CS ASSIGNMENT

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Given the values:-

x	0	1	2	3	4
$f(x)$	1	7	23	55	109

Find $f(0.5)$ & $f(1.5)$ using Newton's forward difference formula.

Sol" $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$y_0 = 1$, $y_1 = 7$, $y_2 = 23$, $y_3 = 55$, $y_4 = 109$

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
x_1	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_2	y_2	$\Delta y_2 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
x_3	y_3	$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
0	1	$7-1=6$	$16-6=10$	$16-10=6$	$6-6=0$
1	7	$23-7=16$	$32-16=16$	$22-16=6$	
2	23	$55-23=32$	$54-32=22$		
3	55	$109-55=54$			
4	109				

From Newton's forward interpol" formula:-

$$f(x) = f(x_0) + u\Delta f(x_0) + u(u-1)\frac{\Delta^2 f(x_0)}{2!} + \frac{u(u-1)(u-2)\Delta^3 f(x_0)}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 f(x_0)}{4!}$$

where $u = \frac{x - x_0}{h}$

$$f(x) = f(x_0) + (x-x_0)\frac{\Delta f(x_0)}{1!} + (x-x_0)(x-x_1)\frac{\Delta^2 f(x_0)}{2!} + (x-x_0)(x-x_1)(x-x_2)\frac{\Delta^3 f(x_0)}{3!} + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\frac{\Delta^4 f(x_0)}{4!}$$

$$f(x) = 1 + (x-0)\frac{6}{1!} + (x-0)(x-1)\frac{10}{2} + (x-0)(x-1)(x-2)\frac{6}{3!} + 0$$

$$f(x) = 1 + 6x + 5x(x-1) + x(x-1)(x-2)$$
$$= 1 + 6x + 5x^2 - 5x + x^3 - 3x^2 + 2x$$
$$= x^3 + 2x^2 + 3x + 1$$

$\therefore f(x) = x^3 + 2x^2 + 3x + 1$

$\rightarrow f(0.5) = (0.5)^3 + 2(0.5)^2 + 3(0.5) + 1$
$= 0.125 + 0.5 + 1.5 + 1$
$= 3.125$

$\rightarrow f(1.5) = (1.5)^3 + 2(1.5)^2 + 3(1.5) + 1$
$= 3.375 + 4.5 + 4.5 + 1$
$= 13.375$