Digital Logic and Circuit
Paper Code: CS-102

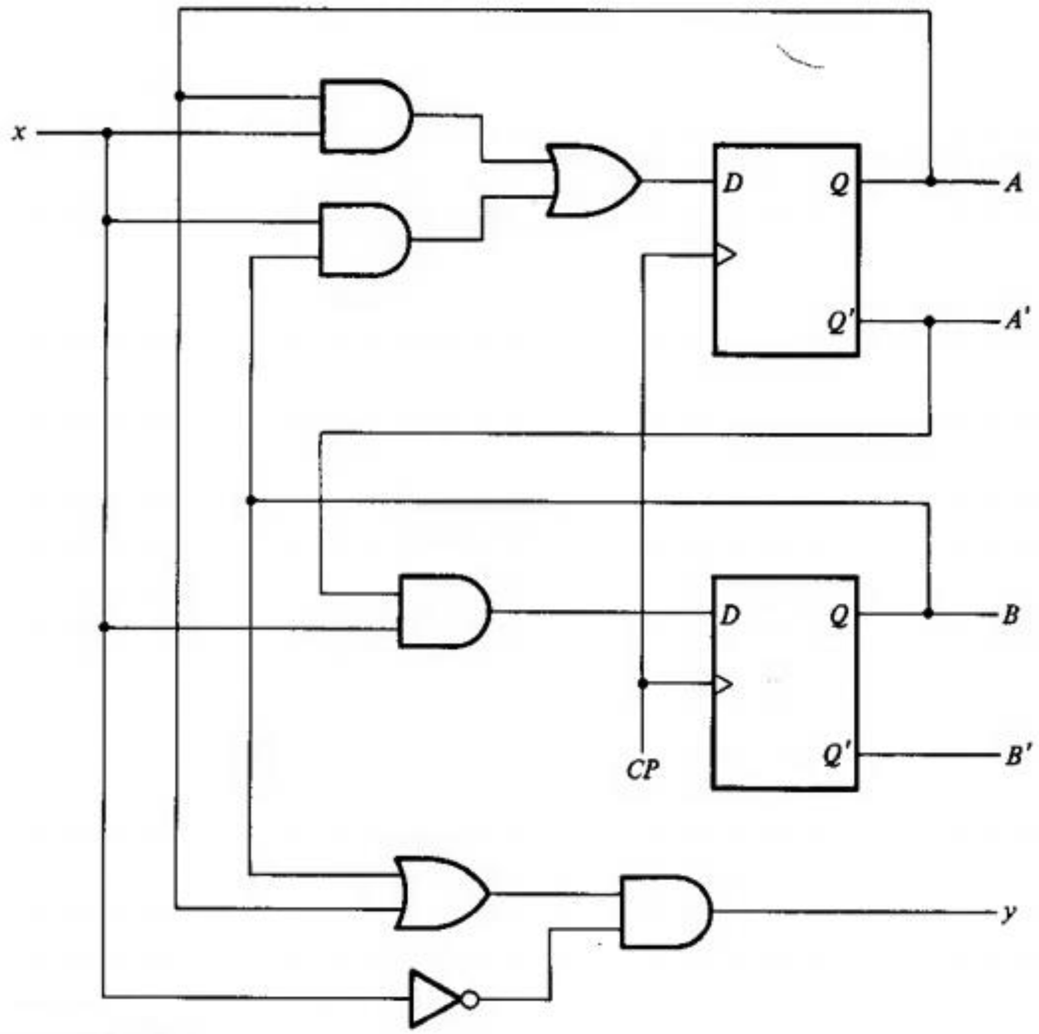
Outline

- **Sequential Circuit**
- **Analysis of clocked sequential circuits**
- **State Table**
- **State Diagram**

ANALYSIS OF CLOCKED SEQUENTIAL CIRCUITS

- The behavior of a sequential circuit is determined from the inputs, the outputs, and the state of its flip-flops.
- The outputs and the next state are both a function of the inputs and the present state. The analysis of a sequential circuit consists of obtaining a table or a diagram for the time sequence of inputs, outputs, and internal states.
- It is also possible to write Boolean expressions that describe the behavior of the sequential circuit.
- However, these expressions must include the necessary time sequence, either directly or indirectly.
- A logic diagram is recognized as a clocked sequential circuit if it includes flip-flops.
- The flip-flops may be of any type and the logic diagram may or may not include combinational circuit gates.

