(d) Once the balancing weights have been adjusted, these should not be disturbed during the whole experiment.

QUESTIONS

(i) What is an inertia table and why is it so called ?

(ii) How does the twisting couple of the wire produce angular acceleration in the inertia table ?

(iii) Define moment of inertia and state the laws of parallel and perpendicular axes.

(iv) A circular disc and a circular ring of same mass and radius roll down an inclined plane. Which one will reach the bottom first ?

(v) What is the function of the concentric groove on the inertia table ?

Exp. No. 3(A) Acceleration due to gravity by compound pendulum

Object

To determine the acceleration due to gravity by compound pendulum.

Apparatus

Bar pendulum, knife-edge, a stop-watch and a metre scale.

The bar pendulum is a uniform metal bar MN having a series of holes (about 5 mm diameter) drilled along its length, symmetrically at equal distances on either side of its centre of gravity G (fig. 3.3).

Theory

Let the bar pendulum of mass m be suspended from S. Let it be displaced through a small angle θ from its mean position of rest (fig. 3.1). If the distance of its c.g. from the point of suspension be 'l', the restoring couple τ acting on it will be given by

$$\tau = mg \times l \sin\theta$$
$$= mgl\theta, \text{ (}\theta\text{ being small).}$$

This couple gives rise to an angular acceleration $\frac{d^2\theta}{dt^2}$ in the body. If I_s be the moment of inertia of the body about a horizontal axis passing through the point of suspension S, the equation of motion of the pendulum will be given by

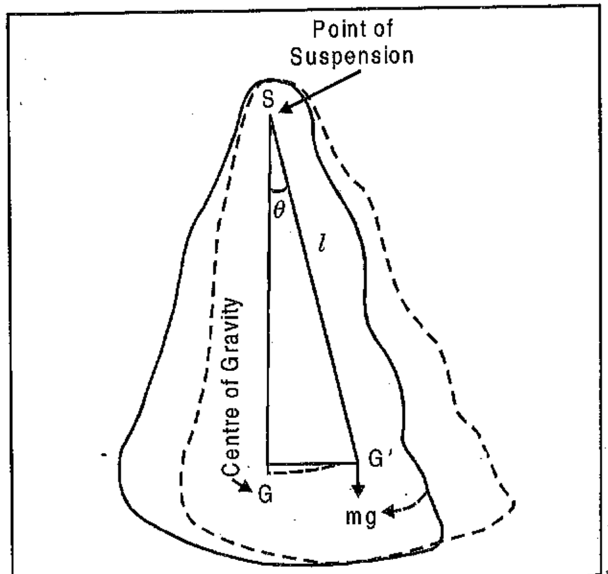

Fig. 3.1

$$I_S \cdot \frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\therefore \quad \frac{d^2\theta}{dt^2} = -\frac{mgl\theta}{I_S} = -\frac{mgl\theta}{I_G + m(SG)^2}$$

$$= -\frac{mgl\theta}{mk^2 + m(SG)^2} \quad (k \text{ being the radius of gyration about c.g.})$$

$$= -\left(\frac{gl}{k^2 + l^2}\right)\theta = -\mu^2\theta$$

Hence the motion is simple harmonic and the period of oscillation is given by

$$T = \frac{2\pi}{\mu} = 2\pi\sqrt{\frac{k^2 + l^2}{lg}} = 2\pi\sqrt{\frac{(k^2/l) + l}{g}} = 2\pi\sqrt{\frac{L}{g}}.$$

In the above expression 'L' is known as the length of the equivalent simple pendulum.

Also $$T^2 = 4\pi^2 \, (l^2 + k^2)/lg$$

or $$l^2 - \frac{gT^2}{4\pi^2} l + k^2 = 0.$$

$$\therefore \quad l = \frac{gT^2}{8\pi^2} \pm \sqrt{\frac{g^2T^4}{64\pi^2} - k^2} = x \pm y$$

where $$x = \frac{gT^2}{8\pi^2} \text{ and } y = \sqrt{\frac{g^2T^4}{64\pi^2} - k^2}.$$

Therefore, there are two values of l at distances $(x + y)$ and $(x - y)$ from c.g. and on the same side of it for which the time period is the same.

