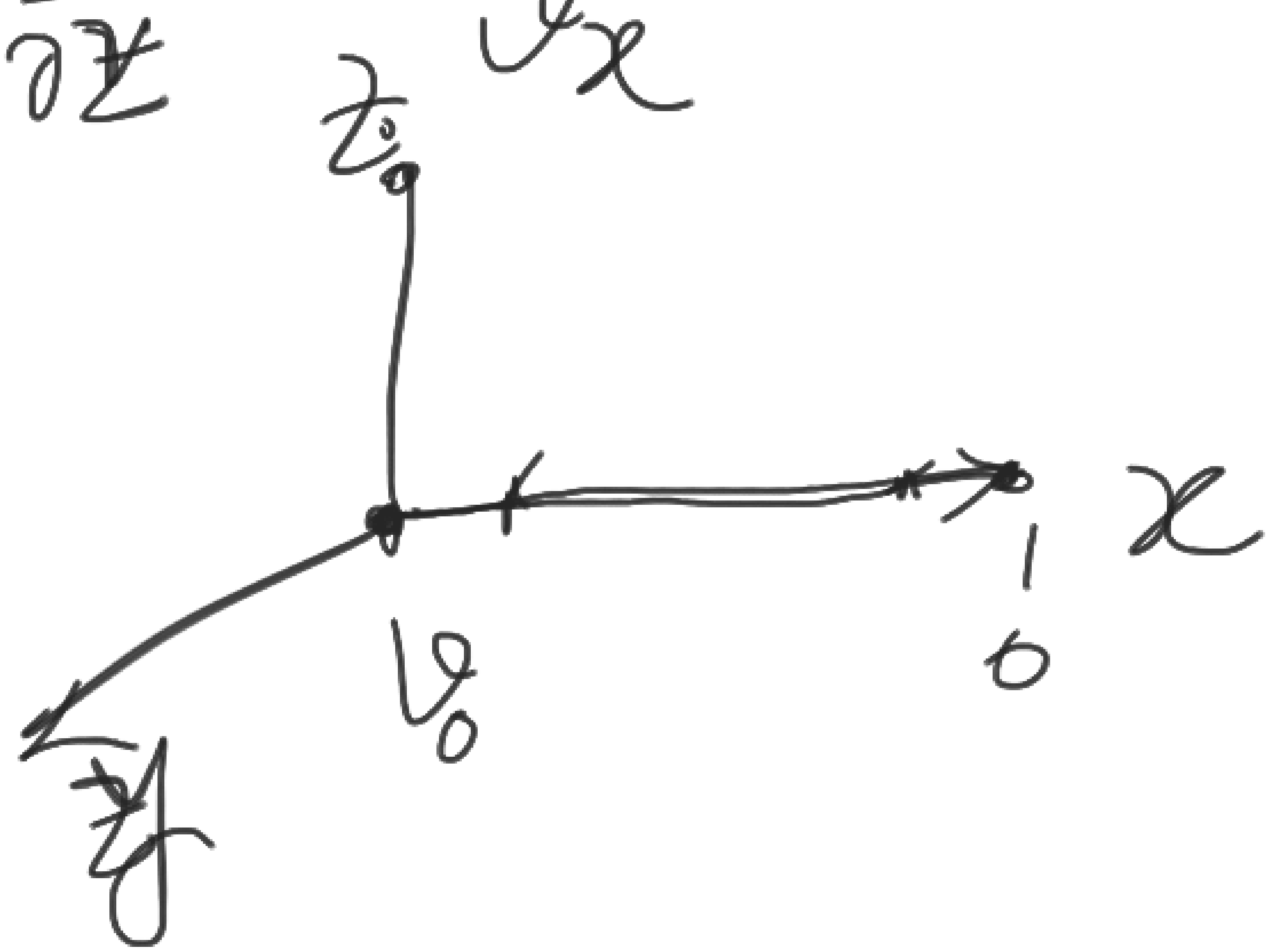
