2020년 1학기 수학2 동영상강의 노트(1) 학과: ______________ 학번: ______________ 성명: ______________

Chapter 12 Vectors and the Geometry of Space

Section 12.1 Three-Dimensional Coordinate Systems

■ a three dimensional rectangular coordinate system

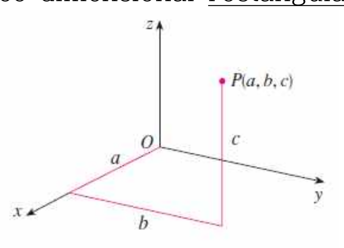


FIGURE 4

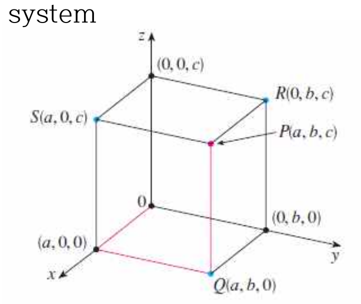


FIGURE 5

■ Plot the points $(-4, 3, -5)$ and $(3, -2, -6)$.

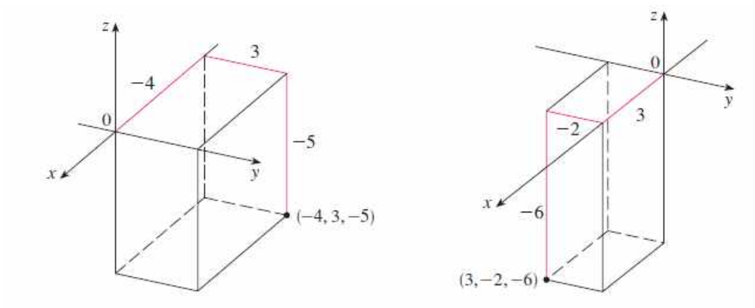


FIGURE 6

Example 1 What surface in $\mathbb{R}^3$ are represented by the following equations?

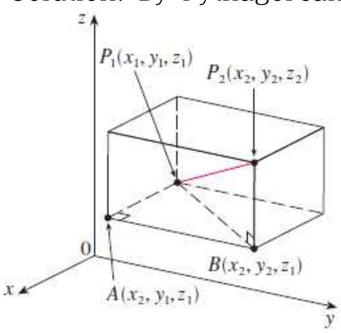
(a) $z = 3$ (b) $y = 5$

■ **Distance Formula in $\mathbb{R}^3$**

The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Solution. By Pythagorean Theorem, we obtain



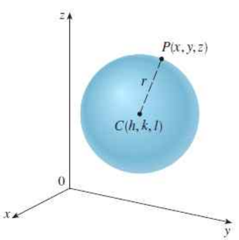
$$\begin{aligned}|P_1P_2|^2 &= |P_1B|^2 + |BP_2|^2 \\ &= (|P_1A|^2 + |AB|^2) + |BP_2|^2 \\ &= |x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\end{aligned}$$

Therefore, $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Example 4 The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is

$|PQ| =$ ______________________________

■ **Equation of a Sphere**



An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

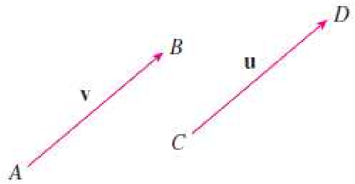
In particular, the center is the **origin**, then an equation of the sphere is

____

Example 6 Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere, and find its center and radius.

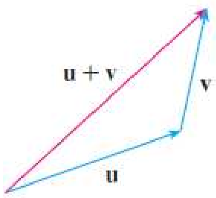
Section 12.2 Vectors

- The term **vector** is used by scientists to indicate a <u>quantity</u> that has both **<u>magnitude</u>** and **<u>direction</u>**.
  (ex.) displacement, velocity or force



- A vector is represents by an **arrow**(or a directed line segment):
  the length of the <u>arrow</u> = the magnitude of the <u>vector</u>
  the direction of the <u>arrow</u> = the direction of the <u>vector</u>
- we denote a vector by <u>a letter in boldface($\mathbf{v}$)</u> or <u>the letter $\vec{v}$</u>.
- The two vectors $\mathbf{u}$ and $\mathbf{v}$ **equal** ($\mathbf{u}=\mathbf{v}$) if the vector $\mathbf{u}$ has the <u>same length</u> and the <u>same direction</u> as $\mathbf{v}$ even through it is in a different position.
- The **zero vector**, denoted by **0** (or $\vec{0}$), has <u>length 0</u>.
  (It is the only vector with no specific direction.)

