1·1. Definition. A *body* is a portion of matter occupying finite space. It has consequently a finite volume and a finite mass. If a body does not change its shape and size when subjected to external forces, it is called a *rigid body*. Accordingly the distance between any two points of a rigid body is invariable.

A *particle* is a body of infinitely small dimensions. It may be regarded as a mathematical point endowed with mass.

1·2. Force. A *force* is that which, acting on a body, changes, or has a tendency to change, the state of rest or of uniform motion of the body. Associated with the idea of a force are the following :

(i) a point of application,

(ii) a direction,

and (iii) a magnitude.

Since a straight line has both magnitude and direction, a force can conveniently be represented by a straight line through the point of application. Very often we represent a force acting at O in the direction OA, by a straight line, say BC, parallel to OA and of suitable length. In such cases it must be remembered that BC represents only the magnitude and direction of the force, and not its line of action.

1·3. Equilibrium. If a number of forces acting on a body keep it at rest, they are said to be in *equilibrium*.

Two forces acting at a point of a body can be in equilibrium only if they are equal in magnitude and act in opposite directions. This we take as an axiom.

1·4. Action and reaction. When a body A presses against a body B with a force F at a point P, then F may be called the *action* of A on B. The body B on the other hand exerts an equal and opposite force F on the body A. This is known as the *reaction*. It is assumed therefore that action and reaction are always equal and opposite.

1·5. Statics. The science which deals with the action of forces on bodies is known as *mechanics*. *Statics* is that branch of mechanics which deals with forces in equilibrium.

The forces on a body with which we are chiefly concerned in Statics can be classified as follows :

(i) *Tension* or *Thrust*, which comes into play when a force is applied by means of a string or rod.

(ii) *Attraction* or *Repulsion*, which is the force with which one material body attracts or repels another. An example of attraction is gravitation, and an example of repulsion is the force exerted between two bodies having similar electric charges.

(iii) *Reaction*, which comes into play when one body is in contact with another.

1·6. Tension and thrust. The tension in a string connecting two particles (or bodies) has a tendency to bring the particles (or bodies) together. Therefore if a string be considered as divided at any point P of it into two parts A and B, the tensions at P in the parts A and B are directed respectively towards B and A.

Just the reverse is the case with thrust applied by a rod which keeps the two particles (or bodies) at its ends from coming together. Consequently the thrusts at a point P in the two parts A and B, say, in which a rod is divided at P, are directed towards A and B respectively.

1·7. Vectors and scalars. Of the quantities we come across, some are associated with direction and some are not. Thus when we say that a certain mass is 5 kg. we have given all the necessary information about the mass. When, however, we say, that the velocity of a particle is 15 km. per second, the information is not complete unless we know in what direction the particle is moving. Clearly 6 km. an hour due north is a different thing from 6 km. an hour due west. We are thus led to what are called *scalar* and *vector* quantities.

A *scalar quantity*, or briefly a *scalar*, has magnitude only; *e.g.* mass, temperature, quantity of heat and electric charge. To specify a scalar we need a unit quantity of the same type, and the ratio n which the quantity bears to this unit. The number n is called the measure of the quantity in terms of the chosen unit.

A *vector quantity*, or briefly a *vector*, has magnitude and is related to a definite direction in space. *Force, velocity* and *acceleration* are *vectors*. The simplest type of vector is that whose magni-

2·1. Resultant of two forces. We first take the case of two forces P_1 and P_2 meeting at a point A.

If we can find a force R acting at A such that the effect of R is the same as the combined effect of the forces P_1 and P_2, then R is said to be the *resultant* of the forces P_1 and P_2. Also, P_1 and P_2 are known as the *components* of R.

2·2. Parallelogram of forces. The determination of the resultant R of the two forces P_1 and P_2 each passing through A is embodied in the following principle known as the *parallelogram of forces* :

If two forces acting at a point are represented in magnitude and direction by the sides of a parallelogram drawn from that point, then their resultant is represented by the diagonal of the parallelogram drawn through that point.

Thus if the forces P_1 and P_2 are represented in magnitude and direction by AB and AD (Fig. 2), then AC, the diagonal through A of the parallelogram $ABCD$, represents the resultant R of P_1 and P_2.

