# **Graph Representation of Code**

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### SVF : Static Value-Flow Analysis Framework for Source Code

A scalable, precise and on-demand interprocedural program dependence analysis framework for both sequential and multithreaded programs.

- The SVF project
  - Publicly available since early 2015 and actively maintained: http://svf-tools.github.io/SVF.
  - Implemented on top of LLVM compiler (the latest version 10.0.0) with over 100 KLOC C/C++ code and 530+ stars with 30+ contributors and over 1K commits on Github.
  - Invited for a plenary talk in EuroLLVM 2016, and awarded an ICSE 2018 Distinguished Paper, an SAS Best Paper 2019 and an OOPSLA 2020 Distinguished Paper.

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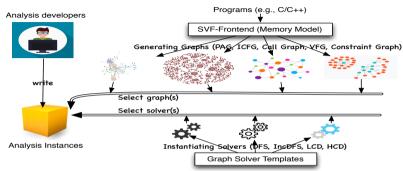
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- Value-Flow Analysis: resolves both control and data dependence.
  - Does the information generated at program point A flow to another program point B along some execution paths?
  - Can function F be called either directly or indirectly from some other function F'?
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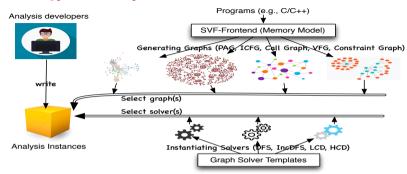
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  - Is there an unsafe memory access that may trigger a bug or security risk?
- Key features of SVF
  - Sparse: compute and maintain the data-flow facts where necessary
  - Selective: support mixed analyses for precision and efficiency trade-offs.
  - On-demand : reason about program parts based on user queries.

### **SVF: Design Principle**



- Serving as an open-source foundation for building practical static source code analysis
  - Bridge the gap between research and engineering
  - Minimize the efforts of implementing sophisticated analysis (extendable, reusable, and robust via layers of abstractions)
  - Support developing different analysis variants (flow-, context-, heap-, field-sensitive analysis) in a sparse and on-demand manner.

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  - Support developing different analysis variants (flow-, context-, heap-, field-sensitive analysis) in a sparse and on-demand manner.
- Client applications:
  - Static bug detection (e.g., memory leaks, null dereferences, use-after-frees and data-races)
  - Accelerate dynamic analysis (e.g., Google's Sanitizers and AFL fuzzing)

## **Graph Representation of Code**

- What is a graph representation of code?
  - Representing a program's control-flow (i.e., execution order) and/or data-flow (variable definition and use relations) using nodes and edges of a graph.

## **Graph Representation of Code**

- What is a graph representation of code?
  - Representing a program's control-flow (i.e., execution order) and/or data-flow (variable definition and use relations) using nodes and edges of a graph.
- Why a graph representation?
  - Abstracting code from low-level complicated instructions
  - Applying general graph algorithms
  - Easy to maintain and extend

### **Call Graph**

```
define i32 @main() #0 {
1 entry:
2%a1 = alloca i8, alian 1
                                             Program calling relations between methods
3%b1 = alloca i8, align 1
4 %a = alloca i8*, align 8
5%b = alloca i8*, align 8
6 store i8* %a1, i8** %a, alian 8
7 store i8* %b1, i8** %b, align 8
8 call void @swap(i8** %a, i8** %b)
gret i32 0
 define void @swap(i8** %p. i8** %a) #0
                                                    main
                                                                                     swap
10entry:
11%0 = load i8** %p, alian 8
12\%1 = load i8** \%a. alian 8
                                                               Call Graph
13store i8* %1, i8** %p, align 8
14store i8* %0, i8** %q, align 8
15ret void
```

