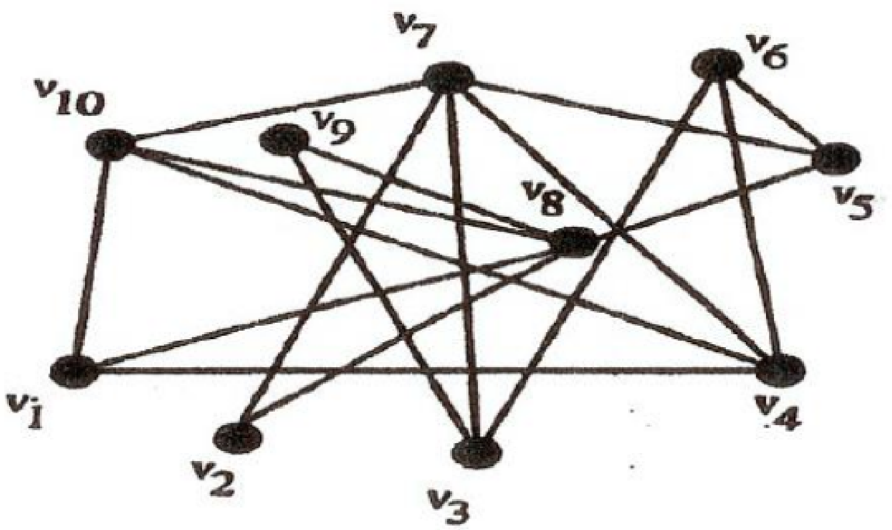
Homework Exercise 1

[1]
Partition the graph given below using the Kernighan-Lin algorithm.



Ans.) Step 1 – Initialization (any random partition)

Let A = {1,2,3,9,10} and B = {4,5,6,7,8} (cut size = 10)

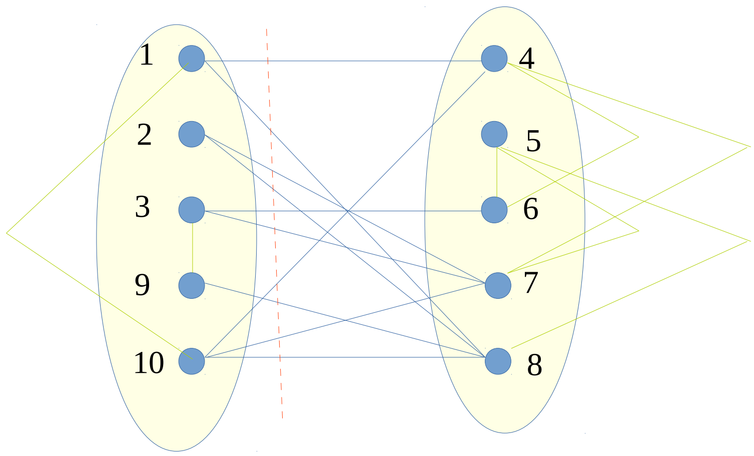


Figure 1 : Depicting initial random partition.

A' = A = {1,2,3,9,10} and B' = B = {4,5,6,7,8}

Step 2: Compute initial D values:

$D_1 = E_1 - I_1 = 2 - 1 = 1$

$D_2 = E_2 - I_2 = 2 - 0 = 2$

$D_3 = E_3 - I_3 = 2 - 1 = 1$

$D_9 = E_9 - I_9 = 1 - 1 = 0$

$D_{10} = E_{10} - I_{10} = 3 - 1 = 2$

$D_4 = E_4 - I_4 = 2 – 2 = 0$

$D_5 = E_5 - I_5 = 0 – 3 = -3$

$D_6 = E_6 - I_6 = 1 – 2 = -1$

$D_7 = E_7 - I_7 = 3 – 2 = 1$

$D_8 = E_8 - I_8 = 4 – 1 = 3$

Step 3: Compute gains (for each possible unlocked pair)

