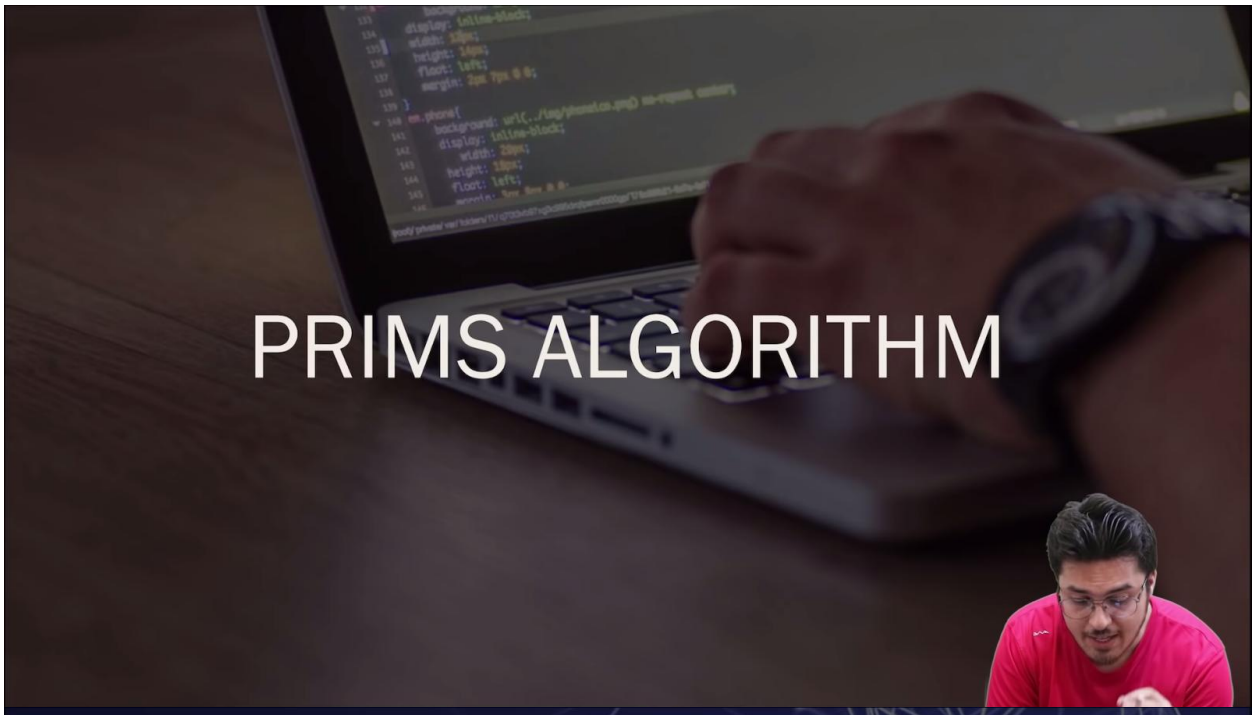
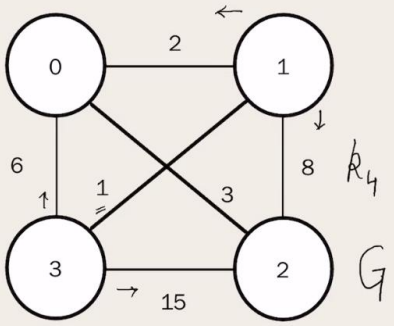


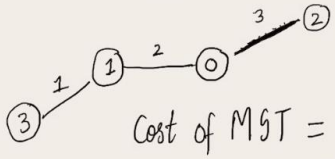
What is a Spanning Tree?

- A subgraph of a graph G is a graph whose vertices and edges are subsets of the original graph G.
- A **Connected subgraph** 'S' of Graph G(V, E) is said to be a **Spanning tree** of graph G iff (if and only if):
  1. *All vertices of G must be present in S*
  2. *No of edges in S should be V-1*
- The cost of the spanning tree is the sum of the weights of all the edges in the tree.
- A minimum spanning tree is the spanning tree with minimum cost

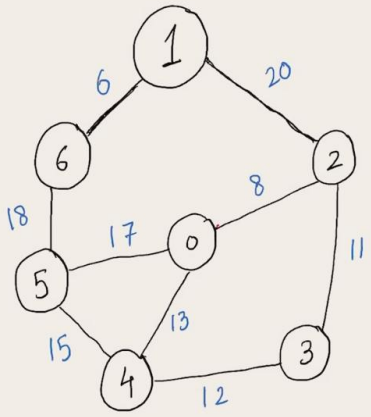
Prims Algorithm

- Prim's Algorithm use Greedy approach to find the minimum spanning tree.
- We start with any node and start creating a MST
- In Prim's Algorithm we grow the spanning tree from a starting position until n-1 edges are formed (or n nodes are covered)