### **Call Graph**

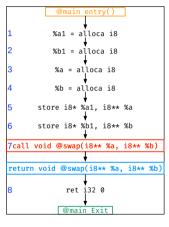
```
define i32 @main() #0 {
1 entry:
2%a1 = alloca i8, alian 1
3%b1 = alloca i8, align 1
                                              - each node represents a program method
4 %a = alloca i8*, align 8
5%b = alloca i8*, align 8
                                              - each edge represents a calling relation
6 store i8* %a1, i8** %a, alian 8
                                                    between two program methods
7 store i8* %b1, i8** %b, align 8
8 call void @swap(i8** %a, i8** %b)
gret i32 0
                                                    caller
                                                                                  callee
 define void @swap(i8** %p. i8** %a) #0
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# **Control Flow Graph**

Program execution order between instructions.

- Intra-procedural control-flow graph: control-flow graph within a program method.
- Inter-procedural control-flow graph: control-flow graph across program methods.

### **Intra-procedural Control Flow Graph**



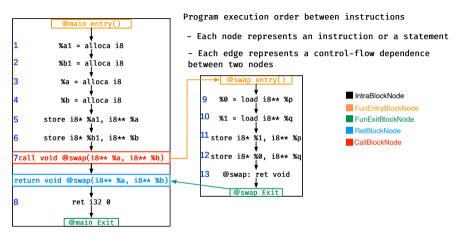
Program execution order between instructions

- Each node represents an instruction or a statement
- Each edge represents a control-flow dependence between two nodes



https://github.com/svf-tools/SVF/wiki/Analyze-a-Simple-C-Program#4-interprocedural-control-flow-graph

### **Inter-procedural Control Flow Graph (ICFG)**



https://github.com/svf-tools/SVF/wiki/Analyze-a-Simple-C-Program#4-interprocedural-control-flow-graph

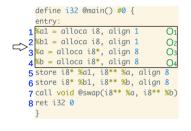
# **Constraint Graph (or Program Assignment Graph)**

- Constraint Graph represents the assignment constraints between variables at the instruction level.
- Constraint Graph and Program Assignment Graph (PAG) are essentially the same.
- The difference is that PAG can not be changed while you can add edges or nodes on the Constraint Graph to perform constraint solving.

### **Constraint Graph (or Program Assignment Graph)**

Program Assignment relation between two variables

- each node represent a pointer or an object
- each edge represents two nodes dependence or constraint relation











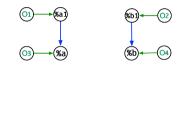
alloca instruction allocates typed integer 8 bytes of memory object as O1 O2 O3 O4

→ Address

Program Assignment relation between two variables

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- each edge represents two nodes dependence or constraint relation

```
define i32 @main() #0 {
    entry:
    1 %a1 = alloca i8, align 1 O1
    2 %b1 = alloca i8, align 1 O2
    3 %a = alloca i8*, align 8 O3
    4 %b = alloca i8*, align 8 O4
    5 store i8* %a1, i8** %a, align 8
    6 store i8* %b1, i8** %b, align 8
    7 call void @swap(i8** %a, i8** %b)
    8 ret i32 0
}
```



alloca instruction allocates typed integer 8 bytes of memory object as O1 O2 O3 O4



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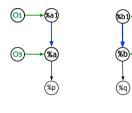
4 %b = alloca i8*, align 8 O4

5 store i8* %a1, i8** %a, align 8
6 store i8* %b1, i8** %b, align 8

call void @swap(i8** %a, i8** %b)

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}
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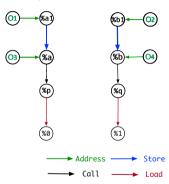
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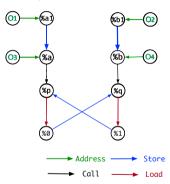
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define void @swap(i8** %p, i8** %q) #0
   {
    entry:
    9%0 = load i8** %p, align 8
    10%1 = load i8** %q, align 8
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}
```