$g_{14} = D_1 + D_4 - 2C_{14} = +1 + 0 – 2(1) = -1$

$g_{15} = D_1 + D_5 - 2C_{15} = +1 + (-3) – 2(0) = -2$

$g_{16} = D_1 + D_6 - 2C_{16} = +1 + (-1) – 2(0) = 0$

$g_{17} = D_1 + D_7 - 2C_{17} = +1 + (+1) – 2(0) = +2$

$g_{18} = D_1 + D_8 - 2C_{18} = +1 + (+3) – 2(1) = +2$

$g_{34} = D_3 + D_4 - 2C_{34} = +1 + 0 – 2(0) = +1$

$g_{35} = D_3 + D_5 - 2C_{35} = +1 + (-3) – 2(0) = -2$

$g_{36} = D_3 + D_6 - 2C_{36} = +1 + (-1) – 2(1) = -2$

$g_{37} = D_3 + D_7 - 2C_{37} = +1 + (+1) – 2(1) = 0$

$g_{38} = D_3 + D_8 - 2C_{38} = +1 + (+3) – 2(0) = +4$

$g_{24} = D_2 + D_4 - 2C_{24} = +2 + 0 – 2(0) = +2$

$g_{25} = D_2 + D_5 - 2C_{25} = +2 + (-3) – 2(0) = -1$

$g_{26} = D_2 + D_6 - 2C_{26} = +2 + (-1) – 2(0) = +1$

$g_{27} = D_2 + D_7 - 2C_{27} = +2 + (+1) – 2(1) = +1$

$g_{28} = D_2 + D_8 - 2C_{28} = +2 + (+3) – 2(1) = +3$

$g_{94} = D_9 + D_4 - 2C_{94} = 0 + 0 – 2(0) = 0$

$g_{95} = D_9 + D_5 - 2C_{95} = 0 + (-3) – 2(0) = -3$

$g_{96} = D_9 + D_6 - 2C_{96} = 0 + (-1) – 2(0) = -1$

$g_{97} = D_9 + D_7 - 2C_{97} = 0 + (+1) – 2(0) = +1$

$g_{98} = D_9 + D_8 - 2C_{98} = 0 + (+3) – 2(1) = +1$

$g_{10,4} = D_{10} + D_4 - 2C_{10,4} = +2 + 0 – 2(1) = 0$

$g_{10.5} = D_{10} + D_5 - 2C_{10.5} = +2 + (-3) – 2(0) = -1$

$g_{10,6} = D_{10} + D_6 – 2C_{10,6} = +2 + (-1) – 2(0) = +1$

$g_{10.7} = D_{10} + D_7 - 2C_{10,7} = +2 + (+1) – 2(1) = +1$

$g_{10,8} = D_{10} + D_8 - 2C_{10,8} = +2 + (+3) – 2(1) = +3$

The largest g value is $g_{38} = +4$

=> **Interchange** 3 and 8 & **lock** $(a_1 , b_1) = (3,8)$

A' = A' – {3} = {1,2,9,10}

B' = B' – {8} = {4,5,6,7}   both of which are not empty

Step 4: Update D values of node connected to vertices (3,8)

$D_6' = D_6 + 2C_{68} - 2C_{63} = -1 + 2(0) – 2(1) = -3$

$D_7' = D_7 + 2C_{78} - 2C_{73} = +1 + 2(0) – 2(1) = -1$

$D_9' = D_9 + 2C_{93} - 2C_{98} = 0 + 2(1) – 2(1) = 0$

$D_1' = D_1 + 2C_{13} - 2C_{18} = +1 + 2(0) – 2(1) = -1$

$D_2' = D_2 + 2C_{23} - 2C_{28} = +2 + 2(0) – 2(1) = 0$

$D_5' = D_5 + 2C_{58} - 2C_{53} = -3 + 2(1) – 2(0) = -1$

$D_{10}' = D_{10} + 2C_{10,3} - 2C_{10,8} = +2 + 2(0) – 2(1) = 0$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{14} = D_1 + D_4 - 2C_{14} = -1 + 0 – 2(1) = -3$

$g_{15} = D_1 + D_5 - 2C_{15} = -1 + (-1) – 2(0) = -2$

$g_{16} = D_1 + D_6 - 2C_{16} = -1 + (-3) – 2(0) = -4$

$g_{17} = D_1 + D_7 - 2C_{17} = -1 + (-1) – 2(0) = -2$

$g_{24} = D_2 + D_4 - 2C_{24} = 0 + 0 – 2(0) = 0$

$g_{25} = D_2 + D_5 - 2C_{25} = 0 + (-1) – 2(0) = -1$

$g_{26} = D_2 + D_6 - 2C_{26} = 0 + (-3) – 2(0) = -3$

$g_{27} = D_2 + D_7 - 2C_{27} = 0 + (-1) – 2(1) = -3$

$g_{94} = D_9 + D_4 - 2C_{94} = 0 + 0 – 2(0) = 0$

$g_{95} = D_9 + D_5 - 2C_{95} = 0 + (-1) – 2(0) = -1$

$g_{96} = D_9 + D_6 - 2C_{96} = 0 + (-3) – 2(0) = -3$

$g_{97} = D_9 + D_7 - 2C_{97} = 0 + (-1) – 2(0) = -1$

$g_{10,4} = D_{10} + D_4 - 2C_{10,4} = 0 + 0 – 2(1) = -2$

$g_{10,5} = D_{10} + D_5 - 2C_{10,5} = 0 + (-1) – 2(0) = -1$

$g_{10,6} = D_{10} + D_6 - 2C_{10,6} = 0 + (-3) – 2(0) = -3$

$g_{10,7} = D_{10} + D_7 - 2C_{10,7} = 0 + (-1) – 2(1) = -3$

Two values are equal; arbitrarily choose $g_{24} = 0$

=> **Interchange** 2 and 4 & **lock** $(a_2 , b_2) = (2,4)$

A' = A' – {2} = {1,9,10}

B' = B' – {4} = {5,6,7} both of which are not empty

Update D values of node connected to vertices (2,4) so that new D values are:

$D_7' = D_7 + 2C_{74} - 2C_{72} = -1 + 2(1) – 2(1) = -1$

$D_1' = D_1 + 2C_{12} - 2C_{14} = -1 + 2(0) – 2(1) = -3$

$D_6' = D_6 + 2C_{64} - 2C_{62} = -3 + 2(1) – 2(0) = -1$

$D_{10}' = D_{10} + 2C_{10,2} - 2C_{10,4} = 0 + 2(0) – 2(1) = -2$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{15} = D_1 + D_5 - 2D_{15} = -3 + (-1) - 2(0) = -4$