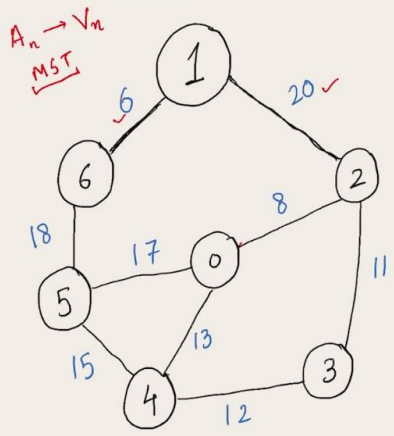
Cost of MST = 1 + 2 + 3
= 6



Graph G

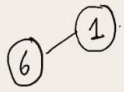
Not included MST
V = {0, 2, 3, 4, 5, 6}

Included in MST
A = {1}



MST:

Graph G



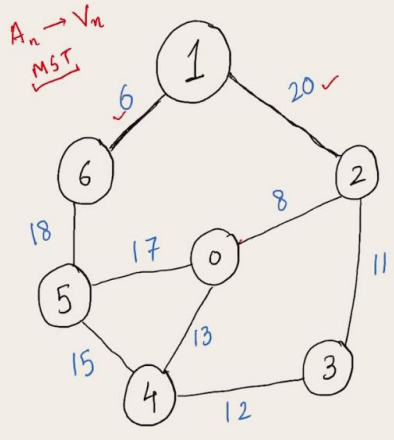
Not included MST ↖ $V = \{0, 2, 3, 4, 5, 6\}$ ✓

↗ Included in MST $A = \{1\}$ ✓

Options ?

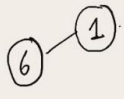
$1 \rightarrow 2 = 20$ ✗

$1 \rightarrow 6 = 6$ ✓



MST:

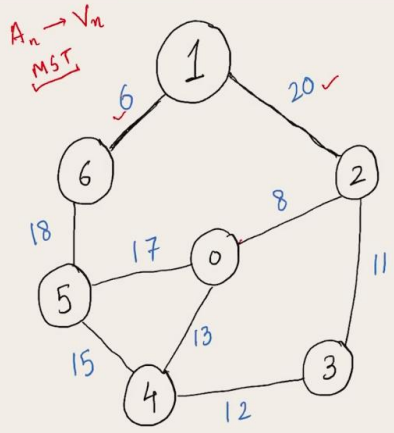
Graph G



Not included MST ↖ $V = \{0, 2, 3, 4, 5, 6\}$ ✓

↗ Included in MST $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$



$A_n \rightarrow V_n$

MST

Not included MST ↖

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

Options

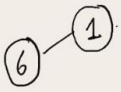
$1 \rightarrow 2 = 20$ ✗

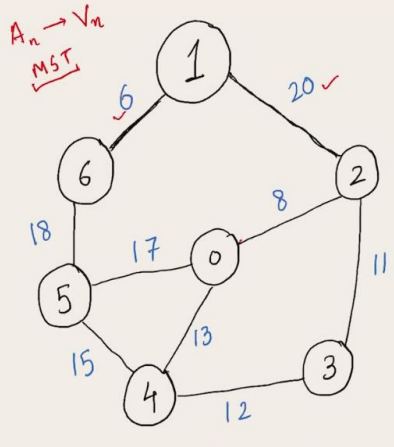
$6 \rightarrow 5 = 18$ ✓

↗ Included in MST

MST:

Graph G





$A_n \rightarrow V_n$

MST

Not included MST ↖

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

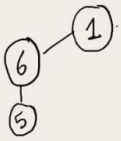
$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

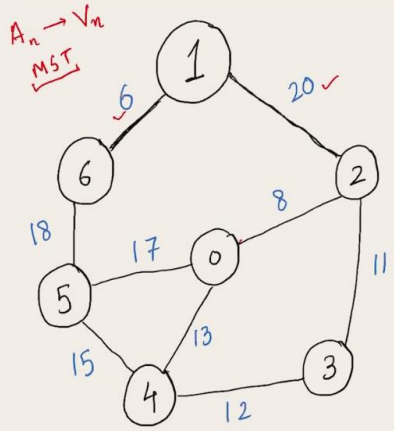
$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

↗ Included in MST

MST:

Graph G





Not included MST ↖ ↗ Included in MST

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

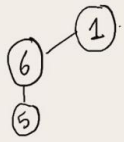
options :

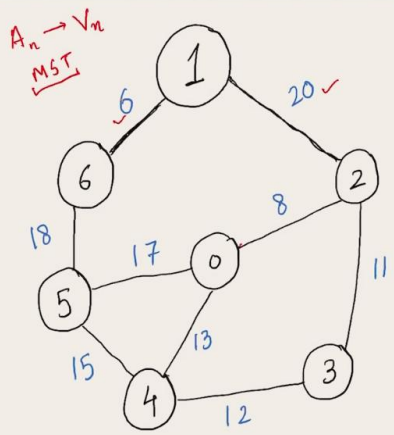
$1 \rightarrow 2 = 20$ ✗

$5 \rightarrow 0 = 17$ ✗

$5 \rightarrow 4 = 15$ ✓

MST: Graph G





→ Not included MST ↖ ↗ Included in MST

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

$V = \{0, 2, 3\}$ $A = \{1, 4, 5, 6\}$

options ?

