Assignment

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1) Solve the follow. linear system of eqn using LU decompn method.

$$y + z = 2$$
$$2x + 3z = 5$$
$$x + y + z = 3$$

1)
$$x + y + z = 3 \quad \text{---} (1)$$
$$2x + 3z = 5 \quad \text{---} (2)$$
$$y + z = 2 \quad \text{---} (3)$$

$$AX = B$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

Let $A = LU$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12} + U_{22} & l_{21}U_{13} + U_{23} \\ l_{31}U_{11} & l_{31}U_{12} + l_{32}U_{22} & l_{31}U_{13} + l_{32}U_{23} + U_{33} \end{bmatrix}$$

$\Rightarrow U_{11} = 1, \quad U_{12} = 1, \quad U_{13} = 1$

$\Rightarrow l_{21}U_{11} = 2 \Rightarrow l_{21} = 2$

$\Rightarrow l_{31}U_{11} = 0 \Rightarrow l_{31} = 0$

$\Rightarrow l_{21}U_{12} + U_{22} = 0 \Rightarrow U_{22} = -2$

$l_{21}u_{13} + u_{23} = 3 \Rightarrow u_{23} = 1$

$l_{31}u_{12} + l_{32}u_{22} = 1 \Rightarrow l_{32} = -1/2$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1 \Rightarrow u_{33} = 3/2$

So, $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix}$

& $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3/2 \end{bmatrix}$

Now, $AX = B$ & $A = LU$

$\Rightarrow LUX = B$

Let $UX = Y \Rightarrow LY = B$

$\rightarrow LY = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

$y_1 = 3 \quad \Rightarrow 2 \times 3 + y_2 = 5 \rightarrow y_2 = -1$

$\Rightarrow -\frac{1}{2}(-1) + y_3 = 2 \rightarrow y_3 = 3/2$

Now, $UX = Y$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 3/2 \end{bmatrix}$$

$x_3 = 1$ $\Rightarrow -2x_2 + 1 = -1$

$x_2 = 1$

$\Rightarrow x_1 + 1 + 1 = 3 \Rightarrow x_1 = 1$

So, the sol^n is $x_1 = 1, x_2 = 1, x_3 = 1$

(2) Solve the follow. linear system of eq^n using LU decompos^n method

$$7x - 2y + z = 12 \quad \text{--- (1)}$$
$$14x - 7y - 3z = 17 \quad \text{--- (2)}$$
$$-7x + 11y + 18z = 5 \quad \text{--- (3)}$$

(2) $AX = B$

$$A = \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 17 \\ 5 \end{bmatrix}$$

Let $A = LU$

$$A = \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$\Rightarrow l_{11} = 7, \quad l_{21} = 14, \quad l_{31} = -7$

$\Rightarrow l_{11}u_{12} = -2 \Rightarrow u_{12} = -2/7$

$l_{11}u_{13} = 1 \Rightarrow u_{13} = 1/7$

$l_{21}u_{12} + l_{22} = -7 \longrightarrow l_{22} = -3$

$l_{31}u_{12} + l_{32} = 11 \longrightarrow l_{32} = 9$

$l_{21}u_{13} + l_{22}u_{23} = -3 \longrightarrow u_{23} = 5/3$

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 18 \longrightarrow l_{33} = 4$

$$L = \begin{bmatrix} 7 & 0 & 0 \\ 14 & -3 & 0 \\ -7 & 9 & 4 \end{bmatrix} \quad \& \quad U = \begin{bmatrix} 1 & -2/7 & 1/7 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, $AX = B$ & $A = LU$

$\rightarrow LUX = B$

Let $UX = Y \rightarrow LY = B$

$\rightarrow LY = B$

$$\begin{bmatrix} 7 & 0 & 0 \\ 14 & -3 & 0 \\ -7 & 9 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 17 \\ 5 \end{bmatrix}$$

$y_1 = \frac{12}{7}$ $\quad 14 \cdot \frac{12}{7} - 3y_2 = 17$

$y_2 = 7/3$

$-7 \cdot \frac{12}{7} + 9 \cdot \frac{7}{3} + 4y_3 = 5$

$y_3 = -1$

Now, $UX = Y$

$$\begin{bmatrix} 1 & -2/7 & 1/7 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12/7 \\ 7/3 \\ -1 \end{bmatrix}$$

$\therefore x_3 = -1$

$\Rightarrow x_2 - \frac{5}{3} = \frac{7}{3}$

$\therefore x_2 = +4$

$\Rightarrow x_1 + \frac{8}{7} - \frac{1}{7} = \frac{12}{7}$

$\hookrightarrow x_1 = 3$

So, the sol^n is $3, +4, -1$