$g_{16} = D_1 + D_6 - 2D_{16} = -3 + (-1) – 2(0) = -4$

$g_{17} = D_1 + D_7 - 2D_{17} = -3 + (-1) – 2(0) = -4$

$g_{95} = D_9 + D_5 - 2D_{95} = 0 + (-1) – 2(0) = -1$

$g_{96} = D_9 + D_6 - 2D_{96} = 0 + (-1) – 2(0) = -1$

$g_{97} = D_9 + D_7 - 2D_{97} = 0 + (-1) – 2(0) = -1$

$g_{10,5} = D_{10} + D_5 - 2D_{10,5} = -2 + (-1) – 2(0) = -3$

$g_{10,6} = D_{10} + D_6 - 2D_{10,6} = -2 + (-1) – 2(0) = -3$

$g_{10,7} = D_{10} + D_7 - 2D_{10,7} = -2 + (-1) – 2(1) = -5$

Three values are equal to -1 (max gain); arbitrarily choose $g_{95} = -1$

=> **Interchange** 9 and 5 & **lock** $(a_3 , b_3) = (9,5)$

$A' = A' – \{9\} = \{1,10\}$

$B' = B' – \{5\} = \{6,7\}$ both of which are not empty

Update D values of node connected to vertices (9,5) so that new D values are:

$D_6 = D_6' + 2C_{65} - 2C_{69} = -1 + 2(1) – (0) = +1$

$D_7 = D_7' + 2C_{75} - 2C_{79} = -1 + 2(1) – (0) = +1$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{16} = D_1 + D_6 - 2D_{16} = -3 + (+1) – 2(0) = -2$

$g_{1,7} = D_1 + D_7 - 2D_{17} = -3 + (+1) – 2(0) = -2$

$g_{10,6} = D_{10} + D_6 - 2D_{10,6} = -2 + (+1) – 2(0) = -1$

$g_{10,7} = D_{10} + D_7 - 2D_{10,7} = -2 + (+1) – 2(1) = -3$

The largest g value is $g_{10,6} = -1$

=> **Interchange** 10 and 6 & **lock** $(a_4 , b_4) = (10,6)$

$A' = A' – \{10\} = \{1\}$

$B' = B' – \{6\} = \{7\}$ both of which are not empty

Update D values of node connected to vertices (10,6) so that new D values are:

$D_1 = D_1' + 2C_{1,10} - 2C_{16} = -3 + 2(1) - 2(0) = -1$

$D_7 = D_7' + 2C_{76} - 2C_{7,10} = 1 + 2(0) – 2(1) = -1$

New gain with $D_1 \leftarrow D_1'$, $D_7' \leftarrow D_7'$

$g_{17} = D_1 + D_7 - 2D_{17} = -1 + (-1) – 2(0) = -2$ => $(a_5 , b_5) = (1,7)$

=> The largest g value is $g_{1,7}$ = -2

=> **Interchange** 1 and 7 & **lock** (a_4 , b_4) = (1,7)

A' = A' – {1} = {}

B' = B' – {7} = {}   both of which are empty

Step 5 – Determine the # of moves to take:

g_1 = +4

$g_1 + g_2$ = +4 + 0 = +4

$g_1 + g_2 + g_3$ = +4 + 0 + (-1) = +3

$g_1 + g_2 + g_3 + g_4$ = +4 + 0 + (-1) + (-1) = +2

$g_1 + g_2 + g_3 + g_4 + g_5$ = +4 + 0 + (-1) + (-1) + (-2) = 0

Hence, the value of k for max G is 1

X = {a_1} = {3}, Y = {b_1} = {8}

Move X to B, Y to A => A = {1,2,8,9,10}, B = {3,4,5,6,7} (cut size = 6)

where $Gain_k = Gain_1$ = +4

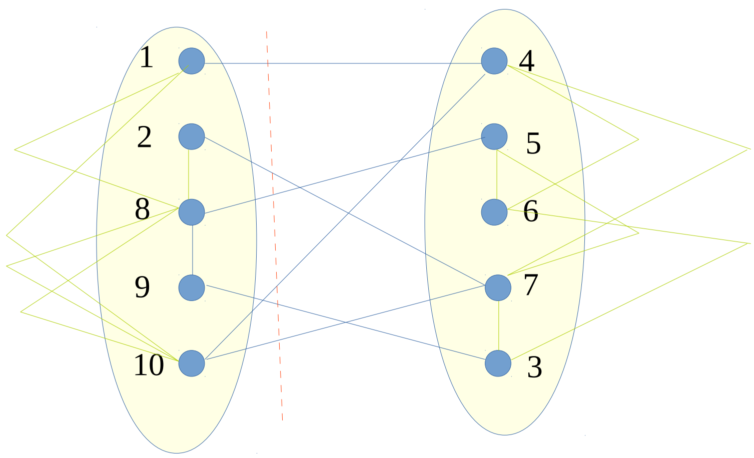


Figure 2 : Depicting partitioning
after first pass.

Since, $Gain_k$ is greater than 0, repeating the whole process untill $Gain_k$ is less than or equal to zero.

