CS Assignment

SURAJ KUMAR YADAV
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1) find the Eigen values & vectors of the follow, matrix.

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

1) Characteristic eqn $|A - \lambda I| = 0$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (5-\lambda)(2-\lambda) - 4 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$$\Rightarrow \lambda = 1, 6$$

So, the eigen values $\Rightarrow$ 1 & 6.

$\rightarrow$ Eigen vector corresponding to eigen value = 1

$$AX = \lambda X$$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$5x_1 + 4x_2 = x_1 \rightarrow 4(x_1 + x_2) = 0$$

$$x_1 + 2x_2 = x_2 \rightarrow x_1 + x_2 = 0$$

These eq^{n}s are not independent.
It's simplest form is $x_1 + x_2 = 0$
or $x_1 = -x_2$

So, eigen vectors for eigen value '1' :-

$$E = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$, where t is parameter.

→ Eigen vector corresponding to eigen value = 6

$$AX = \lambda X$$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 6 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$5x_1 + 4x_2 = 6x_1 \rightarrow 4x_2 = x_1$$
$$x_1 + 2x_2 = 6x_2 \rightarrow x_1 = 4x_2$$

These eq^{n}s are not independent.
It's simplest form is $x_1 = 4x_2$

So, eigen vectors for eigen value '6' :-

$$E = t\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$, where t is parameter.

2) Find the largest Eigen values & corresponding vectors of the follow. matrix using power method.

$$A = \begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 10 \end{bmatrix}$$

Let the initial eigen vector, $x^{(0)} = [1, -1, 1]'$

1st iteration :-

$$A x^{(0)} = \begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 13 \\ -14 \\ 13 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 13 \\ -14 \\ 13 \end{bmatrix} = 14 \begin{bmatrix} 0.9285 \\ -1 \\ 0.9285 \end{bmatrix}$$

$$\lambda^{(1)} = 14 \quad \& \quad x^{(1)} = [0.9285, -1, 0.9285]'$$

2nd iteration :-

$$A x^{(1)} = \begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 10 \end{bmatrix} \begin{bmatrix} 0.9285 \\ -1 \\ 0.9285 \end{bmatrix} = \begin{bmatrix} 12.2142 \\ -13.7142 \\ 12.2142 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 12.2142 \\ -13.7142 \\ 12.2142 \end{bmatrix} = 13.7142 \begin{bmatrix} 0.8906 \\ -1 \\ 0.8906 \end{bmatrix}$$

$$\lambda^{(2)} = 13.7142 \quad \& \quad x^{(2)} = [0.8906, -1, 0.8906]'$$

3rd iteration :-

$$A x^{(2)} = \begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 10 \end{bmatrix} \begin{bmatrix} 0.8906 \\ -1 \\ 0.8906 \end{bmatrix} = \begin{bmatrix} 11.7968 \\ -13.5625 \\ 11.7968 \end{bmatrix}$$

$$\cancel{\lambda^{(3)}} \rightarrow \begin{bmatrix} 11.7968 \\ -13.5625 \\ 11.7968 \end{bmatrix} = 13.5625 \begin{bmatrix} 0.8698 \\ -1 \\ 0.8698 \end{bmatrix}$$

$\lambda^{(3)} = 13.5625$ & $x^{(3)} = [0.8698, -1, 0.8698]'$

4th iteration:-

$\lambda^{(4)} = 13.4792$ & $x^{(4)} = [0.8582, -1, 0.8528]'$

5th iteration:-

$\lambda^{(5)} = 13.4328$ & $x^{(5)} = [0.8516, -1, 0.8516]'$

6th iteration:-

$\lambda^{(6)} = 13.4066$ & $x^{(6)} = [0.8479, -1, 0.8479]'$

7th iteration:-

$\lambda^{(7)} = 13.3918$ & $x^{(7)} = [0.8458, -1, 0.8458]'$

8th iteration:-

$\lambda^{(8)} = 13.3834$ & $x^{(8)} = [0.8446, -1, 0.8446]'$

9th iteration:-

$\lambda^{(9)} = 13.3786$ & $x^{(9)} = [0.8439, -1, 0.8439]'$

10th iteration:-

$\lambda^{(10)} = 13.3759$ & $x^{(10)} = [0.8435, -1, -0.8435]'$

11th iteration:-

$\lambda^{(11)} = 13.3743$ & $x^{(11)} = [0.8433, -1, 0.8433]'$

12th iteration :-

$\lambda^{(12)} = 13.3734$ & $x^{(12)} = [8.8432, -1, 0.8432]$

13th iteration :-

$\lambda^{(13)} = 13.3729$ & $x^{(13)} = [0.8431, -1, 0.8431]$

So, the largest Eigen value, $\lambda = 13.3729 \approx 13.37$

& corresponding eigen vector $= [0.8431, -1, 0.8431]$
$\approx [0.84, -1, 0.84]$