BPT 401

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1)

1) Show that the phase vel. of the de Broglie waves of an particle of mass m & de Broglie wavelength λ is given by

$$V_p = c\sqrt{1+\left(\frac{mc\lambda}{h}\right)^2}$$

And compare the phase & group vel. of an $e^{\ominus}$ whose de-Broglie wavelength is exactly 1×10^{-13} m.

We know wave vel. $(V_p) = \nu\lambda$ — (1) $\nu \to$ frequency. $\lambda \to$ wavelength

For photon, $E = h\nu$

$$\nu = \frac{E}{h} \quad \text{—— (2)}$$

Using (2) in (1):-

$$V_p = \frac{E.\lambda}{h} \quad \text{—— (3)}$$

We know, de-broglie wavelength $(\lambda) = \frac{h}{p}$

$$~~p\alpha~~ \quad \frac{\lambda}{h} = \frac{1}{p} \quad \text{—— (4)}$$

Using (4) in (3):-

$$V_p = \frac{E}{p} \quad \text{—— (5)}$$

2)

For relativistic electron, Energy momentum is given by

$$E^2 = p^2c^2 + m^2c^4$$

$$\therefore E = \sqrt{p^2c^2 + m^2c^4} \quad \text{———} (6)$$

Using (6) in (5):-

$$V_p = \frac{\sqrt{p^2c^2 + m^2c^4}}{p} = \sqrt{\frac{p^2c^2 + m^2c^4}{p^2}}$$

$$V_p = \sqrt{c^2 + \frac{m^2c^4}{p^2}} = c\sqrt{1 + \frac{m^2c^2}{p^2}}$$

from (4):-

$$V_p = c\sqrt{1 + m^2c^2\left(\frac{d^2}{h^2}\right)}$$

$$\boxed{V_p = c\sqrt{1 + \left(\frac{mcd}{h}\right)^2}}$$

For $d = 10^{-13}$ m

$$V_p = 3 \times 10^8 \sqrt{1 + \left(\frac{9.1 \times 10^{-31} \times 3 \times 10^8 \times 10^{-13}}{6.636 \times 10^{-34}}\right)^2}$$

$$\boxed{V_p = 1.00085c}$$

& we know, $V_g = \frac{c^2}{V_p}$

~~$V_g = 1.00085c$~~

3)

$$V_g = \frac{e^x}{(1.00035)c}$$

$$\boxed{V_g = 0.99915\ c.}$$

2(a) The maximum wavelength for photoelectric emission in tungsten is 230nm. What wavelength of light must be used in order for e^- with a max. energy of 1.5eV to be ejected?

(a) Here, $\lambda_0 = 230nm$

We know, work function $(\phi) = \frac{hc}{\lambda_0} = \frac{1240eV}{230}$

$= 5.4eV$

K. E max. $= 1.5eV$

$\Rightarrow$ Einstein Relation,

$E = K.E_{max.} + \phi$

$E = 1.5 + (5.4)$

$E = 6.9\ eV$

If wavelength be λ, then

$$E = \frac{hc}{\lambda}$$

$$6.9eV = \frac{1240}{\lambda}$$

$$\boxed{\lambda = 179.7\,nm}$$

4)

b) The dist. b/w adjacent atomic planes in calcite is 0.3 nm. Find the smallest angle of Bragg Scattering for 0.03 nm X-rays.

b) Given $d = 0.3\ nm$

$\lambda = 0.3\ \text{Å}$

We know, Bragg's law

$$2d\sin\theta = n\lambda$$

for $n = 1$

$$2d\sin\theta = \lambda$$

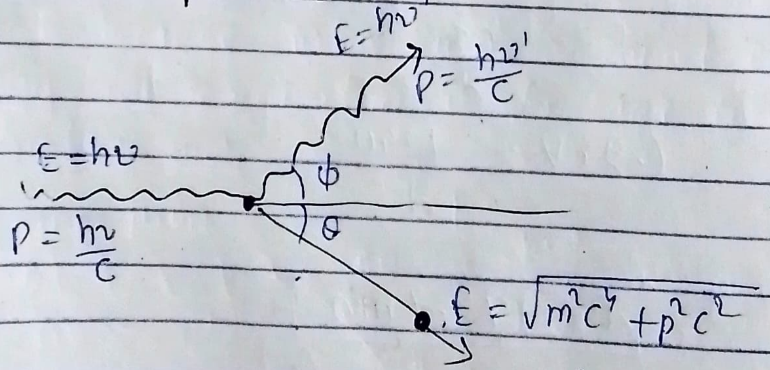
$$\sin\theta = \frac{\lambda}{2d}$$

$$= \frac{0.3 \times 10^{-10}}{2 \times 0.3 \times 10^{-9}}$$

$$\sin\theta = 0.05$$

$$\boxed{\theta = 2.86^\circ}$$

3) (a) A photon whose energy equals the rest energy of the e^- undergoes a compton collision with an e^-. If the e^- moves off at an angle 40° with the original photon dir^n, what is the energy of the scattered photon?



