BPT-201 (semester II)
Topic: Blackbody Radiation-part 4
(Wien's Displacement Law)

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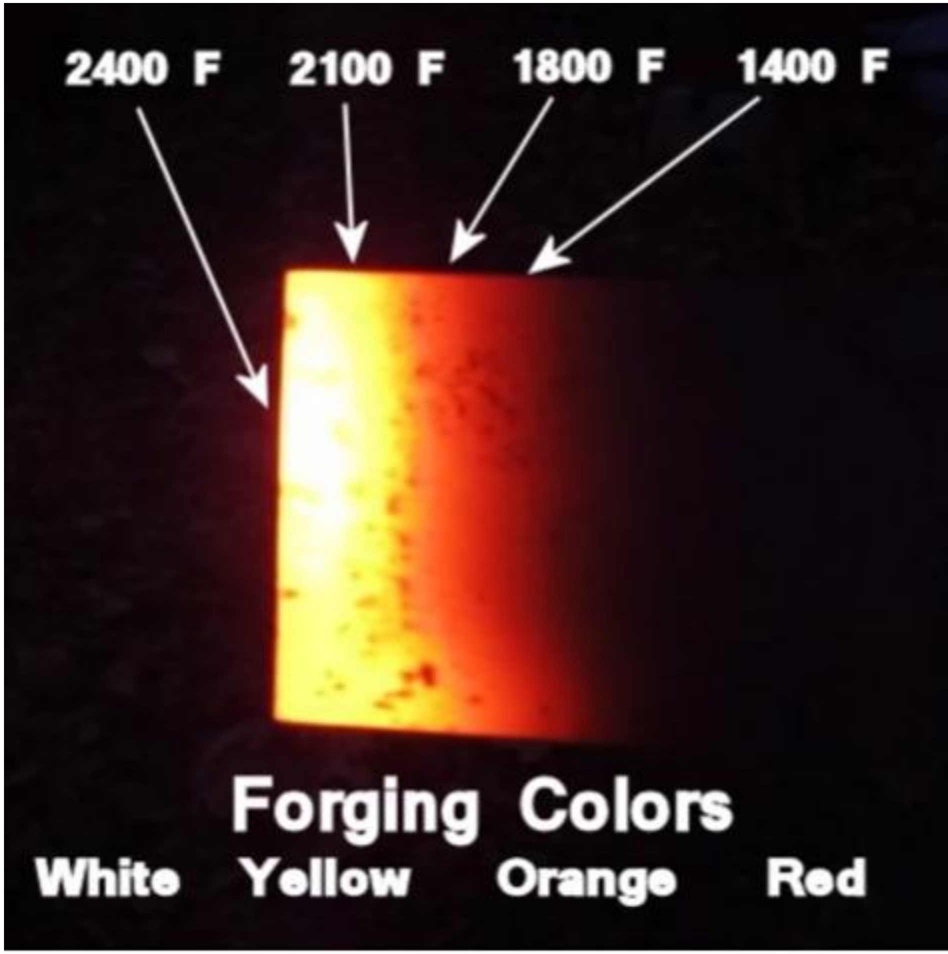
Wien's Displacement Law

- You must have seen an iron rod being heated in fire.
- It changes its color as its temperature changes.
- Figure indicates how the color of heated object changes with temperature.
- It has been observed that λ_{max} (the wavelength, for which the intensity is maximum, is a function of temperature
- Relation between T and λ can be given as
- $\lambda_{max} T = 2.898 \times 10^{-3}$ m-K