Similarly, on inverting the pendulum, we shall get another two values of l on the other side of c.g.

Thus, we have four points A, B, C, D in the same straight line with c.g., two on either side of it for which the time period of the pendulum is the same (fig. 3.2).

$$\therefore \quad AC = BD = (x + y) + (x - y) = 2x$$

$$= 2\left(\frac{gT^2}{8\pi^2}\right) = \frac{gT^2}{4\pi^2} = L.$$

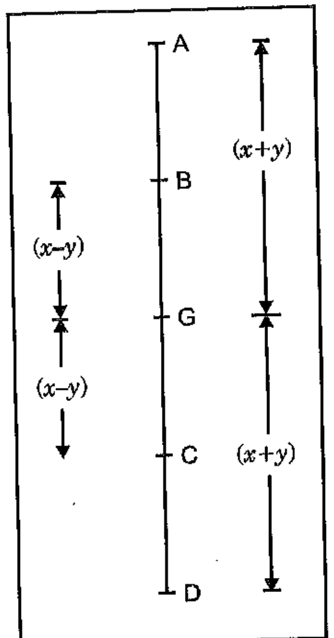

Fig. 3.2

Proceudre

(i) The bar pendulum is suspended about the knife-edge nearest to one end and its time period of oscillation is determined with the stop-watch. The process is repeated by supporting the bar on the knife-edge (fig. 3.2) successively through different holes from M to G (leaving one or two near the c.g.), the distance of the knife-edge from c.g. of the bar being noted in each case.

(ii) The same is repeated by suspending from the holes on other side of the c.g.

(iii) A graph is next plotted between the distance from the c.g. along x-axis and the time period along y-axis. The graph (fig. 3.4) will consist of two symmetrical curves* about a line through the c.g. of the pendulum.

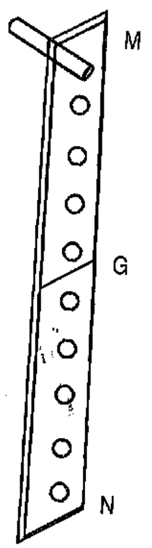

Fig. 3.3

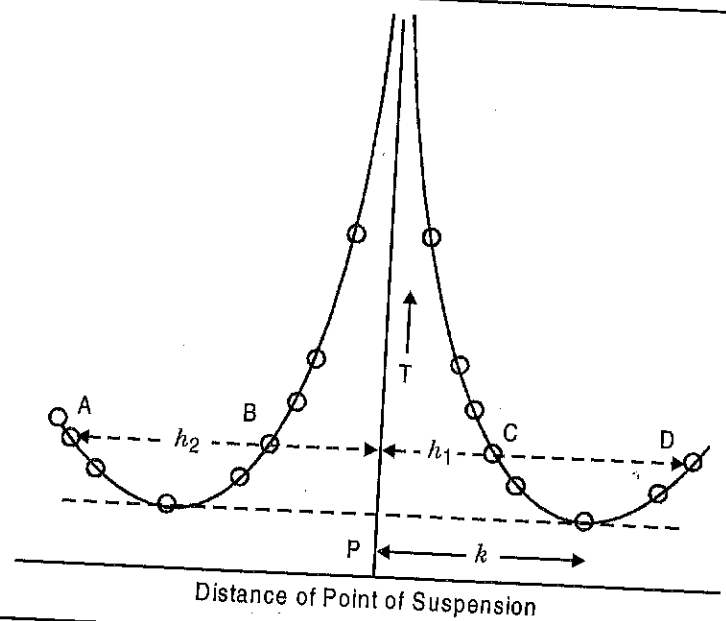

Fig. 3.4

*We have established that

$$T = 2\pi\sqrt{\frac{l+k^2/l}{g}} = \frac{2\pi}{\sqrt{g}}\left[l+\frac{k^2}{l}\right]^{1/2} \qquad ...(1)$$

Differentiating the above with respect to 'l'

$$\frac{dT}{dl} = \frac{2\pi}{\sqrt{g}}\frac{1}{2}\left[l+\frac{k^2}{l}\right]^{-1/2} \times \left[1-\frac{k^2}{l^2}\right] \qquad ...(2)$$

(i) For minimum or maximum time period, $\frac{dT}{dl} = 0$

or, $(1 - k^2/l^2) = 0$ which give $l = \pm k$.