Sequential-Circuit Example

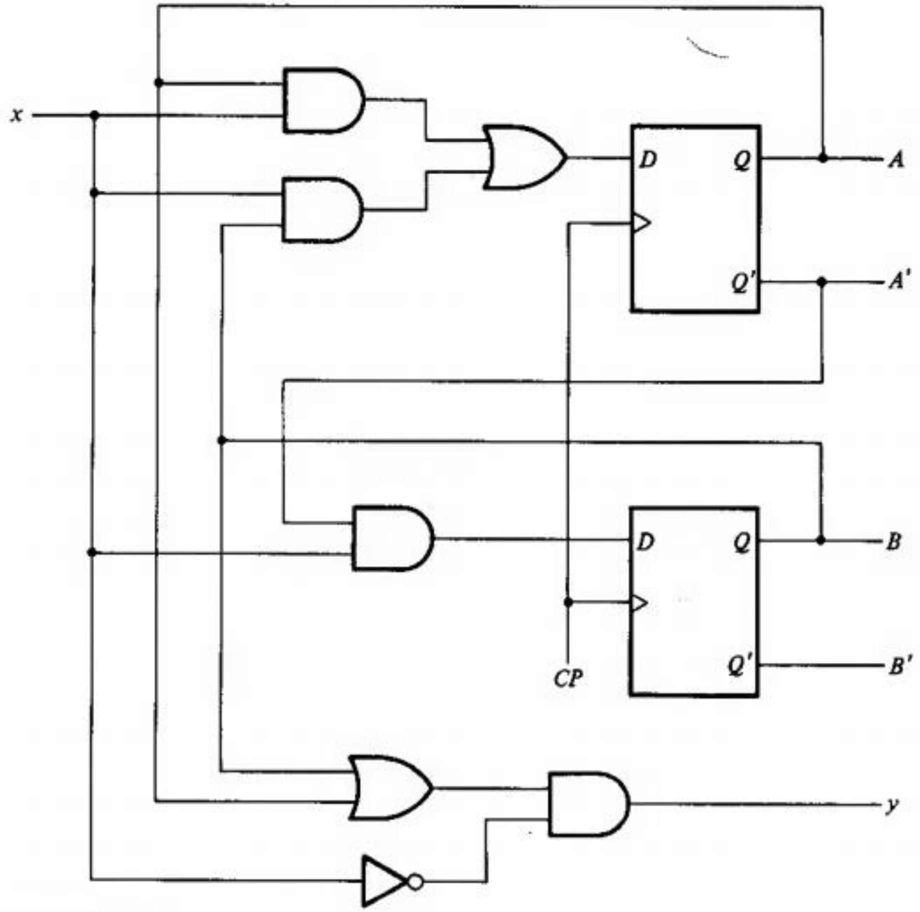


Sequential-Circuit Example

The circuit consists of two D flip-flops A and B, an input x, and an output y. Since the D inputs determine the flip-flops’ next state, it is possible to write a set of next-state equations for the circuit:

$$A(t + 1) = A(t)x(t) + B(t)x(t)$$

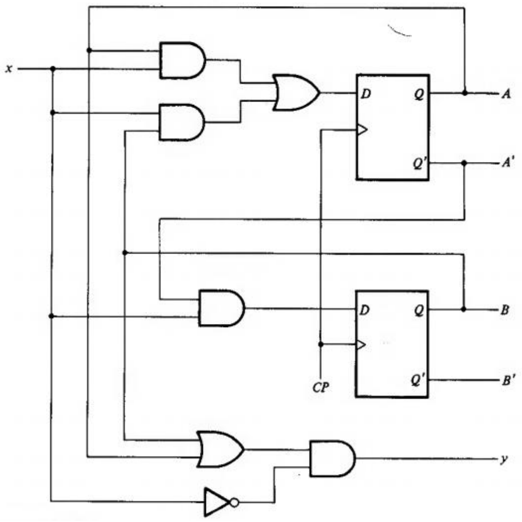
$$B(t + l) = A'(t)x(t)$$



- A state equation is an algebraic expression that specifies the condition for a flip-flop state transition.
- The left side of the equation denotes the next state of the flip-flop and the right side of the equation is a Boolean expression that specifies the present state and input conditions that make the next state equal to 1.
- Since all the variables in the Boolean expressions are a function of the present state, we can omit the designation (t) after each variable for convenience.
- The previous equations can be expressed in more compact form as follows:

A(t + 1) = Ax + Bx

B(t + 1) = A'x



The Boolean expressions for the next state can be derived directly from the gates that
form the combinational-circuit part of the sequential circuit.

