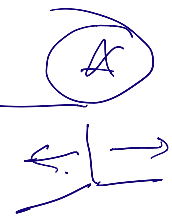
Estimation of constants A and b

1 Estimation of 'A'

$$f_c = N_t A^3 e^{-bc^2}$$



This has spherical symmetry

$$\iiint_{-\infty}^{\infty} f_c \, du\, dv\, dw = N_t$$

$$\iiint_{-\infty}^{\infty} \cancel{N_t} A^3 e^{-bc^2} \, du\, dv\, dw = \cancel{N_t}$$

$$A^3 \iiint_{-\infty}^{\infty} e^{-b(u^2+v^2+w^2)} \, du\, dv\, dw = 1$$

$$A^3 \left[\int_{-\infty}^{\infty} e^{-bu^2} du \int_{-\infty}^{\infty} e^{-bv^2} dv \int_{-\infty}^{\infty} e^{-bw^2} dw \right] = 1$$

$$I = \int_0^{\infty} e^{-bu^2} du = \int_0^{\infty} e^{-bv^2} dv = \int_0^{\infty} e^{-bw^2} dw$$

$$A^3 I^3 = 1 \Rightarrow I = 1/A$$

or $A = 1/I$

$$\boxed{I = \int_{-\infty}^{\infty} e^{-bu^2}\, du = \sqrt{\frac{\pi}{b}}}$$

$$A = \frac{1}{I} = \sqrt{\frac{b}{\pi}}$$

II) Estimation of 'b'

$$\underset{(n, kT)}{P} = 2m \sum_{u=0}^{\infty} n_u u^2.$$

n = number of molecules per cc

where n_u = number of mole per cc. in velocity range $u \leftrightarrow u + du$

$$n_u = \int_0^{\infty} f(u)\, du$$

$$P = 2m \int_0^{\infty} f(u)\, u^2\, du$$

$$= 2m \int_0^{\infty} n A e^{-bu^2} u^2 \, du$$

$$= 2mnA \int_0^{\infty} e^{-bu^2} u^2 \, du$$

$$\boxed{\int_0^{\infty} e^{-bu^2} u^2 \, du = \frac{1}{4} \sqrt{\frac{\pi}{b^3}}}$$

$$P = 2mn \; \frac{1}{4} \sqrt{\frac{\pi}{b^3}} \cdot \sqrt{\frac{b}{\pi}}$$

$$P = \frac{2mn}{4} \sqrt{\frac{1}{b^2}} = \frac{mn}{2b}$$

$$\boxed{b = \frac{m}{2 \cdot kT}} \Rightarrow A = \sqrt{\frac{b}{\pi}}$$

$$\boxed{A = \sqrt{\frac{m}{2\pi kT}}}$$

$$\boxed{\rho = N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mc^2/2kT}}$$

ρ → Number of specified (i.e. having velocity c) molecules in the unit volume

So dN_c = number of molecules in the velocity range c to c+dc

$$\boxed{dN_c = \rho \quad 4\pi c^2\, dc}$$

$$\boxed{dN_c = 4\pi . N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mc^2/2kT} c^2\, dc}$$

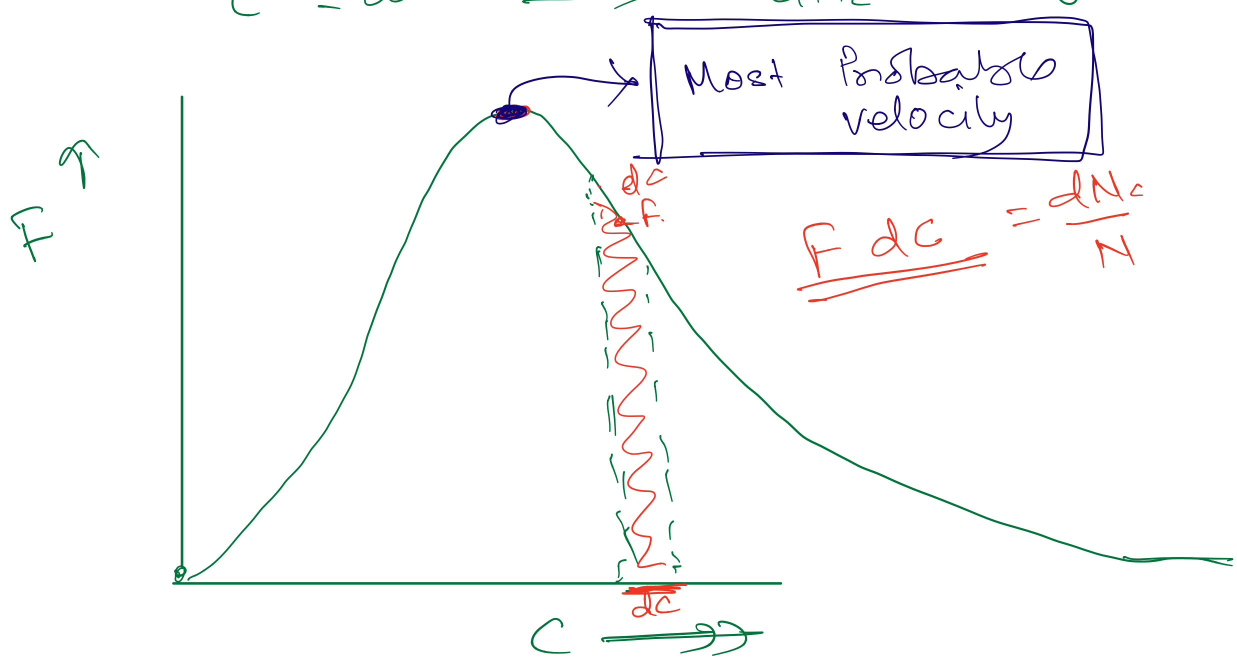
↑ Maxwell's Velocity Distribution.

$$\frac{dN_c}{N} = \underbrace{4\pi \cdot \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mc^2/2kT} c^2}_{F} dc$$

$$\boxed{\frac{dN_c}{N} = F\, dc}$$

$c = 0 \implies dN_c = 0$

$c = \infty \implies dN_c = 0$



Next Class

1. Average velocity
2. Most Probable velocity
3. Root mean velocity