Step 1: Initial partition same as partitioning achieved (A = {1,2,8,9,10}, B = {3,4,5,6,7})

Step 2: Compute initial D values:

$D_1 = E_1 - I_1$ = 1 - 2 = -1

$D_2 = E_2 - I_2$ = 1 - 1 = 0

$D_8 = E_8 - I_8$ = 1 - 4 = -3

$D_3 = E_3 - I_3$ = 1 – 2 = -1

$D_4 = E_4 - I_4$ = 2 – 2 = 0

$D_5 = E_5 - I_5$ = 1 – 2 = -1

$D_9 = E_9 - I_9 = 1 - 1 = 0$

$D_{10} = E_{10} - I_{10} = 2 - 2 = 0$

$D_6 = E_6 - I_6 = 0 - 3 = -3$

$D_7 = E_7 - I_7 = 2 - 3 = -1$

Step 3: Compute gains (for each possible unlocked pair)

$g_{13} = D_1 + D_3 - 2C_{13} = -1 + (-1) - 2(0) = -2$

$g_{14} = D_1 + D_4 - 2C_{14} = -1 + (0) - 2(1) = -3$

$g_{15} = D_1 + D_5 - 2C_{15} = -1 + (-1) - 2(0) = -2$

$g_{16} = D_1 + D_6 - 2C_{16} = -1 + (-3) - 2(0) = -4$

$g_{17} = D_1 + D_7 - 2C_{17} = -1 + (-1) - 2(0) = -2$

$g_{83} = D_8 + D_3 - 2C_{83} = -3 + (-1) - 2(0) = -4$

$g_{84} = D_8 + D_4 - 2C_{84} = -3 + (0) - 2(0) = -3$

$g_{85} = D_8 + D_5 - 2C_{85} = -3 + (-1) - 2(1) = -6$

$g_{86} = D_8 + D_6 - 2C_{86} = -3 + (-3) - 2(0) = -6$

$g_{87} = D_8 + D_7 - 2C_{87} = -3 + (-1) - 2(0) = -4$

$g_{23} = D_2 + D_3 - 2C_{23} = 0 + (-1) - 2(0) = -1$

$g_{24} = D_2 + D_4 - 2C_{24} = 0 + (0) - 2(0) = 0$

$g_{25} = D_2 + D_5 - 2C_{25} = 0 + (-1) - 2(0) = -1$

$g_{26} = D_2 + D_6 - 2C_{26} = 0 + (-3) - 2(0) = -3$

$g_{27} = D_2 + D_7 - 2C_{27} = 0 + (-1) - 2(1) = -3$

$g_{93} = D_9 + D_3 - 2C_{93} = 0 + (-1) - 2(1) = -3$

$g_{94} = D_9 + D_4 - 2C_{94} = 0 + (0) - 2(0) = 0$

$g_{95} = D_9 + D_5 - 2C_{95} = 0 + (-1) - 2(0) = -1$

$g_{96} = D_9 + D_6 - 2C_{96} = 0 + (-3) - 2(0) = -3$

$g_{97} = D_9 + D_7 - 2C_{97} = 0 + (-1) - 2(0) = -1$

$g_{10,3} = D_{10} + D_3 - 2C_{10,3} = 0 + (-1) - 2(0) = -1$

$g_{10.4} = D_{10} + D_4 - 2C_{10.4} = 0 + (0) - 2(1) = -2$

$g_{10,5} = D_{10} + D_5 - 2C_{10,5} = 0 + (-1) - 2(0) = -1$

$g_{10.6} = D_{10} + D_6 - 2C_{10,6} = 0 + (-3) - 2(0) = -3$

$g_{10,7} = D_{10} + D_7 - 2C_{10,7} = 0 + (-1) - 2(1) = -3$

Three values are equal to -1 (max gain); arbitrarily choose $g_{24} = 0$

=> **Interchange** 2 and 4 & **lock** $(a_1, b_1) = (2,4)$

$A' = A' - \{2\} = \{1,8,9,10\}$

$B' = B' - \{4\} = \{3,5,6,7\}$ both of which are not empty

Step 4: Update D values of node connected to vertices (2,4)

$D_7' = D_7 + 2C_{74} - 2C_{72} = -1 + 2(1) - 2(1) = -1$

$D_8' = D_8 + 2C_{82} - 2C_{84} = -3 + 2(1) - 2(0) = -1$

$D_1' = D_1 + 2C_{12} - 2C_{14} = -1 + 2(0) – 2(1) = -3$

$D_6' = D_6 + 2C_{64} - 2C_{62} = -3 + 2(1) – 2(0) = -1$

$D_{10}' = D_{10} + 2C_{10,2} - 2C_{10,4} = 0 + 2(0) – 2(1) = -2$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{13} = D_1 + D_3 - 2C_{13} = -3 + (-1) – 2(0) = -4$

$g_{15} = D_1 + D_5 - 2C_{15} = -3 + (-1) – 2(0) = -4$

$g_{16} = D_1 + D_6 - 2C_{16} = -3 + (-1) – 2(0) = -4$

$g_{17} = D_1 + D_{7\,-}\, 2C_{17} = -3 + (-1) – 2(0) = -4$

$g_{83} = D_8 + D_3 - 2C_{83} = -1 + (-1) – 2(0) = -2$

$g_{85} = D_8 + D_5 - 2C_{85} = -1 + (-1) – 2(1) = -4$

$g_{86} = D_8 + D_6 - 2C_{86} = -1 + (-1) – 2(0) = -2$

$g_{87} = D_8 + D_7 - 2C_{87} = -1 + (-1) – 2(0) = -2$

$g_{93} = D_9 + D_3 - 2C_{93} = 0 + (-1) – 2(1) = -3$

$g_{95} = D_9 + D_5 - 2C_{95} = 0 + (-1) – 2(0) = -1$

$g_{96} = D_9 + D_6 - 2C_{96} = 0 + (-1) – 2(0) = -1$

$g_{97} = D_9 + D_7 - 2C_{97} = 0 + (-1) – 2(0) = -1$

$g_{10,3} = D_{10} + D_3 - 2C_{10,3} = -2 + (-1) – 2(0) = -3$

$g_{10,5} = D_{10} + D_5 - 2C_{10,5} = -2 + (-1) – 2(0) = -3$

$g_{10,6} = D_{10} + D_6 - 2C_{10,6} = -2 + (-1) – 2(0) = -3$

$g_{10,7} = D_{10} + D_7 - 2C_{10,7} = -2 + (-1) – 2(1) = -5$

Three values are equal; arbitrarily choose $g_{95} = -1$

=> **Interchange** 9 and 5 & **lock** $(a_2 , b_2) = (9,5)$

A' = A' – {9} = {1,8,10}

B' = B' – {5} = {3,6,7} both of which are not empty

Update D values of node connected to vertices (2,4) so that new D values are:

