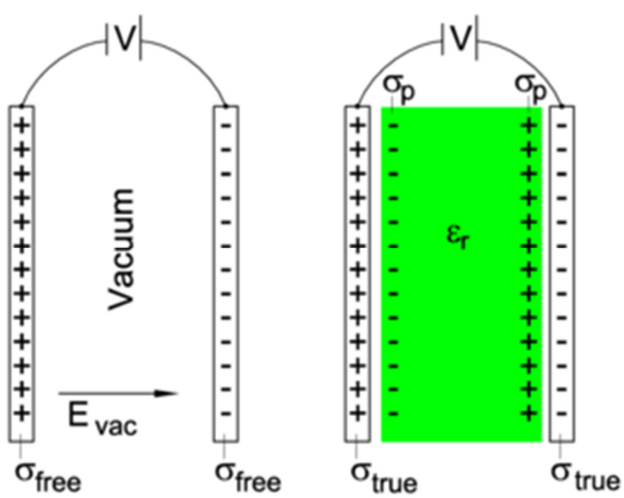
Displacement Vector and Charge density: D

In the special case of a parallel-plate capacitor, often used to study and exemplify problems in electrostatics, the electric displacement D has an interesting interpretation. In that case D (the magnitude of vector **D**) is equal to the *true surface charge density* σ_{true} (the surface density on the plates of the right-hand capacitor in the figure). In this figure two parallel-plate capacitors are shown that are identical, except for the matter between the plates: on the left no matter (vacuum), on the right a dielectric. Note in particular that the plates are held at the same voltage difference V and have the same area A. When the capacitor on the right discharges, it will deliver the total charge $Q_{\text{true}} = A{\cdot}\sigma_{\text{true}}$. The one on the left will produce $Q_{\text{free}} = A{\cdot}\sigma_{\text{free}}$.



To explain that $D = \sigma_{\text{true}}$, we recall that the relative permittivity may be defined as the ratio of the capacitances of two parallel-plate capacitors, (capacitance is total charge on the plates divided by voltage difference). Namely, the ratio of the capacitance C of a capacitor filled with dielectric to the capacitance C_{vac} of an identical capacitor in vacuum,

$$\epsilon_{\mathrm{r}} \equiv \frac{C}{C_{\mathrm{vac}}} = \frac{Q_{\mathrm{true}}}{V}\left[\frac{Q_{\mathrm{free}}}{V}\right]^{-1} = \frac{Q_{\mathrm{true}}}{Q_{\mathrm{free}}} = \frac{\sigma_{\mathrm{true}}}{\sigma_{\mathrm{free}}} \quad \Longrightarrow \quad \sigma_{\mathrm{true}} = \epsilon_{\mathrm{r}}\sigma_{\mathrm{free}},$$

where we used again that the charge Q is $A \cdot \sigma$. Clearly, the charge density on the plates increases by a factor ε_r when the dielectric is inserted in-between the plates. This means that the external source (of voltage V) must deliver a current during this insertion (it must move negative charge—electrons—from the positive plate to the negative plate).

The extra charge on the plates is compensated by the *polarization* of the dielectric, that is, the build-up of a positive polarization surface charge density σ_p on the side of the negative plate and a negative surface charge density on the positive side. The total charge is conserved, for instance on the side of the positively charged plate:

$$\sigma_{\mathrm{free}} = \sigma_{\mathrm{true}} - \sigma_{\mathrm{p}}.$$

(Here the minus sign appears because the polarization charge density σ_p is negative on the positive side of the capacitor).

Assuming that the plates are very much larger than the distance between the plates, we may apply the following formula for E_{vac} (the magnitude of the vector $\mathbf{E}_{vac}$),

$$E_{\mathrm{vac}} = \frac{\sigma_{\mathrm{free}}}{\epsilon_0}.$$

(This electric field strength does not depend on the distance of a field point to the plates: the electric field between the plates is *homogeneous*.) Now

$$D \equiv \epsilon_0\epsilon_{\mathrm{r}}E_{\mathrm{vac}} = \epsilon_{\mathrm{r}}\sigma_{\mathrm{free}} = \sigma_{\mathrm{true}},$$

hence D depends only on the charge delivered upon discharge, $D = Q_{true} / A$.

