


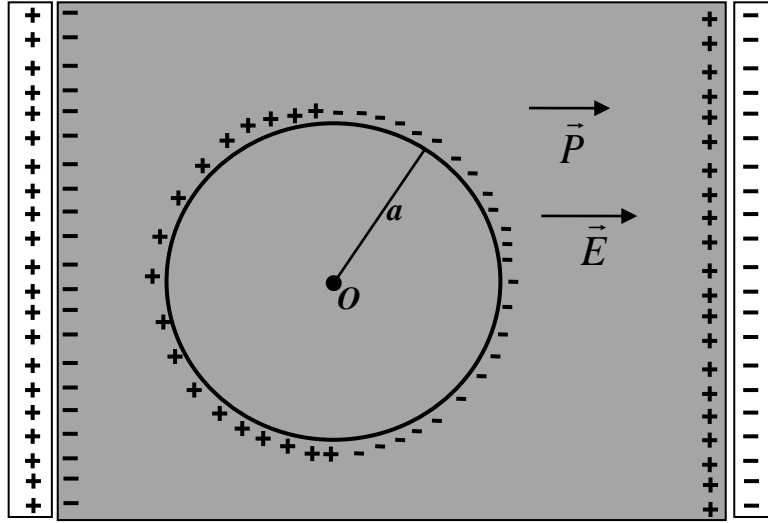
Molecular field in a dielectric: Lorentz Local field

The electric field which is responsible for polarising a molecule of a dielectric is called the molecular field or the polarizing field. If the dielectric is a gas, then the polarising field is simply the external applied electric field, which is known as the macroscopic electric field. In the case of solid or liquid dielectrics, the actual field acting on a molecule of the dielectric is different from the external field. It includes not only the external field but also the field produced by all polarized molecular dipoles in the dielectric excluding the molecule under consideration (because it will not be polarized by its own field). This field is known as the local electric field acting on a molecule and is responsible for the polarization of this particular molecule. Lorentz derived a relation for a non-polar isotropic dielectric. This relation is known as Lorentz's relation for the Molecular field or simple Lorentz Local field.

Derivation of Lorentz Local field

Let us consider a uniform electric polarization of a dielectric between the plates of a capacitor as shown in Figure given below . Draw a sphere of radius ' a ' around a particular molecule inside the dielectric such that it is big enough to contain a large number of molecules but small as compared to distance d between the capacitor plates (i.e., $a \ll d$). Thus, the dielectric outside this sphere can be treated as a continuum, while the molecules inside the sphere can be treated as individual dipoles. The molecule under consideration (i.e., molecules at the center of the sphere) is influenced by the following fields:

- (i) \vec{E}_0 due to free charges of the capacitor plates.
- (ii) \vec{E}_1 due to bound charges of dielectric surfaces facing the capacitor plates.
- (iii) \vec{E}_2 due to bound charges of the polarized molecules on the surface of the sphere.
- (iv) \vec{E}_3 due to the polarized molecules inside the sphere. In many practical cases, it can be ignored due to extremely small contribution.



Thus, the effective field on a dielectric molecule is given by

$$\vec{E}_m = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 \quad \text{--- (1)}$$

The field produced by the surfaces charges on the capacitor plates is

$$E_0 = \frac{\sigma_f}{\epsilon_0} \quad \text{--- (2)}$$

Where σ_f be surface charge density of free charges presented on the capacitor plates. In vector form, Eq. (2) becomes

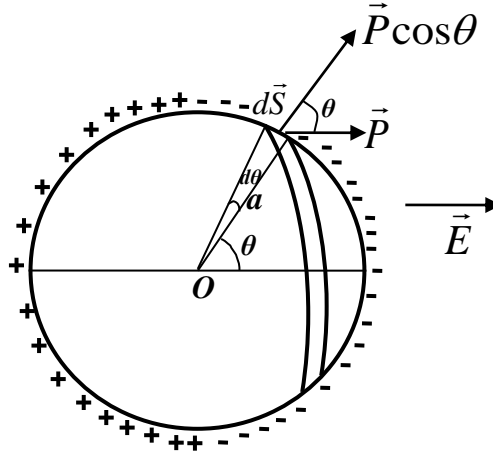
$$\vec{E}_0 = \frac{\vec{D}}{\epsilon_0} \quad \text{--- (3)}$$

The field produced by the surfaces charges of dielectric surfaces facing the capacitor plates is

$$E_1 = \frac{\sigma_b}{\epsilon_0} \quad \text{--- (4)}$$

Where σ_b be surface charge density of bound charges of the dielectric. In scalar form, Eq. (4) becomes

$$\vec{E}_1 = -\frac{\vec{P}}{\epsilon_0} \quad \text{--- (5)}$$



Let us now evaluate the E_2 , the field at O due to the bound charges on the surface of the sphere.

