Transformées-Devoir Maison

Groupe 2

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Problem 1

On pose

$$f : t = \begin{cases} 0, & t < 0 \\ 1, & 0 \leq t \leq \pi \\ -1, & \pi < t \leq 2\pi \\ 0, & t > 2\pi \end{cases}$$

1)

Pour $x \in \mathbb{R}$, on a:

$$\begin{aligned}
\hat{f}(x) &= \int_{-\infty}^{+\infty} f(t)e^{-itx}dt \\
&= \int_0^{\pi} e^{-itx}dt - \int_{\pi}^{2\pi} e^{-itx}dt \\
&= \frac{1}{-ix}[e^{-itx}]_0^{\pi} - \frac{1}{-ix}[e^{-itx}]_{\pi}^{2\pi} \\
&= \frac{i}{x}(e^{-i\pi x} - 1) - \frac{i}{x}(e^{-2i\pi x} - e^{-i\pi x}) \\
&= \frac{i}{x}e^{-i\pi x}(2 - e^{i\pi x} - e^{-i\pi x}) \\
&= \frac{i}{x}e^{-i\pi x}(2 - cos(\pi x) - isin(\pi x) - cos(\pi x) + isin(\pi x)) \\
&= \frac{i}{x}e^{-i\pi x}(2 - 2cos(\pi x)) \\
&= 2i\pi e^{-i\pi x}\frac{1 - cos(\pi x)}{\pi x} \\
&= 2i\pi e^{-i\pi x}\frac{sin^2(\frac{\pi x}{2})}{\frac{\pi x}{2}} \\
&= 2i\pi e^{-i\pi x}sin(\frac{\pi x}{2})sin_c(\frac{\pi x}{2})
\end{aligned}$$

2)

Si $\hat{f}$ était intégrable, on aurait par la formule d'inversion de Fourrier, pour prèsque tout $t \in \mathbb{R}$, on a

$$f(t) = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{f}(x)e^{itx}dx = \frac{1}{2\pi}F(\hat{f})(-t)$$

Alors f serait égale prèsque partout à une fonction continue.

$$\lim_{t \to 0^+} f(t) \neq \lim_{t \to 0^-} f(t)$$

Donc

$$\hat{f} \notin \mathbb{L}(\mathbb{R})$$

3)

La formule de Parseval:

$$\begin{aligned}
\int_{\mathbb{R}} |f(t)|^2 dt = 2\pi &= \frac{1}{2\pi}\int_{\mathbb{R}} |\hat{f}(x)|^2 dx \\
&= \frac{1}{2\pi}\int_{\mathbb{R}} |2i\pi e^{-i\pi x} sin(\frac{\pi x}{2}) sin_c(\frac{\pi x}{2})|^2 dx \\
&= \frac{1}{2\pi}\int_{\mathbb{R}} 4\pi^2 sin^2(\frac{\pi x}{2}) sin_c^2(\frac{\pi x}{2}) dx \\
&= 2\pi \int_{\mathbb{R}} \frac{sin^4(\frac{\pi x}{2})}{(\frac{\pi x}{2})^2} dx \\
&\overset{s=\frac{\pi x}{2}}{=} 2\pi \int_{\mathbb{R}} \frac{sin^4(s)}{s^2}\frac{2}{\pi} ds \\
&= 8\int_0^{+\infty} \frac{sin^4(s)}{s^2} ds
\end{aligned}$$

D'où

$$\int_0^{+\infty} \frac{sin^4(s)}{s^2} ds = \frac{\pi}{4}$$

On utilise Mupad,

Code Mupad

```
int(((sin(x))^4)/(x^2), x = 0..infinity)
```

On a :

```
int(((sin(x))^4)/(x^2), x=0..infinity)
```

$$\frac{\pi}{4}$$

4)

