B.Sc. III Semester

Numerical Computing

Paper code: CS-103

Suggested Readings:

1. Numerical Methods by **S.R.K Lyenger & R.K. Jain.**

2. Introductory methods of Numerical analysis by **S.S. Sastry**.

Course Content

- Error in Numerical Calculations.
- Solution of Algebraic and Transcendental Equations.
- Interpolation.
- Numerical Differentiation.
- Numerical Integration.
- System of Linear equations.
- Eigen Value problem.
- Numerical solution of ordinary differential equations.

Introduction to Numerical Methods

- For the Second-order polynomial equation:

$$ax^2 + bx + c = 0$$

analytical solution:

$$x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$$

- For many types of problems, such as a 5th-order polynomial, a closed-form or analytical solution does not exist. Then the <u>iterative</u>, or <u>numerical</u>, approach must be used.

Characteristics of Numerical Methods

1. The solution procedure is iterative, with the accuracy of the solution improving with each iteration.
2. The solution procedure provides only an approximation to the true, but unknown, solution.
3. An initial estimate of the solution may be required.
4. The algorithm is simple and can be easily programmed.
5. The solution procedure may occasionally diverge from rather than converge to the true solution.

Exact and Approximate numbers

- There are two kinds of numbers: exact and approximate numbers.

  Example- 1, 2, 3, ½, 3/2 etc.

- Approximate numbers are those that represents the number to a certain degree of accuracy

  Example-approximate value of π is 3.1416 or for better approximation we can use 3.14154.

Significant digits

- The digits that are used to express a number are called significant digits

Example 1- 3.1416, 0.66667 and 4.0687 contain five significant digits.

Example 2- 0.0023 has two significant digits.

Example 3- 0.00145, 0.000145 & 0.0000145 has three significant digits.

Note –In case of ambiguity scientific notation should be used

Example- $25600 = 2.56 \times 10^4 = 2.560 \times 10^4 = 2.5600 \times 10^4$ have 3, 4 and 5 significant digits respectively.

Round off

To round off a number to n significant digits, discard all the digits to the right of n^{th} digit, and if the discarded number is

1. Less than half unit in the nth place, leave the nth digit unaltered.
2. greater than half unit in the nth place, increase the nth digit by unity.
3. Exactly half a unit in the nth place , increase the nth digit by unity if it is odd, otherwise leave it unchanged.

Round off to 4 significant digit

Example 1: 1.6583 $\longrightarrow$ 1.658

Example 2: 30.0567 $\longrightarrow$ 30.06

Example 3: 0.859378 $\longrightarrow$ 0.8594

Example 4: 3.14159 $\longrightarrow$ 3.142

Error in Computation

- In numerical analysis, you will get numerical solution to a particular problem.

- So for the desired solution, you have to ensure what are errors and what are the sources of errors.

Errors Types

- In general, errors can be classified based on their sources as non-numerical and numerical errors.

- Non-numerical errors:

  (1) modeling errors: generated by assumptions and limitations.

  (2) blunders and mistakes: human errors

  (3) uncertainty in information and data

Source of Numerical errors:

(1) *round-off errors*: due to a limited number of significant digits.

(2) *truncation errors*: due to the truncated terms e.g. infinite Taylor series

Numerical Errors

- **Absolute errors**
- **Relative errors**
- **percentage errors**

Error= True value (X)- approximate value (X1)

- Absolute error (E^A) = |Error|
- Relative Error (E^R) = E^A /true value
- Percentage Error (E^P) = $E^R \times 100$

If ΔX be a number such that

$$|X1 - X| \leq \Delta X$$

Then ΔX is a upper limit on the magnitude of the absolute error.

Example: If the number X is rounded to N decimal places then

$$\Delta X = 1/2\ (10^{-N})$$

If X= 0.51 and is correct to 2 decimal places then $\Delta X = 0.005$

$E^R = 0.005/0.51 = 0.98$

Example: Numerical Errors Analysis

$$x^3 - 3x^2 - 6x + 8 = 0$$

The initial estimate $x_0 = 2$

$$x_1 = \sqrt{3x_0 + 6 - \frac{8}{x_0}} = 2.828427$$

error: $e = x_1 - x_0 = 0.828427$

$$x_2 = \sqrt{3x_1 + 6 - \frac{8}{x_1}} = ??$$

Thank You