It is of some interest to note that the polarization vector **P** (pointing from minus to plus polarization charges, i.e., parallel to $\mathbf{E}_{vac}$) has magnitude P equal to the polarization surface charge density σ_p. Indeed, the magnitudes of the three parallel vectors are related by (in SI units),

$$P \equiv D - \epsilon_0 E_{\rm vac} = \sigma_{\rm true} - \sigma_{\rm free} = \sigma_{\rm p}.$$

Dielectric Polarization P

Dielectric polarization occurs when a dipole moment is formed in an insulating material because of an externally applied electric field. When a current interacts with a dielectric (insulating) material, the dielectric material will respond with a shift in charge distribution with the positive charges aligning with the electric field and the negative charges aligning against it. By taking advantage of this response, important circuit elements such as capacitors can be made.

Dielectric polarization is the term given to describe the behavior of a material when an external electric field is applied on it. A simple picture can be made using a capacitor as an example. The figure below shows an example of a dielectric material in between two conducting parallel plates. The charges in the material will have a response to the electric field caused by the plates.

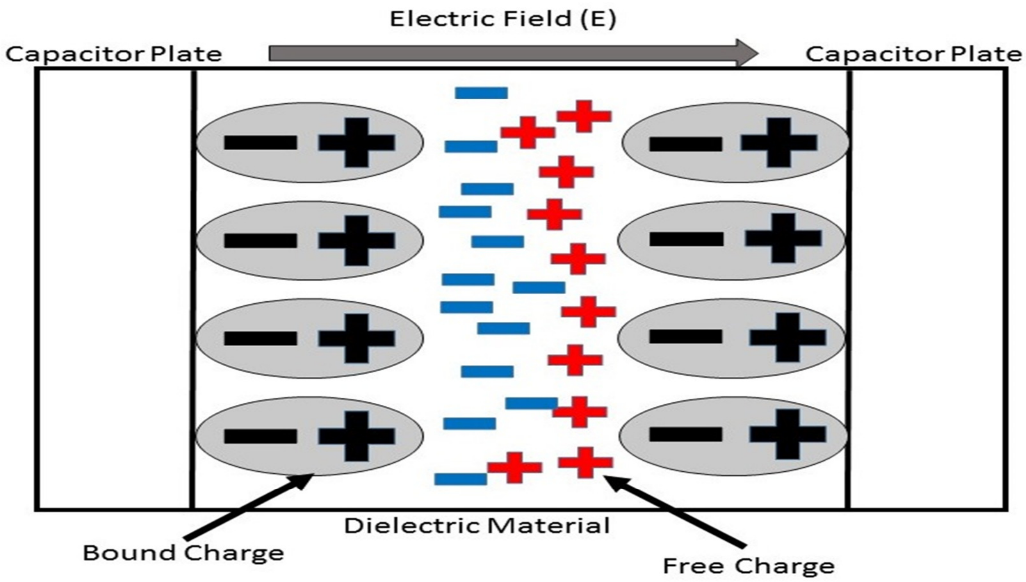


Fig.1The bound charges are the charges that are touching the capacitor plates, while the free charges usually float around in the material, but for this case, they are aligned with the bound charges.

Using the capacitor model, it is possible to define the relative permittivity or the dielectric constant of the material by setting its relative permittivity equivalent to the ratio of the measured capacitance and the capacitance of a test capacitor, which is also equal to the absolute permittivity of the material divided by the permittivity of vacuum.

Ionic Polarization

Ionic polarization is a mechanism that contributes to the relative permittivity of a material. This type of polarization typically occurs in ionic crystal elements such as NaCl, KCl, and LiBr. There is no net polarization inside these materials in the absence of an external electric field because the dipole moments of the negative ions are canceled out with the positive ions. However, when an external field is applied, the ions become displaced, which leads to an induced polarization. Figure 2 shows the displacement of ions due to this external electric field.