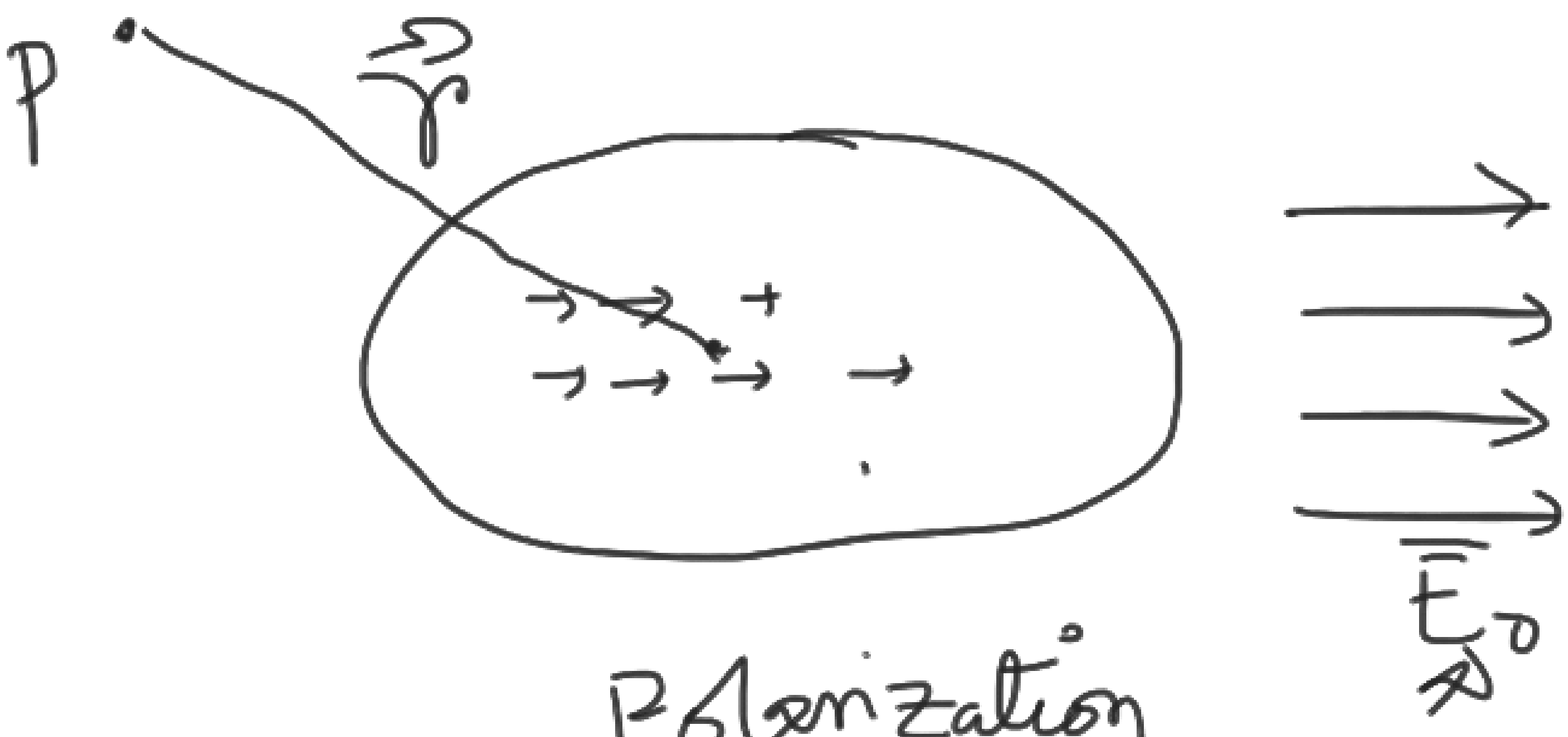
BPT-401

