NEWTON-RALPH METHOD

① Find the real root of the eqⁿ $3x - \cos x - 1 = 0$ using Newton Raphson method, correct to 3 decimal places.

② Obtain cube root of 12 correct to four decimal places by Newton Raphson method

① $f(x) = 3x - \cos x - 1$

Taking initial approximation as $x_0 = 1$

We have $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

Using Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(3x_n - \cos x_n - 1)}{(3 + \sin x_n)}, \quad n = 0, 1, 2, \ldots$$

Starting with $x_0 = 1$

we get :-

$x_1 = 0.620016$

$x_2 = 0.607121$

$x_3 = 0.607102$

The exact value correct to 3 decimal places is 0.607102

(2) According to question :-

$f(x) = x^3 - 12$

Taking inital approximation as $x_0 = 2$

We have $f(x) = x^3 - 12$

$f'(x) = 3x^2$

Using Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 12)}{(3x_n^2)}, \quad n = 0, 1, 2, \ldots$$

Starting with $x_0 = 2$

we get :-

$x_1 = 2.333333$

$x_2 = 2.90249$

$x_3 = 2.289429$

$x_4 = 2.289429$

The exact value correct to 6 decimal places is 2.289429