$$A_{measure} = SA_{real} + B_{bias}$$

$$where\ A_{measure} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix},\ S = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{pmatrix},\ A_{real} = \begin{pmatrix} a_{rx} \\ a_{ry} \\ a_{rz} \end{pmatrix},\ B = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\Rightarrow A_{real} = S^{-1}\left(A_{measure} - B_{bias}\right) = K\left(A_{measure} - B_{bias}\right) = K\bar{A}_{measure}$$

$$where\ K = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{pmatrix},\ \bar{A}_{measure} = \begin{pmatrix} \bar{a}_x \\ \bar{a}_y \\ \bar{a}_z \end{pmatrix} = \begin{pmatrix} a_x - b_x \\ a_y - b_y \\ a_z - b_z \end{pmatrix}$$

$$\left(A_{real}\right)_k = \begin{pmatrix} a_{rx} \\ a_{ry} \\ a_{rz} \end{pmatrix}_k = \begin{pmatrix} k_{11}\bar{a}_{x,k} + k_{12}\bar{a}_{y,k} + k_{13}\bar{a}_{z,k} \\ k_{12}\bar{a}_{x,k} + k_{22}\bar{a}_{y,k} + k_{23}\bar{a}_{z,k} \\ k_{13}\bar{a}_{x,k} + k_{23}\bar{a}_{y,k} + k_{33}\bar{a}_{z,k} \end{pmatrix}$$

$$\begin{aligned} X &= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{pmatrix}^T \\ &= \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{22} & k_{23} & k_{33} & b_x & b_y & b_z \end{pmatrix}^T \end{aligned}$$

$$\sqrt{a_{rx}{}^2 + a_{ry}{}^2 + a_{rz}{}^2} = g \Rightarrow a_{rx}{}^2 + a_{ry}{}^2 + a_{rz}{}^2 = g^2$$

$$\begin{aligned} &e(X) = a_{rx}{}^2 + a_{ry}{}^2 + a_{rz}{}^2 - g^2 \\ &= A\bar{a}_x^2 + B\bar{a}_y^2 + C\bar{a}_z^2 + D\bar{a}_x\bar{a}_y + E\bar{a}_y\bar{a}_z + F\bar{a}_z\bar{a}_x - g^2 \\ &= \left(k_{11}^2 + k_{12}^2 + k_{13}^2\right)\bar{a}_x^2 + \left(k_{12}^2 + k_{22}^2 + k_{23}^2\right)\bar{a}_y^2 + \left(k_{13}^2 + k_{23}^2 + k_{33}^2\right)\bar{a}_z^2 \\ &\quad + 2\left(k_{11}k_{12} + k_{12}k_{22} + k_{13}k_{23}\right)\bar{a}_x\bar{a}_y + 2\left(k_{12}k_{13} + k_{22}k_{23} + k_{23}k_{33}\right)\bar{a}_y\bar{a}_z + 2\left(k_{11}k_{13} + k_{12}k_{23} + k_{13}k_{33}\right)\bar{a}_z\bar{a}_x - g^2 \end{aligned}$$

<u>*Gauss – Newton Method*</u>

$$E(X) = \sum_{k=1}^{N} e_k^2(X) = e^T e,\ where\ e = \begin{pmatrix} e_1 & e_2 & \cdots & e_N \end{pmatrix}^T$$

$$X_{k+1} = X_k - \alpha\left(\nabla^2 E(X)\right)^{-1} \nabla E(X)$$

$$\Rightarrow X_{k+1} = X_k - \alpha\left(\sum_{k=1}^{N} \nabla e_k^T \nabla e_k\right)^{-1}\left(\sum_{k=1}^{N} e_k \nabla e_k^T\right)$$

$$\Rightarrow X_{k+1} = X_k - \alpha\left(\frac{\sum_{k=1}^{N} e_k \nabla e_k^T}{\sum_{k=1}^{N} \nabla e_k^T \nabla e_k}\right)$$