$D_7' = D_7 + 2C_{75} - 2C_{79} = -1 + 2(1) – 2(0) = +1$

$D_8' = D_8 + 2C_{89} - 2C_{85} = -1 + 2(1) – 2(1) = -1$

$D_6' = D_6 + 2C_{65} - 2C_{69} = -1 + 2(1) – 2(0) = +1$

$D_3' = D_3 + 2C_{35} - 2C_{39} = -1 + 2(0) – 2(1) = -3$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{13} = D_1 + D_3 - 2D_{13} = -3 + (-3) - 2(0) = -6$

$g_{16} = D_1 + D_6 - 2D_{16} = -3 + (+1) – 2(0) = -2$

$g_{17} = D_1 + D_7 - 2D_{17} = -3 + (+1) – 2(0) = -2$

$g_{83} = D_8 + D_3 - 2D_{83} = -1 + (-3) – 2(0) = -4$

$g_{86} = D_8 + D_6 - 2D_{86} = -1 + (+1) – 2(0) = 0$

$g_{87} = D_8 + D_7 - 2D_{87} = -1 + (+1) – 2(0) = 0$

$g_{10,3} = D_{10} + D_3 - 2D_{10,3} = -2 + (-3) – 2(0) = -5$

$g_{10,6} = D_{10} + D_6 - 2D_{10,6} = -2 + (+1) – 2(0) = -1$

$g_{10,7} = D_{10} + D_7 - 2D_{10,7} = -2 + (+1) – 2(1) = -3$

Two values are equal to 0 (max gain); arbitrarily choose $g_{86} = 0$

=> **Interchange** 8 and 6 & **lock** $(a_3 , b_3) = (8,6)$

$A' = A' – \{8\} = \{1,10\}$

$B' = B' – \{6\} = \{3,7\}$ both of which are not empty

Update D values of node connected to vertices (8,6) so that new D values are:

$D_1 = D_1' + 2C_{18} - 2C_{16} = -3 + 2(1) – 2(0) = -1$

$D_{10} = D_{10}' + 2C_{10,8} – 2C_{10,6} = -2 + 2(1) – 2(0) = 0$

$D_3 = D_3' + 2C_{36} - 2C_{38} = -3 + 2(1) – (0) = -1$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{13} = D_1 + D_3 - 2D_{13} = -1 + (-1) – 2(0) = -2$

$g_{1,7} = D_1 + D_7 - 2D_{17} = -1 + (+1) – 2(0) = 0$

$g_{10,3} = D_{10} + D_3 - 2D_{10,3} = 0 + (-1) – 2(0) = -1$

$g_{10,7} = D_{10} + D_7 - 2D_{10,7} = 0 + (+1) – 2(1) = -1$

The largest g value is $g_{1,7} = 0$

=> **Interchange** 1 and 7 & **lock** $(a_4 , b_4) = (1,7)$

$A' = A' – \{1\} = \{10\}$

$B' = B' – \{7\} = \{3\}$ both of which are not empty

Update D values of node connected to vertices (1,7) so that new D values are:

$D_{10} = D_{10}' + 2C_{10,1} – 2C_{10,7} = 0 + 2(1) - 2(1) = 0$

$D_3 = D_3' + 2C_{37} - 2C_{31} = -1 + 2(1) – 2(0) = +1$

New gain with $D_1 \leftarrow D_1'$, $D_7' \leftarrow D_7'$

$g_{10,3} = D_{10} + D_3 - 2D_{10,3} = 0 + (+1) – 2(0) = +1$ => $(a_5 , b_5) = (10,3)$

The largest g value is $g_{10,3} = +1$

=> **Interchange** 10 and 3 & **lock** $(a_5 , b_5) = (10,3)$

A' = A' – {10} = {}

B' = B' – {3} = {} both of which are empty

Step 5 – Determine the # of moves to take:

$g_1 = 0$

$g_1 + g_2 = 0 + -1 = -1$

$g_1 + g_2 + g_3 = 0 + (-1) + 0 = -1$

$g_1 + g_2 + g_3 + g_4 = 0 + (-1) + 0 + 0 = -1$

$g_1 + g_2 + g_3 + g_4 + g_5 = 0 + (-1) + 0 + 0 + (+1) = 0$

Hence, the value of k for max G is 1

Since, no gain is achieved over the previous solution, no exchange is done

Therefore, the final solution is A = {1,2,8,9,10}, B = {3,4,5,6,7}

Final cut size = 6 (Initial cut size was 10)

