CSC321 HW 5 2018.3.16

Weidong An

1. $z^{(t)} = Ux^{(t)} + Wh^{(t-1)} + b_h$

$h^{(t)} = \phi(z^{(t)})$

$r^{(t)} = Vh^{(t)} + b_y$

$y^{(t)} = \phi(r^{(t)})$

$$\phi(z) = \begin{cases} 1, & z > 0 \\ 0, & z \le 0 \end{cases}$$

By hint, let

$$U = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad b_h = \begin{bmatrix} -0.5 \\ -1.5 \\ -2.5 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \qquad b_y = -0.5$$

2. (a) $\overline{h^{(t)}} = \overline{i^{(t+1)}} \frac{\partial i^{(t+1)}}{\partial h^{(t)}} + \overline{f^{(t+1)}} \frac{\partial f^{(t+1)}}{\partial h^{(t)}} + \overline{o^{(t+1)}} \frac{\partial o^{(t+1)}}{\partial h^{(t)}}$

$$+ \overline{g^{(t+1)}} \frac{\partial g^{(t+1)}}{\partial h^{(t)}}$$

$$= \overline{i^{(t+1)}}\, i^{(t+1)} \left(1 - i^{(t+1)}\right) w_{ih} +$$

$$\overline{f^{(t+1)}}\, f^{(t+1)} \left(1 - f^{(t+1)}\right) w_{fh} +$$

$$\overline{o^{(t+1)}}\, o^{(t+1)} \left(1 - o^{(t+1)}\right) w_{oh} +$$

$$\overline{g^{(t+1)}} \left(1 - \tanh^2\left(w_{gx} x^{(t)} + w_{gh} h^{(t)}\right)\right) w_{gh}$$

$$\overline{c^{(t)}} = \overline{c^{(t+1)}} \frac{\partial c^{(t+1)}}{\partial c^{(t)}} + \overline{h^{(t)}} \frac{h^{(t)}}{\partial c^{(t)}}$$

$$= \overline{c^{(t+1)}}\, f^{(t+1)} + \overline{h^{(t)}}\, o^{(t)} \left(1 - \tanh^2\left(c^{(t)}\right)\right)$$

$$\overline{g^{(t)}} = \overline{c^{(t)}} \frac{\partial c^{(t)}}{g^{(t)}} = \overline{c^{(t)}}\, i^{(t)}$$

$$\overline{o^{(t)}} = \overline{h^{(t)}} \frac{\partial h^{(t)}}{\partial o^{(t)}} = \overline{h^{(t)}} \tanh\left(c^{(t)}\right)$$

$$\overline{f^{(t)}} = \overline{c^{(t)}} \frac{\partial c^{(t)}}{\partial f^{(t)}} = \overline{c^{(t)}}\, c^{(t-1)}$$

$$\overline{i^{(t)}} = \overline{c^{(t)}} \frac{\partial c^{(t)}}{\partial i^{(t)}} = \overline{c^{(t)}}\, g^{(t)}$$

(b) $$\overline{w_{ix}} = \sum_t \overline{i^{(t)}} \frac{\partial i^{(t)}}{\partial w_{ix}}$$

$$= \sum_t \overline{i^{(t)}} \, i^{(t)} \left(1 - i^{(t)}\right) x^{(t)}$$