Stop Condition :

$$\max\left\{\left|2\frac{X_k - X_{k-1}}{X_k + X_{k-1}}\right|\right\} < \varepsilon$$

$$\nabla E(X) = 2\left(\nabla e\right)^T e$$

$$= 2\begin{pmatrix} \frac{\partial e_1}{\partial x_1} & \frac{\partial e_1}{\partial x_2} & \cdots & \frac{\partial e_1}{\partial x_9} \\ \frac{\partial e_2}{\partial x_1} & \frac{\partial e_2}{\partial x_2} & \cdots & \frac{\partial e_2}{\partial x_9} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial e_N}{\partial x_1} & \frac{\partial e_N}{\partial x_2} & \cdots & \frac{\partial e_N}{\partial x_9} \end{pmatrix}_{N\times 9}^T \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}_{N\times 1} = 2\begin{pmatrix} \frac{\partial e_1}{\partial x_1} & \frac{\partial e_2}{\partial x_1} & \cdots & \frac{\partial e_N}{\partial x_1} \\ \frac{\partial e_1}{\partial x_2} & \frac{\partial e_2}{\partial x_2} & \cdots & \frac{\partial e_N}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial e_1}{\partial x_9} & \frac{\partial e_2}{\partial x_9} & \cdots & \frac{\partial e_N}{\partial x_9} \end{pmatrix}_{9\times N} \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}_{N\times 1}$$

$$= 2\sum_{k=1}^{N} e_k \begin{pmatrix} \frac{\partial e_k}{\partial x_1} & \frac{\partial e_k}{\partial x_2} & \cdots & \frac{\partial e_k}{\partial x_9} \end{pmatrix}^T = 2\sum_{k=1}^{N} e_k \nabla e_k^T$$

$$\nabla^2 E(X) = \nabla\left(\nabla E(X)\right) = \nabla\left(2\sum_{k=1}^{N} e_k \nabla e_k^T\right) = 2\sum_{k=1}^{N}\left(\nabla e_k^T \nabla e_k + e_k \nabla^2 e_k\right) \approx 2\sum_{k=1}^{N} \nabla e_k^T \nabla e_k$$

$$= 2\sum_{k=1}^{N}\left(\begin{pmatrix} \frac{\partial e_k}{\partial x_1} \\ \frac{\partial e_k}{\partial x_2} \\ \vdots \\ \frac{\partial e_k}{\partial x_9} \end{pmatrix}_{9\times 1} \begin{pmatrix} \frac{\partial e_k}{\partial x_1} & \frac{\partial e_k}{\partial x_2} & \cdots & \frac{\partial e_k}{\partial x_9} \end{pmatrix}_{1\times 9} + e_k \begin{pmatrix} \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_1} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_1} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_1} \\ \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_2} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_2} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_9} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_9} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_9} \end{pmatrix}\right)_{9\times 9}$$

$$= 2\sum_{k=1}^{N}\left(\begin{pmatrix} \frac{\partial e_k}{\partial x_1}\frac{\partial e_k}{\partial x_1} & \frac{\partial e_k}{\partial x_2}\frac{\partial e_k}{\partial x_1} & \cdots & \frac{\partial e_k}{\partial x_9}\frac{\partial e_k}{\partial x_1} \\ \frac{\partial e_k}{\partial x_1}\frac{\partial e_k}{\partial x_2} & \frac{\partial e_k}{\partial x_2}\frac{\partial e_k}{\partial x_2} & \cdots & \frac{\partial e_k}{\partial x_9}\frac{\partial e_k}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial e_k}{\partial x_1}\frac{\partial e_k}{\partial x_9} & \frac{\partial e_k}{\partial x_2}\frac{\partial e_k}{\partial x_9} & \cdots & \frac{\partial e_k}{\partial x_9}\frac{\partial e_k}{\partial x_9} \end{pmatrix} + e_k \begin{pmatrix} \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_1} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_1} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_1} \\ \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_2} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_2} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_1}\frac{\partial e_k}{\partial x_9} & \frac{\partial}{\partial x_2}\frac{\partial e_k}{\partial x_9} & \cdots & \frac{\partial}{\partial x_9}\frac{\partial e_k}{\partial x_9} \end{pmatrix}\right)_{9\times 9}$$