Date - 22/03/2021

$$\bar{\nabla} \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$





10^{23}

Polarization

$\vec{P}$ = dipole moment per unit volume

dipole

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{3(\vec{r}\cdot\vec{p})\vec{r} - \vec{p}}{r^5}$$

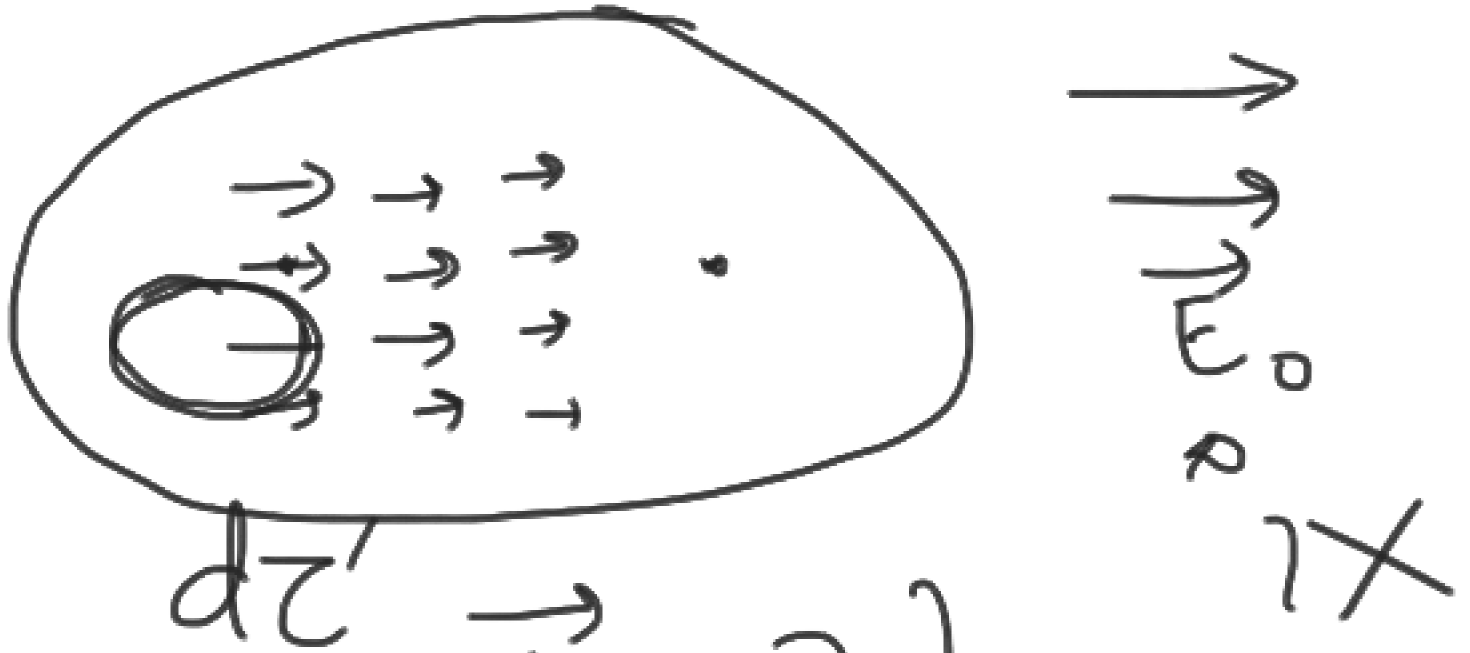
What is field due to polarised material?

E_1 ?

E, ?? ∞ $\cdot P$

Potential due to single dipole

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}\cdot\vec{p}}{r^2} \Rightarrow (1)$$



$\vec{P} = ??$

Volume element $d\tau'$

Total Dipole moments in $d\tau'$
$= \vec{P} \cdot d\tau'$

Potential due to dipoles in volume $d\tau'$

$$V(\vec{r}) = \int dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\hat{r} \cdot (\vec{P}\, d\tau')}{r^2} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\hat{r} \cdot \vec{P}(\vec{r})}{r^2}\, d\tau'$$

$$\vec{\nabla}'\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2}$$

Now, $$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \vec{P} \cdot \vec{\nabla}'\left(\frac{1}{r}\right) d\tau$$

