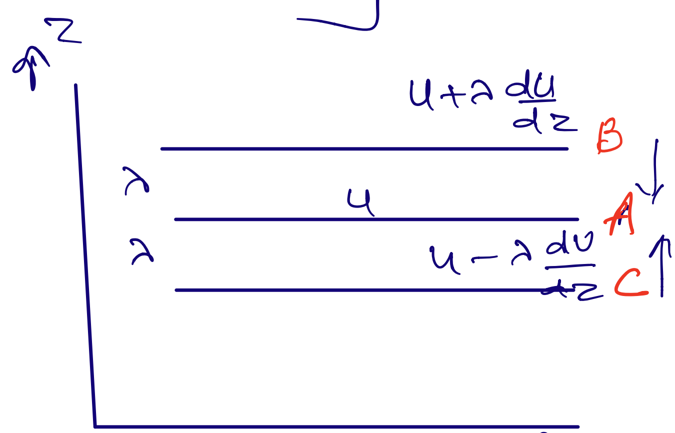
Transport Phenomena – Part 2.

Deducing the formulas for

1. viscosity
2. Thermal conductivity
3. Diffusion.

—x—x—x—.

(1) Viscosity : Elementary Treatment.



Number of molecules per unit volume crossing the area dA of layer 'A' in unit time (1 sec.)

$$= \frac{1}{6} n \bar{c}\, dA$$

momentum transferred due to layer 'B' to layer 'A'.

$$\frac{1}{6} n \bar{c}\, dA \; m\left(u + \lambda \frac{du}{dz}\right)$$

Similarly momentum transferred due to layer 'C' to layer 'A'

$$= \frac{1}{6} n \bar{c}\, dA \; m\left(u - \lambda \frac{du}{dz}\right)$$

Net transfer of momentum per sec. downward through the layer dA.

$$= m \frac{1}{6} n \bar{c}\, dA \left[\left(u + \lambda \frac{du}{dz}\right) - \left(u - \lambda \frac{du}{dz}\right)\right]$$

$$= \frac{m n \bar{c}\, dA}{6} \; 2 \cdot \lambda \frac{du}{dz}$$

$$= \frac{1}{3} m n \bar{c} \lambda \frac{du}{dz} \cdot dA$$

$$F = \eta \, dA \cdot \frac{du}{dz}$$

$$\boxed{\eta = \frac{1}{3} m n \bar{c} \lambda}$$

$$= \frac{1}{3} \rho \bar{c} \lambda$$

gas density

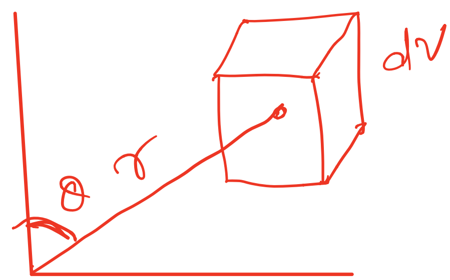
~~Viscosity~~ - Detailed Analysis.

Let us consider a general entity = H

- Value of parameter at $z=0$ is H

value of parameter at same layer above

$$= H + \frac{\partial H}{\partial z} \cdot r\cos\theta$$



- similarly at equal distance below the layer $H - \frac{\partial H}{\partial z} \cdot r\cos\theta$.

—x—

Number of molecules having velocity betwee c to c+dc in the dV volume = $dn_c\, dV$

- Molecules in dV volume suffer from collisions and come out in all possible.

— If P_c is the collision probability of two molecules then the number of collision in dt time is

$$\frac{1}{2} P_c \, dn_c \, dV \, c\,dt$$

— Because each collision will result in 2 paths so number of paths

$$= P_c \, dn_c \, dV \, c\,dt.$$

— The molecules are coming out of dV in all possible direction (sphere)

- We have to calculate the number of molecules coming towards the the layer under consideration will depend ~~upon~~ upon $d\omega = \frac{dA \cos\theta}{r^2}$

so number of molecules coming towards dA

$$\frac{d\omega}{4\pi} \cdot P_c \, dn_c \, dV \, c\,dt.$$

— Few of them leave the path while reaching dA

- So number of molecules reaching dA will be $= \frac{dA \cos\theta}{4\pi r^2} P_c \, dn_c \, dV \, c \, dt \, e^{-r/\lambda}$

$$= \frac{dA \cos\theta}{4\pi \cancel{r^2}} \, c \, dn_c \, \cancel{r^2} \sin\theta \, d\theta \, d\phi \, \frac{e^{-r/\lambda}}{\lambda} \, dr \, dt$$

— Transport of the physical entity downward

$$T\downarrow = \frac{dA\,dt}{4\pi} \int_{c=0}^{\infty} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \int_{r=0}^{\infty} c \, dn_c \, \sin\theta \cos\theta \, d\theta \, d\phi \, \frac{e^{-r/\lambda}}{\lambda} \left(H + r\cos\theta \frac{dH}{dz} \right) dr$$

→ Similarly the upwards transfer of entity from lower layer

$$T\uparrow = \frac{dA\,dt}{4\pi} \int_{c=0}^{\infty} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \int_{r=0}^{\infty} c \, dn_c \, \sin\theta \cos\theta \, d\theta \, d\phi \, \frac{e^{-r/\lambda}}{\lambda} \left(H - r\cos\theta \frac{dH}{dz} \right) dr$$

so the net transfer of entity through the layer of area dA $= T\downarrow - T\uparrow$

$$= 2\,\frac{dH}{dz}\,\frac{dA\,dt}{4\pi}\int_{c=0}^{\infty} c\,dn_c \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} \frac{r e^{-r/\lambda}}{\lambda}\,dr.$$

$$\underline{\underline{B}} = T\downarrow - T\uparrow = \frac{1}{3}\,dA\,dt\,\frac{dH}{dz}\,\lambda n\bar{c}$$

For Viscosity Analysis

$$H = \underline{\underline{mu}} \qquad \frac{dH}{dz} = m\,\frac{du}{dz}$$

$B = F$

$$F = \frac{1}{3}\,dA\,dt \cdot m\,\frac{du}{dz}\,\lambda n\bar{c}.$$

F per unit time

$$\eta\,\cancel{dA}\,\cancel{\frac{du}{dz}} = \frac{1}{3}\,mn\lambda\bar{c}\,\cancel{\frac{du}{dz}}\cdot\cancel{dA}.$$

$$\boxed{\eta = \frac{1}{3}\,mn\lambda\bar{c}} = \frac{1}{3}\,\rho\lambda\bar{c}$$

For thermal Conductivity

Here $H \to E$ so $\frac{dH}{dz} = \frac{dE}{dz}$

Thermal energy per unit time $= \frac{1}{3} \lambda n \bar{c} \, dA \, \frac{dE}{dz}$

But the thermal energy flowing per second per unit area.

$$Q = K \cdot dA \, \frac{dT}{dz}.$$

$$K \cdot \cancel{dA} \, \frac{dT}{\cancel{dz}} = \frac{1}{3} n \bar{c} \lambda \, \cancel{dA} \, \frac{dE}{\cancel{dz}}.$$

$$K = \frac{1}{3} n \bar{c} \lambda \, \frac{dE}{dT}$$

If m be the mass C_v the specific heat at constant volume.

$$dE = m C_v \, dT \Rightarrow \frac{dE}{dT} = m C_v.$$

$$\boxed{K = \frac{1}{3} m n \bar{c} \lambda c_v}$$

$$\boxed{K = \eta c_v}$$