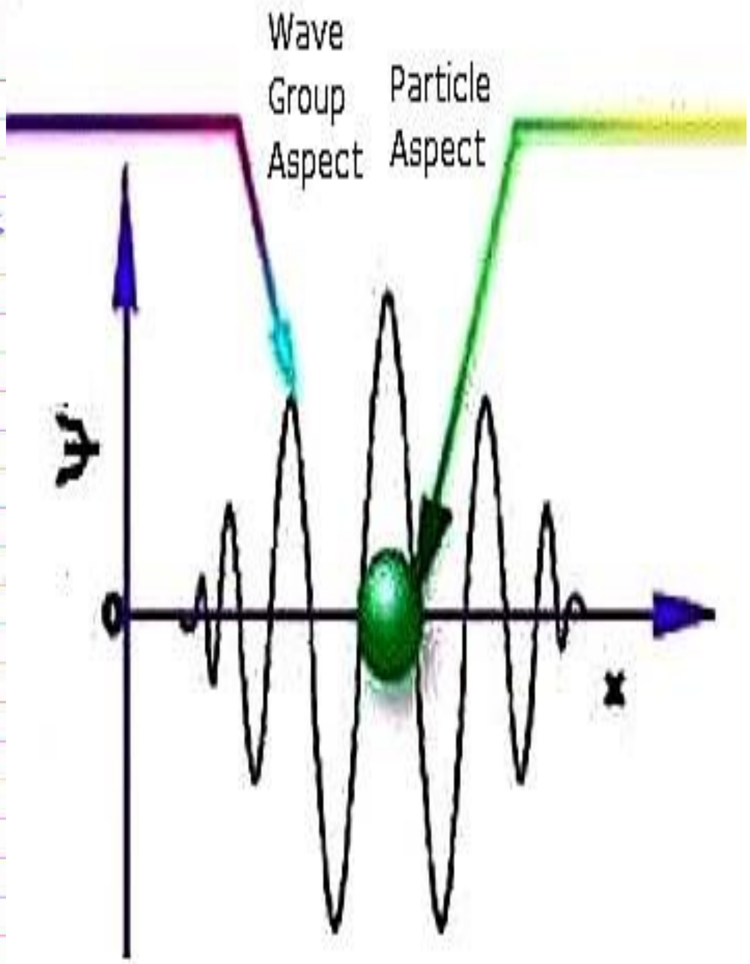
$$V_p = \nu \lambda.$$

So

$$y = A \cos 2\pi \left(\nu t - \frac{x}{\lambda}\right)$$

Angular frequency $\omega = 2\pi\nu$

Wave formula $k = \frac{2\pi}{\lambda} = \frac{\omega}{V_p}$



Wave formula.