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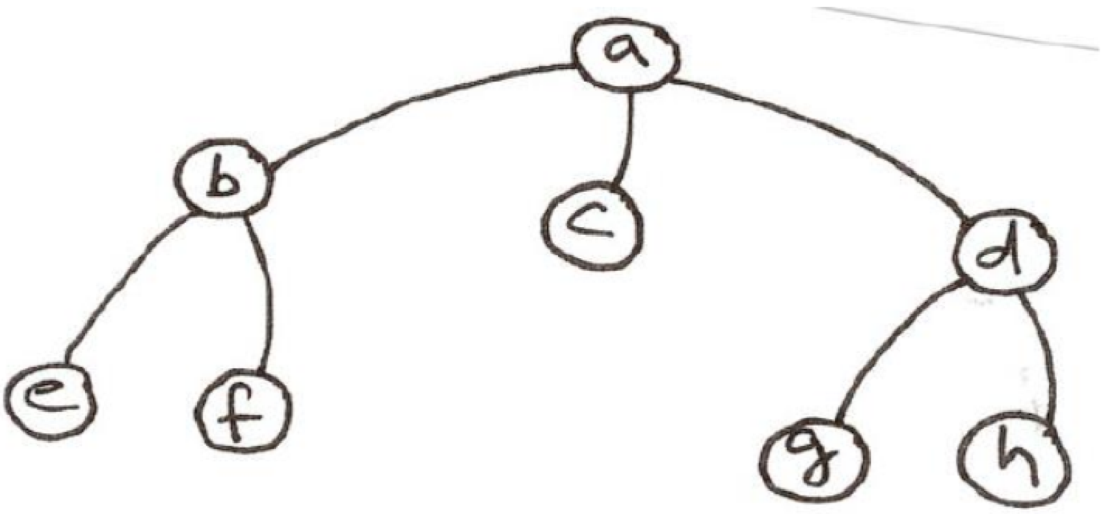
[2]
Consider a tree with 8 nodes (shown below). Apply the Kernighan-Lin algorithm to this graph. As the initial partition let A = {a, b, c, d} and B = {e, f, g, h}.



Ans.) Step 1 – Initialization

Let $A = \{a,b,c,d\}$ and $B = \{e,f,g,h\}$ (cut size = 4)

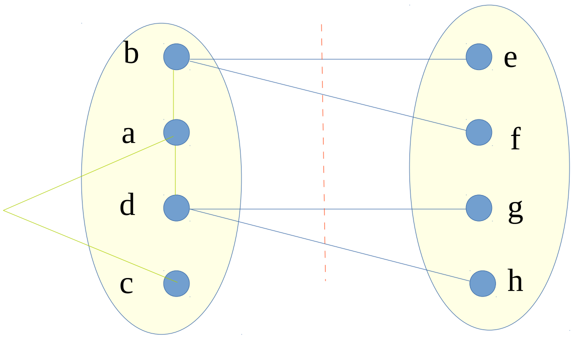


Figure 1 : Depicting initial partition.

$A' = A = \{a,b,c,d\}$ and $B' = B = \{e,f,g,h\}$

Step 2: Compute initial D values:

$D_a = E_a - I_a = 0 - 3 = -3$

$D_b = E_b - I_b = 2 - 1 = 1$

$D_c = E_c - I_c = 0 - 1 = -1$

$D_d = E_d - I_d = 2 - 1 = 1$

$D_e = E_e - I_e = 1 – 0 = 1$

$D_f = E_f - I_f = 1 – 0 = 1$

$D_g = E_g - I_g = 1 – 0 = 1$

$D_h = E_h - I_h = 1 – 0 = 1$

Step 3: Compute gains (for each possible unlocked pair)

$g_{ae} = D_a + D_e - 2C_{ae} = -3 + 1 – 2(0) = -2$

$g_{af} = D_a + D_f - 2C_{af} = -3 + 1 – 2(0) = -2$

$g_{ag} = D_a + D_g - 2C_{ag} = -3 + 1 – 2(0) = -2$

$g_{ah} = D_a + D_h - 2C_{ah} = -3 + 1 – 2(0) = -2$

$g_{be} = D_b + D_e - 2C_{be} = +1 + 1 – 2(1) = 0$

$g_{bf} = D_b + D_f - 2C_{bf} = +1 + 1 – 2(1) = 0$

$g_{bg} = D_b + D_g - 2C_{bg} = +1 + 1 – 2(0) = +2$

$g_{bh} = D_b + D_h - 2C_{bh} = +1 + 1 – 2(0) = +2$

$g_{ce} = D_c + D_e - 2C_{ce} = -1 + 1 – 2(0) = 0$

$g_{cf} = D_c + D_f - 2C_{cf} = -1 + 1 – 2(0) = 0$

$g_{cg} = D_c + D_g - 2C_{cg} = -1 + 1 – 2(0) = 0$

$g_{ch} = D_c + D_h - 2C_{ch} = -1 + 1 – 2(0) = 0$

$g_{de} = D_d + D_e - 2C_{de} = +1 + 1 – 2(0) = +2$

$g_{df} = D_d + D_f - 2C_{df} = +1 + 1 – 2(0) = +2$

$g_{dg} = D_d + D_g - 2C_{dg} = +1 + 1 – 2(1) = 0$

$g_{dh} = D_d + D_h - 2C_{dh} = +1 + 1 – 2(1) = 0$

Four values are equal, aribtirarily choose $g_{bg} = +2$

=> **Interchange** b and g & **lock** $(a_1 , b_1) = (b,g)$

$A' = A' – \{b\} = \{a,c,d\}$

$B' = B' – \{g\} = \{e,f,h\}$ both of which are not empty

Step 4: Update D values of node connected to vertices (b,g)