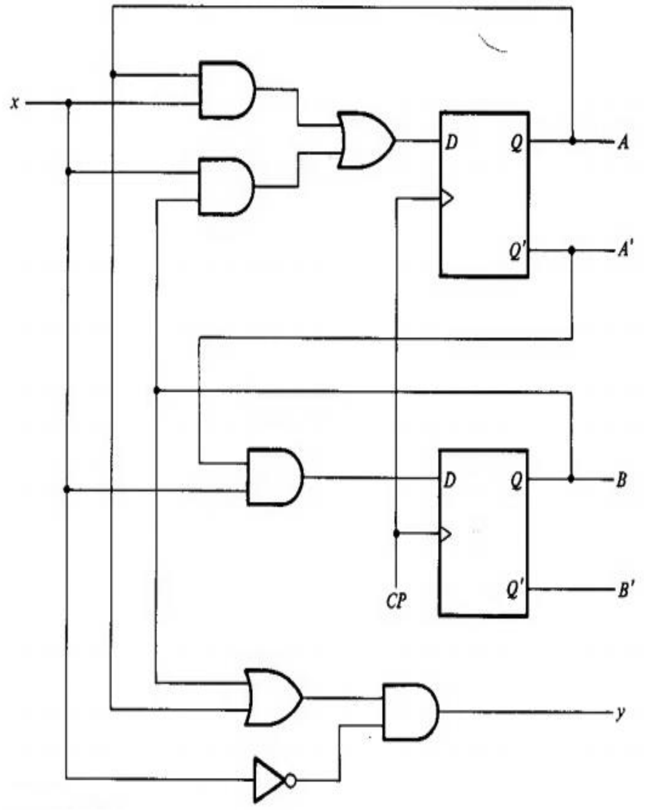
The outputs of the combinational circuit are applied to the D inputs of the flip-flops. The D input values determine the next state.

Similarly, the present- state value of the output can be expressed algebraically as follows:

$$y(t) = [A(t) + B(t)]x'(t)$$

Removing the symbol (t) for the present state, we obtain the output Boolean function:

$$y = (A + B)x'$$



State Table

- The time sequence of inputs, outputs, and flip-flop states can be enumerated in a state table.
- The state table for the circuit shown in previous slide is in Table 1 . The table consists of four sections labeled present state, input , next state , and output .
- The present state section shows the states of flip-flops A and B at any given time f.
- The input section gives a value of x for each possible present state.
- The next-state section shows the states of the flip-flops one clock period later at time t + 1. The output section gives the value of y for each present state.
- The derivation of a state table consists of first listing all possible binary combinations of present state and inputs. In this case, we have eight binary combinations from 000 to 111. The next-state values are then determined from the logic diagram or from the state equations.
- The next state of flip-flop A must satisfy the state equation

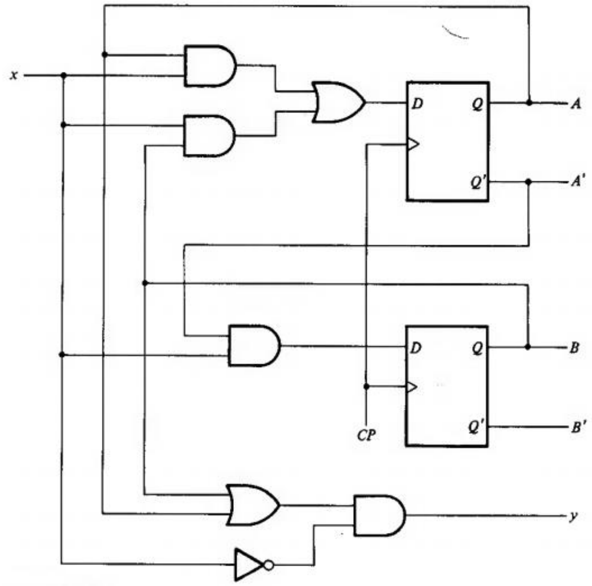
$$A(t + 1) = Ax + Bx$$

State table of same example

State Table

Present State A	Present State B	Input x	Next State A	Next State B	Output y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Table-1



$A(t + 1) = Ax + Bx$

B{t + 1) = $A'x$

$y = (A + B)x'$

Another way of representing state table

Second Form of the State Table

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
AB	AB	AB	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

- In general, a sequential circuit with m flip-flops and n inputs needs 2^{m+n} rows in the state table. The binary numbers from 0 through $2^{m+n} - 1$ are listed under the present- state and input columns.
- The next-state section has m columns, one for each flip-flop. The binary values for the next state are derived directly from the state equations.
- The output section has as many columns as there are output variables.
- Its binary value is derived from the circuit or from the Boolean function in the same manner as in a truth table.
- Note that the examples here, use only one input and one output variable, but, in general, a sequential circuit may have two or more inputs or outputs.

