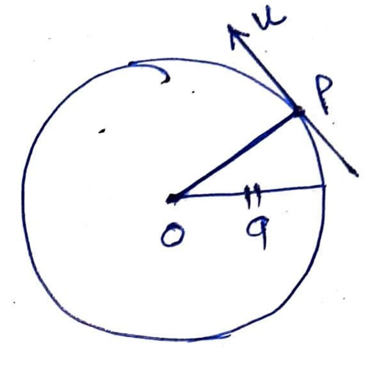
Ex 1(E)

Q8 A point is describing a circle of radius a in such a way that the tangential acceleration is K times the normal acceleration. If its speed at a certain point is u, prove that it will return to the same point after a time

$$\frac{a}{Ku}\left(1-e^{-2\pi K}\right)$$



Let at time $t=0$, particle be at point P.

at point P, $t=0$, $S=0$, $V=u$

It has a tangential velocity $\frac{ds}{dt}=u$ at P.

tangential Acceleration $= K\{\text{normal acceleration}\}$

$$\frac{d^2s}{dt^2} = K\frac{V^2}{a} \qquad \rho = a = \text{radius of Curvature.}$$

$$\frac{d}{dt}\left\{\frac{ds}{dt}\right\} = \frac{dV}{dt} = \frac{dV}{ds}\cdot\frac{ds}{dt} = \cancel{v}\frac{dV}{ds} = K\frac{V^{\cancel{2}}}{a}$$

$$\frac{dv}{ds} = \frac{Kv}{a} \Rightarrow \frac{dv}{v} = \frac{K}{a}ds$$

Integrating both sides $\int\frac{dv}{v} = \int\frac{K}{a}ds \Rightarrow \log v = \frac{K}{a}s + c_1$

At initial point P, $S=0$, $v=u$

$$\log u = \frac{K}{a}\cdot 0 + u \Rightarrow u = \log u$$

$$\Rightarrow \log v = \frac{K}{a}s + \log u \Rightarrow v = ue^{\frac{K}{a}s}$$

tangential velocity $v = \frac{ds}{dt} = ue^{K/a\,s}$

$$\frac{ds}{e^{K/a\,s}} = u\,dt \Rightarrow \int e^{-K/a\,s}ds = ut + c_2$$

$$\left(\frac{e^{-K/a\,s}}{-\frac{K}{a}}\right) = ut + c_2 \Rightarrow -\frac{a}{K}e^{-K/a\,s} = ut + c_2 \qquad \text{At } t=0,\ s=0$$

$$-\frac{a}{K} = c_2$$

$$-\frac{a}{K}e^{-K/a\,s} = ut - \frac{a}{K} \Rightarrow ut = \frac{a}{K}\left[1-e^{-K/a\,s}\right]$$ the time taken by complet one round, the distance covered $s=2\pi a$

$$T = \frac{a}{uK}\left[1-e^{-\frac{K}{a}\cdot 2\pi a}\right\} = \frac{a}{Ku}\left\{1-e^{-2\pi K}\right\}$$

Ans.