$D_a' = D_a + 2C_{ab} - 2C_{ag} = -3 + 2(1) – 2(0) = -1$

$D_e' = D_e + 2C_{eg} - 2C_{eb} = +1 + 2(0) – 2(1) = -1$

$D_f' = D_f + 2C_{fg} - 2C_{fb} = +1 + 2(0) – 2(1) = -1$

$D_d' = D_d + 2C_{db} - 2C_{dg} = +1 + 2(0) – 2(1) = -1$

Assing $D_x = D_x'$, repeat step 3 and step 4:

$g_{ae} = D_a + D_e - 2C_{ae} = -1 + (-1) – 2(0) = -2$

$g_{af} = D_a + D_f - 2C_{af} = -1 + (-1) – 2(0) = -2$

$g_{ah} = D_a + D_h - 2C_{ah} = -1 + (+1) – 2(0) = 0$

$g_{ce} = D_c + D_e - 2C_{ce} = -1 + (-1) – 2(0) = -2$

$g_{cf} = D_c + D_f - 2C_{cf} = -1 + (-1) – 2(0) = -2$

$g_{ch} = D_c + D_h - 2C_{ch} = -1 + (+1) – 2(0) = 0$

$g_{de} = D_d + D_e - 2C_{de} = -1 + (-1) – 2(0) = -2$

$g_{df} = D_d + D_f - 2C_{df} = -1 + (-1) – 2(0) = -2$

$g_{dh} = D_d + D_h - 2C_{dh} = -1 + (+1) – 2(1) = -2$

Two values are equal; arbitrarily choose $g_{ah} = 0$

=> **Interchange** a and h & **lock** $(a_2 , b_2) = (a,h)$

A' = A' – {a} = {c,d}

B' = B' – {h} = {e,f} both of which are not empty

Update D values of node connected to vertices (a,h) so that new D values are:

$D_c' = D_c + 2C_{ca} - 2C_{ch} = -1 + 2(1) – 2(0) = +1$

$D_d' = D_d + 2C_{da} - 2C_{dh} = -1 + 2(1) – 2(1) = -1$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{ce} = D_c + D_e - 2D_{ce} = +1 + (-1) - 2(0) = 0$

$g_{cf} = D_c + D_f - 2D_{cf} = +1 + (-1) – 2(0) = 0$

$g_{de} = D_d + D_e - 2D_{de} = -1 + (-1) – 2(0) = -2$

$g_{df} = D_d + D_f - 2D_{df} = -1 + (-1) – 2(0) = -2$

Two values are equal to 0 (max gain); arbitrarily choose $g_{ce} = 0$

=> **Interchange** c and e & **lock** $(a_3, b_3) = (c,e)$

A' = A' – {c} = {d}

B' = B' – {e} = {f}   both of which are not empty

Update D values of node connected to vertices (c,e) so that new D values are:

None of the D values are updated

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{df} = D_d + D_f - 2D_{df} = -1 + (-1) - 2(0) = -2$

The largest g value is $g_{df} = -2$

=> **Interchange** d and f & **lock** $(a_4, b_4) = (d,f)$

A' = A' – {d} = {}

B' = B' – {f} = {}   both of which are empty

Step 5 – Determine the # of moves to take:

$g_1 = +2$

$g_1 + g_2 = +2 + 0 = +2$

$g_1 + g_2 + g_3 = +2 + 0 + 0 = +2$

$g_1 + g_2 + g_3 + g_4 = +2 + 0 + 0 + (-2) = 0$

Hence, the value of k for max G is 1

$X = \{a_1\} = \{b\}$, $Y = \{b_1\} = \{g\}$

Move X to B, Y to A => A = {a,c,d,g}, B = {b,e,f,h} (cut size = 2)

where $Gain_k = Gain_1 = +2$

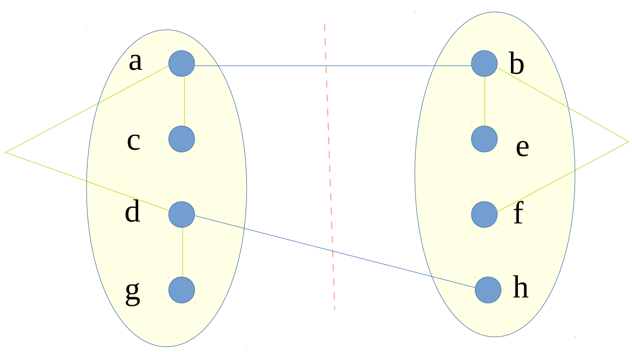


Figure 2 : Depicting partitioning after first pass.

Since, $Gain_k$ is greater than 0, repeating the whole process untill $Gain_k$ is less than or equal to zero.

Step 1: Initial partition same as partitioning achieved (A = {a,c,d,g}, B = {b,e,f,h})

Step 2: Compute initial D values:

$D_a = E_a - I_a = 1 - 2 = -1$

$D_b = E_b - I_b = 1 - 2 = -1$

$D_c = E_c - I_c = 0 - 1 = -1$

$D_d = E_d - I_d = 1 - 2 = -1$

$D_e = E_e - I_e = 0 - 1 = -1$

$D_f = E_f - I_f = 0 - 1 = -1$

$D_g = E_g - I_g = 0 - 1 = -1$

$D_h = E_h - I_h = 1 - 0 = 1$

Step 3: Compute gains (for each possible unlocked pair)