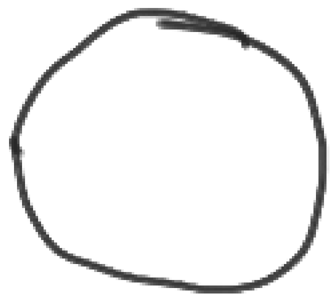
$$\vec{\nabla}'\left(\frac{\vec{P}}{r}\right) = \frac{1}{r}\left(\vec{\nabla}' \cdot \vec{P}\right) + \vec{P} \cdot \vec{\nabla}'\left(\frac{1}{r}\right)$$

$$\Rightarrow \vec{P} \cdot \vec{\nabla}'\left(\frac{1}{r}\right) = \vec{\nabla}'\left(\frac{\vec{P}}{r}\right) - \frac{1}{r}\left(\vec{\nabla}' \cdot \vec{P}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\int_V \bar\nabla'\left(\frac{\vec P}{r}\right)d\tau' - \int_V \frac{1}{r}\left(\bar\nabla'\cdot\vec P\right)d\tau'\right]$$

$$\Downarrow$$

Using divergence theorem



$$V = \frac{1}{4\pi\varepsilon_0}\oint_S \frac{1}{r}\underbrace{\vec P\cdot d\bar a'}_{q} - \frac{1}{4\pi\varepsilon_0}\int_V \frac{1}{r}\underbrace{\left(\bar\nabla'\cdot\vec P\right)}_{\rho_b = -\bar\nabla\cdot\vec P}d\tau'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma_b\, da'}{r}$$ where $\underline{\sigma_b = \vec P\cdot\hat n}$

$$V = \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\sigma_b}{r}\, da' + \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_b}{r}\, d\tau'$$

