Digital Logic and Circuit
Paper Code: CS-102

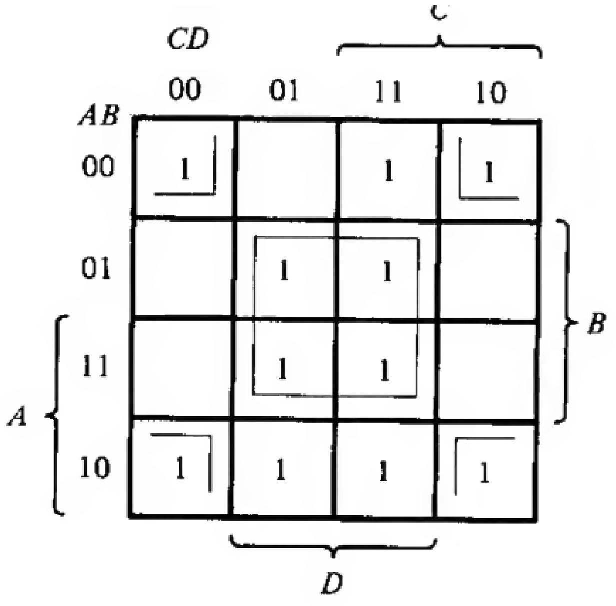
Outline

- Simplification of Boolean Functions Tabulation method
  - Determination of prime implicants

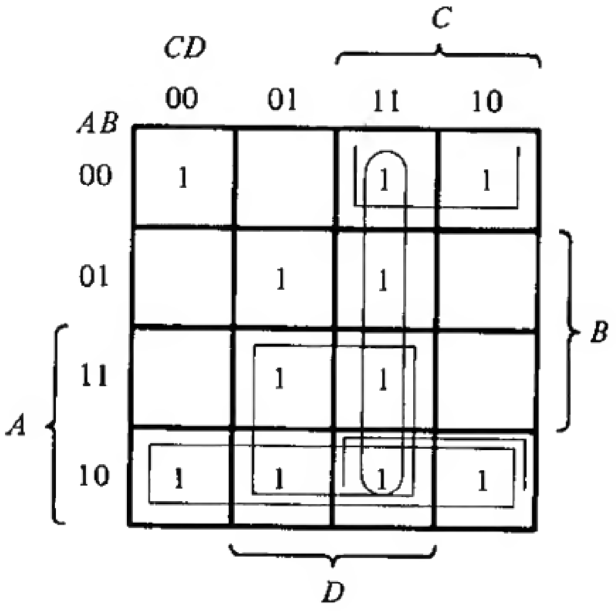
Prime implicants and essential prime implicants

A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.

If a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential.



(a) Essential prime implicants BD and $B'D'$



(b) Prime implicants CD, $B'C$, AD, and AB'

Tabulation method

- The map method of simplification is convenient as long as the number of variables does not exceed five or six.
- As the number of variables increases, the excessive number of squares prevents a reasonable selection of adjacent squares. The obvious disadvantage of the map is that it is essentially a trial-and-error procedure that relies on the ability of the human user to recognize certain patterns.
- For functions of six or more variables, it is difficult to be sure that the best selection has been made.
- The tabulation method overcomes this difficulty. It is a specific step-by-step procedure that is guaranteed to produce a simplified standard-form expression for a function.
- It can be applied to problems with many variables and has the advantage of being suitable for machine computation.

DETERMINATION OF PRIME IMPLICANTS

- The starting point of the tabulation method is the list of minterms that specify the function.
- The first tabular operation is to find the prime implicants by using a matching process. This process compares each minterm with every other minterm. If two minterms differ in only one variable, that variable is removed and a term with one less literal is found.
- This process is repeated for every minterm until the exhaustive search is completed. The matching-process cycle is repeated for those new terms just found. Third and further cycles are continued until a single pass through a cycle yields no further elimination of literals.
- The remaining terms and all the terms that did not match during the process comprise the prime implicants.

Simplify the following Boolean function by using Tabulation method

$$F(A, B, C, D) = \sum(0, 1, 2, 8, 10, 11, 14, 15)$$

Step 1 .

- Group binary representation of the minterms according to the number of 1 's contained, as shown in **Table, column (a)**.
- This is done by grouping the minterms into five sections separated by horizontal lines.
- The first section contains the number with no 1 's in it.
- The second section contains those numbers that have only one 1. The third, fourth, and fifth sections contain those binary numbers with two, three, and four 1 's, respectively.

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

Solution (Cont..)

F(A, B , C, D) = $\sum$(0, 1, 2, 8, 10, 11, 14, 15)

Step 2 .

- Any two minterms that differ from each other by only one variable canbe combined, and the unmatched variable removed. Two minterm numbers fit into this category if they both have the same bit value in all positions except one.
- The minterms in one section' are compared with those of the next section down only, because two terms differing by more than one bit cannot match.
- The minterm in the first section is compared with each of the three minterms in the second section. If any two numbers are the same in every position but one, a check is placed to the right of both minterms to show that they have been used.
- The resulting term, together with the decimal equivalents, is listed in column (b) of the table.