Let $d\vec{S}$ is the surface element on the sphere with polar coordinates (a, θ) as shown in the above figure. The component of electric polarization P normal to $d\vec{S}$ is $P \cos \theta$. The induced charge density over $d\vec{S}$ is, therefore, $d\sigma = P \cdot \cos \theta \cdot dS$ and the field at O due to this charge is

$dE_2 = \frac{1}{4\pi\epsilon_0} \frac{d\sigma}{a^2} = \frac{1}{4\pi\epsilon_0} \frac{P \cdot \cos \theta \cdot dS}{a^2}$. The field will be represented by a vector directed from O to

$d\vec{S}$. The component of this field in the direction of \vec{E} is

$$dE_2 \cdot \cos \theta = \left(\frac{1}{4\pi\epsilon_0} \frac{P \cdot \cos \theta \cdot dS}{a^2} \right) \cdot \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{P \cdot \cos^2 \theta \cdot dS}{a^2}.$$

Now suppose that the geometry of the surface element $d\vec{S}$ is the ring-shaped element of radius $a \sin \theta$ and width $(a \cdot d\theta)$, on the surface of the sphere. The area of this element is

$$dS = 2\pi(a \sin \theta)(a \cdot d\theta) = 2\pi a^2 \sin \theta \cdot d\theta$$

$$\therefore dE_2 \cdot \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{P \cdot \cos^2 \theta \cdot (2\pi a^2 \sin \theta \cdot d\theta)}{a^2} = \frac{P}{2\epsilon_0} \cdot \cos^2 \theta \cdot \sin \theta \cdot d\theta$$

By symmetry, the contribution to this field comes only from the components of the field from each surface element parallel to \vec{P} , i.e.,

$$\text{or } E_2 = 2 \int_0^{\pi/2} dE_2 \cdot \cos \theta = 2 \int_0^{\pi/2} \frac{P}{2\epsilon_0} \cdot \cos^2 \theta \cdot \sin \theta \cdot d\theta$$

$$\text{or } E_2 = \frac{P}{\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

$$\text{or } E_2 = \frac{P}{\epsilon_0} \left[-\frac{\cos^3 \theta}{3} \right]_0^{\pi/2}$$

$$\text{or } E_2 = \frac{P}{3\epsilon_0} \quad \text{--- (6)}$$

In vector form, Eq. (6) takes the form

$$\vec{E}_2 = \frac{\vec{P}}{3\epsilon_0} \quad \text{--- (7)}$$

Using Eqs. (3), (5) and (7) in Eq. (1), we get

$$\vec{E}_m = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} + \frac{\vec{P}}{3\epsilon_0}$$

$$\text{or } \vec{E}_m = \frac{\vec{D}}{\epsilon_0} + \frac{\vec{P}}{\epsilon_0} \left(\frac{1}{3} - 1 \right)$$

$$\text{or } \vec{E}_m = \frac{(\epsilon_0 \vec{E} + \vec{P})}{\epsilon_0} - \frac{2}{3} \frac{\vec{P}}{\epsilon_0} \quad [\because \vec{D} = \epsilon_0 \vec{E} + \vec{P}]$$

$$\text{or } \vec{E}_m = \vec{E} + \frac{\vec{P}}{\epsilon_0} - \frac{2}{3} \frac{\vec{P}}{\epsilon_0}$$

$$\text{or } \vec{E}_m = \vec{E} + \frac{\vec{P}}{\epsilon_0} \left(1 - \frac{2}{3} \right)$$

$$\text{or } \vec{E}_m = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$$

This equation is known as Lorentz equation for a non-polar isotropic dielectric.

Clausius-Mossotti relation

According to Lorentz equation, the electric field experienced by a molecule of a non-polar isotropic dielectric can be expressed as

$$\vec{E}_m = \vec{E} + \frac{\vec{P}}{3\epsilon_0} \quad \text{--- (1)}$$

Suppose that \vec{p}_m is the induced dipole moment of the molecule under consideration due to this field, then we have

$$\vec{p}_m = \alpha \vec{E}_m \quad \text{--- (2)}$$

Where α is the polarizability of the molecule.

Using Eq. (1) in Eq. (2), we obtain

$$\vec{p}_m = \alpha \left(\vec{E} + \frac{\vec{P}}{3\epsilon_0} \right) \quad \text{--- (3)}$$

Let 'n' be the number of molecules per unit volume in the dielectric material, then by definition, the polarization vector is expressed as

$$\vec{P} = n \vec{p}_m \quad \text{--- (4)}$$

Using Eq. (3) in Eq. (4), we obtain

$$\vec{P} = n\alpha \left(\vec{E} + \frac{\vec{P}}{3\epsilon_0} \right)$$

$$\text{or} \quad \vec{P} = n\alpha \vec{E} + \frac{n\alpha}{3\epsilon_0} \vec{P}$$

$$\text{or} \quad \vec{P} - \frac{n\alpha}{3\epsilon_0} \vec{P} = n\alpha \vec{E}$$

$$\text{or} \quad \vec{P} \left(1 - \frac{n\alpha}{3\epsilon_0} \right) = n\alpha \vec{E} \quad \text{--- (5)}$$