$$\frac{\partial e}{\partial x_1}=\frac{\partial e}{\partial k_{11}}=2\left(k_{11}\bar{a}_x^2+k_{12}\bar{a}_x\bar{a}_y+k_{13}\bar{a}_x\bar{a}_z\right)=2\bar{a}_x\left(k_{11}\bar{a}_x+k_{12}\bar{a}_y+k_{13}\bar{a}_z\right)$$

$$\frac{\partial e}{\partial x_2}=\frac{\partial e}{\partial k_{12}}=2\left(k_{12}\left(\bar{a}_x^2+\bar{a}_y^2\right)+\left(k_{11}+k_{22}\right)\bar{a}_x\bar{a}_y+k_{13}\bar{a}_y\bar{a}_z+k_{23}\bar{a}_z\bar{a}_x\right)$$

$$\frac{\partial e}{\partial x_3}=\frac{\partial e}{\partial k_{13}}=2\left(k_{13}\left(\bar{a}_x^2+\bar{a}_z^2\right)+k_{23}\bar{a}_x\bar{a}_y+k_{12}\bar{a}_y\bar{a}_z+\left(k_{11}+k_{33}\right)\bar{a}_z\bar{a}_x\right)$$

$$\frac{\partial e}{\partial x_4}=\frac{\partial e}{\partial k_{22}}=2\left(k_{22}\bar{a}_y^2+k_{12}\bar{a}_x\bar{a}_y+k_{23}\bar{a}_y\bar{a}_z\right)=2\bar{a}_y\left(k_{12}\bar{a}_x+k_{22}\bar{a}_y+k_{23}\bar{a}_z\right)$$

$$\frac{\partial e}{\partial x_5}=\frac{\partial e}{\partial k_{23}}=2\left(k_{23}\left(\bar{a}_y^2+\bar{a}_z^2\right)+k_{13}\bar{a}_x\bar{a}_y+\left(k_{22}+k_{33}\right)\bar{a}_y\bar{a}_z+k_{12}\bar{a}_z\bar{a}_x\right)$$

$$\frac{\partial e}{\partial x_6}=\frac{\partial e}{\partial k_{33}}=2\left(k_{33}\bar{a}_z^2+k_{23}\bar{a}_y\bar{a}_z+k_{13}\bar{a}_z\bar{a}_x\right)=2\bar{a}_z\left(k_{13}\bar{a}_x+k_{23}\bar{a}_y+k_{33}\bar{a}_z\right)$$

$$\frac{\partial e}{\partial x_7}=\frac{\partial e}{\partial b_x}=-2\left(A\bar{a}_x+\frac{1}{2}D\bar{a}_y+\frac{1}{2}F\bar{a}_z\right)$$

$$=-2\left(\left(k_{11}^2+k_{12}^2+k_{13}^2\right)\bar{a}_x+\left(k_{11}k_{12}+k_{12}k_{22}+k_{13}k_{23}\right)\bar{a}_y+\left(k_{11}k_{13}+k_{12}k_{23}+k_{13}k_{33}\right)\bar{a}_z\right)$$

$$\frac{\partial e}{\partial x_8}=\frac{\partial e}{\partial b_y}=-2\left(\frac{1}{2}D\bar{a}_x+B\bar{a}_y+\frac{1}{2}E\bar{a}_z\right)$$

$$=-2\left(\left(k_{11}k_{12}+k_{12}k_{22}+k_{13}k_{23}\right)\bar{a}_x+\left(k_{12}^2+k_{22}^2+k_{23}^2\right)\bar{a}_y+\left(k_{12}k_{13}+k_{22}k_{23}+k_{23}k_{33}\right)\bar{a}_z\right)$$

$$\frac{\partial e}{\partial x_9}=\frac{\partial e}{\partial b_z}=-2\left(\frac{1}{2}F\bar{a}_x+\frac{1}{2}E\bar{a}_y+C\bar{a}_z\right)$$