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 –	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

Solution (Cont..)

$F(A, B, C, D) = \sum(0, 1, 2, 8, 10, 11, 14, 15)$

Step 2 .

In this case, m_0 (0000) combines with m_1 (0001) to form (000_).

This combination is equivalent to the algebraic operation

$$m_o+m_1=w'x'y'z'+w'x'y'z=w'x'y'$$

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 0	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

Solution (Cont..)

F(A, B , C, D) = $\sum(0, 1, 2, 8, 10, 11, 14, 15)$

Step 3 .

- The terms of column (b) have only three variables. A 1 under the variable means it is unprimed, a 0 means it is primed, and a dash means the variable is not included in the term.
- The searching and comparing process is repeated for the terms in column (b) to form the two-variable terms of column (c).
- Again, terms in each section need to be compared only if they have dashes in the same position. Note that the term **(000_)** does not match with any other term. Therefore, it has no check mark at its right.

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 0	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

(c)

	w x y z
0, 2, 8, 10	– 0 – 0
0, 8, 2, 10	– 0 – 0
10, 11, 14, 15	1 – 1 –
10, 14, 11, 15	1 – 1 –

Solution (Cont..)

F(A, B , C, D) = $\sum$(0, 1, 2, 8, 10, 11, 14, 15)

Step-3

- The decimal equivalents are written on the left-hand side of each entry for identification purposes.
- The comparing process should be carried out again in column (c) and in subsequent columns as long as proper matching is encountered. In the present example, the operation stops at the third column.

(b)

	w	x	y	z	
0, 1	0	0	0	–	
0, 2	0	0	–	0	√
0, 8	–	0	0	0	√
2, 10	–	0	1	0	√
8, 10	1	0	–	0	√
10, 11	1	0	1	–	√
10, 14	1	–	1	0	√
11, 15	1	–	1	1	√
14, 15	1	1	1	–	√

(c)

	w	x	y	z
0, 2, 8, 10	–	0	–	0
0, 8, 2, 10	–	0	–	0
10, 11, 14, 15	1	–	1	–
10, 14, 11, 15	1	–	1	–

Solution (Cont..)

F(A, B , C, D) = ∑(0, 1, 2, 8, 10, 11, 14, 15)

Step-4

- The unchecked terms in the table form the prime implicants. In this example, we have the term w'x'y' (000—) in column (b), and the terms x 'z' (—0-0) and wy (1-1-) in column (c).
- Note that each term in column (c) appears twice in the table, and as long as the term forms a prime implicant, it is necessary to use the same term twice. The sum of the prime implicants gives a simplified expression for the function.
- This is because each checked term in the table has been taken into account by an entry of a simpler term in a subsequent column.
- Therefore, the unchecked entries (prime implicants) are the terms left to formulate the function.

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 –	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

(c)

	w x y z
0, 2, 8, 10	– 0 – 0
0, 8, 2, 10	– 0 – 0
10, 11, 14, 15	1 – 1 –
10, 14, 11, 15	1 – 1 –

Determination of Prime Implicants

(a)

	w x y z	
0	0 0 0 0	√
1	0 0 0 1	√
2	0 0 1 0	√
8	1 0 0 0	√
10	1 0 1 0	√
11	1 0 1 1	√
14	1 1 1 0	√
15	1 1 1 1	√

(b)

	w x y z	
0, 1	0 0 0 –	
0, 2	0 0 – 0	√
0, 8	– 0 0 0	√
2, 10	– 0 1 0	√
8, 10	1 0 – 0	√
10, 11	1 0 1 –	√
10, 14	1 – 1 –	√
11, 15	1 – 1 1	√
14, 15	1 1 1 –	√

(c)

	w x y z
0, 2, 8, 10	– 0 – 0
0, 8, 2, 10	– 0 – 0
10, 11, 14, 15	1 – 1 –
10, 14, 11, 15	1 – 1 –

Solution (Cont..)

