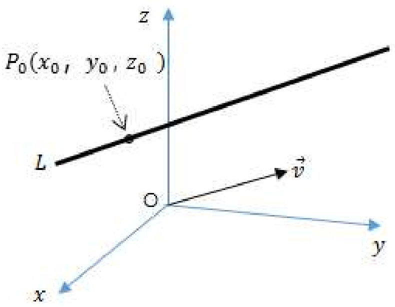
Section 12.5 Equations of Lines and Planes

■ Lines

Equation for a line L through the **point** $P_0\,(x_0, y_0, z_0)$ and **parallel** to the direction **vector** $\mathbf{v} = \langle a, b, c \rangle$:



(1) vector equation

$$\mathbf{r} = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

(2) parametric equations

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

(3) symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad \text{when } a,b,c \neq 0$$

※ when $a = 0$, $x = x_0$ & $\dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

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※ when $b = 0$, $y = y_0$ & $\dfrac{x - x_0}{a} = \dfrac{z - z_0}{c}$

※ when $c = 0$, $z = z_0$ & $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b}$

Example 1 (a) Find a vector equation and parametric equations for the line that passes through the point $(5, 1, 3)$ and is parallel to $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

(b) Find two other points on the line.

Example 2 (a) Find parametric equations and symmetric equations for the line that passes through $A(2, 4, -3)$ and $B(3, -1, 1)$.

(b) At what point does this line intersect the xy-plane?

(c) Find the vector equation for the **line segment** from $A(2, 4, -3)$ to $B(3, -1, 1)$.

Remark1. Equation for a line L that passes through $P_0\,(x_0, y_0, z_0)$ and $P_1\,(x_1, y_1, z_1)$:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

Remark2. The line segment from r_0 to r_1 :

$$r(t) = (1 - t)r_0 + tr_1, 0 \leq t \leq 1$$

Example 3 Show that the lines L_1 and L_2 with parametric equations

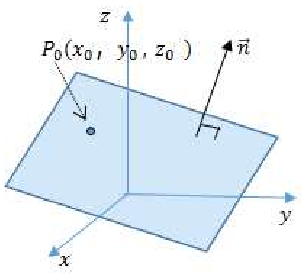
$$L_1 : x = 1+t \qquad y = -2 + 3t \qquad z = 4 - t$$

$$L_2 : x = 2s \qquad y = 3 + s \qquad z = -3 + 4s$$

are **skew lines**; that is, they do not intersect and are not parallel(and therefore do not lie in the same plane).

■ **Planes**

Equation of the **plane** through point $P_0\,(x_0, y_0, z_0)$ and perpendicular to **vector** $\mathrm{n} = \langle a, b, c \rangle$:



$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

This orthogonal vector n called a **normal vector**.

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Example 4 Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\mathrm{n} = \langle 2, 3, 4 \rangle$.
Find the intercepts and sketch the plane.

Remark3. If a, b, and c are not all 0, then the linear equation $ax + by + cz + d = 0$ represents a plane with normal vector $\langle a, b, c \rangle$.

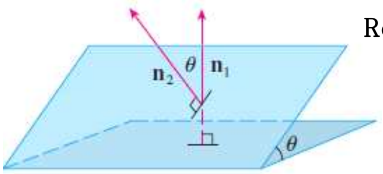
ex) Sketch the graph of $x + 2y + 3z - 12 = 0$.

Example 5 Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$.

Example 6 Find the point at which the line parametric equations $x = 2 + 3t$, $y = -4t$, $z = 5 + t$ intersects the plane $4x + 5y - 2z = 18$.

Remark3. Two planes are **parallel** if their normal vectors are parallel.

ex.) The planes $x+2y-3z=4$ and $2x+4y-6z=3$ are parallel.

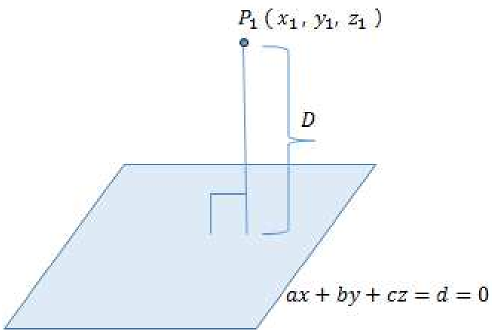


Remark4. If two planes are not parallel, then they intersect in a straight line and the angle between the two planes is defined as the acute angle between their normal vectors.

Example 7 (a) Find the angle between the planes $x+y+z=1$ and $x-2y+3z=1$.
(b) Find the symmetric equations for the line of intersection of these two planes.

■ Distance

Example 8 The distance D from a **point** $P_1(x_1, y_1, z_1)$ to the **plane** $ax+by+cz+d=0$ is



$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Solution.

Example 9 Find the distance between the parallel planes $10x+2y-2z=5$ and $5x+y-z=1$.