BPT-401

Date-08/03/2021

The operator $\bar{\nabla}$:

$T(x, y, z)$

$$\bar{\nabla} T = \left(\hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}\right) \checkmark \quad \text{gradient}$$

where

$$\bar{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$\bar{\nabla}$ = vector operator

$\bar{A}$, $\bar{A}a$, $\bar{A} \times \bar{B}$

$\bar{A} \cdot \bar{B}$,

(i) On a scalar function T: $\bar{\nabla} T$ (The gradient of T)

(ii) On a vector function $\bar{v}$; via dot product

$\bar{\nabla} \cdot \bar{v}$ (the divergence)

(iii) " " " " $\bar{v}$; via cross product

$\bar{\nabla} \times \bar{v}$ (the curl)

Gradient

Scalar function $T(x, y, z)$

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)$$

$$= (\bar{\nabla} T) \cdot d\bar{l}$$

where, $\bar{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$

Geometrical interpretation :

$\bar{\nabla} T$ has magnitude & direction

$$dT = \bar{\nabla} T \cdot d\bar{l} = |\bar{\nabla} T| \, |d\bar{l}| \cos\theta$$

θ angle between $\bar{\nabla} T$ & $d\bar{l}$

let say fix magnitude of $|d\bar{l}|$

θ - vary $\rightarrow$ to find dT is maximum

$\theta = 0$ then $dT = max$

$\bar{\nabla} T$ Slope of in that direction is the magnitude.

$\underline{\bar{\nabla} T = 0}$, $dT = 0$ for small displacement about (x, y, z)

The divergence

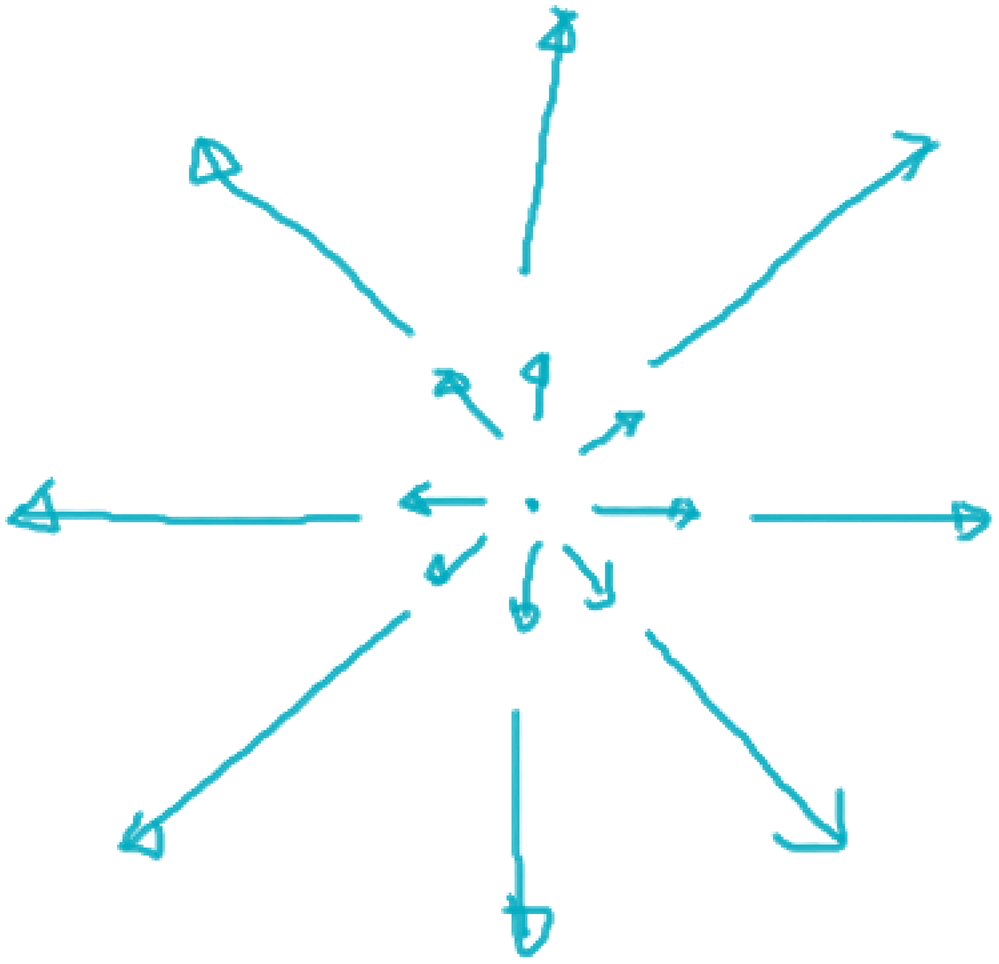
$$\bar{\nabla} \cdot \bar{v} = \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \cdot \left(v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \right)$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

