Superconductivity: General Aspects ☆

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Nomenclature

e electron charge
g conductance
T_c superconducting transition temperature
Δ superconducting gap
μ chemical potential

Introduction

Superconductivity is a very exciting and unusual macroscopic quantum phenomenon. Different superconducting devices are continuing to find more and more applications around the world. There is no doubt that in the future, these applications will become even more widespread. In order to understand how these devices work, one should know the general aspects of superconductivity. A lot of detailed information on the subject can be found in the corresponding specialized articles of this encyclopedia. This article deals with only the main results of the Bardeen–Cooper–Schrieffer (BCS) theory and the Ginzburg–Landau theory as well as those aspects of superconductivity which are not mentioned in the other articles such as unconventional superconductivity, inhomogeneous states (also known as Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) states), superconducting fluctuations, pinning of vortices. A special article is devoted to the manifestation of superconductivity in various nonmetallic systems.

Conventional Superconductors

A major success of low-temperature physics was achieved with the introduction of the notion of quasiparticles by Landau. According to the Landau Fermi-liquid theory, the properties of a many-body interacting system at low temperatures are determined by the spectrum of some low-energy, long-living excitations (called quasiparticles). Another milestone of the many-body theory is the mean field approximation (MFA). The BCS theory of superconductivity is a good example of the use of both the quasiparticle description and MFA. In 1956, L. Cooper found that even a weak attraction between particles in a degenerated Fermi liquid led to the formation of bound states, now called Cooper pairs. Soon after this discovery Bardeen, Cooper, and Schrieffer proposed a microscopic theory of superconductivity as a theory describing Bose condensation of Cooper pairs. N. N. Bogolyubov succeeded in solving the problem of superconductivity by the method of approximate second quantization and L. P. Gor'kov proposed a solution of the problem in the framework of the Green functions formalism. This method permitted the use of the well-known Feynman diagram technique from the quantum field theory of superconductivity. Using these methods, Gor'kov demonstrated that the phenomenological Ginzburg–Landau equations followed from the BCS theory in the limit $T \to T_c$ (i.e., in the vicinity of the transition). According to the BCS theory, the excitation spectrum $E(p)$ in a superconductor has a gap proportional to the amplitude of the Bose-condensate of Cooper pairs:

$$E(p) = \sqrt{[\varepsilon(p) - \mu]^2 + \Delta^2(p)}$$

where $\varepsilon(p)$ is the excitation spectrum in the normal state, $\Delta(p)$ is the superconducting gap, and μ is the chemical potential. In conventional superconductors, the gap has the same symmetry as the one of the lattices and in some systems does not depend on the momentum. In the latter case, both the gap and the transition temperature do not depend upon the density of nonmagnetic impurities (this statement is known as the Anderson theorem).

Unconventional Superconductors

In some systems, the effective electron–electron interaction is strongly repulsive at short distances but becomes attractive at large distances. In this case, an unconventional pairing can take place, with the gap having a symmetry group smaller than the one of the

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lattices: for example, in a tetragonal high-T_c superconductor, the angular dependence of the gap has the form

$$\Delta_d(p) \propto p_x^2 - p_y^2$$

In the isotropic case, such a dependence corresponds to the orbital momentum $l=2$ (d-wave superconductivity). In the anisotropic case, the momentum is not a good quantum number anymore. The corresponding label just indicates the number of zeros of the function $\Delta(p)$. For example, the d-wave gap function has four zeros corresponding to the momenta $p_x = \pm p_y$. Nonmagnetic impurities destroy unconventional superconductivity if $k_B T_{c0} \cdot \tau_{tr} > \hbar$, where τ_{tr} is the transport scattering time and T_{c0} is the transition temperature in the pure system.

Inhomogeneous State

In the usual BCS state, electrons forming a Cooper pair have opposite momenta and the total momentum of a pair is zero. In ferromagnetic superconductors or layered superconductors in a strong magnetic field, the densities and Fermi-momenta of electrons with opposite spins are different. In such conditions, it is more favorable for each electron to stay close to its Fermi surface and for Cooper pairs to have nonzero total momenta, unlike in the usual BCS state. In such systems, the condensate wave function varies in space and the ground state is inhomogeneous, which may lead to the formation of a crystalline structure (LOFF state). The LOFF state is believed to exist in heavy fermion systems such as $CeCoIn_5$, $CeRu_2$, UPd_2Al_3, and in some organic compounds. The LOFF state also appears in superconductor–ferromagnet–superconductor junctions. In these systems, the wave function of a Cooper pair oscillates inside the ferromagnet and, for a certain thickness of the layer, may have opposite signs on different sides of the ferromagnetic layer. As a result, the Josephson energy of the contact changes sign and has two minima, corresponding to the phase difference $\pm\pi$. Such a device, called π-junction, has two degenerate states and can be used as a qubit for quantum computation.

