CS ASSIGNMENT

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1) Find y at $x = 0.1$ & $x = 0.2$ correct to 3 decimal places, given $y' - 2y = 3e^x$, $y(0) = 0$

1) $$\frac{dy}{dx} - 2y = 3e^x$$

$$\frac{dy}{dx} = 2y + 3e^x$$

$x_0 = 0$, $y_0 = 0$, $h = 0.1$ (let)

with initial cond^n $y(0) = 0$

Let's take taylor series second order method :-

$$y(x_{i+1}) = y_i + \frac{h y_i'}{1!} + \frac{h^2}{2!} y_i''$$

$$y(x_{i+1}) = y_i + 0.1 y_i' + \frac{(0.1)^2}{2} y_i''$$

$$y(x_{i+1}) = y_i + 0.1 y_i' + 0.005 y_i''$$

We have, $y' = 2y + 3e^x$
$y'' = 2y' + 3e^x$

$x_0 = 0$, $y_0 = 0$
$y_0' = 3$ & $y_0'' = 9$

$$y(0.1) \simeq y_1 = y_0 + 0.1 y_0' + 0.005 y_0''$$
$$= 0 + 0.3 + 0.045$$
$$= 0.345$$

with $x_1 = 0.1$, $y_1 = 0.345$
$y_1' = 2(0.345) + 3e^{0.1} = 4.005$

$y_1'' = 2(4.005) + 3e^{0.1}$
$y_1'' = 11.325$

$y(0.2) = y_1 + 0.1y_1' + 0.005y_1''$
$= 0.345 + 0.1(4.005) + 0.005(11.325)$
$= 0.345 + 0.400 + 0.056$
$= 0.801$

2) Use ~~taly~~ Taylor series method of order four to solve $y' = x^2 + y^2$, $y(0) = 1$
for $x \in [0, 0.4]$ with $h = 0.2$

$\frac{dy}{dx} = x^2 + y^2$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$

Taylor Series 4th order method :-

$y(x_{i+1}) = y_i + \frac{h y_i'}{1!} + \cancel{\frac{h^2}{2!}} + \frac{h^2}{2!} y_i'' + \frac{h^3}{3!} y_i''' + \frac{h^4}{4!} y_i''''$

$y(x_{i+1}) = y_i + (0.2) y_i' + \frac{(0.2)^2}{2!} y_i'' + \frac{(0.2)^3}{3!} y_i''' + \frac{(0.2)^4}{4!} y_i''''$
$= y_i + 0.2y_i' + 0.02y_i'' + 0.0013 y_i''' + 0.00006 y_i''''$

We have $y' = x^2 + y^2$
$y'' = 2x + 2y.y'$
$y''' = 2 + 2((y')^2 + y.y'')$
$y'''' = 2[2y'y'' + y'y'' + yy''']$

$x_0 = 0$, $y_0 = 1$

$y_0' = 1$	$y_0''' = 8$
$y_0'' = 2$	$y_0'''' = 28$

$$y(0.2) \simeq y_1 = y_0 + 0.2y_0' + 0.02y_0'' + 0.0013y_0''' + 0.00006y_0''''$$
$$= 1 + (0.2)1 + (0.02)2 + (0.0013)8 + (0.00006)28$$
$$= 1.25208$$

with $x_1 = 0.2$, $y_1 = 1.25208$

$$y_1' = (0.2)^2 + (1.25208)^2 = 1.6077$$
$$y_1'' = 2(0.2) + 2(1.25208)(1.6077) = 4.4259$$
$$y_1''' = 2 + 2[(1.6077)^2 + (1.25208)(4.4259)] = 18.2525$$
$$y_1'''' = 2[3(1.6077)(4.4259) + (1.25208)(18.2525)] = 88.4002$$

$$y(0.4) = y_2 = y_1 + 0.2y_1' + 0.02y_1'' + 0.0013y_1''' + 0.00006y_1''''$$
$$= 1.25208 + (0.2)(1.6077) + (0.02)(4.4259) + (0.0013)(18.2525) + (0.00006)(88.4002)$$
$$= 1.62067$$

③ Find an approx. to $y(1.6)$, for the initial value problem
$$y' = x + y^2, \quad y(1) = 1$$
using Euler method with $h = 0.1$ & $h = 0.2$

$h = 0.2$

$$y' = x + y^2, \quad x_0 = 1, \; y_0 = 1$$

By Euler's method:-

$$y(x_n) = y_{n-1} + h f(x_{n-1}, y_{n-1})$$
$$y(x_n) = y_{n-1} + 0.2 (x_{n-1} + y_{n-1}^2)$$

with $x_0 = 1$, $y_0 = 1$

$$y(1.2) = y_1 = y_0 + 0.2[x_0 + y_0^2]$$
$$y(1.2) = y_1 = 1 + 0.2(1 + 1^2)$$
$$\Rightarrow y(1.2) = 1.4$$

with $x_1 = 1.2$, $y_1 = 1.4$

$$y(1.4) = y_2 = y_1 + 0.2[x_1 + y_1^2]$$

$$y(1.4) = y_2 = 1.4 + 0.2\left(1.2 + (1.4)^2\right)$$
$$= \cancel{2.72}\ 2.032$$

with $x_2 = 1.4$, $y_2 = \cancel{2.72}\ 2.032$

$$y(1.6) = y_3 = \cancel{2.72}\ 2.032 + 0.2\left(1.4 + (2.032)^2\right)$$
$$= 3.13278$$

$$y(1.6) = 3.13278$$

$\Rightarrow$ $h = 0.1$

$y' = x + y^2$, $x_0 = 1$, $y_0 = 1$

$$y(x_n) = y_{n-1} + h\, f(x_{n-1}, y_{n-1})$$

with $x_0 = 1$, $y_0 = 1$

$$y(1.1) = 1 + (0.1)(1 + 1^2) = 1.2$$

with $x_1 = 1.1$, $y_1 = 1.2$

$$y(1.2) = 1.2 + (0.1)\left(1.1 + (1.2)^2\right) = 1.254$$

with $x_2 = 1.2$, $y_2 = 1.254$

$$y(1.3) = 1.254 + 0.1\left[1.2 + (1.354)^2\right] = 1.657$$

with $x_3 = 1.3$, $y_3 = 1.657$

$$y(1.4) = 1.657 + 0.1\left[1.3 + (1.657)^2\right] = 2.061$$

with $x_4 = 1.4$, $y_4 = 2.061$

$$y(1.5) = 2.061 + 0.1\left[1.4 + (2.061)^2\right] = 2.6257$$

with $x_5 = 1.5$, $y_5 = 2.6257$

$$y(1.6) = 2.6257 + 0.1\left[1.5 + (2.6257)^2\right] = 3.4651$$

$$y(1.6) = 3.4651$$

(4) Given the initial value problem

$$y' = 2x + \cos y, \quad y(0) = 1$$

Show that it is sufficient to use Euler method with step length $h = 0.2$ to compute $y(0.2)$ with an error less than 0.05.

(4) $y' = 2x + \cos y$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$

By Euler's method:-

$$y(x_n) = y_{n-1} + h\, f(x_{n-1}, y_{n-1})$$

$$y(x_n) = y_{n-1} + 0.2\, f(x_{n-1}, y_{n-1})$$

~~y~~ with $x_0 = 0$, $y_0 = 1$

$$y(0.2) = 1 + 0.2\left[2(1) + \cos(1)\right]$$

$$y(0.2) = 1 + 0.2(2 + 0.5403)$$

$$y(0.2) = 1.5080$$