On a vu en TD. On pose pour $t \in \mathbb{R}$:

$$H(t) = e^{-|t|}$$

Alors on a:

$$\hat{H}(x) = \frac{2}{1+x^2}$$

Si $f * g = H$, avec $g \in \mathbb{L}^1(\mathbb{R})$, on a :

$$\hat{f}\hat{g} = \hat{H}$$

Donc,

$$\hat{g}(x) = \frac{2}{1+x^2}\frac{1}{2i\pi e^{-i\pi x} sin(\frac{\pi x}{2}) sin_c(\frac{\pi x}{2})}$$

$$\lim_{x \to \pm\infty} \hat{g}(x) \neq 0$$

D'où, aucune solution $g \in \mathbb{L}^1(\mathbb{R})$

Problem 2

Soit

$$f = [7, 7, 5, 5, 3, 3, 1, 1]$$

1)

En utilisant la "transformee de Walsh rapide":

	0	1	2	3	4	5	6	7
f	7	7	5	5	3	3	1	1
regroup2 par2	14	0	10	0	6	0	2	0
regroup4 par4	24	0	4	0	8	0	4	0
regroup8 par8	32	0	8	0	16	0	0	0

$$W(f) = [32, 0, 8, 0, 16, 0, 0, 0]$$

2)

En compressant à 25%, on a $W_{25\%}(f) = [32, 0, 0, 0, 16, 0, 0, 0]$

	0	1	2	3	4	5	6	7
$W_{25\%}(f)$	32	0	0	0	16	0	0	0
regroup2 par2	32	32	0	0	16	16	0	0
regroup4 par4	32	32	32	32	16	16	16	16
regroup8 par8	48	48	48	48	16	16	16	16

$$f^{25\%} = [6, 6, 6, 6, 2, 2, 2, 2]$$

En compressant à 50%, on a $W_{50\%}(f) = [32, 0, 8, 0, 16, 0, 0, 0]$

	0	1	2	3	4	5	6	7
$W_{50\%}(f)$	32	0	8	0	16	0	0	0
regroup2 par2	32	32	8	8	16	16	0	0
regroup4 par4	40	40	24	24	16	16	16	16
regroup8 par8	56	56	40	40	24	24	8	8

$$f^{50\%} = [7, 7, 5, 5, 3, 3, 1, 1]$$

On peut voir que $f^{50\%} = f$.

Problem 3

Soit

$$A = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 3 & 1 & 3 & 1 \\ 1 & 3 & 1 & 3 \\ 3 & 1 & 3 & 1 \end{bmatrix}$$

1)

En utilisant l'algorithme rapide sur les lignes et les colonnes de A:

1^{eme}ligne:

$$\begin{bmatrix} 4 & -2 & 4 & -2 \\ 4 & 2 & 4 & 2 \\ 4 & -2 & 4 & -2 \\ 4 & 2 & 4 & 2 \end{bmatrix}$$

2^{eme}ligne:

$$\begin{bmatrix} 8 & -4 & 0 & 0 \\ 8 & 4 & 0 & 0 \\ 8 & -4 & 0 & 0 \\ 8 & 4 & 0 & 0 \end{bmatrix}$$

1^{eme}colume:

$$\begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 16 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \end{bmatrix}$$

2^{eme}colume:

$$\begin{bmatrix} 32 & 0 & 0 & 0 \\ 0 & -16 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2)

En compressant à 25%, on a $W_2(A)_{25\%} = \begin{bmatrix} 32 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. On a:

1^{eme}ligne:

$$\begin{bmatrix} 32 & 32 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2^{eme}ligne:

$$\begin{bmatrix} 32 & 32 & 32 & 32 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1^{eme}colume:

$$\begin{bmatrix} 32 & 32 & 32 & 32 \\ 32 & 32 & 32 & 32 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2^{eme}colume:

$$\begin{bmatrix} 32 & 32 & 32 & 32 \\ 32 & 32 & 32 & 32 \\ 32 & 32 & 32 & 32 \\ 32 & 32 & 32 & 32 \end{bmatrix}$$

On divise par 16, on a : $\omega_{25\%} = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$