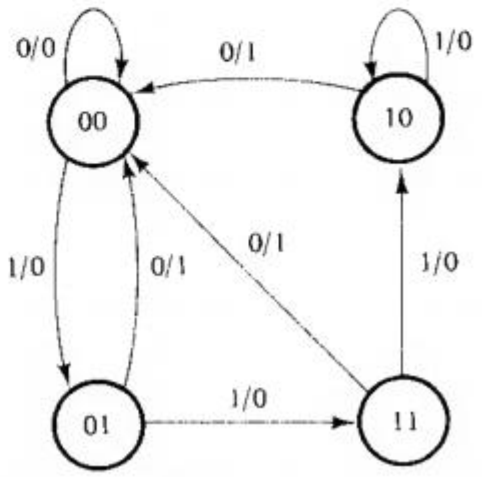
State Diagram

- The information available in a state table can be represented graphically in a state diagram.

- In this type of diagram, a state is represented by a circle, and the transition between states is indicated by directed lines connecting the circles.

State Diagram of the same example

- The state diagram provides the same information as the state table and it can be obtained directly from the state table.
- The directed lines are labeled with two binary numbers separated by a slash. The input value during the present state is labeled first and the number after the slash gives the output during the present state.



Second Form of the State Table

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
AB	AB	AB	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

- There is no difference between a state table and a state diagram except in the manner of representation.
- The state table is easier to derive from a given logic diagram and the state diagram follows directly from the state table.
- The state diagram gives a pictorial view of state transitions and is the form suitable for human interpretation of the circuit operation.

Characteristic Tables

- The analysis of a sequential circuit with flip-flops other than the D type is complicated because the relationship between the inputs of the flip-flop and the next state is not straightforward.
- This relationship is best described by means of a characteristic table rather than a state equation.

Characteristic Tables

Flip-Flop Characteristic Tables

JK Flip-Flop

J	K	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

RS Flip-Flop

S	R	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Unpredictable

D Flip-Flop

D	$Q(t+1)$	
0	0	Reset
1	1	Set

T Flip-Flop

T	$Q(t+1)$	
0	$Q(t)$	No change
1	$Q'(t)$	Complement

- Here the next state is defined as a function of the inputs and present state.
- Q(t) refers to the present state prior to the application of a pulse. Q(t + 1) is the next state one clock period later.
- Note that the clock-pulse input is not listed in the characteristic table, but is implied to occur between time t and t + 1

Analysis with JK and Other Flip-Flops

- It was shown previously that the next-state values of a sequential circuit with D flipflops can be derived directly from the next- state equations.
- When other types of flipflops are used, it is necessary to refer to the characteristic table. The next-state values of a sequential circuit that uses any other type of flip-flop such as JK , RS , or T can be derived by following a two-step procedure:

1. Obtain the binary values of each flip-flop input function in terms of the present state and input variables.

2. Use the corresponding flip-flop characteristic table to determine the next state.

Example

JK Flip-Flop

J	K	Q(t + 1)	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

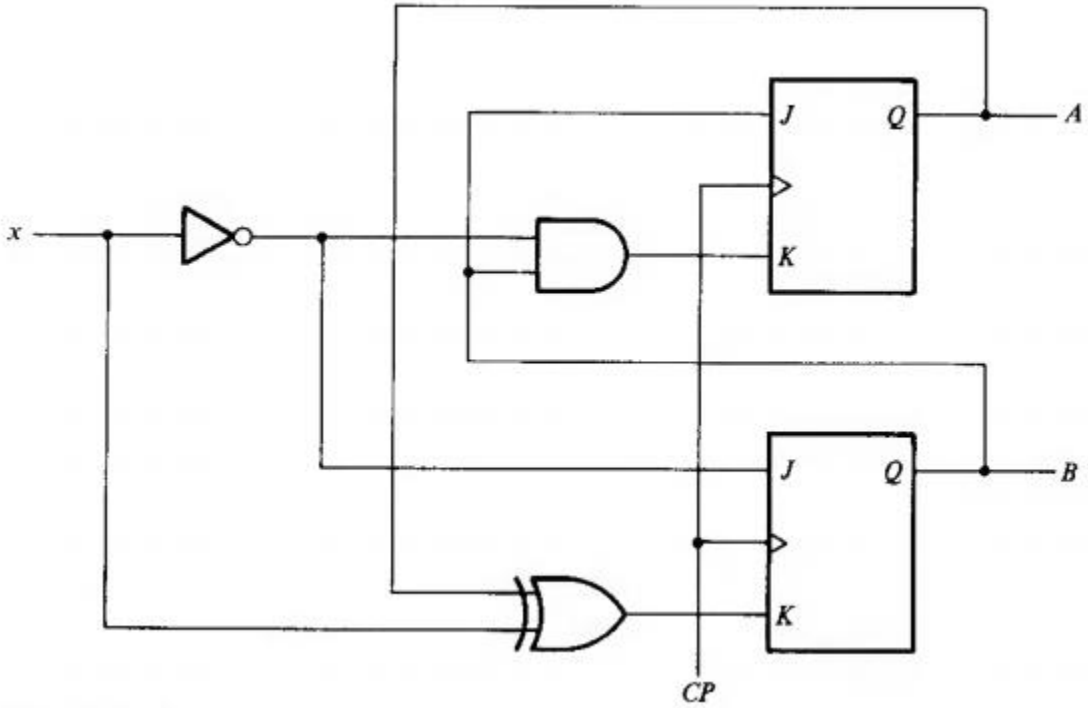
To illustrate this procedure, consider the sequential circuit with two JK flip-flops A and B and one input x, as shown in Figure.