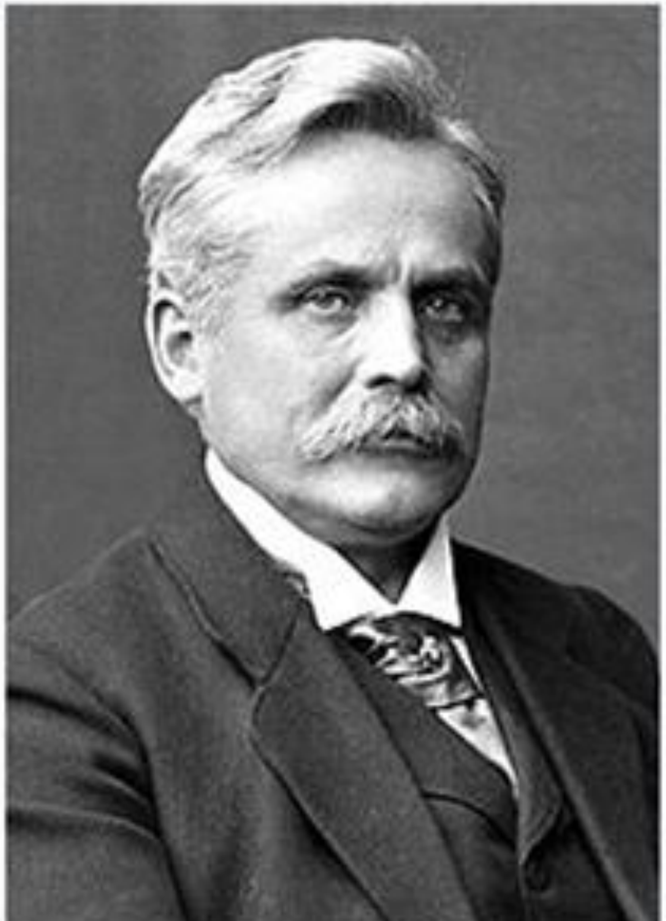


Enjoy this video to observe the changes

https://youtu.be/LRPvJQieBTQ

Wilhelm Carl Werner Otto Fritz Franz Wien (German: 13 January 1864 – 30 August 1928) - a German Nobel Laureate (1911) who, in 1893, gave theories related to heat

Wilhelm Wien



Taken from https://en.wikipedia.org/wiki/Wilhelm_Wien

Born	Wilhelm Carl Werner Otto Fritz Franz Wien 13 January 1864 Gaffken near Fischhausen, Province of Prussia
Died	30 August 1928 (aged 64) Munich, Germany
Nationality	German
Alma mater	University of Göttingen University of Berlin
Known for	Blackbody radiation Wien's displacement law
Awards	Nobel Prize for Physics (1911)
	Scientific career
Fields	Physics
Institutions	University of Giessen University of Würzburg University of Munich RWTH Aachen
Doctoral advisor	Hermann von Helmholtz
Doctoral students	Gabriel Holtsmark Eduard Rüchardt

Proof of Wien's Displacement Law

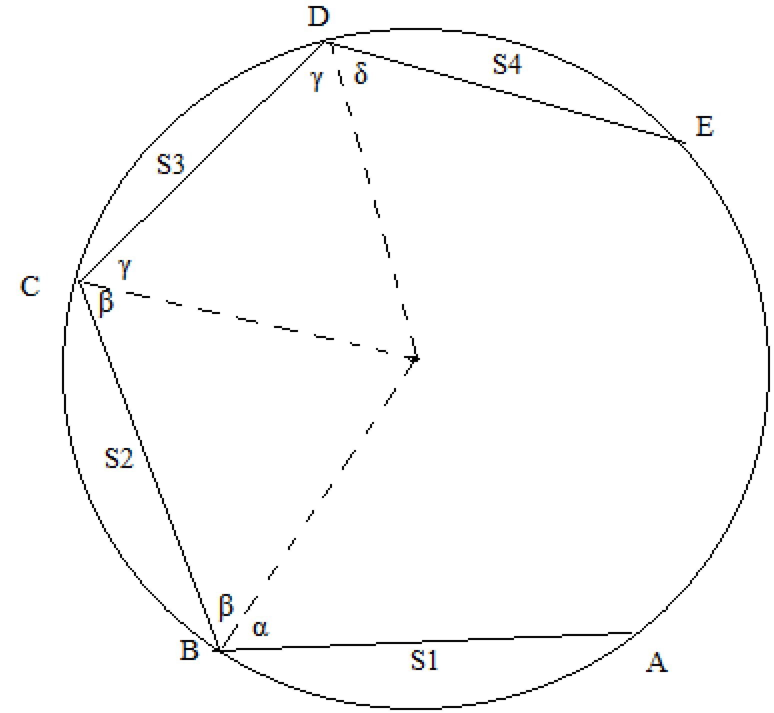
- Let us carry out an imaginary experiment, using a spherical enclosure with perfectly and diffusively reflecting wall
- The wall is capable of moving outward
- Let the enclosure is filled with black body radiation characterized by temperature T
- Now allow it to expand through reversible adiabatic process
- Let the velocity of wall is v (<<c) which is much less than c
- Due to reversible adiabatic process the temperature of enclosure will fall.
- Let the new temperature is given by T'