The expression for the polarization vector \vec{P} in terms of electric field strength \vec{E} is given by

$$\vec{P} = \epsilon_0 (K - 1) \vec{E} \quad \text{--- (6)}$$

Using Eq. (6) in Eq. (5), we obtain

$$[\epsilon_0 (K - 1) \vec{E}] \left(1 - \frac{n\alpha}{3\epsilon_0} \right) = n\alpha \vec{E}$$

$$\text{or} \quad [\epsilon_0 (K - 1)] \left(1 - \frac{n\alpha}{3\epsilon_0} \right) = n\alpha$$

$$\text{or} \quad (K - 1) \left(1 - \frac{n\alpha}{3\epsilon_0} \right) = \left(\frac{n\alpha}{\epsilon_0} \right)$$

$$\text{or} \quad (K - 1) - (K - 1) \left(\frac{n\alpha}{3\epsilon_0} \right) = \left(\frac{n\alpha}{\epsilon_0} \right)$$

$$\text{or} \quad (K - 1) = (K - 1) \left(\frac{n\alpha}{3\epsilon_0} \right) + \left(\frac{n\alpha}{\epsilon_0} \right) \text{ or } (K - 1) = \left[\frac{(K - 1)}{3} + 1 \right] \left(\frac{n\alpha}{\epsilon_0} \right)$$

$$\text{or} \quad (K - 1) = \left[\frac{(K - 1 + 3)}{3} \right] \left(\frac{n\alpha}{\epsilon_0} \right) \text{ or } (K - 1) = \left[\frac{(K + 2)}{3} \right] \left(\frac{n\alpha}{\epsilon_0} \right)$$

$$\text{Hence } \frac{(K - 1)}{(K + 2)} = \frac{n\alpha}{3\epsilon_0} \quad \text{--- (A)}$$

This equation is known as the Clausius-Mossotti relation.

If M is the molecular weight of the dielectric material, ρ is its density and N_A is Avogadro's number, then we have

$$n = \frac{N_A \rho}{M}$$

Hence another form of the Clausius-Mossotti relation is

$$\frac{M}{\rho} \frac{(K - 1)}{(K + 2)} = \frac{N_A \alpha}{3\epsilon_0} \quad \text{--- (B)}$$

Physical significance

Clausius-Mossotti relation relates the microscopic property polarizability α to the macroscopic property dielectric constant K . It is applicable to gases and non-polar liquids. The expression (B) indicates that the L.H.S. is constant if the model under consideration for the polarization is valid. The experimental verification of this prediction of Clausius-Mossotti relation has been observed successfully for various gases over a quite large range of density.

Consequences of Clausius-Mossotti relation

(i) According to electromagnetic theory, the refractive index μ of a transparent dielectric material can be expressed as the square root of the dielectric constant K , i.e.,

$$\mu = \sqrt{K}$$

or $\mu^2 = K$

Then Clausius-Mossotti relation given by expression (B) takes the form

$$\frac{M}{\rho} \frac{(\mu^2 - 1)}{(\mu^2 + 2)} = \frac{N_A \alpha}{3\epsilon_0} = \text{Constant}$$

This result represents the dependence of the refractive index μ on the density of the medium. It is interesting to note that this law holds good for many substances.

(ii) For most of the gases and liquids, the polarizability is very small, therefore $(n\alpha/3\epsilon_0)$ is lesser than 1. Thus, Clausius-Mossotti relation is valid for such gases and liquids. However, in case of few crystalline solids, $(n\alpha/3\epsilon_0) \rightarrow 1$. In this situation, $K \rightarrow \infty$. Such dielectrics are known as ferroelectrics and they exhibit the property the of spontaneous polarization analogous to ferromagnetic materials.

(iii) For mono-atomic gases, the polarizability α is expressed as

$$\alpha = 4\pi\epsilon_0 R^3$$

Where R is the radius of the atom.

Using the value of α in Clausius-Mossotti relation, we have

$$\frac{(K - 1)}{(K + 2)} = \frac{4\pi\epsilon_0 R^3 n}{3\epsilon_0}$$

or $R^3 = \frac{3}{4\pi n} \frac{(K - 1)}{(K + 2)}$

or
$$R^3 = \frac{(K-1)}{4\pi n} \quad [\because K \rightarrow 1]$$

Thus, knowing the value of K and calculating the value of n at a constant temperature and pressure, the value of radius of a molecule can be determined.

Experimental verification of Clausius-Mossotti relation

The Clausius-Mossotti relation has been verified experimentally for various gases like hydrogen etc. Since α is a constant for a particular gas, therefore $\frac{1}{n} \cdot \frac{(K-1)}{(K+2)}$ must be a constant. The value of “ n ” was varied by changing the pressure of hydrogen gas, and the dielectric constant K was measured at various pressures. It was found that the value of $\frac{1}{n} \cdot \frac{(K-1)}{(K+2)}$ was independent of the pressure. This verifies experimentally the Clausius-Mossotti relation.