$F(A, B, C, D) = \sum(0, 1, 2, 8, 10, 11, 14, 15)$

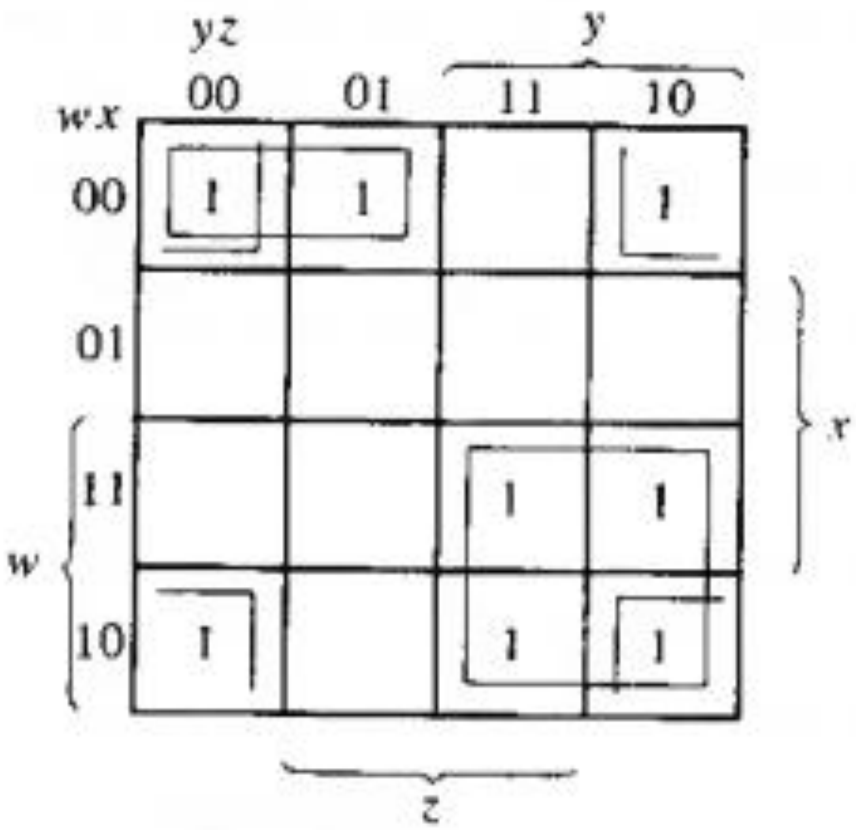
the sum of prime implicants gives the minimized function in sum of products:

$$F = w'x'y' + x'z' + wy$$

Note: It is worth comparing this answer with that obtained by the map method.

Solution (Cont..)

F(A, B , C, D) = $\sum$(0, 1, 2, 8, 10, 11, 14, 15)



The simplified expression in SOP

F = w'x'y' + x'z' + wy

Improvement in Tabulation method

- The tedious manipulation that one must undergo when using the tabulation method is reduced if the comparing is done with decimal numbers instead of binary.
- A method will now be shown that uses subtraction of decimal numbers instead of the comparing and matching of binary numbers.
- We note that each 1 in a binary number represents the coefficient multiplied by a power of 2.
- When two minterms are the same in every position except one, the minterm with the extra 1 must be larger than the number of the other minterm by a power of 2.
- Therefore, two minterms can be combined if the number of the first minterm differs by a power of 2 from a second larger number in the next section down the table.

Solve the following Boolean function

$F(A, B, C, D) = \sum(0, 1, 2, 8, 10, 11, 14, 15)$

As shown in Table, column (a), the minterms are arranged in sections as before, except that now only the decimal equivalents of the minterms are listed.

(a)	
0	√
1	√
2	√
8	√
10	√
11	√
14	√
15	√

➢The process of comparing minterms is as follows:

➢Inspect every two decimal numbers in adjacent sections of the table.

➢If the number in the section below is greater than the number in the section above by a power of 2 (i.e. , 1. 2, 4, 8, 16, etc.), check both numbers to show that they have been used, and write them down in column (b).

➢The pair of numbers transferred to column (b) includes a third number in parentheses that designates the power of 2 by which the numbers differ.

➢The number in parentheses tells us the position of the dash in the binary notation.

➢The results of all comparisons of column (a) are shown in column (b).

(a)		(b)	
0	√	0, 1 (1)	
		0, 2 (2)	√
1	√	0, 8 (8)	√
2	√		
8	√	2, 10 (8)	√
		8, 10 (2)	√
10	√		
		10, 11 (1)	√
11	√	10, 14 (4)	√
14	√		
		11, 15 (4)	√
15	√	14, 15 (1)	√

- The comparison between adjacent sections in column (b) is carried out in a similar fashion, except that only those terms with the same number in parentheses are compared.
- The pair of numbers in one section must differ by a power of 2 from the pair of numbers in the next section. And the numbers in the next section below must be greater for the combination to take place.
- In column (c), write all four decimal numbers with the two numbers in parentheses designating the positions of the dashes.

(b)			(c)	
0, 1	(1)		0, 2, 8, 10	(2, 8)
0, 2	(2)	√	0, 2, 8, 10	(2, 8)
0, 8	(8)	√		
2, 10	(8)	√	10, 11, 14, 15	(1, 4)
8, 10	(2)	√	10, 11, 14, 15	(1, 4)
10, 11	(1)	√		
10, 14	(4)	√		
11, 15	(4)	√		
14, 15	(1)	√		

- The prime implicants are those terms not checked in the table.
- These are the same as before, except that they are given in decimal notation. To convert from decimal notation to binary, convert all decimal numbers in the term to binary and then insert a dash in those positions designated by the numbers in parentheses.
- Thus 0 1 (1) is converted to binary as 0000, 0001; a dash in the first position of either number results in (000-).
- Similarly, 0, 2, 8, 10 (2, 8) is converted to the binary notation from 0000, 0010, 1000, and 1010, and a dash inserted in positions 2 and 8, to result in (-0-0).
- 10,11,14,15 (1, 4) is converted to the binary notation from 1010. 1011, 1110, 1111 and a dash inserted in positions 1 and 4, to result in (1-1-).

$F=w'x'y'+ x'y'+wy$

Suggested Reading

- M. Morris Mano, Digital Logic and Computer Design, PHI.

Thank you