$$=-2\left(\left(k_{11}k_{13}+k_{12}k_{23}+k_{13}k_{33}\right)\bar{a}_x+\left(k_{12}k_{13}+k_{22}k_{23}+k_{23}k_{33}\right)\bar{a}_y+\left(k_{13}^2+k_{23}^2+k_{33}^2\right)\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{11}}=2\left(\bar{a}_x^2\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{12}}=2\left(\bar{a}_x\bar{a}_y\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{13}}=2\left(\bar{a}_z\bar{a}_x\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{22}}=0$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{23}}=0$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial k_{33}}=0$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial b_x}=-2\left(2k_{11}\bar{a}_x+k_{12}\bar{a}_y+k_{13}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial b_y}=-2\left(k_{12}\bar{a}_x\right)$$

$$\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{11}}\frac{\partial e}{\partial b_z}=-2\left(k_{13}\bar{a}_x\right)$$

$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{11}}=2\left(\bar{a}_x\bar{a}_y\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{12}}=2\left(\bar{a}_x^2+\bar{a}_y^2\right)$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{13}}=2\left(\bar{a}_y\bar{a}_z\right)$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{22}}=2\left(\bar{a}_x\bar{a}_y\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{23}}=2\left(\bar{a}_z\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial k_{33}}=0$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial b_x}=-2\left(2k_{12}\bar{a}_x+\left(k_{11}+k_{22}\right)\bar{a}_y+k_{23}\bar{a}_z\right)$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial b_y}=-2\left(\left(k_{11}+k_{22}\right)\bar{a}_x+2k_{12}\bar{a}_y+k_{13}\bar{a}_z\right)$$
$$\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{12}}\frac{\partial e}{\partial b_z}=-2\left(k_{23}\bar{a}_x+k_{13}\bar{a}_y\right)$$

$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{11}}=2\left(\bar{a}_x\bar{a}_z\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{12}}=2\left(\bar{a}_y\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{13}}=2\left(\bar{a}_x^2+\bar{a}_z^2\right)$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{22}}=0$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{23}}=2\left(\bar{a}_x\bar{a}_y\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial k_{33}}=2\left(\bar{a}_z\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial b_x}=-2\left(2k_{13}\bar{a}_x+k_{23}\bar{a}_y+\left(k_{11}+k_{33}\right)\bar{a}_z\right)$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial b_y}=-2\left(k_{23}\bar{a}_x+k_{12}\bar{a}_z\right)$$
$$\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{13}}\frac{\partial e}{\partial b_z}=-2\left(\left(k_{11}+k_{33}\right)\bar{a}_x+k_{12}\bar{a}_y+2k_{13}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{11}}=0$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{12}}=2\left(\bar{a}_x\bar{a}_y\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{13}}=0$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{22}}=2\left(\bar{a}_y^2\right)$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{23}}=2\left(\bar{a}_y\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial k_{33}}=0$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial b_x}=-2\left(k_{12}\bar{a}_y\right)$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial b_y}=-2\left(k_{12}\bar{a}_x+2k_{22}\bar{a}_y+k_{23}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{22}}\frac{\partial e}{\partial b_z}=-2\left(k_{23}\bar{a}_y\right)$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{11}}=0$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{12}}=2\left(\bar{a}_z\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{13}}=2\left(\bar{a}_x\bar{a}_y\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_2}$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{22}}=2\left(\bar{a}_y\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{23}}=2\left(\bar{a}_y^2+\bar{a}_z^2\right)$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial k_{33}}=2\left(\bar{a}_y\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial b_x}=-2\left(k_{13}\bar{a}_y+k_{12}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial b_y}=-2\left(k_{13}\bar{a}_x+2k_{23}\bar{a}_y+\left(k_{22}+k_{33}\right)\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{23}}\frac{\partial e}{\partial b_z}=-2\left(k_{12}\bar{a}_x+\left(k_{22}+k_{33}\right)\bar{a}_y+2k_{23}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{11}}=0$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{12}}=0$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{13}}=2\left(\bar{a}_z\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{22}}=0$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{23}}=2\left(\bar{a}_y\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_3}$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial k_{33}}=2\left(\bar{a}_z^2\right)$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial b_x}=-2\left(k_{13}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial b_y}=-2\left(k_{23}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial k_{33}}\frac{\partial e}{\partial b_z}=-2\left(k_{13}\bar{a}_x+k_{23}\bar{a}_y+2k_{33}\bar{a}_z\right)$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{11}}=-2\left(2k_{11}\bar{a}_x+k_{12}\bar{a}_y+k_{13}\bar{a}_z\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{12}}=-2\left(2k_{12}\bar{a}_x+\left(k_{11}+k_{22}\right)\bar{a}_y+k_{23}\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{13}}=-2\left(2k_{13}\bar{a}_x+k_{23}\bar{a}_y+\left(k_{11}+k_{33}\right)\bar{a}_z\right)=\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{22}}=-2\left(k_{12}\bar{a}_y\right)=\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{23}}=-2\left(k_{13}\bar{a}_y+k_{12}\bar{a}_z\right)=\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial k_{33}}=-2\left(k_{13}\bar{a}_z\right)=\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_7}$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial b_x}=2\left(k_{11}^2+k_{12}^2+k_{13}^2\right)=2A$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial b_y}=2\left(k_{11}k_{12}+k_{12}k_{22}+k_{13}k_{23}\right)=D$$