where, $\sigma_b = \vec{P}\cdot\hat{n}$ and ~~bo~~ $\rho_b = \nabla\cdot\vec{P}$

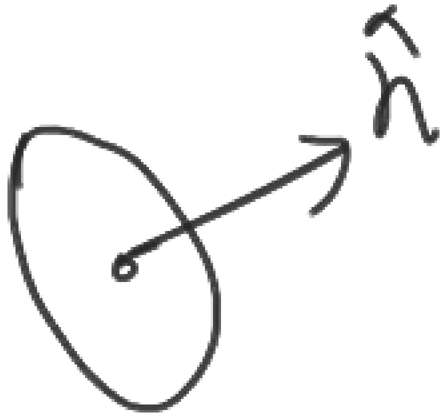
$\Downarrow$ =

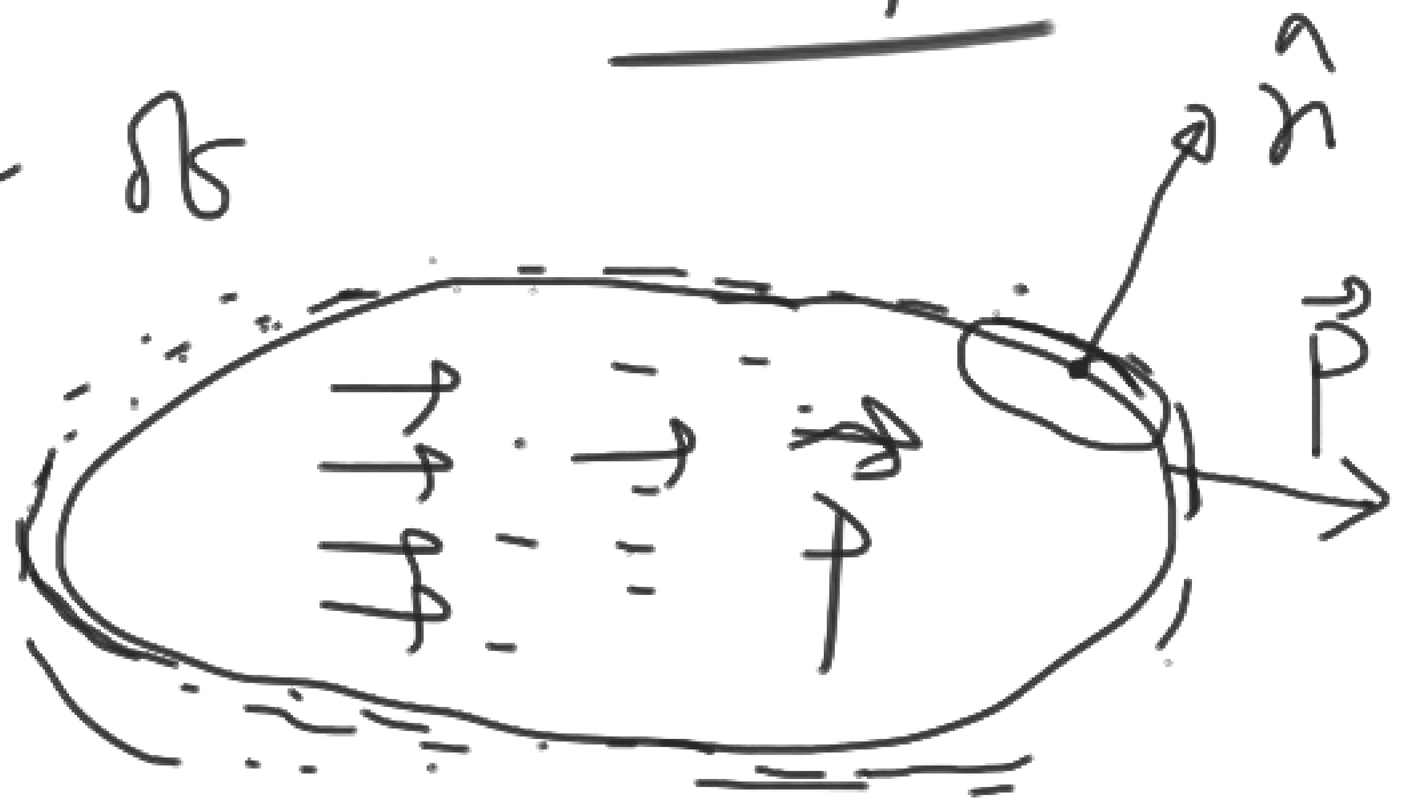
bound surface charge density