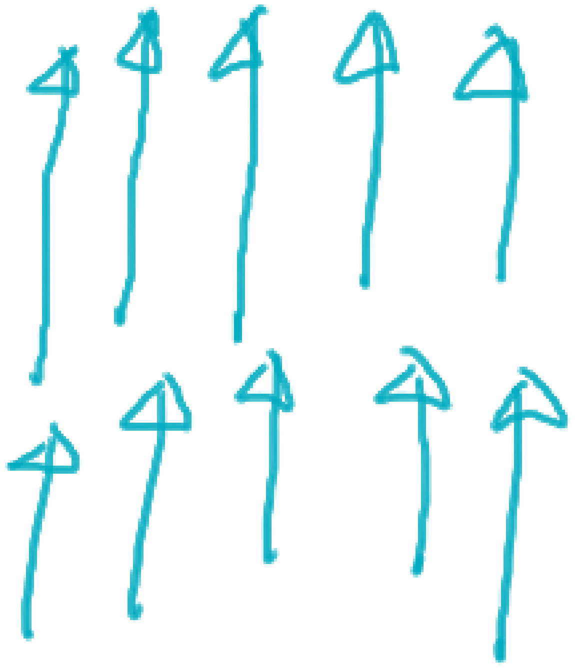
'diverge' $\rightarrow$ spreads out

$\vec{v}$ is spreading out from a point in question

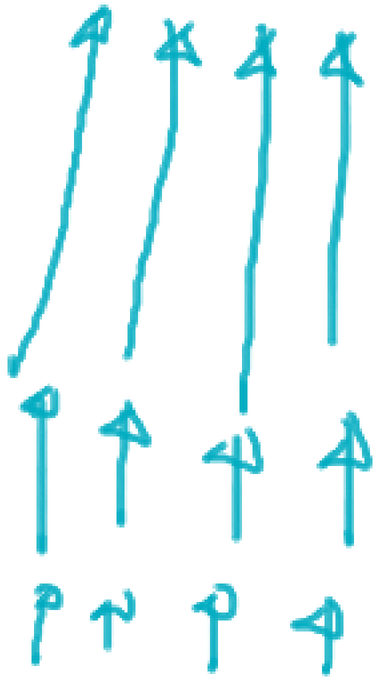
$\cancel{\frac{dt}{dx}}$ $\cancel{\frac{dt}{dn}}$