Fluctuations (Paraconductivity)

Phenomena that cannot be described within the quasiparticle method or the MFA are called fluctuations. In bulk samples of traditional superconductors, the critical temperature T_c sharply divides the superconducting and normal phases. Such a behavior of the physical characteristics of superconductors is in perfect agreement with both the Ginzburg–Landau phenomenological theory and the BCS microscopic theory of superconductivity. Both of these theories can be derived in the framework of the mean field approximation. In the BCS theory, only the Cooper pairs in the Bose-condensate are considered. The fluctuation theory deals with Cooper pairs out of the condensate. Fluctuation phenomena manifest themselves much stronger in disordered low-dimensional systems. This is because the fluctuation region in disordered superconducting films, which is determined by the resistance per unit square, is typically much wider than in bulk samples. What is even more important is that fluctuation effects exist beyond the critical region and affect not only thermodynamic quantities but also the kinetic ones. The phenomenon, which is now known as paraconductivity, is the decrease of the resistance of a superconducting sample above the transition temperature (i.e., in the normal phase) due to the appearance of fluctuating Cooper pairs. The fluctuation conductance g of a superconducting film reads:

$$g - g_n = \frac{e^2}{16h} \frac{T_c}{T - T_c}$$

Note that in high-temperature, organic, amorphous, and low-dimensional superconducting systems being studied presently, the fluctuation effects strongly differ from those in traditional superconductors. The transition turns out to be much more smeared out. The appearance of superconducting fluctuations above the critical temperature leads to precursor effects of the superconducting phase occurring in the normal phase, sometimes far from the transition. The conductivity, heat capacity, diamagnetic susceptibility, sound attenuation, and other properties may change considerably in the vicinity of the transition. A strong dependence of superconducting fluctuations on the temperature and magnetic field permits one to definitely separate the fluctuation effects from other contributions and to use them as a source of information about the microscopic parameters of a material. The account for fluctuations has become a necessary part in the design of superconducting devices.

Recent studies show the fluctuation phenomena is significant in high-temperature superconductors (high-T_c). The small coherence lengths give rise to high degree of fluctuation which makes the fluctuation effects obviously observed in the high-T_c compared with conventional superconductors (Mishonov and Penev, 2011). Research in high-T_c superconductor is focused on energy-saving applications. It has been proven that the interactions between electrons that cause antiferromagnetic interactions is related to high-T_c superconductivity. However, there are many complex behaviors among the electrons that have made it difficult to reach a consensus on the mechanism involved to explain why such superconductivity occurs in these materials (Donaldson, 2013). The high-T_c superconductors could hold exciting potential for new applications in power generation and transmission (superconducting cable; transformer; superconducting magnetic energy storage), transport, information technology, and medicine (Shiohara *et al.*, 2013; Donaldson, 2013).

Pinning of Vortices

The greatest success of the Ginzburg–Landau theory was the explanation of Shubnikov's phase by A. A. Abrikosov in 1957. In this phase, superconductivity and a magnetic field can peacefully coexist. Abrikosov suggested that the magnetic field can penetrate a superconductor along vortex lines, which can form a perfect lattice structure. In homogeneous superconductors, an applied current causes a drift of the vortex lattice and dissipation. When a transport current j_{tr} flows through a superconductor, the Lorentz force $F_{\mathrm{L}} = (\Phi_0/c)j_{\mathrm{tr}}$ exerted on each vortex line appears immediately ($\Phi_0 = hc/2e$ is the flux quantum). In homogeneous superconductors, the vortices start moving due to this force. The corresponding flow of the magnetic flux induces the electric field $E = (v/c) \cdot B$ and energy dissipation. In order to preserve superconductivity, one should prevent the motion of vortices by introducing a dry friction force. This can be done by creating local inhomogeneous regions (pinning centers), which would pin the vortex lattice. Such centers may appear due to structural inhomogeneities of the initial crystalline lattice (dislocations, accumulations of impurities, etc.). In the theory of collective pinning, even small structural fluctuations can qualitatively change the properties of the vortex structure. A single weak center causes only a weak elastic deformation, but the collective effect of a large number of weak centers destroys the lattice. The long-range lattice order disappears, while the short-range order survives only at distances smaller than a correlation length L_{c}. Each region of size L_{c} finds its equilibrium in a local minimum of the random pinning potential. A force is needed in order to move the pinned regions from the energetically favorable positions. The appearance of the frictional force leads to a finite critical current and hysteresis.