bound volume charge density

$\Downarrow$ imaginary charge on the surface of S

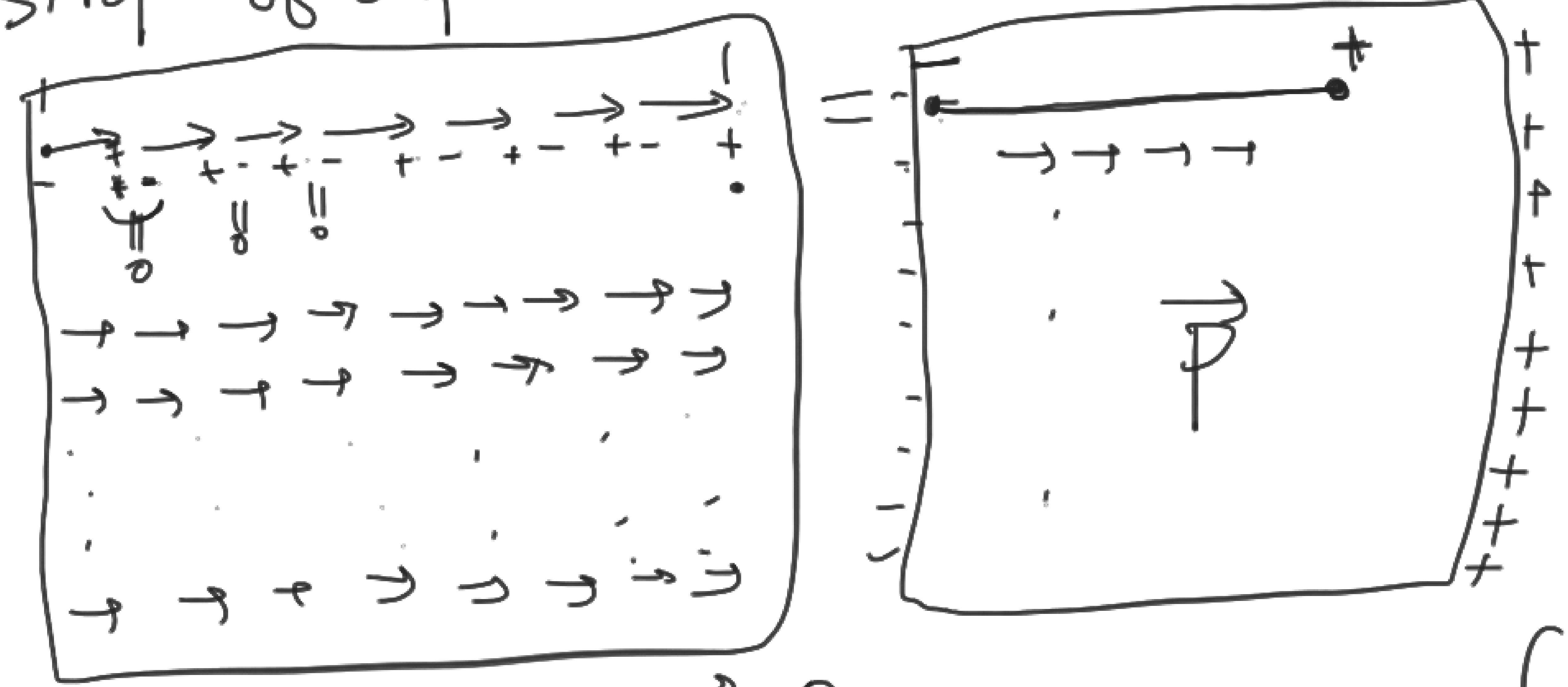
$$\sigma_b = \vec{P}\cdot\hat{n}$$





Physical interpretation of bound charges :