(+ve) divergence



Zero divergence



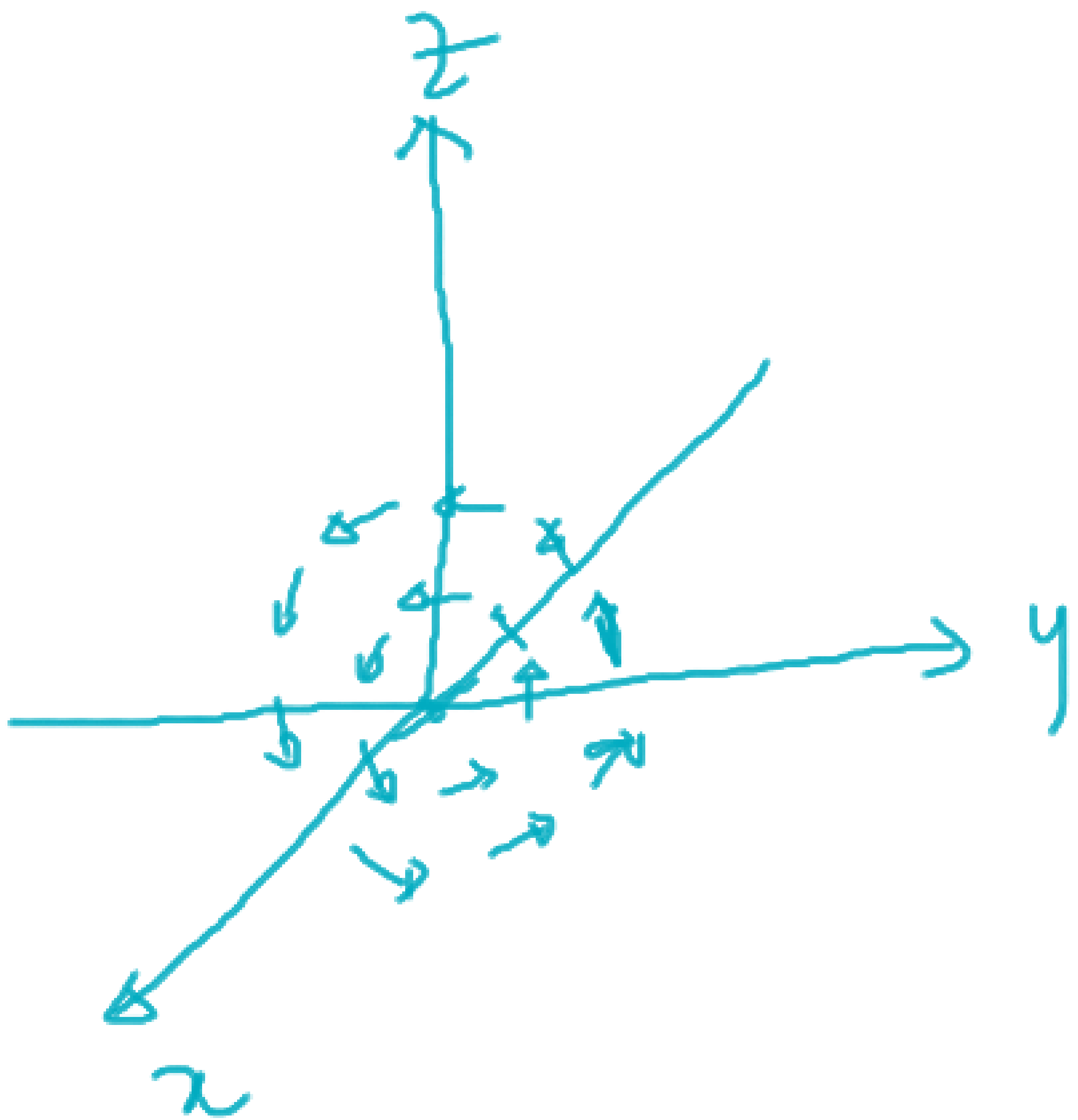
(+ve divergence)

