Martrix Project

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Question:14

A line drawn through the point p=(4,7) cuts the circle $X^2 + Y^2 = 9$at the points A and B.Find PA.PB.

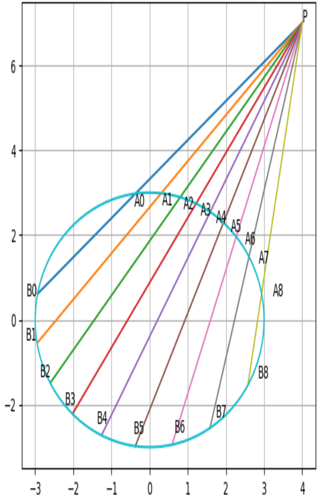
Matrix Transformation

- Circle equation:$X^TX = 9$
- Matrix form of a line through a given point P: X=P+mλ
- i.e; X=$\begin{bmatrix} 4 \\ 7 \end{bmatrix} + m\lambda$
- Let the line through given point cut the circle at the points A,B.
- $A^TA = 9$ and $B^TB = 9$
  A=P+mλ_1 and B=P+mλ_2
- So,
  $(p + m\lambda)^T(P + m\lambda) = 9$ is a quadratic equation in λ with roots λ_1 and λ_2.
- On expanding
  $||m||\lambda^2 + 2P^Tm\lambda + ||P||^2 - 9 = 0$

Condition for line to intersect the circle

- the roots of the quadratic equation are $\lambda_1 = -P^T + \sqrt{(P^T m)^2 + 9 - ||P||^2}$ and $\lambda_2 = -P^T - \sqrt{(P^T m)^2 + 9 - ||P||^2}$
- condition for line to intersect the circle: $(P^T m)^2 >= ||P||^2 - 9$ $(P^T m)^2 >= 56$

Figure


P
A0
A1
A2
A3
A4
A5
A6
A7
A8
B0
B1
B2
B3
B4
B5
B6
B7
B8
6
4
2
0
−2
−3
−2
−1
0
1
2
3
4

Solution

- $PA.PB = ||P - A||.||P - B||$
  $PA.PB = ||m\lambda_1||.||m\lambda_2|| = \lambda_1\lambda_2||m||^2$
- Obtained quadratic equation
  $||m||\lambda^2 + 2P^T m\lambda + ||P||^2 - 9 = 0$
- Product of roots:
  $\lambda_1\lambda_2 = \frac{||P||^2-9}{||m||^2}$
- rearranging the equation
  PA.PB=$|(||P||^2 - 1)|$
- PA.PB=(65-9)=56

Conclusion

- PA.PB=56=Constant
- Any line which passes through the point has same PA.PB

The End