Problems :

(1) For any square matrix A, A^tA and AA^t are symmetric matrices.

(2) Let A, B be skew Hermitian matrices with $AB = BA$. Is the matrix AB Hermitian or skew Hermitian ?

(3) Prove that: A, B are square matrices;

(i) If A has a zero row, so does AB.

(ii) If B has a zero column, so does AB.

(iii) Any matrix with a zero row or zero column can not be invertible.

(4) Let A and B be matrices of the same size;

(i) Show that, if $AX = 0$, $\forall X$, then $A \equiv 0$.

(ii) Show that if $AX = BX$, $\forall X$, then $A = B$.

(5) (i) Find example of a 2×2 matrix A (other than O and I) s.t. $A^2 = A$.

(ii) Find example of a 2×2 matrix A (other than O) s.t. $A^2 = 0$.

(iii) Is it necessary that $AB = AC \Rightarrow B = C$ $\underline{(A \neq 0)}$

(6) <u>True / False</u>

(i) Let A be a square matrix s.t. $AA = A$. Then A is the identity.

(ii) Let A and B invertible matrices with $A^2 = I$, $B^2 = I$. <u>Then</u> $(AB)^{-1} = BA$

(iii) If A and B are invertible matrices, A+B is also invertible.

(iv) If A, B and AB are symmetric then $AB = BA$.

(v) If A and B are symmetric and of the same size, AB is also symmetric

(vi) If A is invertible and symmetric, then A^{-1} is also symmetric.

(7.) Let $A = [a_{ij}]_{n \times n}$, the trace of A, denoted as $tr(A)$ is defined as $tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$.

(i) $tr(A+B) = tr(A) + tr(B)$

(ii) $tr(AB) = tr(BA)$

(iii) Does there exist matrices A and B s.t. $AB - BA = cI$, for some $c \neq 0$.

(8) A matrix A is called nilpotent if $A^K = 0$ for some K.

Show that a nilpotent matrix is not invertible.