$g_{ae} = D_a + D_e - 2C_{ae} = -1 + (-1) - 2(0) = -2$

$g_{af} = D_a + D_f - 2C_{af} = -1 + (-1) - 2(0) = -2$

$g_{ab} = D_a + D_b - 2C_{ab} = -1 + (-1) - 2(1) = -4$

$g_{ah} = D_a + D_h - 2C_{ah} = -1 + 1 - 2(0) = 0$

$g_{ge} = D_g + D_e - 2C_{be} = -1 + (-1) - 2(0) = -2$

$g_{gf} = D_g + D_f - 2C_{bf} = -1 + (-1) - 2(0) = -2$

$g_{gb} = D_g + D_b - 2C_{gb} = -1 + (-1) - 2(0) = -2$

$g_{gh} = D_g + D_h - 2C_{gh} = -1 + 1 - 2(0) = 0$

$g_{ce} = D_c + D_e - 2C_{ce} = -1 + (-1) - 2(0) = -2$

$g_{cf} = D_c + D_f - 2C_{cf} = -1 + (-1) - 2(0) = -2$

$g_{cb} = D_c + D_b - 2C_{cb} = -1 + (-1) - 2(0) = -2$

$g_{ch} = D_c + D_h - 2C_{ch} = -1 + 1 - 2(0) = 0$

$g_{de} = D_d + D_e - 2C_{de} = -1 + (-1) - 2(0) = -2$

$g_{df} = D_d + D_f - 2C_{df} = -1 + (-1) - 2(0) = -2$

$g_{db} = D_d + D_b - 2C_{db} = -1 + (-1) - 2(0) = -2$

$g_{dh} = D_d + D_h - 2C_{dh} = -1 + 1 - 2(1) = -2$

Three values are equal to 0 (max gain), aribitrarily choose $g_{ah} = 0$

=> **Interchange** a and h & **lock** $(a_1, b_1) = (a,h)$

A' = A' – {a} = {c,d,g}

B' = B' – {h} = {b,e,f} both of which are not empty

Step 4: Update D values of node connected to vertices (a,h)

$D_b' = D_b + 2C_{bh} - 2C_{ba} = -1 + 2(0) - 2(1) = -3$

$D_c' = D_c + 2C_{ca} - 2C_{ch} = -1 + 2(1) - 2(0) = 1$

$D_d' = D_d + 2C_{da} - 2C_{dh} = -1 + 2(1) - 2(1) = -1$

Assing $D_x = D_x'$, repeat step 3 and step 4:

$g_{ge} = D_g + D_e - 2C_{ge} = -1 + (-1) – 2(0) = -2$

$g_{gf} = D_g + D_f - 2C_{gf} = -1 + (-1) – 2(0) = -2$

$g_{gb} = D_g + D_b - 2C_{gb} = -1 + (-3) – 2(0) = -4$

$g_{ce} = D_c + D_e - 2C_{ce} = 1 + (-1) – 2(0) = 0$

$g_{cf} = D_c + D_f - 2C_{cf} = 1 + (-1) – 2(0) = 0$

$g_{cb} = D_c + D_b - 2C_{cb} = 1 + (-3) – 2(0) = -2$

$g_{de} = D_d + D_e - 2C_{de} = -1 + (-1) – 2(0) = -2$

$g_{df} = D_d + D_f - 2C_{df} = -1 + (-1) – 2(0) = -2$

$g_{db} = D_d + D_b - 2C_{db} = -1 + (-3) – 2(0) = -4$

Two values are equal to 0 (max gain); arbitrarily choose $g_{ce} = 0$

=> **Interchange** c and e & **lock** $(a_2 , b_2) = (c,e)$

A' = A' – {c} = {d,g}

B' = B' – {e} = {b,f}   both of which are not empty

Update D values of node connected to vertices (c,e) so that new D values are:

$D_b' = D_b + 2C_{be} - 2C_{bc} = -3 + 2(1) – 2(0) = -1$

Assing $D_i = D_i'$, repeat step 3 and step 4:

$g_{db} = D_d + D_b - 2D_{db} = -1 + (-1) - 2(0) = -2$

$g_{df} = D_d + D_f - 2D_{df} = -1 + (-1) – 2(0) = -2$

$g_{gb} = D_g + D_b - 2D_{gb} = -1 + (-1) – 2(0) = -2$

$g_{gf} = D_g + D_f - 2D_{gf} = -1 + (-1) – 2(0) = -2$

Four values are equal to -2 (max gain); arbitrarily choose $g_{db} = 0$

=> **Interchange** d and b & **lock** $(a_3 , b_3) = (d,b)$

A' = A' – {d} = {g}

B' = B' – {b} = {f}   both of which are not empty

Update D values of node connected to vertices (d,b) so that new D values are:

None of the D values are updated

Assing $D_x = D_x'$, repeat step 3 and step 4:

$g_{gf} = D_g + D_f - 2D_{gf} = -1 + (-1) – 2(0) = -2$

The largest g value is $g_{gf} = -2$

=> **Interchange** g and f & **lock** $(a_4 , b_4) = (g,f)$

A' = A' – {g} = {}

B' = B' – {f} = {}   both of which are empty

Step 5 – Determine the # of moves to take:

$g_1 = 0$

$g_1 + g_2 = 0 + 0 = 0$

$g_1 + g_2 + g_3 = 0 + 0 + 0 = 0$

$g_1 + g_2 + g_3 + g_4 = 0 + 0 + 0 + (-2) = -2$

Hence, the value of k for max G is 1

Since, no gain is achieved over the previous solution, no exchange is done

Therefore, the final solution is A = {a,c,d,g}, B = {b,e,h,f}

Final cut size = 2 (Initial cut size was 4)