Problem 4

On considere l'equation differentielle:

$$y''(t) + 6y'(t) + 5y(t) = 12e^t$$

avec la condition initiale $y(0) = 1, y'(0) = -3$

$$\mathbb{L}(y)'(z) = zY(z) - y(0) = zY(z) - 1$$

$$\mathbb{L}(y)''(z) = z\mathbb{L}(y)'(z) - y'(0) = z^2Y(z) - z + 3$$

a)

D'apres la tableau de transformee Laplace. On a $\mathbb{L}(e^t) = \frac{1}{z-1}$, en suite:

$$z^2Y(z) - z + 3 + 6(zY(z) - 1) + 5Y(z) = \frac{12}{z-1}$$

On a:

$$Y(z) = \frac{12 + (z-1)(z+3)}{(z+1)(z+5)(z-1)} = \frac{z^2 + 2z + 9}{(z+1)(z+5)(z-1)}$$

$$\lim_{z \to -1}(z+1)Y(z) = -1$$

$$\lim_{z \to -5}(z+5)Y(z) = 1$$

$$\lim_{z \to 1}(z-1)Y(z) = 1$$

D'ou,

$$Y(z) = \frac{-1}{z+1} + \frac{1}{z+5} + \frac{1}{z-1}$$

Donc,

$$y(t) = -e^{-t} + e^{-5t} + e^t$$

b)

On utilise Mupad, On a :

```
u:=solve(ode({y''(t)+6*y'(t)+5*y(t)=12*exp(t),y(0)=1,y'(0)=-3},{y(t)}))
```