### What's next?

- (1) Compile two C programs (swap.c and example.c) into their LLVM IR.
  - A guide can be found here https://github.com/SVF-tools/SVF-Teaching/wiki/CodeGraph#2-llvm-ir-generation
  - Understand the mapping from a C program to its corresponding LLVM IR.
- (2) Generate and visualize the graph representation of LLVM IR (swap.11 and example.11).
  - https://github.com/SVF-tools/SVF-Teaching/wiki/CodeGraph# 3-run-and-debug-your-codegraph
- (3) Write code to iterate nodes and edges of ICFG and PAG and print their contents.
  - https://github.com/SVF-tools/SVF-Teaching/blob/main/CodeGraph/ CodeGraph.cpp#L65-L82
- (4) More about LLVM IR and SVF's graph representation
  - LLVM language manual https://llvm.org/docs/LangRef.html
  - SVF website https://github.com/SVF-tools/SVF

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#### Pointer analysis

- Points-to Analysis: Statically determine the possible runtime values of a pointer at compile-time.
- Alias Analysis: determine whether two pointer dereferences refer to the same memory location.
- e.g., p = &a; p = q;
- p and q both point to a. \*p and \*q are aliases.

#### **Pointer analysis**

Why do we need to learn pointer analysis

Pointer analysis tells us what memory locations code uses or modifies

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$$p = &a p = q; *p = x; y = *q$$

- Good for program understanding, bug detection and compiler optimisations
  - Constant propagation
    - p = 1; \*q = x; r = p;
    - r is a constant value and equals 1, if p and q do not alias each other, otherwise, can not perform constant propagation.

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  - Constant propagation
    - p = 1; \*q = x; r = p;
    - r is a constant value and equals 1, if p and q do not alias each other, otherwise, can not perform constant propagation.
  - Taint analysis
    - p = taintedInput; x = \*q;
    - x is tainted if p and q alias each other.

#### Pointer analysis

An ongoing research topic classified into the following precision dimensions.

- Flow-insensitive analysis:
  - Ignore program execution order
  - A single solution at each program point
- Flow-sensitive analysis:
  - Respect the program execution order
  - A Separate solution at each program point

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- Field-sensitive analysis:
  - Distinguish the fields of an aggregate object (e.g., struct object).

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- Field-sensitive analysis:
  - Distinguish the fields of an aggregate object (e.g., struct object).
- Context-insensitive analysis:
  - Merges all of its calling contexts together when analysing a program method
- Context-sensitive analysis:
  - Distinguishes between different calling contexts of a program method

- The most popular and widely used pointer analysis
- Constraint solving (inclusion-based constraints between program variables)

We will practice a flow-insensitive, field-insensitive and context-insensitive Andersen's analysis through analyzing the PAG (or Constraint Graph) of a program.

SVF transforms LLVM instructions into a PAG (or Constraint Graph).

Node:

Constraint Type

- A pointer: (LLVM Value in pointer type)
- An object: (heap, stack, global, function)

C code

Edge: A Constraint between two nodes

```
Address: p = \&obj \{obj\} \subseteq Pts(p)

Copy: p = q Pts(q) \subseteq Pts(p)

Load: p = *q \forall o \in Pts(q), Pts(o) \subseteq Pts(p)

Store: *p = q \forall o \in Pts(p), Pts(q) \subseteq Pts(o)
```

Constraint rule

A popular inclusion-based pointer analysis (flow-insensitive and field-sensitive). SVF transforms LLVM instructions into a PAG (or Constraint Graph).