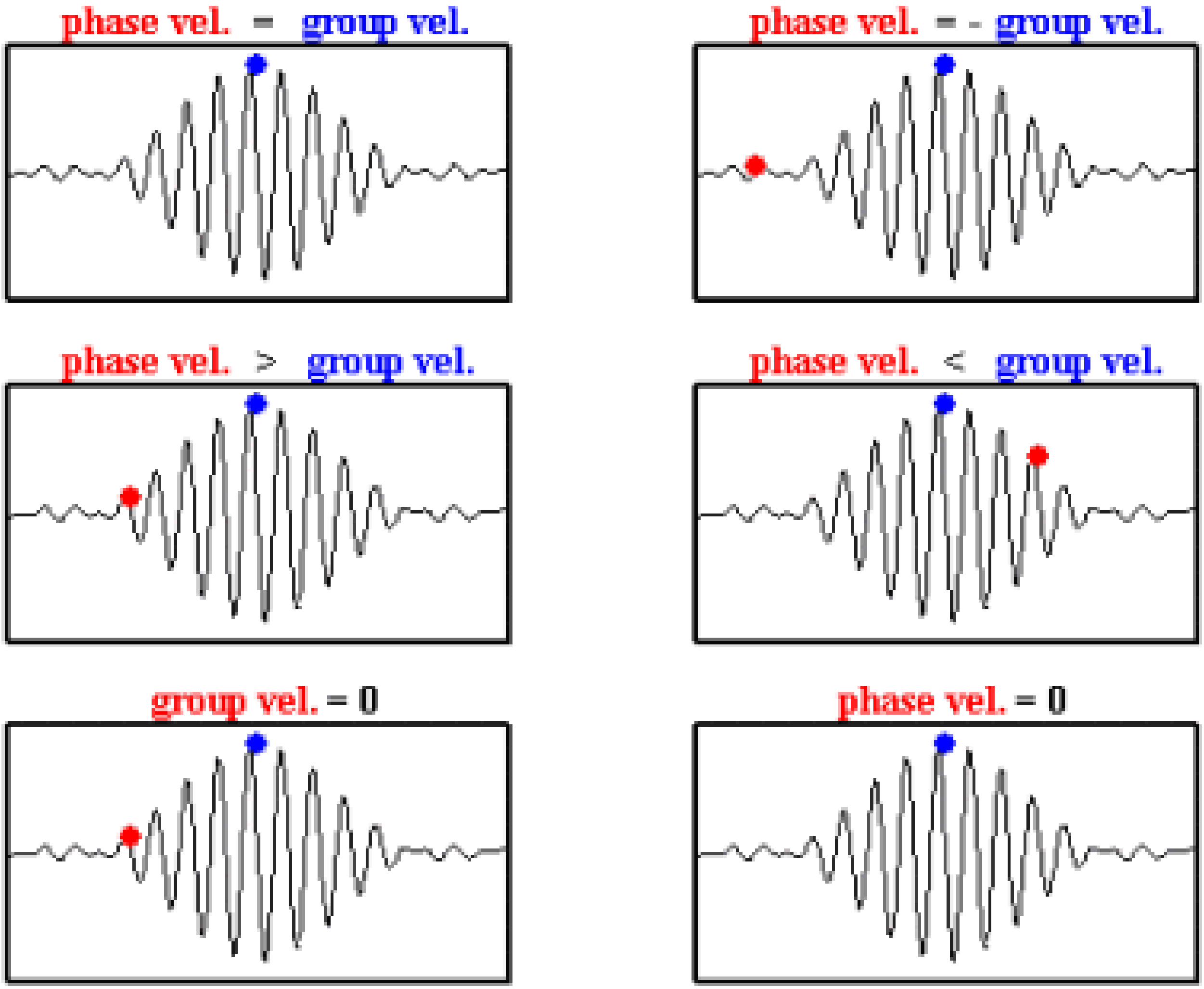
$$Y = A \cos(\omega t - Kx)$$ one dim

$$Y = A \cos(\omega t - K \cdot r)$$ Three dim

Phase and Group velocities:

$$V_p = \frac{\omega}{K}$$

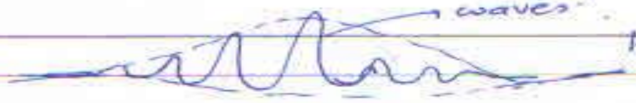
$$V_g = \frac{d\omega}{dK}$$


phase vel. = group vel.
phase vel. = - group vel.
phase vel. > group vel.
phase vel. < group vel.
group vel. = 0
phase vel. = 0

isvr

Matter waves

Wave packet or wave group.



Wave packet

$V_g \rightarrow$ group velocity — wave group travels

Superposition of two waves. (Beat):

$$Y_1 = A\cos(\omega t - kx)$$

$$Y_2 = A\cos[(\omega + \Delta\omega)t - (k+\Delta k)x]$$

$$Y = Y_1 + Y_2 = A\cos\alpha + A\cos\beta$$

$$= A(\cos\alpha + \cos\beta)$$

$$Y = A\left[2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)\right]$$

We know $\cos(-\theta) = \cos\theta$

Now

$$Y = Y_1 + Y_2$$

$$= 2A\cos\tfrac{1}{2}\left[(2\omega + \Delta\omega)t - (2k+\Delta k)x\right]$$

$$\cos\tfrac{1}{2}(\Delta\omega t - \Delta k x)$$

If $\Delta\omega$ and Δk are very small then.

$$2\omega + \Delta\omega \simeq 2\omega$$

$$2k + \Delta k \simeq 2k$$

So

$$Y = 2A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}t\right)$$

Means: A wave of angular frequency ω and wave number k that has superimposed upon it a modulation of angular frequency $\frac{\Delta\omega}{2}$ and wave number of $\Delta k/2$

$$\omega t - kx = 0$$
$$\frac{\omega}{k} = \frac{x}{t} = v_p$$

Phase velocity $v_p = \frac{\omega}{k}$

Group velocity. $v_g = \frac{\Delta \omega}{\Delta k}$

when ω and k are continuous spreads then.

$$v_g = \frac{d\omega}{dk}$$

Group velocity may be less or greater than the phase velocities of its member waves.

like v_g and v_p are same when light passes through the free space.

$$v_g = v \text{ (particle velocity)}$$
$$v_p = \frac{\omega}{k} = \frac{c^2}{v}$$

$v_p > c$ but $v < c$

So $\boxed{v_g < c}$

De' broglie waves therefore doesn't ~~violet~~ violet special relativity. Because v_p has no physical significance because motion of the wave group, not the motion of individual waves that make-up the group, corresponds to the motion of body $v_g < c$.