2)

Conservation of horizontal & vertical momenta give

$\rightarrow h\nu' \cos\phi = h\nu - pc\cos\theta$ — (1)

$\rightarrow h\nu' \sin\phi = pc \sin\theta$ — (2)

$(1)^2 + (2)^2$:

$$(h\nu')^2 (\sin^2\phi + \cos^2\phi) = (h\nu - pc\cos\theta)^2 + (pc\sin\theta)^2$$
$$(h\nu')^2 = (h\nu)^2 - 2h\nu pc\cos\theta + p^2c^2$$
$$2h\nu pc\cos\theta = (h\nu)^2 - (h\nu')^2 + p^2c^2 \quad \text{— (3)}$$

The K.E gained by the $e^{\ominus}$ is $h\nu - h\nu'$ & the initial energy of the photon is $h\nu = mc^2$.
Thus equating the expressions for the total energy of the ~~the~~ $e^{\ominus}$ & substituting

$$K.E + mc^2 = \sqrt{m^2c^4 + p^2c^2}$$
$$(K.E + mc^2)^2 = m^2c^4 + p^2c^2$$
$$p^2c^2 = K.E^2 + 2mc^2 K.E$$
$$p^2c^2 = (h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu') \quad \text{— (4)}$$

Using (4) in (3) :

$$2h\nu pc\cos\theta = (h\nu)^2 - (h\nu')^2 + (h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu')$$
$$2h\nu pc\cos\theta = 4(h\nu)^2 - 4(h\nu)(h\nu')$$
$$pc\cos\theta = 2(h\nu - h\nu')$$
$$p^2c^2\cos^2\theta = 4(h\nu - h\nu')^2 \quad \text{— (5)}$$

Using (4) in (5) :

$$\left((h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu')\right)\cos^2\theta = 4(h\nu - h\nu')^2$$

$$(\cos^2\theta - 4)(h\nu')^2 + 4h\nu(2 - \cos^2\theta)(h\nu') + (h\nu)^2(3\cos^2\theta - 4) = 0$$
— (6)

6)

So, $$h\nu' = \frac{-4h\nu(2-\cos^2\theta) \pm \sqrt{16(h\nu)^2(2-\cos^2\theta)^2 - 4(\cos^2\theta-4)(h\nu)^2(3\cos^2\theta-4)}}{2(\cos^2\theta-4)}$$

$$h\nu' = h\nu\left(\frac{-2(2-\cos^2\theta) \pm \sqrt{4(2-\cos^2\theta)^2 - (\cos^2\theta-4)(3\cos^2\theta-4)}}{\cos^2\theta-4}\right)$$

Given $\theta = 40^\circ$

& we know, initial (rest) energy of photon = 511 KeV

$$h\nu' = 511\left(\frac{-2.826 \pm \sqrt{0.3463}}{-3.413}\right)$$

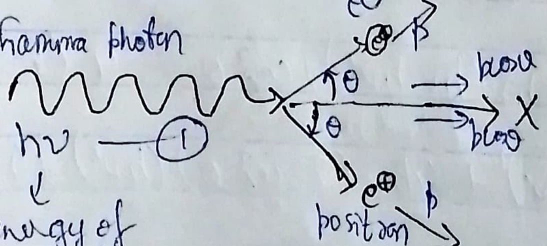
$$h\nu' = 511 \times 0.656$$

$$\boxed{h\nu' = 335 \text{ KeV}}$$

b) Can a single photon break up into pair of production into an $e^{\ominus}$ - positron pair ?

In vaccum, single photon cannot break into pair production.
It requires a material medium or the some other external medium to show pair production.

It is because in vaccum, we cannot conserve energy and momentum at the same time.
In case of vaccum,



$E = h\nu$ — (1)

Total energy of gamma photon

7).

Suppose, a gamma photon is producing e^--positron pair, moving in X-dir^n.

Since, initially gamma photon is only moving in X-dir^n, i.e., no momentum is present in Y-dir^n. So, $e^{\ominus}$ & positron makes equal angle from X-axis to maintain zero momentum in Y-dir^n.

Energy of $e^{\ominus}$-positron pair (E) = $m_oc^2 + m_oc^2 + K.E$

$$E = 2m_oc^2 + K.E$$
$$E = 2\gamma m_o c^2 \quad \text{—— (1)}$$

, K.E → initial K.E of gamma photon

m_oc^2 → rest mass energy of photon

γm_o → relativistic mass

Momentum of photon $= \frac{h\nu}{c}$ —— (3)

Let linear momentum of $e^{\ominus} = p$
& " " " positron $= p$.

Applying energy conservation from (1) & (2)

$$h\nu = 2\gamma m_o c^2 \quad \text{—— (4)}$$

Applying conservation of momentum

$$\frac{h\nu}{c} = 2p\cos\theta$$

, $p = mv$
↳ relatavistic mass

$p = \gamma m_o v$

So, $\frac{h\nu}{c} = 2\gamma m_o v\cos\theta$

$$h\nu = 2\gamma m_o c v\cos\theta$$

5)

$$h\nu = 2\gamma m_0 c^2 \left(\frac{v}{c}\right) \cos\theta \quad \text{--- (5)}$$

Here, $\frac{v}{c} < 1$ & $\cos\theta < 1$] we know

So, (5) become

$$h\nu < 2\gamma m_0 c^2 \quad \text{--- (6)}$$

From eq^n (4) & (6), we met a contradiction.

We met contradiction, when we try to conserve both energy & momentum at same time. So

So, it means this process is not possible i.e. a single photon can not occur in vaccum or ~~vac~~ empty space.

The only way this process is possible is if there could some initial object which could take initial recoil of photon & make eq^n (6) right.

In presence of nucleus it is possible that single photon break up into pair of production.