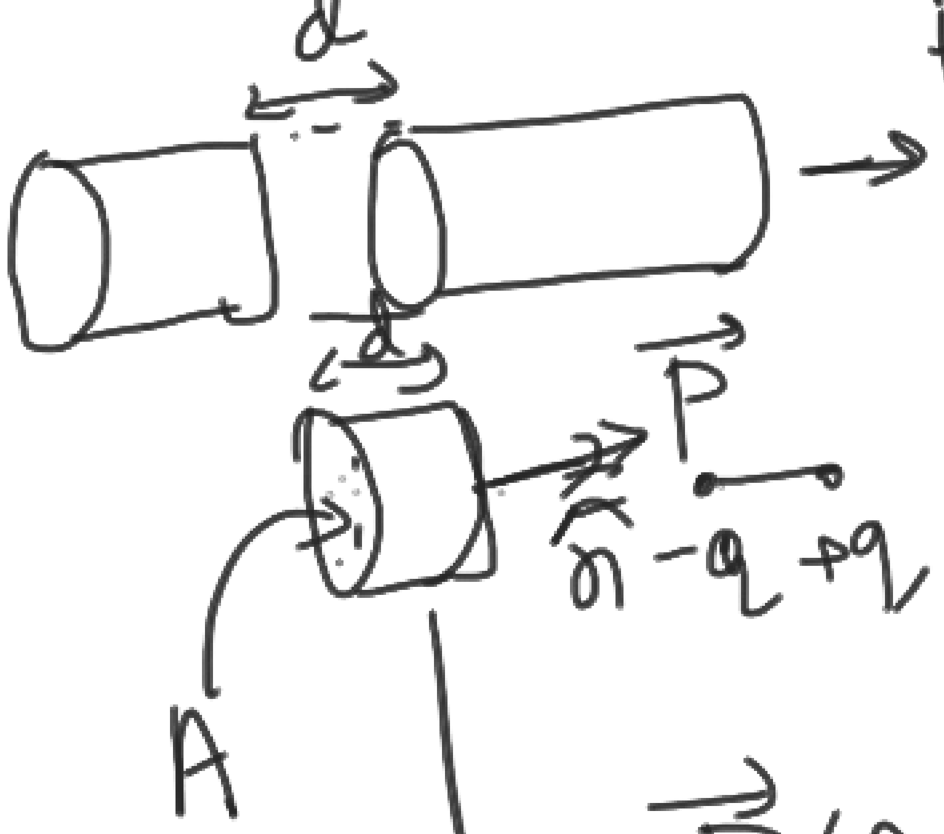
strip of dipoles



$\vec{P}$ $\qquad \sigma_b = \vec{P} \cdot \hat{n}$

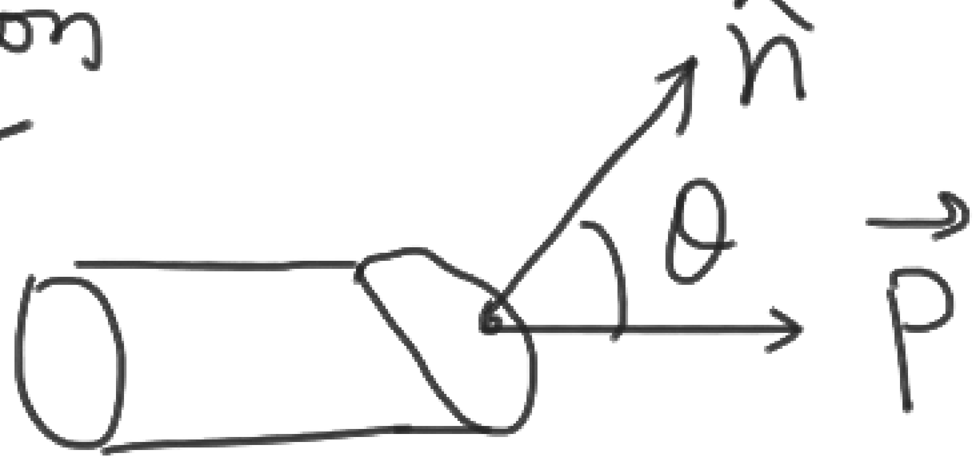
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \sigma_b \, d\bar{a}'$$