$$\left\{e^{-5\,t} - e^{-t} + e^{t}\right\}$$

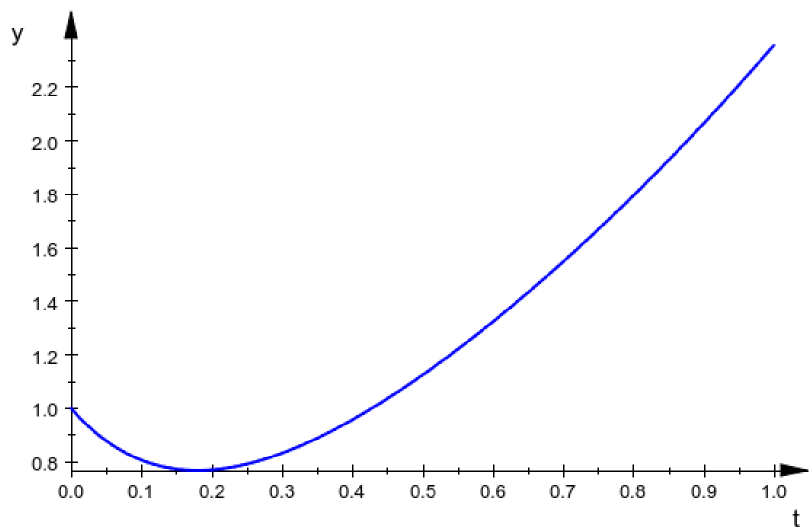
```
u
```

$$\left\{e^{-5\,t} - e^{-t} + e^{t}\right\}$$

```
y:=t->op(u)
```

$$t \rightarrow \mathrm{op}(u)$$

```
plot(plot::Function2d(y(t),t=0..1))
```



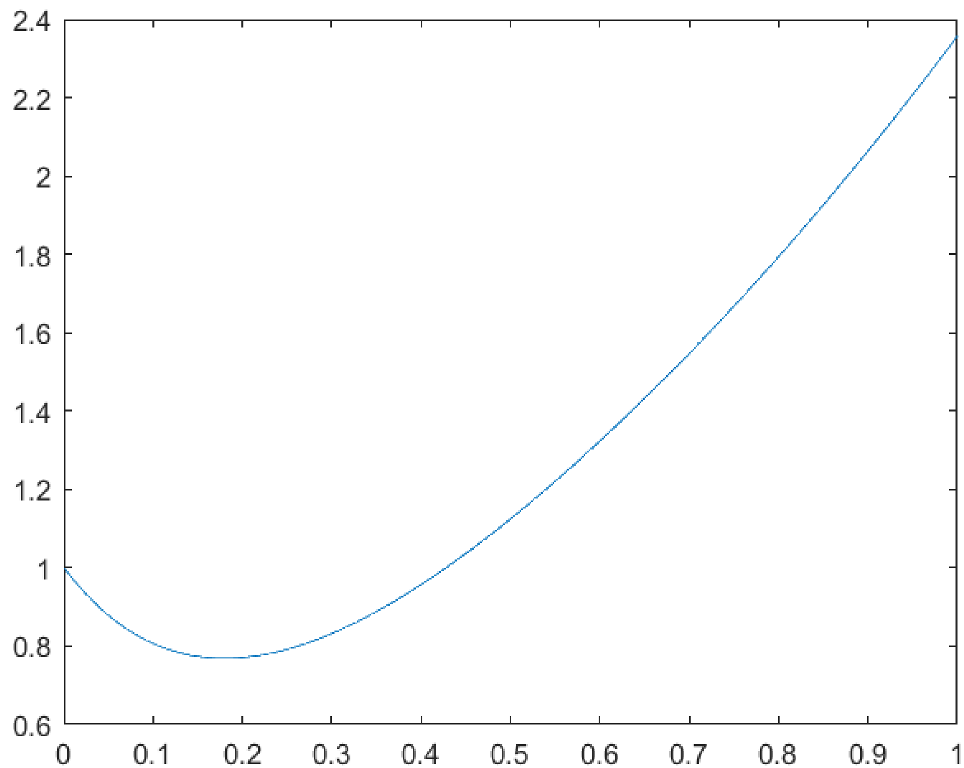
c)

En utilisant Matlab:

```
clear all
clc
y=dsolve('D2y+6*Dy+5*y=12*exp(t)','y(0)=1','Dy(0)=-3')
tt=linspace(0,1,1001)
for i=1:1001
    t=tt(i)
    yy(i)=eval(y)
end
plot(tt,yy)
```

Et on a:

Problem 4 continued on next page. . .


2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Problem 5

1)

Rappele:

$$\hat{f} = A_4 f, \quad avec \quad A_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

2)

Pour $f = [4, 1, 2, 0]$

f	4	1	2	0
$rev(f)$	4	2	1	0
etape 1	6	2	1	1
etape 2	7	2-i	5	2+i

et pour $g = [1, 8, 0, 0]$

g	1	8	0	0
$rev(g)$	1	0	8	0
etape 1	1	1	8	8
etape 2	9	1-8i	-7	1+8i

3)

On pose $p(x) = 4 + x + 2x^2$ et $q(x) = 1 + 8x$

$$\begin{aligned} \hat{p}\hat{q} = \hat{f}\hat{g} &= [7, 2-i, 5, 2+i] \times [9, 1-8i, -7, 1+8i] \\ &= [63, -6-17i, -35, -6+17i] \end{aligned}$$

$\hat{p}\hat{q}$	63	-6-17i	-35	-6+17i
$rev(\hat{p}\hat{q})$	63	-35	-6-17i	-6+17i
etape 1	28	98	-12	-34i
etape 2	16	132	40	64

en suite, on divise par 4, on a:

$$pq(x) = 4 + 33x + 10x^2 + 16x^3$$

En plus,

$$214 \times 81 = pq(10) = 4 + 330 + 1000 + 16000 = 17334$$

Problem 5 continued on next page...