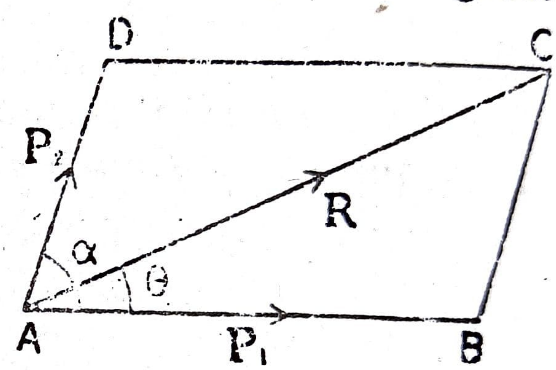

Fig. 2

In the vector notation, the law of the parallelogram of forces can be written as

$$\mathbf{AB}+\mathbf{AD}=\mathbf{AC},$$

or, alternatively as

$$\mathbf{AB}+\mathbf{BC}=\mathbf{AC}.$$

In particular we have $\mathbf{AB}+\mathbf{AD}=2\,\mathbf{AM}$, where M is the middle point of BD.

2·3. Composition and resolution of forces. If the angle BAD be denoted by α (Fig. 2), then,

$$\begin{aligned}AC^2&=AB^2+BC^2-2AB\,.\,BC\cos ABC\\&=AB^2+BC^2-2AB\,.\,BC\cos(180^\circ-\alpha)\\&=AB^2+BC^2+2AB\,.\,BC\cos\alpha.\end{aligned}$$

Hence $$R^2=P_1^2+P_2^2+2P_1 \cdot P_2 \cos \alpha,$$
which gives R when P_1, P_2 and α are known.

By drawing a perpendicular from C on AB, it is easy to see, if θ be the angle between the directions of R and P_1, that

$$\tan \theta=(P_2 \sin \alpha)/(P_1+P_2 \cos \alpha).$$

We may also determine P_1 and P_2 when R, θ and α are known. For, from the triangle ABC,

$$\frac{BA}{\sin BCA}=\frac{BC}{\sin BAC}=\frac{AC}{\sin ABC}.$$

Therefore

$$\frac{P_1}{\sin(\alpha-\theta)}=\frac{P_2}{\sin \theta}=\frac{R}{\sin(180^\circ-\alpha)}=\frac{R}{\sin \alpha}.$$

Thus $$P_1=R\frac{\sin(\alpha-\theta)}{\sin \alpha} \text{ and } P_2=\frac{R \sin \theta}{\sin \alpha}.$$

In case the angle between the lines of action of the forces P_1 and P_2 at A is a right angle, the formulae become simpler. We have in this case,

$$R^2=P_1^2+P_2^2$$

and $$\tan \theta=P_2/P_1.$$

If P_1 and P_2 are given, the above equations give R and θ.

If R is given, the components P_1 and P_2 are called the *resolved parts* of the force R. These are easily seen to be given by

$$P_1=R \cos \theta \text{ and } P_2=R \sin \theta.$$

2·31. Resolved part and component. It is important to observe that the ratio of each resolved part to R is the cosine of the angle between the direction of R and the resolved part. Thus the resolved part along AC of a force Q acting along AB is $Q \cos BAC$.

If a force R is given in magnitude and direction, and two directions AB and AD passing through the point of application of R are given, it must be carefully noted that the components of R along AB, AD are not the same as the resolved parts of R along AB, AD. In fact the component along AB is the force which together with the corresponding component along AD will give a resultant of magnitude R acting along the given direction; whereas the resolved part of R along AB is the force which, with the resolved part of R along the direction at right angles to AB, will give the resultant.

Ex. 1. *ABCD is a quadrilateral, and E the point of intersection of lines joining the middle points of the opposite sides. O is any point. Prove that the resultant of the forces* OA, OB, OC, OD *is equal to* 4 OE.

Let $ABCD$ be the quadrilateral, and let the directions of the forces OA, OB, OC and OD be marked by arrows in the figure (Fig. 3).