The circuit has no outputs and, therefore, the state table does not need an output column.

The circuit can be specified by the following flipflop input functions:

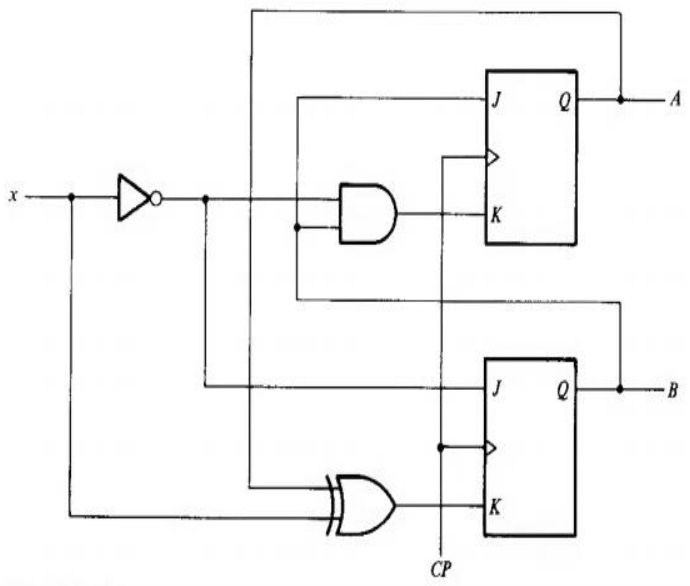
JA=B
KA= Bx‘

JB = x'
KB = A’x + Ax' = A XOR x



# Inputs to flipflops
JA=B JB = x'

KA= Bx‘ KB = A’x + Ax' = A XOR x



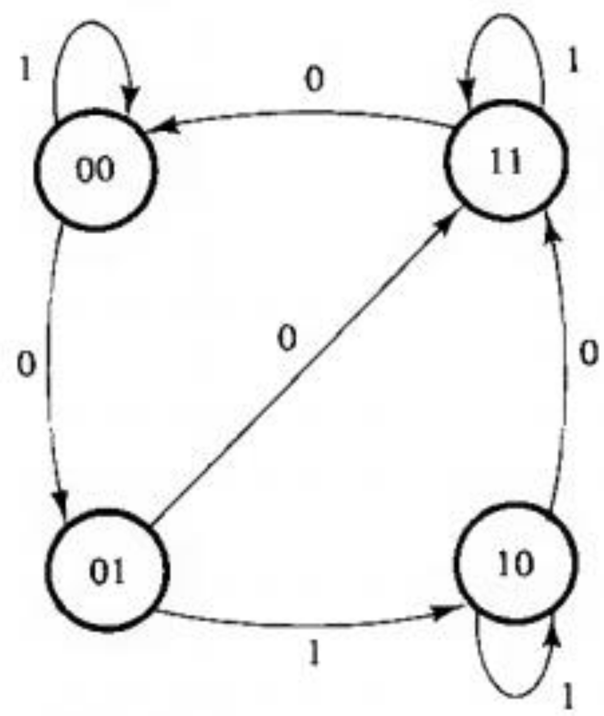
State Table for Sequential Circuit with *JK* flip-Flops

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	*A*	*B*	*JA*	*KA*	*JB*	*KB*
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

***JK* Flip-Flop**

J	*K*	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

State diagram



State Table for Sequential Circuit with *JK* flip-Flops

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	*A*	*B*	*JA*	*KA*	*JB*	*KB*
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

Suggested Reading

- M. Morris Mano, Digital Logic and Computer Design, PHI.

Thank you