Vector eqn of reflected line l_2 in l_1 : $\underline{r} = \begin{pmatrix} -9 \\ 6 \\ -1.5 \end{pmatrix} + \mu\left(\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -9 \\ 6 \\ -1.5 \end{pmatrix}\right)$

$$= \begin{pmatrix} -9 \\ 6 \\ -1.5 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ -8 \\ 5.5 \end{pmatrix}, \mu \in \mathbb{R}$$

$$= \begin{pmatrix} 9 \\ 6 \\ -1.5 \end{pmatrix} + y\begin{pmatrix} 10 \\ -16 \\ 11 \end{pmatrix}, y \in \mathbb{R}$$

12 OB: $\underline{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

$\underline{n} = \begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix}$

$\angle$ between OB and ABCE $= \sin^{-1}\left(\dfrac{\left|\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}\cdot\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}\right|\left|\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix}\right|}\right)$

$= \sin^{-1}\left(\dfrac{18}{\sqrt{1869}}\right)$

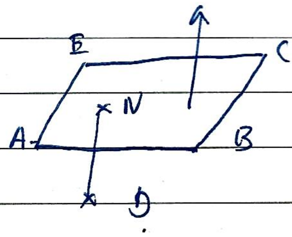
$= 24.60492^\circ$

$= 24.6^\circ$ (1dp).

ii ABCE: $\underline{r}\cdot\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix} = 30$

D(3, −2, 5)

$\vec{OD} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$



$\underline{r}\cdot\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix} = 30$

Let N be the foot of ⊥ from D to plane ABCE,

$\vec{ON} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix}$, for some $\lambda \in \mathbb{R}$

$= \begin{pmatrix} 3-2\lambda \\ -2-6\lambda \\ 5+7\lambda \end{pmatrix}$

Since N lies on ABCE,

$$\begin{pmatrix} 3-2\lambda \\ -2-6\lambda \\ 5+7\lambda \end{pmatrix}\cdot\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix} = 30$$

$-2(3-2\lambda)+(-6)(-2-6\lambda)+7(5+7\lambda)=30$

$-6+4\lambda+12+36\lambda+35+49\lambda=30$

$41+89\lambda=30$

$\lambda=-\frac{11}{89}$

$\overrightarrow{ON}=\begin{pmatrix}3\frac{22}{89}\\-1\frac{23}{89}\\4\frac{12}{89}\end{pmatrix}$

~~distance from D to ABCE =~~

$|\overrightarrow{DN}|=\left|\begin{pmatrix}3\frac{22}{89}\\-1\frac{23}{89}\\4\frac{12}{89}\end{pmatrix}-\begin{pmatrix}3\\-2\\5\end{pmatrix}\right|$

$=1.165997$

$=1.17$ (3sf).

iii DEC: $\underline{r}\cdot\begin{pmatrix}2\\4\\-2\end{pmatrix}=10$ OAB: $\underline{r}\cdot\begin{pmatrix}1\\2\\-1\end{pmatrix}=3$ DEC // OAB

Let point on plane DEC and OAB be M and X respectively.

By observation, $\overrightarrow{OM}=\begin{pmatrix}5\\0\\0\end{pmatrix}$ and $\overrightarrow{OX}=\begin{pmatrix}3\\0\\0\end{pmatrix}$

Shortest distance between DEC and OAB =

$$\frac{\left|\left[\begin{pmatrix}5\\0\\0\end{pmatrix}-\begin{pmatrix}3\\0\\0\end{pmatrix}\right]\cdot\begin{pmatrix}1\\2\\-1\end{pmatrix}\right|}{\left|\begin{pmatrix}1\\2\\-1\end{pmatrix}\right|}=\frac{2}{\sqrt{6}}$$

iv $\cos\theta=\frac{|n_1\cdot n_2|}{|n_1||n_2|}$

$$\angle \text{ b/t DEC and ABCE}=\cos^{-1}\left[\frac{\left|\begin{pmatrix}2\\4\\-2\end{pmatrix}\cdot\begin{pmatrix}-2\\-6\\7\end{pmatrix}\right|}{\left|\begin{pmatrix}2\\4\\-2\end{pmatrix}\right|\left|\begin{pmatrix}-2\\-6\\7\end{pmatrix}\right|}\right]$$

$=\cos^{-1}\frac{42}{\sqrt{2136}}$

$=24.6656°$

$=24.7°$ (1dp).