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  - An object: (heap, stack, global, function)
- Edge: A Constraint between two nodes

Constraint Type C code LLVM IR Constraint Graph

Address: p = obj %p = alloca // obj

obj

pts(p) = {obj}

- Node:
  - A pointer: (LLVM Value in pointer type)
  - An object: (heap, stack, global, function)
- Edge: A Constraint between two nodes

Constraint Type	C code	LLVM IR	Constraint Graph
Address: Copy:	p = obj p = q	%p = alloca // obj %p = bitcast %q	{obj} <b>q p</b>
			pts(p) = pts(q)

- Node:
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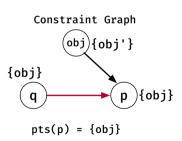
Constraint Type	C code	LLVM IR	Constraint	Graph
Address: Copy:	p = obj p = q	%p = alloca // obj %p = bitcast %q	{obj}	{obj} ▶ p
			pts(p) = pt	cs(q)

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9			
Constraint Type	C code	LLVM IR	Constraint Graph
Address: Copy: Load:	p = obj p = q p = *q	<pre>%p = alloca // obj %p = bitcast %q %q = load %p</pre>	(obj) {obj'} {obj}  q
			$nts(n) = \{ohi\}$

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Constraint Type	C code	LLVM IR
Address:	p = obj	%p = alloca // obj
Copy:	p = q	%p = bitcast %q
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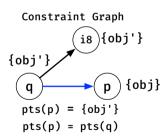


A popular inclusion-based pointer analysis (flow-insensitive and field-sensitive). SVF transforms LLVM instructions into a PAG (or Constraint Graph).

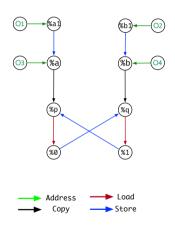
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```
Constraint Type C code

Address: p = obj %p = alloca // obj
Copy: p = q %p = bitcast %q
Load: p = *q %q = load %p
Store: *p = q store %q, %p
```

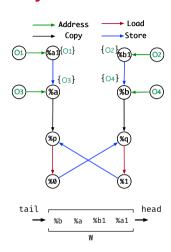


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define i32 @main() #0 {
entry:
%a1 = alloca i8. alian 1
                               // 01
%b1 = alloca i8, alian 1
                               // O2
%a = alloca i8*, alian 8
                               // O3
%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 {
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



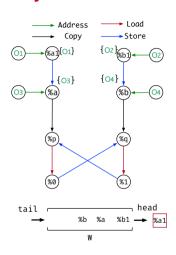
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G = < V.E > // Constraint Graph
  V: a set of nodes in graph
  E: a set of edges in graph
  W: a vector of nodes
 1 foreach address n = &n do
       nts(n) = \{0\}
        W ← W u {n}
 4 while W ≠ Ø do
      p ← select-from(W)
      foreach o \in pts(p) do
         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W ∪ {a}
         foreach load r = *p do
11
             if o→r ∉ E then
12
                   F \leftarrow F \cup \{o \rightarrow r\}
13
                   W ← W u {o}
14
      foreach p \rightarrow x \in E do
15
           pts(x) \leftarrow pts(x) \cup pts(p)
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define void @swap(i8** %p, i8** %a)
#0 S
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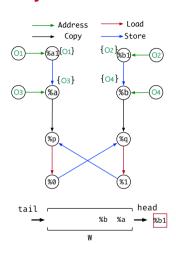
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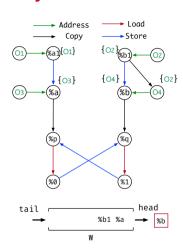
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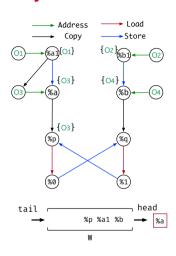
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      p ← select-from(W)
      foreach o \in pts(p) do
         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W u {a}
         foreach load r = *p do
11
             if o→r ∉ E then
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
14
      foreach p \rightarrow x \in E do
15
           pts(x) \leftarrow pts(x) \cup pts(p)
16
          if pts(x) changed then
17
                  W \leftarrow W \cup \{x\}
18
```

```
define i32 @main() #0 {
entry:
%a1 = alloca i8. alian 1
                               // 01
%b1 = alloca i8, alian 1
                               // O2
%a = alloca i8*, alian 8
                               // O3
%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



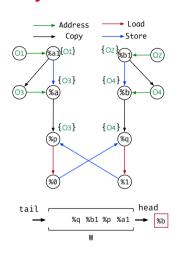
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store i8* %a1, i8** %a, alian 8
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call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



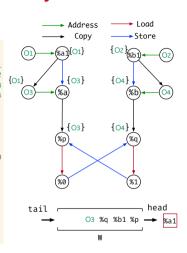
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store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



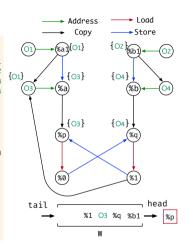
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4 while W ≠ Ø do
     p ← select-from(W)
     foreach o E pts(p) do
         foreach store *p = q do
8
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W u {a}
         foreach load r = *p do
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             if o→r ∉ E then
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                   F \leftarrow F \cup \{o \rightarrow r\}
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call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



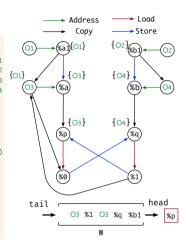
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%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



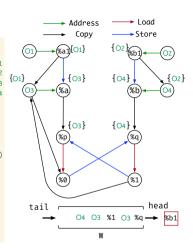
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         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W ∪ {a}
         foreach load r = *p do
11
             if o→r ∉ E then
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
14
      foreach p \rightarrow x \in E do
15
           pts(x) \leftarrow pts(x) \cup pts(p)
16
          if pts(x) changed then
17
                  W \leftarrow W \cup \{x\}
18
```

