STUDY OF FOURIER ANALYSIS

OBJECT: To find out the frequencies and the relative amplitudes of the first few simple harmoic components of a given non-sinusoidal periodic wave.

APPARATUS: A function generator, L-C combination of frequency = 5 to 10 KZ., CRO.

THEORY: The Fourier theorem states that any single valued periodic wave-integral multiple of the frequency of the given waveform. Thus a periodic function :

$$V_{(t)} = V_0 + V_1 \text{ Sin}(\omega t + \phi_1) + V_2 \text{ Sin}(2\omega t + \phi_2) + \ldots + V_n \text{ Sin } (n\omega t + \phi_v) \quad (1)$$

V_0 is the dc level of the given waveform V_1, V_2, ... V_n are the amplitudes of the constituent harmonics respectively with frequencies ω, 2ω .... $n\omega$ and ϕ_1, ϕ_2, ... ϕ_n are their phases. The amplitude V_k is evaluated by integrals

$$V_k = (2/T) \int V(t) \text{ Sin } (k\omega t)\, dt \quad (2a)$$

and $$V_0 = (1/T) \int V(t)\, dt \quad (2b)$$

T being the time period of the given waveform. It is found that

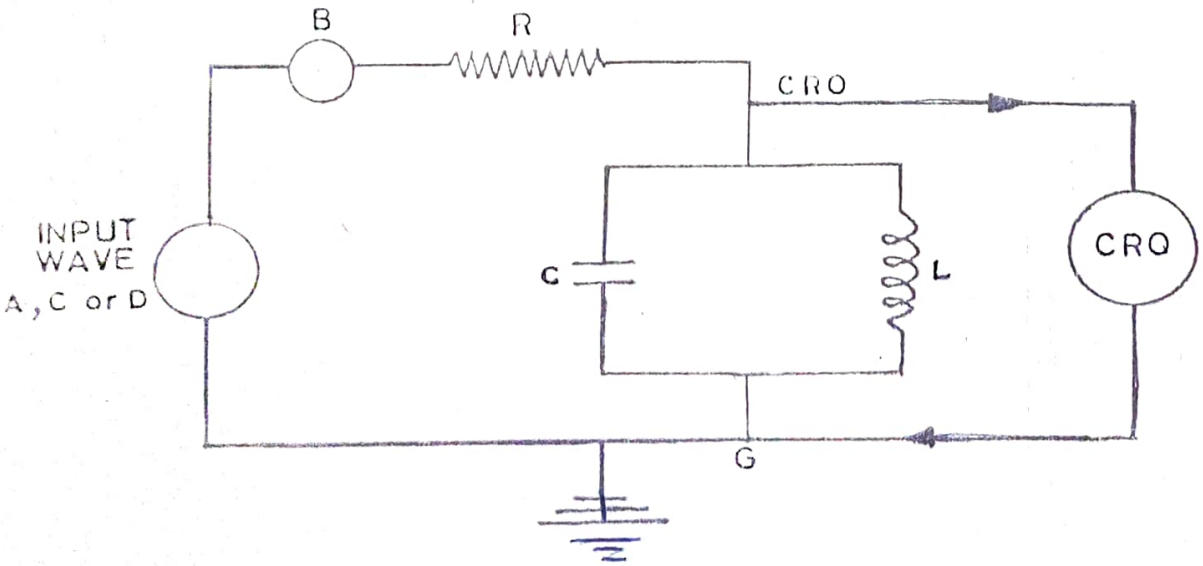
(i) $V(t)$ (square wave) $= (4V/\pi)$ [Sin ωt + (1/3) Sin $3\omega t$ + (1/5) Sin $5\omega t$ + ...] (3)

(ii) $V_{(t)}$ (Triangular) $= (8V/\pi^2)$ [cos ωt + $(1/3^2)$ cos $3\omega t$ + $(1/5^2)$ cos $5\omega t$ + ...] (4)

and

(iii) $V_{(t)}$ (half wave) = const. [cos ωt - 0.425 cos $2\omega t$ - 0.085 cos $4\omega t$ - ...](5)

To analyse the components of any waveform we use the following circuit :



At first sight the straightforward method to perform the experiment seems to be the following. At a given frequency 'f' of the input waveform, if we change the value of 'C' regularly, the LC combinations resonates only when its frequency m = $1/2\pi\sqrt{LC}$] is equal to the frequency of any harmonic (= nf) present in the input waveform. At this situation a sine wave with frequency nf will appear on CRO screen. By changing 'C' the possible values of nf corresponding to various harmonics and their relative amplitudes can be directly read from CRO. However, this procedure will not give the relative amplitudes of various harmonics correctly because the Q-factor (= 2πnf.L/R) of LC combination changes at different 'nf' even though 'L' is kept constant. Thus the response of 'LC' combination changes for different harmonics. So a different procedure is adopted to analyse the harmonics. It is obvious from eqns. (3) and (4) that a harmonic of angular frequency $\omega = 2\pi f$ must be present in the input square and triangular waves of frequencies f, f/3, f/5 etc. respectively with the same relative amplitudes with which f, 3f, 5f exist in these waveforms of frequency f. Therefore in the kit the frequency 'f' of LC combination is kept fixed i.e. L or C are not varied, rather the frequency 'f' of the input waveform is decreased gradually by rotating the caliberated potentiometer knob (TTP). The frequency of the input waveform is calculated by

$$f = [44.4/(1+R)\ \text{KHz} \qquad (6)$$

where R is the reading of TTP multiplied by 10. Whenever LC combination oscillates at a particular position of TTP, a sinewave of frequency 'f' appears on CRO screen. The order of the corresponding harmonic is equal to f upon the frequency f of input waveform and its amplitude can be correctly read directly by the CRO.

PROCEDURE:

(1) Connect power supply with the kit using the provided cords. Switch on the supply.
(2) Connect CRO with the sockets A, C and D one by one. Use toggle switch N to ascertain that the correct waveform is obtained at these sockets
(3) The distortion of the triangular or half wave rectified waveforms can be removed by decreasing the amplitude
(4) Now connect the CRO at the socket marked CRO on the panel of the kit and feed the desired waveform from A, C or D into B which is internally connected with the 'LC' circuit
(5) Change the frequency of the input waveform by rotating the potentiometer TTP slowly from its zero position and adjust it till you obtain a sine wave on CRO with maximum possible amplitude. Read the amplitude of this fundamental harmonic (= f') on CRO and calculate its frequency from eqn. (6)
(6) Now gradually rotate the potentiometer TTP and note its reading whenever you observe sine waves of largest possible amplitudes on CRO. Measure their amplitudes and calculate the corresponding frequency (= f). of the input waveform

OBSERVATIONS AND RESULTS

Wave form	TTP reading x 10 = R	Input frequency f	Harmonic f'/f	Amplitude	Relative amplitude	
					Exptl.	Theor.
Square						
Triangular						
Half wave rectified						

PRECAUTIONS

(1) The frequency of the waveform should be carefully adjusted to obtain the maximum amplitude of the sine wave appearing on CRO

(2) If there are periodic distortions in the waveform, once again agreement with the theory should not be given much importance.

(3) It should be noticed if there are no aperiodic distortions in the input waveform. In such cases Fourier theorem can not be applied.