It can be shown that for $l = \pm k$, d^2T/dl^2 is positive and hence, *the corresponding time period is minimum.*

(ii) From eqn. (1), if $l = 0$ or ∞, $T = \infty$ and is therefore *maximum*. Thus, the nature of the curve is as shown in fig. 3.4.

(iv) A horizontal straight line is drawn which meets the curves at A, B, C and D about which obviously the time period of the bar would be the same. As explained in the theory, AC or BD (preferably the mean of these two) will give the length L of the equivalent simple pendulum. Similarly, the other two straight lines are drawn and the mean value of L is found in each case.

Observations

Measurement of T and l

Least count of stop-watch = ... sec.

	Readings on one side of c.g.									Readings on other side of c.g.								
Hole No.																		
l in cm																		
No. of vibrations																		
Total time in sec.																		
T in sec.																		

Calculations

Straight line drawn $ABCD$	AC = ...	BD = ...	L in cm $= \frac{AC+BD}{2}$	T in sec.	g (cm/sec²) $= 4\pi^2 \frac{L}{T^2}$
$A'B'C'D'$	$A'C'$ =	$B'D'$ =	...	...	...
...	...	...	...	...	...
				Mean	...

% error :

Result

The acceleration due to gravity by compound pendulum (correct to significant figures) at .... = ... cm/sec^2.

Precautions

(i) The knife-edge should be horizontal and sharp. The sharp edge should point towards the centre of gravity of the bar.

(ii) The amplitude of vibration should be small.

(iii) The distance of the sharp edge of the knife-edge and not of middle point of the hole, should be measured from the centre of gravity of the bar.

Important points about the experiment

(*i*) At first, time for 50 oscillations can easily be taken. When the centre of the bar is approached, the time period considerably increases and at the centre, it is infinity. So, after 4 or 5 holes, periods of few number of oscillations should be taken.

(*ii*) An alternative method for finding g may be suggested as follows :

We already know that

$$T = 2\pi\sqrt{\frac{l+(k^2/l)}{g}} \quad \text{or } lT^2 = \frac{4\pi^2 k^2}{g} + \frac{4\pi^2 l^2}{g}.$$

A graph plotted with lT^2 along y-axis and l^2 along x-axis is a straight line. The gradient of this line will be $(4\pi^2/g)$. Hence g is known.

QUESTIONS

(*i*) What is a compound pendulum ? Distinguish between simple pendulum and compound pendulum.

(*ii*) Define 'g' and give its units and dimensions.

(*iii*) Explain the variation of g from place to place.

(*iv*) Distinguish between gravity and gravitation.

(*v*) What are centres of suspension and oscillation ? How are the related ?

(*vi*) Explain the nature of $l - T$ graph in this experiment.

(*vii*) Why do you use a bar pendulum in this experiment ?

(*viii*) What is radius of gyration. How do you measure it from this experiment ?

(*ix*) What are the sources of error in this experiment ?

Exp. No. 3(B) Acceleration due to gravity by Kater's pendulum method

Object

To determine the acceleration due to gravity with Kater's pendulum.

Apparatus

Kater's pendulum, stop-watch and a telescope.

Kater's pendulum (fig. 4.1) is a long rod having two fixed kife-edges, two adjustable weights and two cylinders at the ends of identical shape. One of the cylinders is metallic (heavier) and another is wooden.

Theory

Let the time periods about the two knife-edges be T_1 and T_2 respectively and that l_1 and l_2 be the respective distances of the knife-edges from the c.g. of the pendulum. Then we have