$$\frac{\partial}{\partial x_7}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial b_x}\frac{\partial e}{\partial b_z}=2\left(k_{11}k_{13}+k_{12}k_{23}+k_{13}k_{33}\right)=F$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{11}}=-2\left(k_{12}\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{12}}=-2\left(\left(k_{11}+k_{22}\right)\bar{a}_x+2k_{12}\bar{a}_y+k_{13}\bar{a}_z\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{13}}=-2\left(k_{23}\bar{a}_x+k_{12}\bar{a}_z\right)=\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{22}}=-2\left(k_{12}\bar{a}_x+2k_{22}\bar{a}_y+k_{23}\bar{a}_z\right)=\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{23}}=-2\left(k_{13}\bar{a}_x+2k_{23}\bar{a}_y+\left(k_{22}+k_{33}\right)\bar{a}_z\right)=\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial k_{33}}=-2\left(k_{23}\bar{a}_z\right)=\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_8}$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial b_x}=2\left(k_{11}k_{12}+k_{12}k_{22}+k_{13}k_{23}\right)=D$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial b_y}=2\left(k_{12}^2+k_{22}^2+k_{23}^2\right)=2B$$

$$\frac{\partial}{\partial x_8}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial b_y}\frac{\partial e}{\partial b_z}=2\left(k_{12}k_{13}+k_{22}k_{23}+k_{23}k_{33}\right)=E$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_1}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{11}}=-2\left(k_{13}\bar{a}_x\right)=\frac{\partial}{\partial x_1}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_2}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{12}}=-2\left(k_{23}\bar{a}_x+k_{13}\bar{a}_y\right)=\frac{\partial}{\partial x_2}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_3}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{13}}=-2\left(\left(k_{11}+k_{33}\right)\bar{a}_x+k_{12}\bar{a}_y+2k_{13}\bar{a}_z\right)=\frac{\partial}{\partial x_3}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_4}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{22}}=-2\left(k_{23}\bar{a}_y\right)=\frac{\partial}{\partial x_4}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_5}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{23}}=-2\left(k_{12}\bar{a}_x+\left(k_{22}+k_{33}\right)\bar{a}_y+2k_{23}\bar{a}_z\right)=\frac{\partial}{\partial x_5}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_6}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial k_{33}}=-2\left(k_{13}\bar{a}_x+k_{23}\bar{a}_y+2k_{33}\bar{a}_z\right)=\frac{\partial}{\partial x_6}\frac{\partial e}{\partial x_9}$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_7}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial b_x}=2\left(k_{11}k_{13}+k_{12}k_{23}+k_{13}k_{33}\right)=F$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_8}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial b_y}=2\left(k_{12}k_{13}+k_{22}k_{23}+k_{23}k_{33}\right)=E$$

$$\frac{\partial}{\partial x_9}\frac{\partial e}{\partial x_9}=\frac{\partial}{\partial b_z}\frac{\partial e}{\partial b_z}=2\left(k_{13}^2+k_{23}^2+k_{33}^2\right)=2C$$