

一、填空题:

1. $|A+B| = 2^3(|A|+|B|) = 24$, 2. $\lambda = 1$, $\xi = k(1, 1, \dots, 1)^T$ $k \neq 0, k \in R$

3. $\eta = (1, -1, 2, -1)^T$, 4. $X = (1, 1, -1)^T$, 5. $\begin{cases} x^2 + y^2 = 1 \\ z = 0 \end{cases}$

二、选择题:

1. 选 (D) 2. 选 ($\frac{\pi}{3}$) 3. 选 (B) 4. 选 (A) 5. 选 (C) 注: 选项 (C)

中 $\lambda_i > 0$.

三、计算下列各题:

1. 解: $\because (A^{-1} - E)BA = 6A, \Rightarrow B = 6(A^{-1} - E)^{-1}AA^{-1} = 6(A^{-1} - E)^{-1}$,

$$A^{-1} - E = \begin{pmatrix} -\frac{3}{4} & & \\ & 1 & \\ & & -\frac{1}{2} \end{pmatrix} \Rightarrow B = \begin{pmatrix} -8 & & \\ & 6 & \\ & & -12 \end{pmatrix}$$

2. 解: 类似于试卷一的第四题;

3. 解: 类似于试卷一的第五题,

4. 解: (1) $A = \begin{pmatrix} 1-a & 1+a & 0 \\ 1+a & 1-a & 0 \\ 0 & 0 & 2 \end{pmatrix}$, $\because r(A) = 2 \Rightarrow |A| = 0 \Rightarrow a = 0$.

(2) $|A - \lambda E| = \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = -\lambda(\lambda-2)^2 \Rightarrow \lambda_1 = 0, \lambda_2 = \lambda_3 = 2$,

当 $\lambda = 0$ 时, $A - 0E = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \xi_1 = (-1, 1, 0)^T$

当 $\lambda = 2$ 时, $A - 2E = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\Rightarrow \xi_2 = (1, 1, 0)^T, \xi_3 = (0, 0, 1)^T \quad (\text{这里 } \xi_2, \xi_3 \text{ 已经正交})$$

取 $Q = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix}$, 经过正交变换 $X = QY$, 标准形为 $f = 2y_2^2 + 2y_3^2$.

(3) $f = 8$ 表示圆柱面.

四、解: 类似于试卷一的第六题

五、解: $\because A(\alpha_1, \alpha_2, \alpha_3) = (A\alpha_1, A\alpha_2, A\alpha_3) = (\alpha_1, 2\alpha_2, 3\alpha_3)$

$$\Rightarrow A = (\alpha_1, 2\alpha_2, 3\alpha_3)(\alpha_1, \alpha_2, \alpha_3)^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$