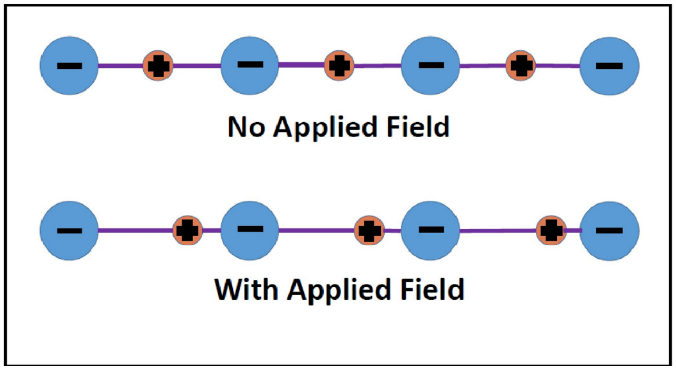


Fig 2. The effect of an external electric field on an ionic material. The positive charges will flow with the field and the negative charges will flow against the field, causing a net average dipole moment per ion to form.

Orientational Polarization

Orientational polarization arises when there is a permanent dipole moment in the material. Materials such as HCl and H_2O will have a net permanent dipole moment because the charge distributions of these molecules are skewed. For example, in a HCl molecule , the chlorine atom will be negatively charged and the hydrogen atoms will be positively charged causing the molecule to be dipolar. The dipolar nature of the molecule should cause a dipole moment in the material, however, in the absence of an electric field, the dipole moment is canceled out by thermal agitation resulting in a net zero dipole moment per molecule. When an electric field is applied however, the molecule will begin to rotate to align the molecule with the field, causing a net average dipole moment per molecule as shown in figure 3.

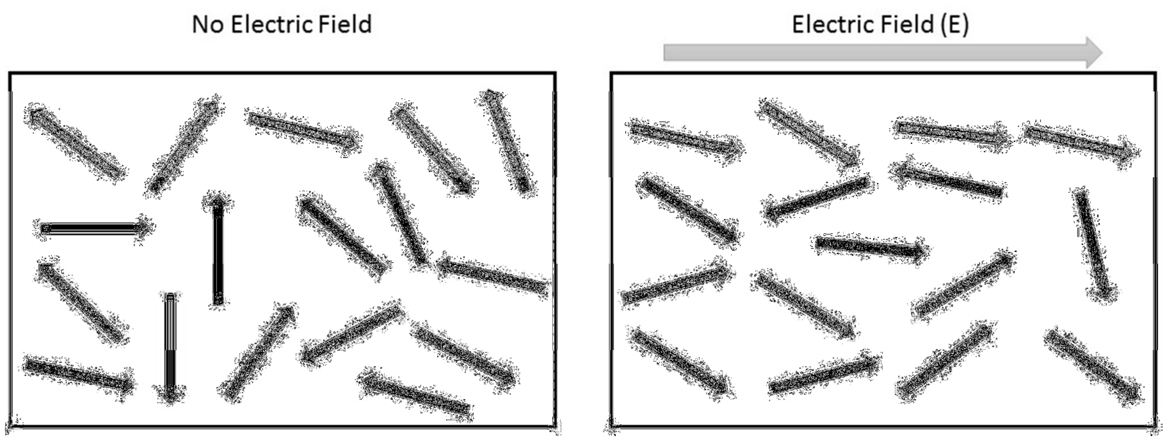


Fig 3. The figure shows how thermally agitated molecules (left) can be made to produce a net dipole moment per ion in the material with an externally applied field (right).

Interfacial Polarization

Interfacial or space charge polarization occurs when there is an accumulation of charge at an interface between two materials or between two regions within a material because of an external field. This can occur when there is a compound dielectric, or when there are two electrodes connected to a dielectric material. This type of electric polarization is different from orientational and ionic polarization because instead of affecting bound positive and negative charges i.e. ionic and covalent bonded structures, interfacial polarization also affects free charges as well. As a result interfacial polarization is usually observed in amorphous or polycrystalline solids. Figure 4 shows an example of how free charges can accumulate in a field, causing interfacial polarization. The electric field will cause a charge imbalance because of the dielectric material's insulating properties. However, the mobile charges in the dielectric will migrate over maintain charge neutrality. This then causes interfacial polarization.