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                               // 01
%b1 = alloca i8, alian 1
                               // O2
%a = alloca i8*, alian 8
%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
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call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



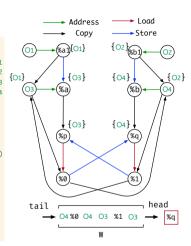
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         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W ∪ {a}
         foreach load r = *p do
             if o→r ∉ E then
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
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store i8* %a1, i8** %a, alian 8
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define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
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store i8* %0. i8** %a. alian 8
ret void
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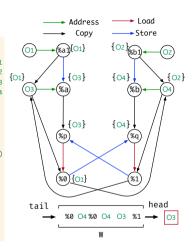
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      p ← select-from(W)
      foreach o \in pts(p) do
         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W ∪ {a}
         foreach load r = *p do
11
             if o→r ∉ E then
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
14
15
      foreach n \rightarrow x \in F do
           pts(x) \leftarrow pts(x) \cup pts(p)
16
17
          if pts(x) changed then
                  W \leftarrow W \cup \{x\}
```

```
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%b = alloca i8*, alian 8
                               // 04
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store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
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\%0 = load i8** \%p, alian 8
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store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



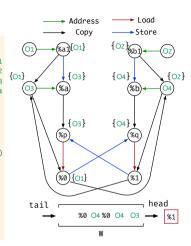
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store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



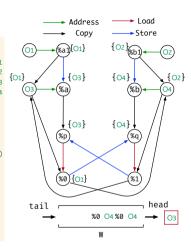
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store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



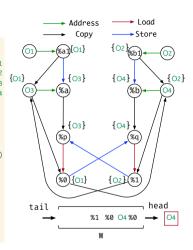
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ret i32 0
define void @swap(i8** %p, i8** %a)
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%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



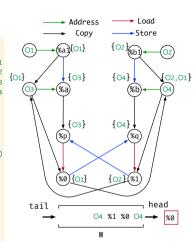
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call void @swap(i8** %a, i8** %b)
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define void @swap(i8** %p, i8** %a)
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store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



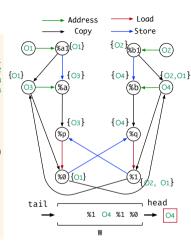
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store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