- Definition of **Vector Addition** $\mathbf{u}+\mathbf{v}$

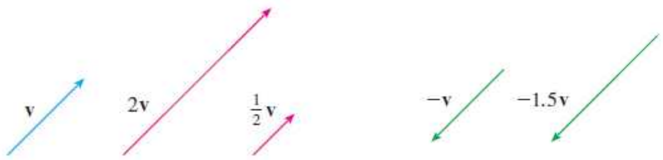


If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{v}$, then the **sum** $\mathbf{u}+\mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.
(This is called the **Triangle Law**.)

ex) the **Parallelogram Law**

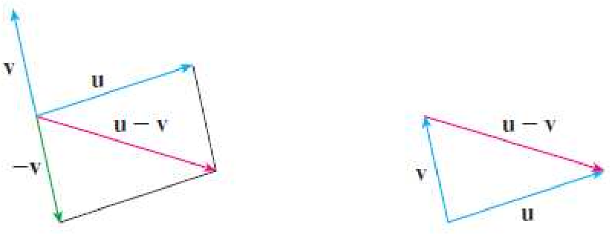
- Definition of Scalar Multiplication
  If c is a scalar(real number) and $\mathbf{v}$ is a vector, then the scalar multiple $c\mathbf{v}$ is a vector whose length=______________ and whose direction =_________________
- the **negative** of $\mathbf{v}$: $-\mathbf{v} = (-1)\mathbf{v}$

ex)



Note: Two nonzero vectors are **parallel** if they are scalar multiples of one another.
(that is, $\mathbf{u}$ and $\mathbf{v}$ are parallel if $\mathbf{u} = c\mathbf{v}$ for some c)

- the vector **difference**: $\mathbf{u}-\mathbf{v} = \mathbf{u}+(-\mathbf{v})$



(a) (b)

Example 2 If $\mathbf{a}$ and $\mathbf{b}$ are the vectors in shown in **Figure 9**, draw $\mathbf{a}-2\mathbf{b}$.
Solution.

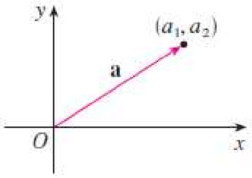
FIGURE 9

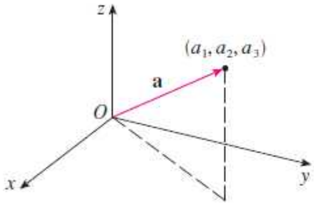
- **Components**

  If we place the **initial point** of a vector a at the **origin** of a rectangular coordinate system, then the **terminal point** of a has **coordinates** of the form (a_1, a_2) or (a_1, a_2, a_3).

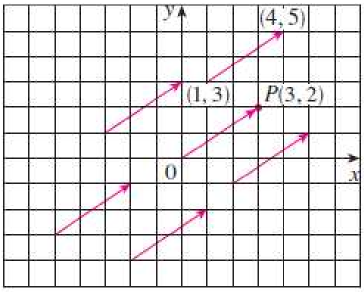
  These coordinates are called the **components** of a and we write

$$\mathbf{a} = \langle a_1, a_2 \rangle \qquad \text{or} \qquad \mathbf{a} = \langle a_1, a_2, a_3 \rangle$$





ex.1) The vectors shown in Figure are all equivalent to the vector a $= \langle 3, 2 \rangle$.



- If the vector a has the **initial** point $A(x_1, y_1, z_1)$ and the **terminal** point $B(x_2, y_2, z_2)$, then a $= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$.

Example 3 Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$.

Answer. $\overrightarrow{AB} =$ ______________________________

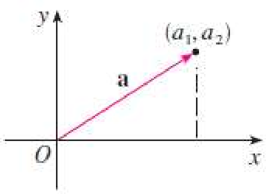
- The **magnitude** or **length** of the vector v is denoted by the symbol $|\mathbf{v}|$ or $\|\mathbf{v}\|$.