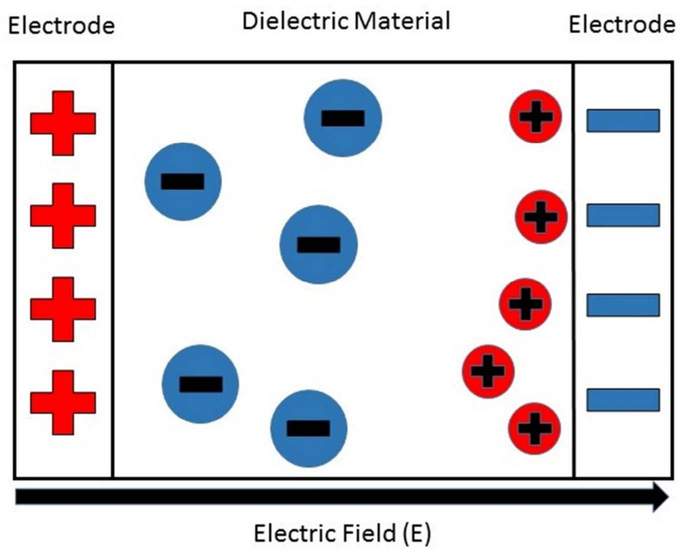


Fig 4. This shows how the free positive charges inside the dielectric material migrate towards the negative charge build-up on the right, caused by the external electric field.

Relation between Three electric vectors (D, E and P):

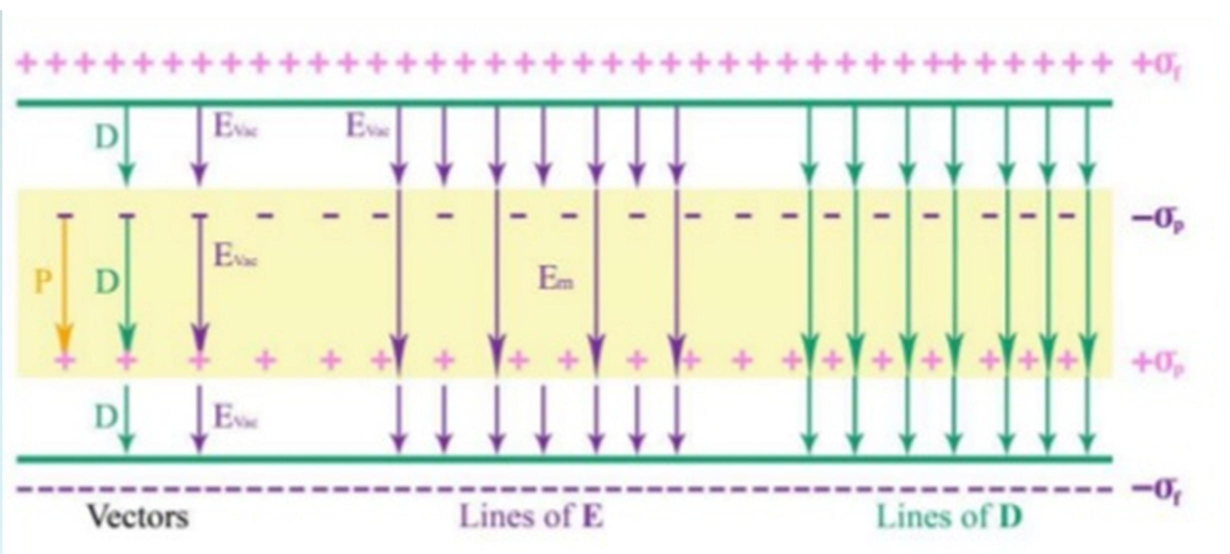
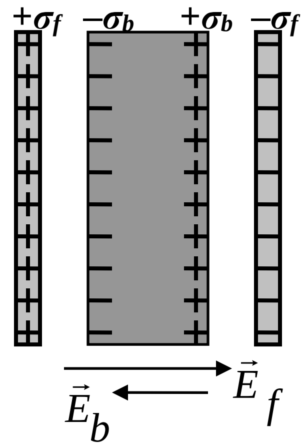