4)

On utilise Matlab pour calculer $(2x^2+x+4)^4(8x+1)^7$. Comme $p^4q^7 \in \mathbb{C}^{16}$, on choix $k=4, n=2^k=16$.

```
clear all
clc
p=[4,1,2,0]
q=[1,8,0,0]
P=fft(p,16)
Q=fft(q,16)
K=(P.^4).*(Q.^7)
k=ifft(K,16)
```

```
k =

   1.0e+09 *

    0.0000    0.0000    0.0004    0.0050    0.0421    0.2243    0.7409    1.4913    2.0006    2.3715    1.8815    1.4852    0.7227    0.3885    0.0965    0.0336
```

C'est a dire:

$$p^4q^7(x) = 10^9(0.0336x^15+0.0965x^14+0.3885x^13+0.7227x^12+1.4852x^11+1.8815x^10+2.3715x^9+2.0006x^8+1.4913x^7$$

$$+0.7409x^6+0.2243x^5+0.0421x^4+0.0050x^3+0.0004x^2+0.0000x+0.0000)$$

D'apres Mupad, on a:

```
p:=4+x+2*x^2;
```

$2\,x^2+x+4$

```
q:=1+8*x;
```

$8\,x+1$

```
k:=p^4*q^7;expand(k)
```

$(8\,x+1)^7\,(2\,x^2+x+4)^4$

$33554432\,x^{15}+96468992\,x^{14}+388497408\,x^{13}+722665472\,x^{12}+1485201408\,x^{11}+1881510912\,x^{10}+2371500928\,x^9+2000553232\,x^8+1491255648\,x^7+740911512\,x^6+224321024\,x^5+42127713\,x^4+4966032\,x^3+359008\,x^2+14592\,x+256$

```
s:=subs(k,[x=10]);
```

47978893234497573608976

```
t:=subs(p^4,[x=10]);
```

2097273616

```
r:=subs(q^7,[x=10]);
```

22876792454961

```
s-t*r;
```

0

Apres la comparaison, on trouve que c'est la meme resultat.

Problem 6

On pose

$$f : t = \begin{cases} 0, & t < 0 \\ 1, & 0 \leq t \leq \pi \\ -1, & \pi < t \leq 2\pi \\ 0, & t > 2\pi \end{cases}$$

$$g = f * f$$

1)

D'apres la definition de la convention, on a:

$$g(t) = f * f = \int_{-\infty}^{+\infty} f(s)f(t-s)ds$$

Quand $s \in (-\infty, 0)$ et $s \in (2\pi, +\infty), f(s) = 0, g(t) = 0$. On a $s \in [0, 2\pi]$
Quand $t - s \in (-\infty, 0)$ et $t - s \in (2\pi, +\infty), f(t-s) = 0, g(t) = 0$
D'ou, quand $t \in (-\infty, 0)$ et $t \in (4\pi, +\infty), g(t) = 0$ et pour $t \in [0, 4\pi]$

$$g(t) = f * f = \int_0^t f(s)f(t-s)ds$$

Pour $t \in [0, \pi]$

$$\begin{aligned} g(t) = f * f &= \int_0^t f(s)f(t-s)ds \\ &= \int_0^t f(s)ds \\ &= t \end{aligned}$$

Pour $t \in [\pi, 2\pi]$

$$\begin{aligned} g(t) = f * f &= \int_0^t f(s)f(t-s)ds \\ &= \int_0^\pi f(s)f(t-s)ds + \int_\pi^t f(s)f(t-s)ds \\ &= \int_0^\pi f(t-s)ds - \int_\pi^t f(t-s)ds \\ &= \int_0^{t-\pi} f(t-s)ds + \int_{t-\pi}^\pi f(t-s)ds - \int_\pi^t f(t-s)ds \\ &= -(t-\pi) + (2\pi - t) - (t - \pi) \\ &= 4\pi - 3t \end{aligned}$$