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 4 while W ≠ Ø do
      p ← select-from(W)
      foreach o \in pts(p) do
         foreach store *p = q do
             if q→o ∉ E then
                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W u {a}
         foreach load r = *p do
11
             if o→r ∉ E then
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
14
      foreach p \rightarrow x \in E do
           pts(x) \leftarrow pts(x) \cup pts(p)
16
17
          if pts(x) changed then
                  W \leftarrow W \cup \{x\}
18
```

```
define i32 @main() #0 {
entry:
%a1 = alloca i8. alian 1
                               // 01
%b1 = alloca i8, alian 1
                               // O2
%a = alloca i8*, alian 8
%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```



```
G = < V.E > // Constraint Graph
  V: a set of nodes in graph
  E: a set of edges in graph
  W: a vector of nodes (WorkList)
1 foreach address p = &o do
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        W ← W u {n}
 4 while W ≠ Ø do
      p ← select-from(W)
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                   W ← W u {a}
         foreach load r = *p do
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             if o→r ∉ E then
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entry:
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                               // 01
%b1 = alloca i8, alian 1
                               // O2
%a = alloca i8*. alian 8
                               // 03
%b = alloca i8*, alian 8
                               // 04
store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```

```
Address
                                → Load
                  Copy
                               → Store
                           {O2}<sub>(2h1)</sub>-
             %a1{O1}
    (01)
{01,02}
               {03}
                            {04}
                                         {02,01}
                {03}
                            {04}
              (%p)
              (%0){01}
      tail
                                     head
                        O3 %1 O4
                         W
```

```
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                   E \leftarrow E \cup \{a \rightarrow o\}
10
                   W ← W u {a}
         foreach load r = *p do
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             if o→r ∉ E then
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                   F \leftarrow F \cup \{o \rightarrow r\}
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call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```

```
Address
                                → Load
                  Copy
                                → Store
                           {O2}<sub>(2h1)</sub>-
             (%a){O1}
    (01)
{01,02}
               {03}
                            {04}
                                         {02,01}
                {03}
                            {04}
              %p
     {01,62}(%0)
      tail
                                      head
                                          О3
                         W
```

```
G = < V.E > // Constraint Graph
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     foreach o \in pts(p) do
         foreach store *p = q do
             if q→o ∉ E then
                  E \leftarrow E \cup \{a \rightarrow o\}
10
                  W ← W ∪ {a}
         foreach load r = *p do
11
             if o→r ∉ E then
12
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                  W ← W u {o}
14
     foreach p \rightarrow x \in E do
          pts(x) \leftarrow pts(x) \cup pts(p)
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store i8* %a1, i8** %a, alian 8
store i8* %b1, i8** %b, alian 8
call void @swap(i8** %a, i8** %b)
ret i32 0
define void @swap(i8** %p, i8** %a)
#0 S
entry:
\%0 = load i8** \%p, alian 8
%1 = load i8** %a, alian 8
store i8* %1, i8** %p, alian 8
store i8* %0. i8** %a. alian 8
ret void
```

```
Address
                                 → Load
                  Copy
                                → Store
                           {O2}<sub>(2h1)</sub>-
             (%a){O1}
    (01)
{01,02}
               {03}
                            {04}
                                         {02,01}
                {03}
                            {04}
              %p
     {01,62}(%0)
      tail
                                      head
                         W
```

```
G = < V.E > // Constraint Graph
  V: a set of nodes in graph
  E: a set of edges in graph
  W: a vector of nodes (WorkList)
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                   W ← W ∪ {a}
         foreach load r = *p do
11
             if o→r ∉ E then
12
13
                   F \leftarrow F \cup \{o \rightarrow r\}
                   W ← W u {o}
14
      foreach p \rightarrow x \in E do
15
           pts(x) \leftarrow pts(x) \cup pts(p)
16
          if pts(x) changed then
17
                  W \leftarrow W \cup \{x\}
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```