- Since the source (wall of enclosure) is moving hence due to Doppler's principle the wavelength will be altered at receiving end
- Hence for oblique incidence we will be get

$$\lambda' = \lambda \frac{c+v}{c-v} \approx \lambda\left(1+\frac{2v}{c}\right) \; (\,because\, v \ll c)$$

- If the incident angle with normal is α then the resolved part of the v in the direction of incident will be v cosθ
- This is the effective velocity which will affect the λ
- Hence in this case Doppler effect will give us

$$\lambda' = \lambda\left(1+\frac{2v\, cos\theta}{c}\right)$$



- Because of diffusive nature of wall the angle of reflection will not be the same as incident angle
- It will continuously change as shown in figure in previous slide
- Let us consider that in unit time it is reflected n times that means chords S_1, S_2.......S_n will be travelled by the radiation in unit time
- i.e. $S_1 + S_2 + \ldots\ldots + S_n = \sum S_i = c$
- from figure we get $\cos\alpha = S_1/2r$, $\cos\beta = S_2/2r$ etc.
- Let us now find the value of wavelength after every reflection
- After first reflection $\lambda_1 = \lambda\left(1 + \frac{2v}{c}\frac{S_1}{2r}\right) = \lambda\left(1 + \frac{vS_1}{cr}\right)$
- After second reflection $\lambda_2 = \lambda_1\left(1 + \frac{vS_2}{cr}\right) = \lambda\left(1 + \frac{vS_1}{cr}\right)\left(1 + \frac{vS_2}{cr}\right)$
- So after n^{th} reflection the final wave length will be

$$\lambda_n = \lambda_f = \lambda_{n-1}\left(1 + \frac{vS_n}{cr}\right) = \lambda\left(1 + \frac{vS_1}{cr}\right)\left(1 + \frac{vS_2}{cr}\right)\ldots\ldots\ldots\ldots\left(1 + \frac{vS_n}{cr}\right)$$

- Considering that sv/rc <1 and their higher multiples will have much lower values hence to get the final wavelength these terms will be neglected
- Hence the final value of wavelength will be given as
- $\lambda_f = \lambda\left(1 + \frac{v}{cr}\sum_{i=1}^{n} S_i\right)$ or $\lambda_f = \lambda\left(1 + \frac{vc}{cr}\right) = \lambda\left(1 + \frac{v}{r}\right)$
- Or $\lambda_f - \lambda = \frac{d\lambda}{dt} = \lambda\left(\frac{v}{r}\right)$ or $\frac{d\lambda}{\lambda} = \frac{v\,dt}{r} = \frac{dr}{dt}\frac{dt}{r} = \frac{dr}{r}$
- i.e. $\frac{d\lambda}{\lambda} = \frac{dr}{r}$ ……………………(A)

- Since the energy with which we have started is now distributed
  - Distributed over a large volume
  - A part of energy is spent in doing external work
- Hence the temperature of the enclosure will change
- Let us use the thermodynamical equations to understand it

- dQ= dU + pdV=0 because process is adiabatic
- We know U=uV and p=u/3 so using these values in above equation we will get

$$d(uV) + \frac{udV}{3} = 0 \; or \; udV + Vdu + \frac{udV}{3}$$

Or

$$\frac{du}{u} + \frac{4dV}{3V} = 0$$

- Where u is energy density

- Integrating it we will get $uV^{4/3}$ = constant
- From Stefan's law $u = \alpha T^4$ and equation for volume $V = \frac{4\pi}{3} r^3$
- We will get $\alpha T^4 \left(\frac{4\pi}{3} r^3\right)^{4/3} = \alpha \frac{4\pi}{3}^{4/3} T^4 r^4 = constant$
- Or $(Tr)^4 = constant \; or \; Tr = constant$ or $\frac{dT}{T} = -\frac{dr}{r}$
- the equation (A) from previous slide we will get

$$\frac{d\lambda}{\lambda} = -\frac{dT}{T}$$

- Which on integrating will result in λT=constant.
- Since this has been calculated for total u so we can say that it is for dominating wavelength and so λ_{max} T= constant
- Constant's value can be easily estimated which comes to be 2.898×10^{-3} m-K

Study Material

- http://galileo.phys.virginia.edu/classes/252/black_body_radiation.html#A%20Note%20on%20Wien%E2%80%99s%20Displacement%20Law
- http://hyperphysics.phy-astr.gsu.edu/hbase/wien.html