$\vec{P} \Rightarrow$ Polarization



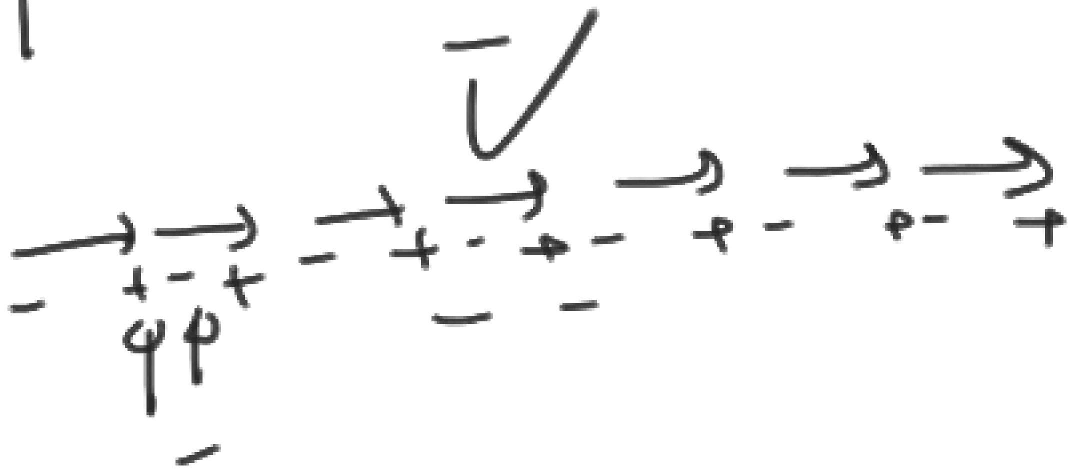
$$q = \vec{P}(A\,d)$$

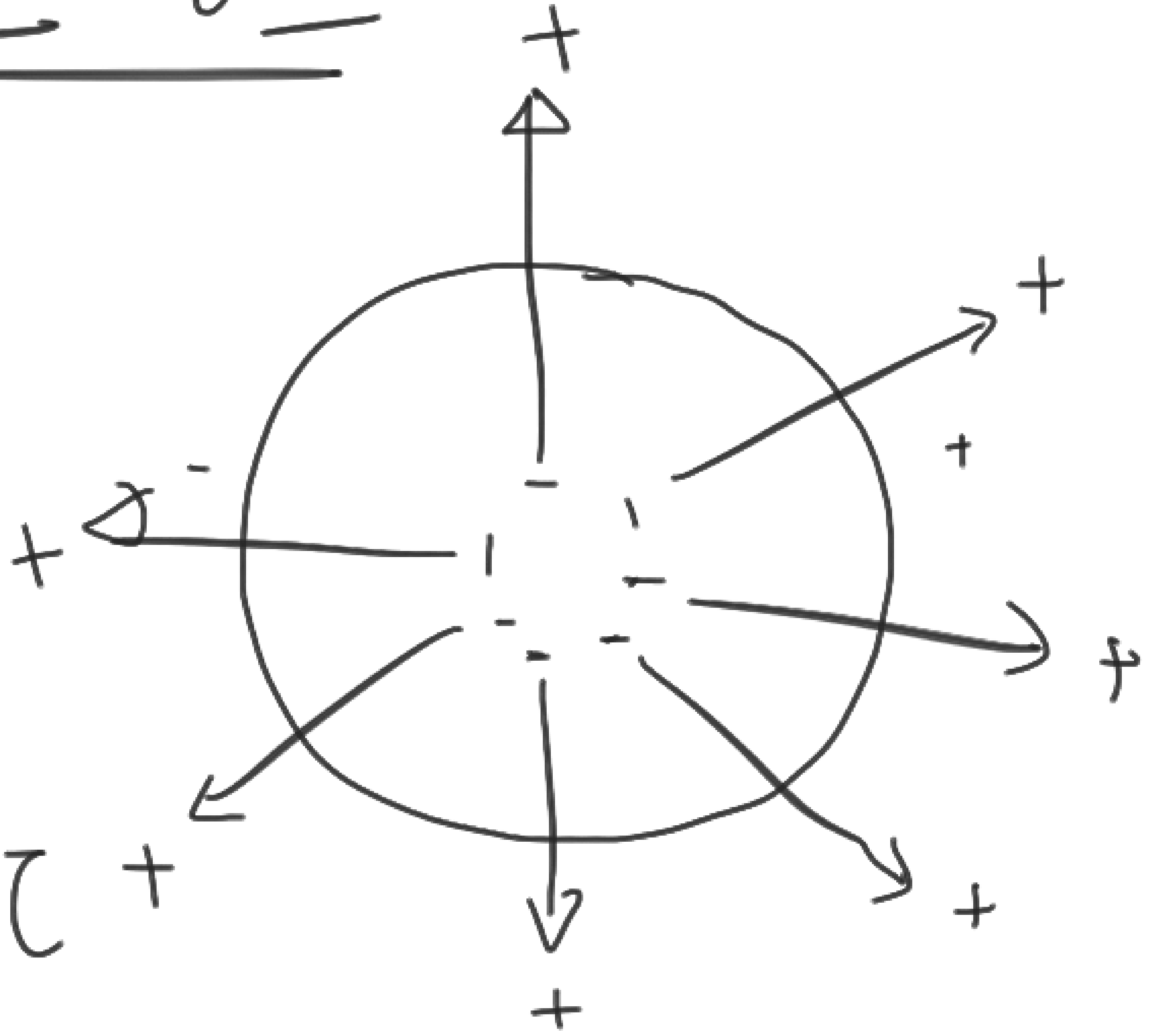
$$\sigma_b = \frac{q}{A} = \vec{P}$$



$$\sigma_b = \frac{q}{A_{end}} = P\cos\theta$$

If polarization $\vec{P}$ is non-uniform





$$\int \rho_b \, d\tau = -\oint_S \vec{P}\cdot d\bar{a}$$

$$= -\int (\bar{\nabla}\cdot\vec{P})\, d\tau$$

$$\boxed{\rho_b = -\bar{\nabla}\cdot\vec{P}}$$