Pour $t \in [2\pi, 3\pi]$

$$\begin{aligned}
g(t) = f * f &= \int_0^t f(s)f(t-s)ds \\
&= \int_0^{\pi} f(s)f(t-s)ds + \int_{\pi}^{2\pi} f(s)f(t-s)ds + \int_{2\pi}^{t} f(s)f(t-s)ds \\
&= \int_0^{\pi} f(t-s)ds - \int_{\pi}^{2\pi} f(t-s)ds \\
&= \int_{t-2\pi}^{\pi} f(t-s)ds - \int_{\pi}^{t-\pi} f(t-s)ds - \int_{t-\pi}^{2\pi} f(t-s)ds \\
&= (\pi - (t-2\pi))(-1) - (t-\pi-\pi)(-1) - (2\pi - (t-\pi)) \\
&= t - 3\pi + t - 2\pi - 2\pi + t - \pi \\
&= 3t - 8\pi
\end{aligned}$$

Pour $t \in [3\pi, 4\pi]$

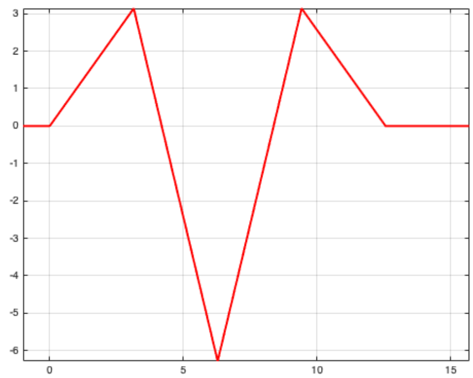
$$\begin{aligned}
g(t) = f * f &= \int_0^t f(s)f(t-s)ds \\
&= \int_0^{\pi} f(s)f(t-s)ds + \int_{\pi}^{2\pi} f(s)f(t-s)ds + \int_{2\pi}^{3\pi} f(s)f(t-s)ds + \int_{3\pi}^{t} f(s)f(t-s)ds \\
&= \int_0^{\pi} f(t-s)ds - \int_{\pi}^{2\pi} f(t-s)ds \\
&= -\int_{\pi}^{2\pi} f(t-s)ds \\
&= -\int_{t-2\pi}^{2\pi} f(t-s)ds \\
&= -(2\pi - (t-2\pi))(-1) \\
&= 4\pi - t
\end{aligned}$$

En conclusion:

$$g : t = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t \leq \pi \\ 4\pi - 3t, & \pi \leq t \leq 2\pi \\ 3t - 8\pi, & 2\pi \leq t \leq 3\pi \\ 4\pi - t, & 3\pi \leq t \leq 4\pi \\ 0, & t > 4\pi \end{cases}$$

```
clear all
clc
x=-1:0.01:5*pi
g=0*(x<0)+x.*(x>=0&x<=pi)+(4*pi-3*x).*(x>=pi&x<=2*pi)+(3*x-8*pi).*(x>=2*pi&x<=3*pi)+(4*pi-x).*(x>=3*pi&x<=4*pi)+0*(x>4*pi)
plot(x,g,'r','linewidth',2)
axis([-1 5*pi -2*pi pi])
grid on
```

Problem 6 continued on next page...



2)

D'apres la definition, $\hat{g} = \hat{f}^2$, d'apres le problem 1.1, on a:

$$\begin{aligned}\hat{g}(x) &= (2i\pi e^{-i\pi x} sin(\frac{\pi x}{2}) sin_c(\frac{\pi x}{2}))^2 \\ &= -4\pi^2 e^{-2i\pi x} sin^2(\frac{\pi x}{2}) sin_c^2(\frac{\pi x}{2})\end{aligned}$$

3)

D'apres le cour. Formule sommatoire de Poisson: g une fonction continue sur $\mathbb{R}$, avec $c = 2\pi$. Alors la série $\sum_{n\in\mathbb{Z}} g(cn)$ est absolument convergente, et on a:

$$\sum_{n\in\mathbb{Z}} g(cn) = \frac{1}{c} \lim_{p\to+\infty} \frac{1}{p+1} \sum_{q=0}^{p} [\sum_{m=-q}^{q} \hat{g}(\frac{2\pi m}{c})]$$

