

## Thermodynamic Scale of temperature or absolute scale of temperature

- Efficiency of reversible engine is independent of substance.
- $\eta$  only depends upon the two-temperature between which engine is working
- Lord Kelvin used this property and defined an **Absolute Scale of temperature**

Let us imagine that a reversible engine is working between two temp  $\theta_1$  and  $\theta_2$

$$\eta_R = \frac{Q_1 - Q_2}{Q_1} = f(\theta_1, \theta_2)$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - \eta_R} = \frac{1}{1 - f(\theta_1, \theta_2)} = f(\theta_1, \theta_2)$$

Now let us now consider engine  
is working between  $\theta_2$  and  $\theta_3$

$$\frac{Q_2}{Q_3} = F(\theta_2, \theta_3)$$

Similarly if it is working  
between  $\theta_1$  and  $\theta_3$

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_3)$$

But we know

$$\checkmark \quad \frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = \frac{Q_1}{Q_3}$$

$$F(\theta_1, \theta_3) = F(\theta_1, \theta_2) \times F(\theta_2, \theta_3)$$

taking log on both two sides  
and differentiating with respect  
to  $\theta_1$  and  $\theta_3$

$$\frac{\partial^2 \ln[F(\theta_1, \theta_3)]}{\partial \theta_1 \partial \theta_3} = 0 \quad \text{--- (A)}$$

Now Integrating equation (A) w.r.t.  $\theta_1$  and  $\theta_3$

$$\ln F(\theta_1, \theta_3) = f_1(\theta_1) + f_3(\theta_3)$$

where  $f_1(\theta_1)$  and  $f_3(\theta_3)$  are arbitrary function of  $\theta_1$  and  $\theta_3$ .

$$\boxed{F(\theta_1, \theta_3) = e^{f_1(\theta_1)} e^{f_3(\theta_3)} = \psi(\theta_1) \phi(\theta_3)}$$

So we will have.

$$\begin{aligned} \psi(\theta_1) \phi(\theta_2) \cdot \psi(\theta_2) \phi(\theta_3) &= \psi(\theta_1) \phi(\theta_3) \\ \phi(\theta_2) &= 1/\psi(\theta_2) \end{aligned}$$

$$F(\theta_1, \theta_2) = \varphi(\theta_1) \phi(\theta_2) = \frac{\varphi(\theta_1)}{\varphi(\theta_1)}$$

$$F(\theta_2, \theta_3) = \dots \frac{\varphi(\theta_2)}{\varphi(\theta_2)}$$

$$F(\theta_1, \theta_3) = \frac{\varphi(\theta_1)}{\varphi(\theta_3)}$$

What is ' $\varphi$ '?

It is any function of  $\theta$  and Lord Kelvin argued that it can be given as an empirical temperature

$$\tau = A \varphi(\theta)$$

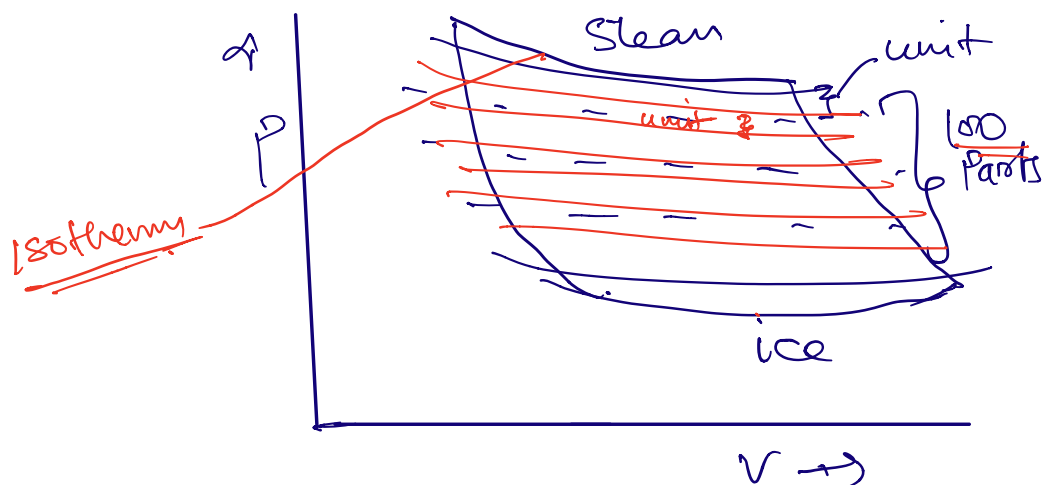
Where  $A$  = arbitrary constant.

So we can have  $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

and so the temperature is defined by this

Zero is the temperature of the sink, where the rejected heat  $Q_2$  is zero.

Unit: to define the unit, Carnot engine is allowed to work between ice and steam temperatures



we divide this curve into  
100 parts and in this way we  
get the unit of here. Absolute  
scale.