Superconductivity in Nonmetallic Systems

Organic Superconductors

W. A. Little has proposed the possibility of a nonphonon pairing mechanism in some organic superconductors. Little's work has stimulated a lot of further theoretical activity in the field of one-dimensional systems. All real organic superconductors are not truly one-dimensional but consist of long chains strongly coupled to each other, which make the system effectively two-dimensional and strongly anisotropic. This may result in an unconventional superconducting pairing and in the appearance of inhomogeneous LOFF states in such systems. High upper critical fields in some organic materials (e.g., $(\mathrm{TMTSF})_2\mathrm{ClO}_4$ or $(\mathrm{TMTSF})_2\mathrm{PF}_6$) support the scenario of the unconventional triplet pairing. An experimental evidence of the LOFF state has been found in the organic compounds $k - (\mathrm{BEDT}-\mathrm{TTF})_2 - \mathrm{Cu(NCS)}_2$ and $\lambda - (\mathrm{BETS})_2\mathrm{FeCl}_4$.

Ceramic Superconductors

Most of the high-T_{c} superconductors are ceramic superconductors.

Superfluidity of Helium

Superfluidity in He_4 is a result of Bose condensation of He_4 atoms themselves. In He_3, the atoms are spin-1/2 particles and the normal state of He_3 is thought to be a Fermi-liquid. Superfluidity in He_3 is the result of Cooper pairing in the fermion system. Cooper pairs in He_3 have a total spin $S = 1$ and orbital moment $l = 1$, which corresponds to the p-wave unconventional superconducting pairing.

Superfluid Model of Atomic Nuclei

Strong interactions between nucleons in heavy nuclei may yield the formation of Cooper pairs. This leads to the gap in the spectrum of excitations in the nuclei, which typically is of the order of $\Delta \sim 2$ MeV, which is much larger than the level spacing. The appearance of the nucleon Bose-condensate leads to parity effects in atomic nuclei. The energy and mass of a nucleus containing an even number of nucleons are smaller than the ones of an odd nucleus:

$$2M_{2N+1} - M_{2N} - M_{2N+2} = 2\Delta$$

A rotating nucleus creates supercurrents, which produce an effect similar to the anomalous diamagnetism in conventional superconductors. As a result, the moment of inertia of a nucleus is smaller than the one of a solid or liquid object with the same mass and geometry.

Superconductivity in Neutron Stars

In neutron stars (pulsars), the density of neutrons may be of the order of their density in heavy atomic nuclei. As a result, Cooper pairs are created and superconductivity can arise. Since pulsars are spinning, the neutron superfluid is threaded with a regular array of rotational vortices. The rotation frequency of the superfluid is proportional to the density of the vortices. As a pulsar's spin rate gradually decreases due to the emission of electromagnetic radiation, the vortices gradually move outwards.

Color Superconductivity

In the core of a neutron star, the density is so high that quark matter can appear. Strongly interacting quarks may form Cooper pairs (it is predicted that the red up-quarks are paired with the green down-ones) yielding the so-called color superconductivity. Since the quarks forming pairs have different charges and slightly different densities, the Fermi surface splits into two and an inhomogeneous crystalline state similar to the LOFF state appears in a shell, where the quark densities satisfy the appropriate conditions. The rotational vortices may be pinned by the LOFF structure leading to the pinning effects. Within this picture, the so-called glitch phenomena (sudden increases of rotation frequencies of a pulsar) are explained as the hopping of vortex bundles from one metastable minimum to another.

Superconductivity and Masses of Elementary Particles

Papers of Nambu and Jona-Lasinio and Vaks and Larkin have introduced the mechanism of elementary particle mass generation via the dynamical symmetry breaking. This spontaneous symmetry breaking is formally very similar to the one occurring in superconductors, with the superconducting gap corresponding to the masses of the elementary particles. In superconductors and in the models considered by Nambu and Jona-Losinio and by Vaks and Larkin, the transition occurs due to the appearance of the Bose-condensate of Cooper pairs, while in the standard model the symmetry breaking is due to the Bose-condensate of the scalar Higgs bosons. Furthermore, the Meissner effect, which is characterized by a penetration length, is the origin, in the elementary particle physics language, of the masses of the gage vector bosons. The masses correspond to the inverse of the penetration length in the conventional theory of superconductivity.

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