En particulier, les séries:

$$\sum_{m=1}^{+\infty} \hat{g}(-m) = \sum_{m=1}^{+\infty} -4\pi^2 \frac{sin^4(\frac{\pi m}{2})}{\frac{\pi^2 m^2}{4}} \quad est \quad convergente$$

$$\sum_{m=0}^{+\infty} \hat{g}(m) = \hat{g}(0) + \sum_{m=1}^{+\infty} -4\pi^2 \frac{sin^4(\frac{\pi m}{2})}{\frac{\pi^2 m^2}{4}} = \sum_{m=1}^{+\infty} -4\pi^2 \frac{sin^4(\frac{\pi m}{2})}{\frac{\pi^2 m^2}{4}} \quad est \quad convergente$$

Donc:

$$\begin{aligned}\sum_{n\in\mathbb{Z}} g(cn) &= \frac{1}{c} \sum_{n\in\mathbb{Z}} \hat{g}(\frac{2\pi n}{c}) \\ \sum_{n\in\mathbb{Z}} g(2\pi n) &= \frac{1}{2\pi} \sum_{n\in\mathbb{Z}} -4\pi^2 \frac{sin^4(\frac{\pi n}{2})}{\frac{\pi^2 n^2}{4}} \\ g(0) + g(2\pi) + g(4\pi) &= \frac{1}{2\pi} \sum_{n\in\mathbb{Z}} -16 \frac{sin^4(\frac{\pi n}{2})}{n^2} \\ \frac{\pi^2}{4} &= \sum_{n=1}^{+\infty} \frac{2}{(2n+1)^2} \quad on \quad a: \sum_{n=1}^{+\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}\end{aligned}$$

Problem 7

on considère pour n entier, $n \geq 0$ le solide V_n défini par le système d'inéquations.

$$\begin{cases} x^2 + y^2 \leq z - n^2 \\ n^2 \leq z \leq n^2 + \frac{2}{2n+1} \end{cases}$$

1)

On pose

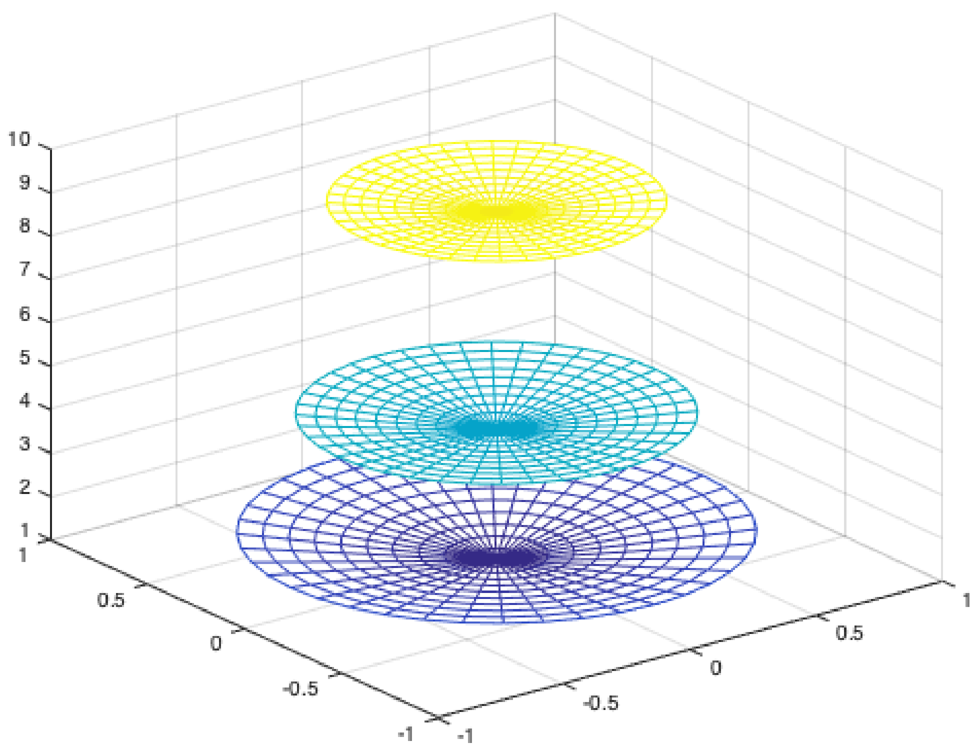
$$V_n = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 \leq z - n^2, n^2 \leq z \leq n^2 + \frac{2}{2n+1}\}$$