Fig. Relation between **D**, **E** and **P**

The relation between $\vec{D}, \vec{E}\ \&\ \vec{P}$

Let us consider the electric polarization of a dielectric slab which is inserted between the plates of a capacitor. Suppose that σ_f be surface charge density of free charges presented on the capacitor plates and σ_b surface charge density of bound charges of the dielectric slab as shown in Figure on RHS. ☛



If $\vec{E}_f$ and $\vec{E}_b$ are the corresponding electric fields strengths of free charges and bound charges, then their magnitudes are given by

$$E_f = \frac{\sigma_f}{\varepsilon_0} \qquad \text{--- (1)}$$

and

$$E_b = \frac{\sigma_b}{\varepsilon_0} \qquad \text{--- (2)}$$

Since these fields are oppositely directed as shown in above Figure (☝) so the net electric field strength is given by

$$E = E_f - E_b \qquad \text{--- (3)}$$

Using Eq. (1) and (2) in (3), we have

$$E = \frac{\sigma_f}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

or

$$\sigma_f - \sigma_b = \varepsilon_0 E \qquad \text{--- (4)}$$

We know that the magnitude of polarization vector $\vec{P}$ and electric displacement vector $\vec{D}$ represents the surface charge density of bound charges and free charges respectively, i.e.,

$$P = \sigma_b \qquad \text{--- (5)}$$

&

$$D = \sigma_f \qquad \text{--- (6)}$$

Using Eq. (5) and (6) in (4), we get

$$D - P = \varepsilon_0 E$$

or

$$D = \varepsilon_0 E + P$$

Invector form, the above equation takes the form

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \text{--- (7)}$$

This is the relation between $\vec{D}, \vec{E}\ \&\ \vec{P}$.

In the above relation, it is important to note that

(i) $\vec{D}$ is related to free charges only. It can be represented by lines of electric displacement just as $\vec{E}$ is represented by lines of forces. The lines of $\vec{D}$ begin and end on the free charges.

(ii) $\vec{P}$ is only related to bound charges. The lines of $\vec{P}$ begin and end on the bound charges.

(iii) $\vec{E}$ is related to both free and bound charges.

Electric susceptibility χ_e: Definition and physical significance

For most linear dielectric materials, the polarization *P* is directly proportional to the electric field strength *E* i.e.,

$$P \propto E \text{ or } P = \chi_e E$$

The proportionality constant χ_e is known as electric susceptibility. The electric susceptibility is always a dimensionless positive number. It indicates the degree of polarization of dielectric material in response to an applied electric field. The greater the electric susceptibility, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material. It is in this way that the electric susceptibility influences the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

The relation between electric susceptibility χ_e and dielectric constant *K*

The electric displacement vector $\vec{D}$ in terms of electric field strength $\vec{E}$ can be expressed as

$$\vec{D} = \varepsilon\vec{E} \qquad \text{--- (8)}$$

Also, the electric susceptibility χ_e of the dielectric is given by the expression

$$\vec{P} = \chi_e \vec{E} \qquad \text{--- (9)}$$

Using (8) and (9) in Eq. (7), we have

$$\varepsilon\vec{E} = \varepsilon_0\vec{E} + \chi_e \vec{E} \qquad \text{--- (10)}$$

Let ε be electric permittivity of the dielectric slab and *K* is the dielectric constant, then

$$\varepsilon = \varepsilon_0 K \qquad \text{--- (11)}$$

Uisng Eq.(11) in (10), we get

$$\varepsilon_0 K\vec{E} = \varepsilon_0\vec{E} + \chi_e \vec{E}$$

or $$K = 1 + \frac{\chi_e}{\varepsilon_0} \qquad \text{--- (12)}$$

This is the relation between the electric susceptibility χ_e and the dielectric constant *K*.

The relation between polarization vector $\vec{P}$ and electric field strength $\vec{E}$

Using (12) in (9), we get

$$\vec{P} = \varepsilon_0 (K - 1)\vec{E}$$

This is the expression for the polarization vector $\vec{P}$ in terms of electric field strength $\vec{E}$.

