

Martrix Analysis

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Question:2

A circle passes through the point $(2,3)$ and $(4,5)$ If the centre lies on the line $(-1\ 4)X + 3 = 0$, then find the radius of the circle.

Matrix Transformation

- ▶ Circle equation: $(X - C)^T(X - C) = R^2$
- ▶ $X^T X - 2X^T C = R^2 - C^T C$ Since $A(2,3), B(4,5)$ lie on circle
- ▶ $A^T A - 2A^T C = R^2 - C^T C$ and $B^T B - 2B^T C = R^2 - C^T C$
- ▶ $A^T A - 2A^T C = B^T B - 2B^T C$
- ▶ so C must lie on line $(A - B)^T C = (A^T A - B^T B)/2$
- ▶ also given c lies on

$$[-1 \ 4]X = -3 \quad (1)$$

- ▶ C is the intersection point of above two lines

Figure

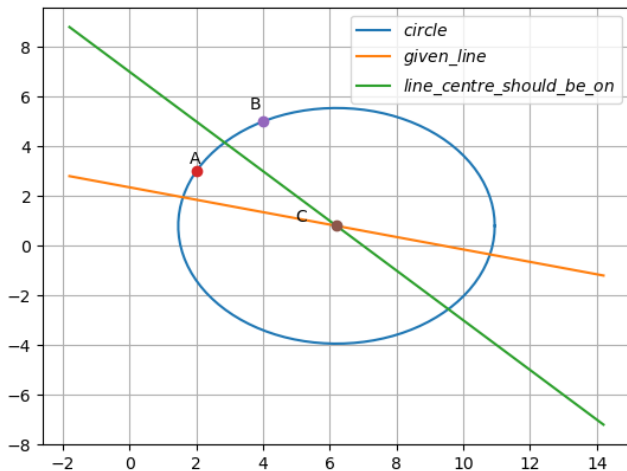


Figure: graph

Solution

- ▶ $((A - B)^T)C = (\|A\|^2 - \|B\|^2)/2$
- ▶ $((A - B)^T)C = (3.605^2 - 6.403^2)/2$
- ▶ $((A - B)^T)C = (13 - 41)/2$
- ▶ $((A - B)^T)C = -14$

$$[-2 \quad -2]C = -14 \quad (2)$$

- ▶ from (1) and (2)
- ▶ $(N^T)C = P$

$$\begin{bmatrix} -1 & 4 \\ -2 & -2 \end{bmatrix} C = \begin{bmatrix} -3 \\ -14 \end{bmatrix} \quad (3)$$

$$C = \begin{bmatrix} -1/5 & -2/5 \\ 1/5 & -1/10 \end{bmatrix} \begin{bmatrix} -3 \\ -14 \end{bmatrix} \quad (4)$$

- ▶ so $C = [+6.2 \quad +0.8]$

Conclusion

- ▶ From solving we get $C=[+6.2 \ +0.8]$
- ▶ $Radius = |C - A| = |C - B|$
- ▶ $Radius=4.741307$

The End