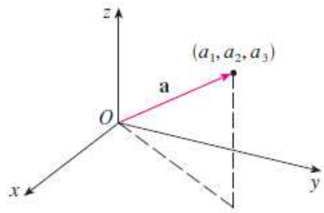
  1) The length of a $= \langle a_1, a_2 \rangle$ is

  $|\mathbf{a}| =$ ________________ .

  2) The length of a $= \langle a_1, a_2, a_3 \rangle$ is

  $|\mathbf{a}| =$ ______________ .



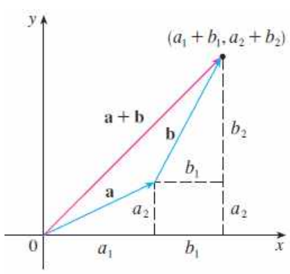


ex.2) (1) If a $= \langle 2, -3 \rangle$, the length of a is $|\mathbf{a}| =$ ________________ .

(2) If a $= \langle 2, -3, 1 \rangle$, the length of a is $|\mathbf{a}| =$ ________________ .

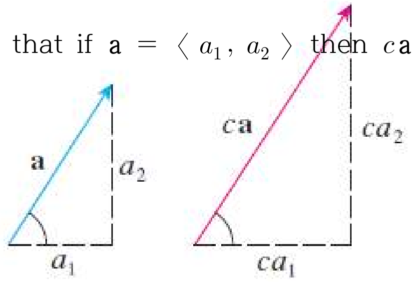
ex.3) Show that if a $= \langle a_1, a_2 \rangle$ and b $= \langle b_1, b_2 \rangle$, then a + b $= \langle a_1 + b_1, a_2 + b_2 \rangle$.

Solution.



ex.4) Show that if $\mathbf{a} = \langle a_1, a_2 \rangle$ then $c\mathbf{a} = \langle c a_1, ca_2 \rangle$.

Solution.



■ **Sum** $\mathbf{a} + \mathbf{b}$ and **Scalar Multiple** $c\mathbf{a}$

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$,

then, $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

$c\mathbf{a} = \langle c a_1, ca_2, ca_3 \rangle$

Example 4 If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$, then

(1) $|\mathbf{a}| =$ ____________________ (2) $\mathbf{a} + \mathbf{b} =$ ______________________

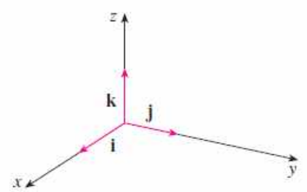
(3) $\mathbf{a} - \mathbf{b} =$ ______________________ (4) $3\mathbf{b} =$ ______________________

(3) $2\mathbf{a} + 5\mathbf{b} =$ ______________________

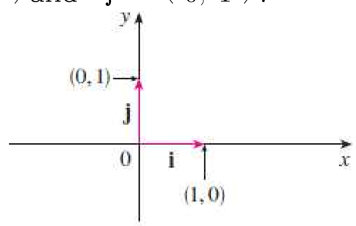
■ the **standard basis vectors** in V_3 are

$\mathbf{i} = \langle 1, 0, 0 \rangle$ $\mathbf{j} = \langle 0, 1, 0 \rangle$ $\mathbf{k} = \langle 0, 0, 1 \rangle$

Similarly, in two-dimensional we define $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.



Standard basis vectors in V_3

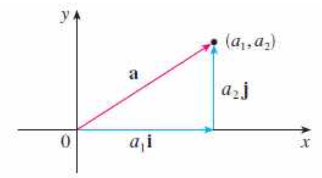


Standard basis vectors in V_2

■ Any vector in V_3 can be expressed in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

and if $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, then $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.

Similarly, in V_2, $\mathbf{a} = \langle a_1, a_2 \rangle = a_1\mathbf{i} + a_2\mathbf{j}$



Example 5 Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} = \langle \qquad\qquad \rangle$

$\mathbf{b} = 4\mathbf{i} + 7\mathbf{k} = \langle \qquad\qquad \rangle$

Then, $2\mathbf{a} + 3\mathbf{b} =$ ____________________

■ A **unit vector** is a vector whose length is 1.

ex.) $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are *all* unit vectors.

■ Theorem. If $\mathbf{a} \neq \mathbf{0}$. then the **unit vector** that has the **same direction** as $\mathbf{a}$ is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Example 6 Find the unit vector in the direction of the vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

Answer. $\mathbf{u} = \left\langle \frac{2}{3}, -\frac{1}{2}, -\frac{2}{3} \right\rangle$