QUESTIONS

(*i*) What are the four elastic constants of a material and how are they related ? Give dimensions and units.
(*ii*) Why do you use threads to suspend the bars and not wires ?
(*iii*) Is there any restriction of the amplitude of vibration for the determination of T_1 ? If so, why ?
(*iv*) Is there any restriction on the angular displacement for torsional vibrations ?
(*v*) How does the bending moment of the wire exerts couple on the bars ?
(*vi*) What is the effect of temperature on the elastic constants ?
(*vii*) What is the advantage of Searle's method of determining elastic constants over other methods of determining Y and η ?

Exp. 8. Modulus of rigidity by Maxwell's needle

Object

To determine the value of the modulus of rigidity of the material of a given wire by dynamical method using a Maxwell's needle.

Apparatus

Maxwell's needle, the given wire, a stop-watch, a metre scale, a screw gauge and a weight box.

Maxwell's needle (fig. 8.1) consists of a hollow brass tube of length, say '*a*' open at both ends suspended horizontally by a vertical wire attached to its middle point.

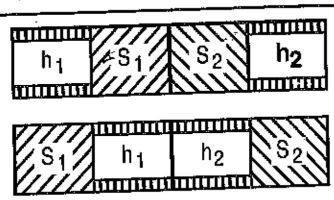

Fig. 8.1

The tube can accommodate four metal cylinders (two hollow and two solid) each of which is of length $a/4$.

Theory

Let the solid cylinders be put in the inner positions and hollow ones in the outer positions and let the combination be slightly rotated in a horizontal plane and released. The needle will execute a S.H.M., the time period T_1 being given by

$$T_1 = 2\pi\sqrt{I_1/C}$$

where C is the couple for unit twist in the wire and I_1 is the moment of inertia of the combination about the wire as axis.

Now $$C = \frac{\pi\eta r^4}{2l}$$

where r, l and η are respectively radius, length and modulus of rigidity of the wire.

Let the hollow and solid cylinders be interchanged so that the two hollow cylinders occupy the central positions. The time period T_2 of the torsional vibration of the combination will be given by

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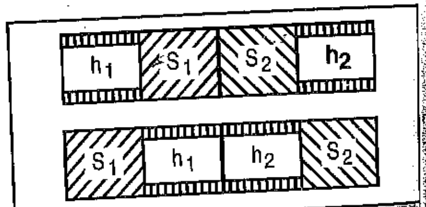

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Let the hollow and solid cylinders be interchanged so that the two hollow cylinders occupy the central positions. The time period T_2 of the torsional vibration of the combination will be given by

$$T_2 = 2\pi\sqrt{I_2/C}$$

where I_2 is the moment of inertia of the combination about the suspension wire.

$$\therefore \quad T_1^2 - T_2^2 = \frac{4\pi^2}{C}(I_1 - I_2) \quad \text{or,} \quad C = \frac{4\pi^2(I_1 - I_2)}{T_1^2 - T_2^2}$$

$$\text{or,} \quad \frac{\pi\eta r^4}{2l} = \frac{4\pi^2(I_1 - I_2)}{T_1^2 - T_2^2} \quad \therefore \quad \eta = \frac{8\pi l\,(I_1 - I_2)}{r^4(T_1^2 - T_2^2)} \qquad ...(1)$$

Let the mass of each hollow cylinder be m_1 and that of solid cylinder be m_2. Clearly the c.g. of the inner and outer cylinders are at distances $(a/8)$ and $(3a/8)$ respectively from the axis of oscillation. Let I_0, I' and I'' be the moment of inertia of hollow tube, hollow cylinders and solid cylinders about the axis of the suspension wire. Applying the theorem of parallel axes, we have,

$$I_1 = I_0 + 2\left[I' + m_2\left(\frac{a}{8}\right)^2 + I'' + m_1\left(\frac{3a}{8}\right)^2\right]$$

and $$I_2 = I_0 + 2\left[I' + m_1\left(\frac{a}{8}\right)^2 + I'' + m^2\left(\frac{3a}{8}\right)^2\right]$$

$$\therefore \quad I_1 - I_2 = \frac{a^2}{4}(m_1 - m_2)$$

Substituting this value of $(I_1 - I_2)$ in (1), we get,

$$\eta = \frac{2\pi l(m_2 - m_1)a^2}{r^4(T_2^2 - T_1^2)}$$

Procedure

(*i*) All the cylinders are placed inside the tube in such a way that the two solid cylinders occupy the central position and the hollow ones in the outer position and that no part of the cylinders project outside the tube.
(*ii*) The time for a known number of oscillations (say 50) is noted with a stop-watch.
(*iii*) The positions of hollow and solid cylinders are then interchanged and the time period is similarly determined.
(*iv*) Masses of the hollow and solid cylinders are determined and the lengths of the suspension wire and the cylinders are measured.
(*v*) Radius of the wire is measured at several places by a screw gauge.

Observations

(*A*) *Readings for the determination of diameter of the wire*

[See Observation table [C] of Expt. No. 6]

(*B*) *Readings for the determination of the periodic times*

Least count of stop-watch = ...sec.

S. No.	No. of Oscillations	Solid cylinders in inner position and hollow cylinders in outer position		Position interchanged	
		Time taken	T_1 in sec	Time taken	T_2 in sec
		Mean		Mean	

[C] (*i*) Length of the hollow tube = ... cm

(*ii*) *Determination of* $(m_2 - m_1)$

Mass of the hollow cylinders = ... gm

mean m_1 = ... gm

Mass of the solid cylinders = ... gm

mean m_2 = ... gm

Calculations

$$\eta = \frac{2\pi l (m_2 - m_1) a^2}{r^4 (T_2^2 - T_1^2)} = \text{... dyne/cm}^2.$$

% *error* :

Result

The value of the modulus of rigidity for the material of the wire (correct to significant figures) = ... dyne/cm^2.

Precautions

(*i*) The Maxwell's needle should always remain horizontal so that the moment of inertia of the hollow tube remains constant throughout the whole experiment.

(*ii*) The experimental wire should be free from kink. It should be fairly thin and long.

(*iii*) The needle should have only torsional oscillations in a horizontal plane.

QUESTIONS

(*i*) Do you get the same value of η (*a*) for a thin and a thick wire (*b*) from statical and dynamical methods ? Give reasons for your answers.

(*ii*) What is the advantage of this method over the statistical method ?

(*iii*) Why is it necessary to keep the needle horizontal ?

(*iv*) Why does the periodic time change when the position of the cylinders is interchanged although the mass of the oscillating system remains unchanged ?