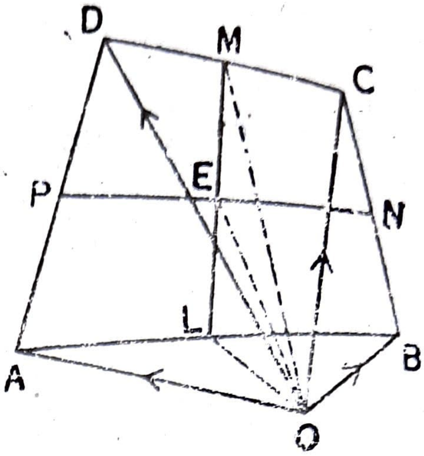

Fig. 3

If L, N, M and P be the middle points of the sides AB, BC, CD and DA respectively, and E the point of intersection of LM and NP, then from geometry it is known that the figure $LNMP$ is a parallelogram, and consequently E is the middle point of LM.

Now

$$OA+OB=2\ OL, \text{ and } OC+OD=2\ OM.$$

Also since E is the middle point of LM, $OL+OM=2\ OE$.

Hence

$$OA+OB+OC+OD=2\ (OL+OM)=4\ OE.$$

Ex. 2. *The resultant of two forces P, Q acting at a certain angle is X, and that of the forces P, R acting at the same angle is also X. The resultant of forces Q, R again acting at the same angle is Y. Prove that*

$$P=(X^2+QR)^{1/2}=\frac{QR(Q+R)}{Q^2+R^2-Y^2}.$$

Prove also that, if $P+Q+R=0$, then $Y=X$.

Suppose that the forces P and Q act at an angle α. Then, since their resultant is X,

$$X^2=P^2+Q^2+2PQ\cos\alpha. \qquad \ldots (1)$$

Also from the remaining data,

$$X^2=P^2+R^2+2PR\cos\alpha, \qquad \ldots (2)$$

and

$$Y^2=Q^2+R^2+2QR\cos\alpha. \qquad \ldots (3)$$

Subtracting (2) from (1), we have

$$0=Q^2-R^2+2P(Q-R)\cos\alpha$$

$$=(Q-R)(Q+R+2P\cos\alpha).$$

But $Q \neq R$, hence $Q+R+2P\cos\alpha=0$, i.e.

$$\cos\alpha=-(Q+R)/2P. \qquad . \quad . \quad . \quad (4)$$

Substituting the value of $\cos\alpha$ in (2), we get

$$X^2=P^2+R^2-R(Q+R), \text{ giving}$$

$$P=(X^2+QR)^{1/2}.$$

If, however, we substitute the value of $\cos\alpha$ in (3), we have

$$Y^2=Q^2+R^2-2QR\,(Q+R)/2P, \text{ giving}$$

$$P=QR(Q+R)/(Q^2+R^2-Y^2).$$

If $P+Q+R=0$, then, by (4), $\cos\alpha=\frac{1}{2}$. In this case (2) and (3) simplify to

$$X^2=P^2-R^2+PR \text{ and } Y^2=Q^2+R^2+QR.$$

Subtracting these, we have

$$X^2-Y^2=P^2-Q^2+R(P-Q)=(P-Q)(P+Q+R)=0.$$

Hence $$X=Y.$$

Examples 2(a)

1. A, B, P, Q, R are five points on a plane. Forces AP, AQ, AR act at A, and forces PB, QB, RB at B. Prove that the resultant of the six forces is $3AB$ in magnitude and direction.

2. O is any point in the plane of the triangle ABC; D, E, F are the middle points of the sides. Prove that the resultant of forces OE, OF, DO is represented by OA.

3. $ABCD$ is a quadrilateral. Show that if four forces represented by AB, AD, CB, CD, be applied at a point, their resultant will be represented by four times the line joining the middle points of the diagonals.

4. The resultant of two forces P_1 and P_2 acting at right angles is R; if P_1 and P_2 be each increased by 3 kg, R is increased by 4 kg, and is now equal to the sum of the original values of P_1 and P_2. Find P_1 and P_2.

5. Two forces act at a point and are such that if the direction of one is reversed, the direction of the resultant is turned through a right angle. Prove that the two forces must be equal