Q5. If the velocity of a point moving in a plane curve varies as the radius of curvature, show that the direction of motion revolves with constant angular velocity. Also, if the angular velocity of the moving point about a fixed origin be constant, show that its transverse acceleration varies as its radial velocity.

Prof: It is given that velocity at any point is proportional to the radius of curvature. $\therefore$ $v = k\rho$ { k = const }

$$\frac{ds}{dt} = k\frac{ds}{d\psi} \text{ (intrinsic form)} \Rightarrow \frac{ds}{dt} = k\frac{ds}{dt}\cdot\frac{dt}{d\psi}$$

$\boxed{\frac{d\psi}{dt} = k}$ the direction of motion, tangent revolves with constant angular velocity.

$$\frac{d\theta}{dt} = k \quad , \quad \text{Transverse acceleration} = \frac{1}{r}\frac{d}{dt}(r^2\dot\theta) = \frac{1}{r}\frac{d}{dt}(kr^2)$$

$$= \frac{k}{r}\cdot 2r\cdot\frac{dr}{dt} = 2k\frac{dr}{dt}$$

Transverse Acceleration $\propto \frac{dr}{dt}$ = radial velocity

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Q6 A point moves in a plane curve so that its tangential acceleration is constant and the magnitudes of the tangential velocity and normal acceleration are in a constant ratio. Show that the intrinsic equation of the path is of the form $s = A\psi^2 + B\psi + C$.

Prof tangential acceleration $= \frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{dv}{dt}$ $\qquad v = \frac{ds}{dt}$

$\boxed{\frac{dv}{dt} = \alpha}$

$$\frac{\left(\frac{ds}{dt}\right)}{v^2/\rho} = \beta \Rightarrow \frac{v}{v^2}\times\rho = \beta = \frac{\rho}{v} = \beta$$

$$\frac{\frac{ds}{d\psi}}{\frac{ds}{dt}} = \beta \Rightarrow \frac{dt}{d\psi} = \beta \Rightarrow \frac{d\psi}{dt} = \frac{1}{\beta} \Rightarrow \boxed{\psi = \frac{1}{\beta}t + c_1}$$

$$\frac{dv}{d\psi}\cdot\frac{d\psi}{dt} = \alpha \Rightarrow \frac{dv}{d\psi} = \alpha\beta \Rightarrow dv = \alpha\beta\, d\psi \Rightarrow \boxed{v = \alpha\beta\psi + c_2}$$

$$\rho = \beta v = \beta(\alpha\beta\psi + c_2) = \alpha\beta^2\psi + c_2\beta$$

$$\rho = \frac{ds}{d\psi} = \alpha\beta^2\psi + c_2\beta \Rightarrow ds = (\alpha\beta^2\psi + c_2\beta)\,d\psi$$

Integrating $\quad s = \alpha\beta^2\frac{\psi^2}{2} + c_2\beta\psi + c_3 \quad$ let $A = \frac{\alpha\beta^2}{2}$, $B = c_2\beta$

$c = c_3$

$$\boxed{s = A\psi^2 + B\psi + c}$$

Q7. A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with constant angular velocity. Show that the intrinsic equation of the path is of the form $s = Ae^{\psi} + B$

tangential Acceleration $\propto$ normal acceleration

$$v\frac{dv}{ds} \propto \frac{v^2}{\rho} \Rightarrow v\frac{dv}{ds} = K\frac{v^2}{\rho}$$

$$\Rightarrow \frac{dv}{ds} = K\frac{v}{\left(\frac{ds}{d\psi}\right)} \quad \left\{ \because \rho = \frac{ds}{d\psi} \right.$$

tangent rotates with constant angular velocity.

$$\frac{d\psi}{dt} = c$$

$$\frac{dv}{ds} = Kv\frac{d\psi}{ds} \Rightarrow dv = Kv\,d\psi$$

$$\Rightarrow \frac{dv}{v} = K\,d\psi \Rightarrow \log v = K\psi + \log c_1 \Rightarrow \boxed{v = c_1 e^{K\psi}}$$

$$\frac{ds}{dt} = c_1 e^{K\psi} \Rightarrow \frac{ds}{d\psi}\cdot\frac{d\psi}{dt} = c_1 e^{K\psi} \Rightarrow \frac{ds}{d\psi}c = c_1 e^{K\psi} \Rightarrow \frac{ds}{d\psi} = \frac{c_1}{c}e^{K\psi}$$

$$s = \frac{c_1}{c}e^{K\psi}.K + c_2 \Rightarrow \boxed{s = Ae^{\psi} + B} \quad A = \frac{c_1 K}{c},\ B = c_2$$

Q8. A particle describes a curve (for which s and ψ vanish simultaneously) with uniform speed v. If the acceleration at any point s be $\frac{v^2 c}{s^2 + c^2}$, prove that curve is catenary.

Proof:

$$v = \frac{ds}{dt} = \text{Const} \qquad \frac{d^2s}{dt^2} = 0,$$

$$\text{Acceleration} = \sqrt{\left(\frac{d^2s}{dt^2}\right)^2 + \left(\frac{v^2}{\rho}\right)^2} = \frac{v^2}{\rho} = \frac{v^2 c}{s^2+c^2} \Rightarrow \frac{1}{\rho} = \frac{c}{s^2+c^2}$$

$$\frac{d\psi}{ds} = \frac{c}{s^2+c^2} \Rightarrow \int d\psi = \int \frac{c}{s^2+c^2}\,ds \Rightarrow \psi = \tan^{-1}(s/c) + K$$

But $s = 0$ then $\psi = 0$ $\Rightarrow$ $0 = 0 + K \Rightarrow K = 0$

$$\psi = \tan^{-1}(s/c) = \frac{s}{c} = \tan\psi \Rightarrow \boxed{s = c\tan\psi}$$

This equation of Catenary.