Question 2

We state the Leibniz determinant of $zI - A$ here for reference:

$$det(zI - A) = \sum_{\sigma \in S_n} \text{sign}(\sigma)(zI - A)_{\sigma(1)1} ... (zI - A)_{\sigma(n)n}$$

where

$$(zI - A)_{\sigma(i)i} = \begin{cases} z - A_{ii} & \sigma(i) = i \\ -A_{\sigma(i)i} & \sigma(i) \neq i \end{cases}$$

The trivial permutation $\sigma = 1$ contributes the terms

$$(z - A_{11}) ... (z - A_{nn})$$

which comprise an nth degree polynomial z^n and an $n-1$th degree polynomial $\sum_{i=1}^{n} -A_{ii} z^{n-1} = -\text{tr}(A) z^{n-1}$, as well as other lower order terms. Observe that for $\sigma \neq 1$, the terms contributed are of degree $\leq n-2$ because these terms must have both at least one i s.t. $\sigma(i) \neq i$, and a corresponding j s.t. $\sigma(j) = i \neq j$ (since it is required that $\exists$ an element which σ sends to i). Hence the only term in $p_A(z) = det(zI - A)$ of degree $n - 1$ is from the $\sigma = 1$ permutation, and the leading coefficient of this term is equal to $-\text{tr}(A)$.