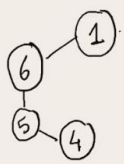
$1 \rightarrow 2 = 20$ ✗

$4 \rightarrow 3 = 12$ ✓

$4 \rightarrow 0 = 13$ ✗

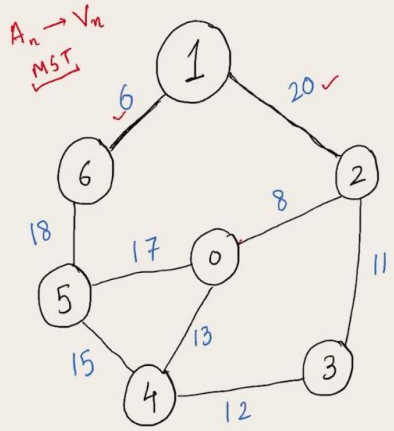
$5 \rightarrow 0 = 17$ ✗

MST: Graph G



$A_n \rightarrow V_n$

MST



→ Not included MST

↗ Included in MST

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

$V = \{0, 2, 3\}$ $A = \{1, 4, 5, 6\}$

$V = \{0, 2\}$ $A = \{1, 3, 4, 5, 6\}$

options :

$1 \rightarrow 2 = 20$

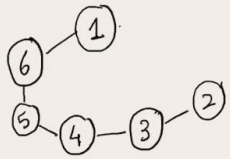
$3 \rightarrow 2 = 11$ ✓

$4 \rightarrow 0 = 13$

$5 \rightarrow 0 = 17$

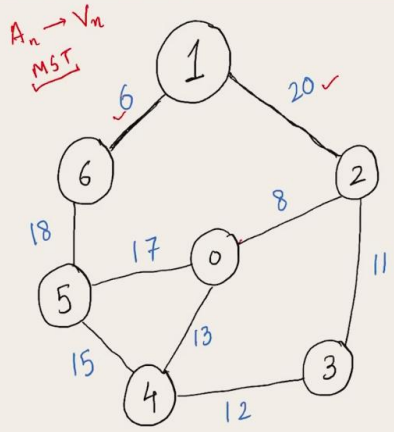
MST:

Graph G



$A_n \rightarrow V_n$

MST



→ Not included MST

↗ Included in MST

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

$V = \{0, 2, 3\}$ $A = \{1, 4, 5, 6\}$

$V = \{0, 2\}$ $A = \{1, 3, 4, 5, 6\}$

$V = \{0\}$ $A = \{1, 2, 3, 4, 5, 6\}$

Options

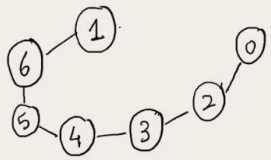
$0 \rightarrow 5 = 17$ ✗

$0 \rightarrow 4 = 13$ ✗

$0 \rightarrow 2 = 8$ ✓

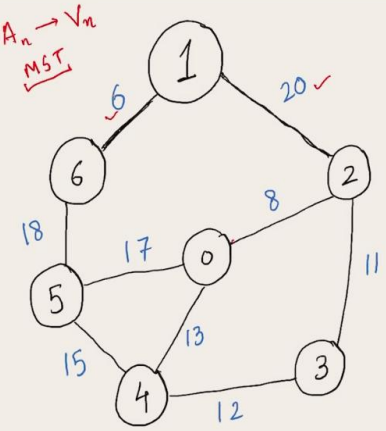
MST:

Graph G



$A_n \rightarrow V_n$

MST



→ Not included MST

↗ Included in MST

$V = \{0, 2, 3, 4, 5, 6\}$ ✓ $A = \{1\}$ ✓

$V = \{0, 2, 3, 4, 5\}$ $A = \{1, 6\}$

$V = \{0, 2, 3, 4\}$ $A = \{1, 5, 6\}$

$V = \{0, 2, 3\}$ $A = \{1, 4, 5, 6\}$

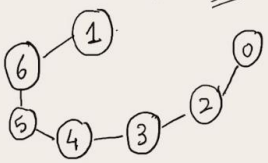
$V = \{0, 2\}$ $A = \{1, 3, 4, 5, 6\}$

$V = \{0\}$ $A = \{1, 2, 3, 4, 5, 6\}$

$V = \{\}$ $A = \{0, 1, 2, 3, 4, 5, 6\}$

MST:

Graph G



Prims Algo Cost

$= 6 + 18 + 15 + 12 + 11 + 8$

$=$ Ans