V_n represente une cone de revolution, le sommet de la quelle est $(0, 0, n^2)$, comprise entre les plans $z = n^2 \quad et \quad z = n^2 + \frac{2}{2n+1}$
En utilisant le théorème de Fubini.

$$\begin{aligned}
V(V_n) &= \int\int\int V_n dxdydz \\
V_n(z) &= \{(x, y) \in \mathbb{R}^2, (x, y, z) \in V_n\} \\
\pi(V_n) &= \{z \in \mathbb{R}, V_n^2 \neq \emptyset\} \\
\pi(V_n) &= [n^2, n^2 + \frac{2}{2n+1}] \\
V_n &= D(0, 0, z) \\
V(V_n) &= \int_{\pi(V_n)} [\int\int_{V_n^2 dxdy}] dz \\
&= \int_{\pi(V_n)} \pi(z - n^2) dz \\
&= [\pi(\frac{z^2}{2} - n^2 z)]_{n^2}^{n^2 + \frac{2}{2n+1}} \\
&= \pi(\frac{(n^2 + \frac{2}{2n+1})^2 - n^4}{2} - n^2(n^2 + \frac{2}{2n+1} - n^2)) \\
&= \frac{\pi}{2}(2\frac{2n^2}{2n+1} + (\frac{2}{2n+1})^2 - 2\frac{2n^2}{2n+1}) \\
&= \frac{\pi}{2}(\frac{2}{2n+1})^2 \\
&= \frac{2\pi}{(2n+1)^2}
\end{aligned}$$

On pose $V = \cup_{n \geq 0} V_n$ En utilisant Matlab, on a(les exemples quand $n = 1, 2, 3$):

```
clear all
clc
n1 = 1
r1 = sqrt(2/(2*n1+1))
h1 = 2/(2*n1+1)
m1 = h1/r1
[R1,A1] = meshgrid(linspace(0,r1,11),linspace(0,2*pi,41))
X1 = R1.*cos(A1)
Y1 = R1.*sin(A1)
Z1 = n1*n1+m1*R1
mesh(X1,Y1,Z1)
hold on
n2 = 2
r2 = sqrt(2/(2*n2+1))
h2 = 2/(2*n2+1)
m2 = h2/r2
[R2,A2] = meshgrid(linspace(0,r2,11),linspace(0,2*pi,41))
X2 = R2.*cos(A2)
Y2 = R2.*sin(A2)
Z2 = n2*n2+m2*R2
mesh(X2,Y2,Z2)
hold on
n3 = 3
r3 = sqrt(2/(2*n3+1))
h3 = 2/(2*n3+1)
m3 = h3/r3
[R3,A3] = meshgrid(linspace(0,r3,11),linspace(0,2*pi,41))
X3 = R3.*cos(A3)
Y3 = R3.*sin(A3)
Z3 = n3*n3+m3*R3
mesh(X3,Y3,Z3)
```



Problem 7 continued on next page. . .

D'apres les figures, il y a pas de croisement parmi les cones. Donc:

$$\begin{aligned}
V &= \sum_{n=0}^{+\infty} V_n \\
&= \sum_{n=0}^{+\infty} \frac{2\pi}{(2n+1)^2} \\
D'apres \quad 6.3, \quad on \quad a: \quad &= 2\pi + \sum_{n=1}^{+\infty} \frac{2\pi}{(2n+1)^2} \\
&= 2\pi + \frac{\pi^3}{4}
\end{aligned}$$