Gauss's law in dielectrics

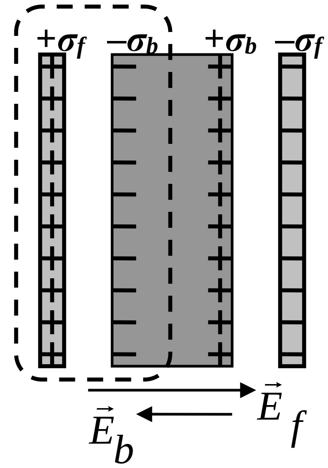
According to Gauss's law in electrostatics, the surface integral of electric displacement vector $\vec{D}$ over a closed surface is equal to the free charge q_f enclosed by it, i.e.,

$$\int_S \vec{D}.d\vec{S} = q_f$$

Proof:

Let us consider a dielectric polarization of a dielectric slab of dielectric constant K, which is inserted between the plates of a parallel plate capacitor. Let q_f and q_b are the free charges and bound charges on the capacitor plates and dielectric slab respectively. Suppose that σ_f be surface charge density of free charges presented on the capacitor plates and σ_b surface charge density of bound charges of the dielectric slab.

The net charge enclosed by the Gaussian surface (shown by dashed lines☛) around the LHS conducting plate of positive charges q_f will be $(q_f - q_b)$.



Hence the Gauss's law for the present system can be written as

$$\int_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}(q_f - q_b) \qquad \text{--- (1)}$$

Suppose that E and E_0 are the magnitudes of electric field strength between the capacitor plates in the absence and presence of the dielectric slab, then we can show that

$$K = \frac{E_0}{E} \qquad \text{--- (2)}$$

If A is the area of the capacitor plates, then the electric field strength between the capacitor plates in the absence of the dielectric slab is given by

$$E_0 = \frac{q_f}{\varepsilon_0 A} \qquad \text{--- (3)}$$

From Eq. (2) and (3), we have

$$K = \frac{q_f}{\varepsilon_0 AE}$$

or $$E = \frac{q_f}{\varepsilon_0 AK} \qquad \text{--- (4)}$$

If $\vec{E}_f$ and $\vec{E}_b$ are the corresponding electric fields strengths of free charges and bound charges, then their magnitudes are given by

$$E_f = \frac{\sigma_f}{\varepsilon_0} \qquad \text{--- (5)}$$

and $$E_b = \frac{\sigma_b}{\varepsilon_0} \qquad \text{--- (6)}$$

Since these field are oppositely directed, so the net electric field strength is given by

$$E = E_f - E_b \qquad \text{--- (7)}$$

Using Eq. (5) and (6) in (7), we have

$$E = \frac{\sigma_f}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

or $$E = \frac{(q_f / A)}{\varepsilon_0} - \frac{(q_b / A)}{\varepsilon_0} \qquad \left[\because \sigma_f = \frac{q_f}{A} \,\&\, \sigma_b = \frac{q_b}{A}\right]$$

or $$E = \frac{q_f}{\varepsilon_0 A} - \frac{q_b}{\varepsilon_0 A} \qquad \text{--- (8)}$$

Comparing Eq. (4) and (8), we obtain

$$\frac{q_f}{\varepsilon_0 AK} = \frac{q_f}{\varepsilon_0 A} - \frac{q_b}{\varepsilon_0 A}$$

or $$\frac{q_b}{\varepsilon_0 A} = \frac{q_f}{\varepsilon_0 A} - \frac{q_f}{\varepsilon_0 AK}$$

or $$q_b = q_f \left(1 - \frac{1}{K}\right) \qquad \text{--- (9)}$$

This is the relation between bound charges and free charges.
Eq. (9) can be written as

$$q_f - q_b = \frac{q_f}{K} \qquad \text{--- (10)}$$

Using Eq. (10) in (1), we get

$$\int_S \vec{E}.d\vec{S} = \frac{q_f}{\varepsilon_0 K}$$

or $$\int_S (\varepsilon_0 K \vec{E}).d\vec{S} = q_f$$

or $$\int_S \vec{D}.d\vec{S} = q_f \qquad \left[\because \vec{D} = \varepsilon_0 K \vec{E}\right]$$

This proves the Gauss's law in dielectrics.