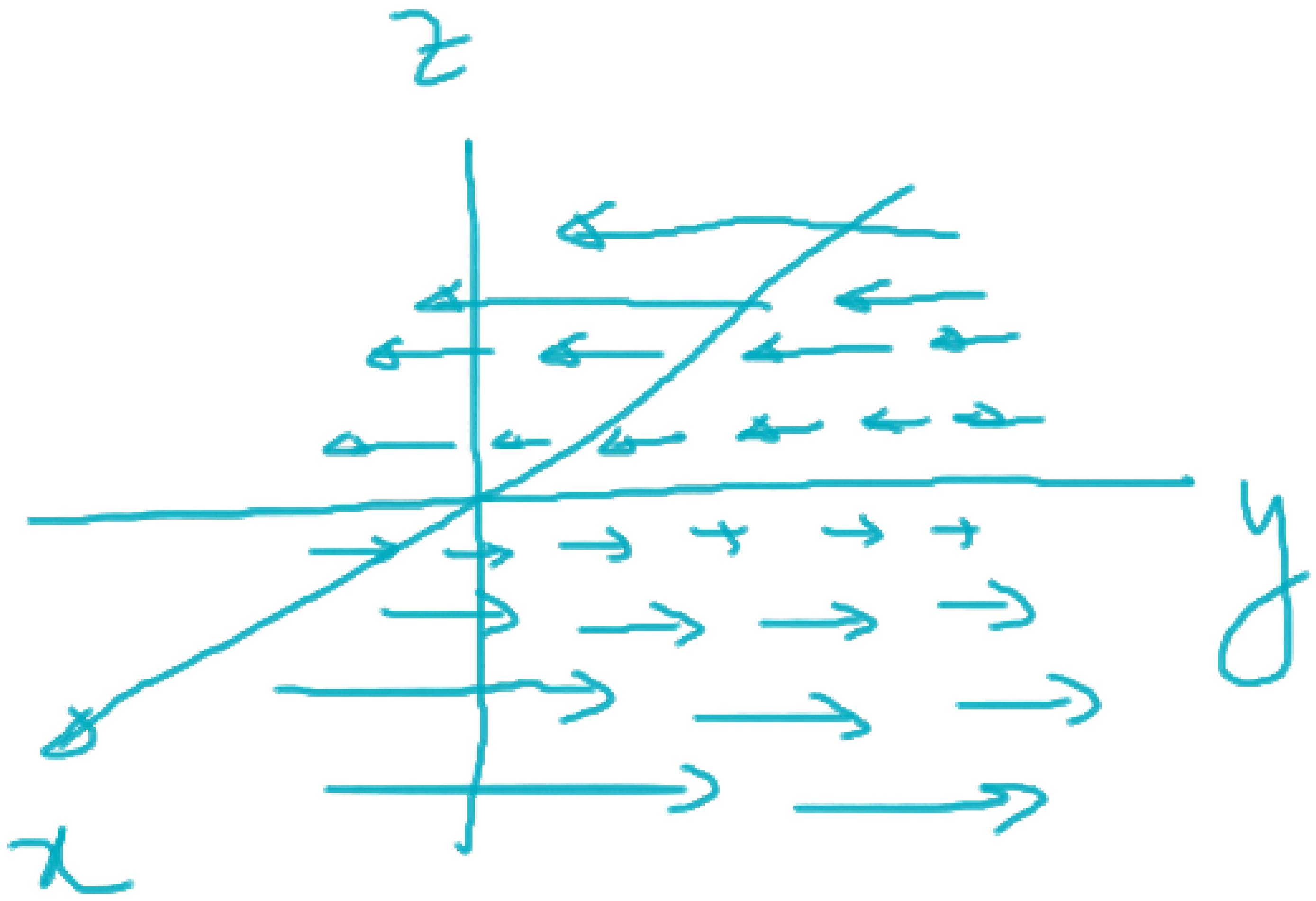
The curl

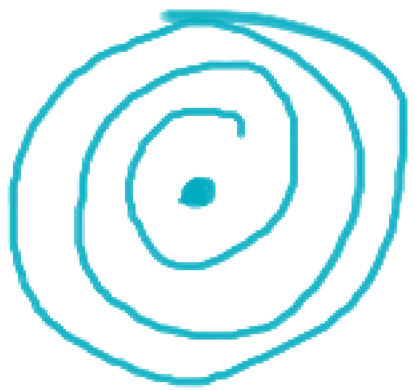
$$\bar{\nabla} \times \bar{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial z}\right) - \hat{y}\left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

The 'curl' now the $\bar{v}$ curls around a point


z
y
x


z
y
x

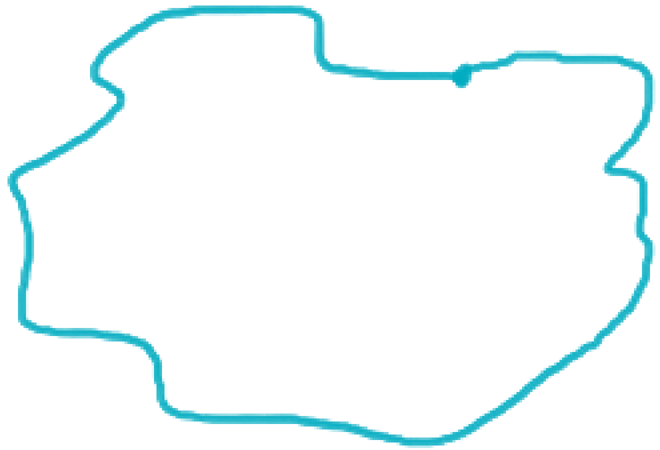


Integral calculus:

line (or path) integrals

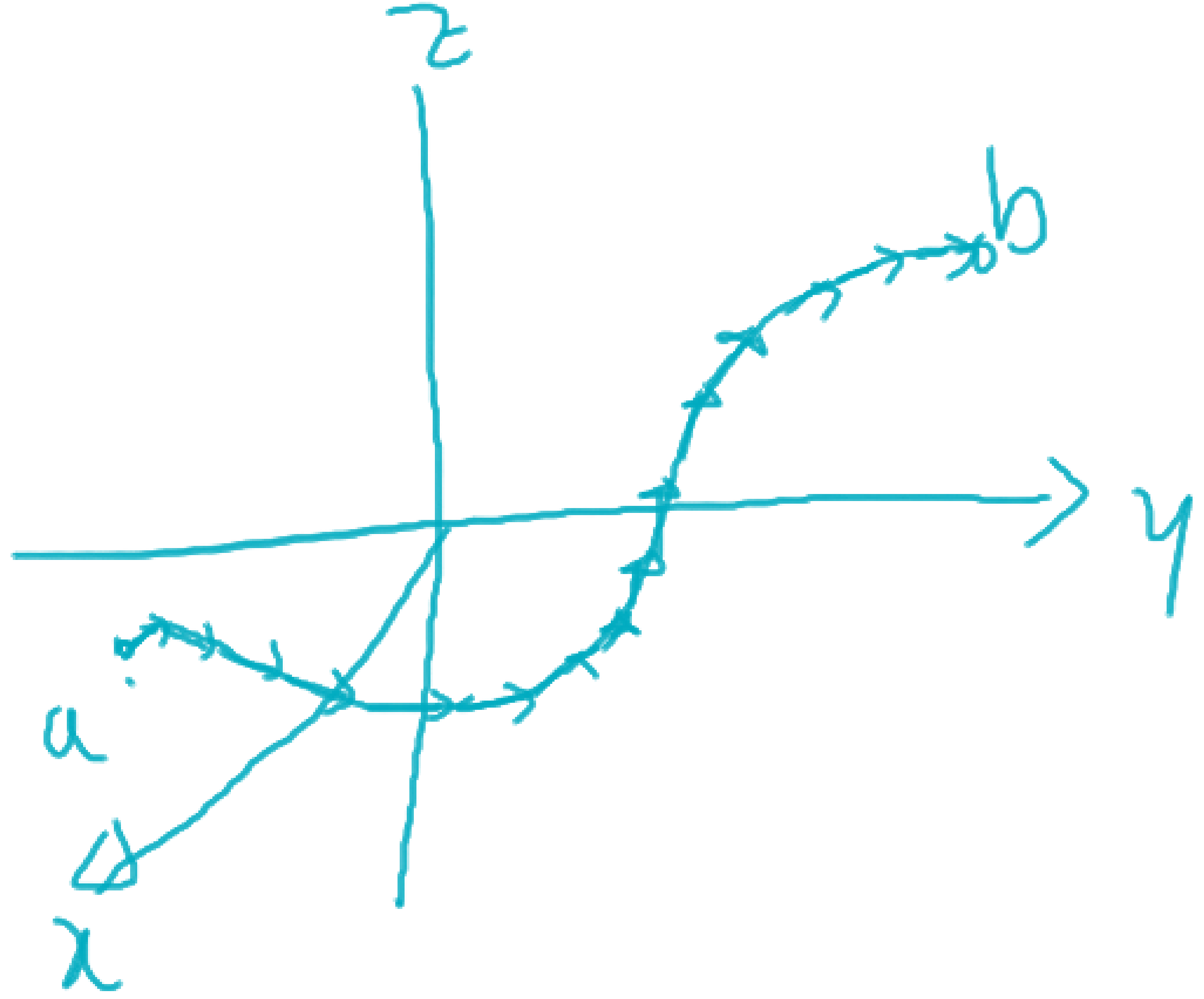
surface " (flux)

volume "



(a) line integrals:

$$\int_{a\,P}^{b} \vec{v}\cdot d\vec{l}$$



closed path, $\oint \vec{v}\cdot d\vec{l}$

$$W = \int \bar{F} \cdot d\bar{l}$$

line