In [4]:
```
import math
from sympy import Symbol, limit, oo, sin, sqrt, solve, factorial, symbols, cos, exp, asin, atan, diff, solve, pi
import numpy as np
```

Предел, непрерывность, ряды

Пример 1

In [4]:
```
from sympy import *
x = Symbol("x")
limit((6*x**2+3*x)/(3*x**2),x,oo)
```

Out[4]: 2

Пример 2

In [14]:
```
limit(sin(x)/x,x,0)
```

Out[14]: 1

In [12]:
```
limit((1+x)**(1/x),x,0)
```

Out[12]: e

In [17]:
```
limit((1+1/x)**x,x,oo)
```

Out[17]: e

Односторонние пределы

Пример 3

In [29]:
```
limit((2**x-1)/(x**2-3*x),x,3,'+')
```

Out[29]: ∞

In [30]:
```
limit((2**x-1)/(x**2-3*x),x,3,'-')
```

Out[30]: $-\infty$

Пример 4

In [33]:
```
from sympy import *
x=Symbol("x")
limit((5**x-5*7**x)/(4*5**x-3*7**x),x,oo)
```

Out[33]: $\frac{5}{3}$

In [34]:
```
from sympy import *
x=Symbol("x")
limit((7*8**x+2*9**x)/(6*8**x-6*9**x),x,-oo)
```

Out[34]: $\frac{7}{6}$

In [35]:
```
limit (sqrt(x*(x+3))-sqrt(x**2+9),x,-oo)
```

Out[35]: $-\frac{3}{2}$

Пример 5

In [36]:
```
from sympy import *
x=Symbol("x")
print(solve(x**3-9*x**2+14*x))
```

```
[0, 2, 7]
```

In [38]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,0,'-')
```

Out[38]: ∞

In [39]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,0,'+')
```

Out[39]: ∞

In [40]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,2,'-')
```

Out[40]: $\frac{1}{2}$

In [42]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,2,'+')
```

Out[42]: $\frac{1}{2}$

In [43]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,7,'-')
```

Out[43]: $\frac{1}{7}$

In [44]:
```
limit(abs((x-2)*(x-7)/(x**3-9*x**2+14*x)),x,7,'+')
```

Out[44]: $\frac{1}{7}$

Пример 6

In [46]:
```
k=limit((1+5*x)/(3+x)/x,x,oo)
print(k)
```

```
0
```

In [47]:
```
b=limit((1+5*x)/(3+x)-k*x,x,oo)
print(b)
```

```
5
```

In [48]:
```
solve(3+x)
```

Out[48]: [-3]

In [51]:
```
limit ((1+5*x)/(3+x),x,-3, '-')
```

Out[51]: ∞

In [52]:
```
limit ((1+5*x)/(3+x),x,-3, '+')
```

Out[52]: $-\infty$

Пример 7

In [54]:
```
from sympy import *
import math as m
x=symbols('x')
limit(1/factorial(x)/(1/factorial(x+1)),x,oo)
```

Out[54]: ∞

Пример 8

In [6]:
```
import math
from sympy import*
x=Symbol("x")
limit (x**x/factorial(x)/((x+1)**(x+1)/factorial(x+1)),x,oo)
```

Out[6]: e^{-1}

Пример 9

In [7]:
```
import sympy
x = symbols('x')
func = sin(x)
x0=0
print((func).series(x, x0, 10))
```

```
x - x**3/6 + x**5/120 - x**7/5040 + x**9/362880 + O(x**10)
```

In [8]:
```
import sympy
x = sympy.symbols('x')
func = cos(x)
x0=0
print((func).series(x, x0, 10))
```

```
1 - x**2/2 + x**4/24 - x**6/720 + x**8/40320 + O(x**10)
```

In [11]:
```
import sympy
x = sympy.symbols('x')
func = exp(x)
x0=0
print((func).series(x, x0, 10))
```

```
1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 + x**6/720 + x**7/5040 + x**8/40320 + x**9/362880 + O(x**10)
```

In [12]:
```
func=asin(x)
X0=0
func.series(x,x0,10)
```

Out[12]:
$$x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + \frac{35x^9}{1152} + O\left(x^{10}\right)$$

Примеры решения задач

Вычислите предел последовательности $\lim\limits_{n\to\infty}\left(\frac{6n^2+1}{7n^2-3n+9}\right)$.

In [64]:
```
import sympy
n=sympy.symbols('n')
limit((6*n**2+1)/(7*n**2-3*n+9),n,oo)
```

Out[64]:
$$\frac{6}{7}$$

Вычислите предел последовательности $\lim\limits_{n\to\infty}\left(\frac{-7n+10}{\sqrt{9n^2+10n}}\right)$.

In [65]:
```
import sympy
n=sympy.symbols('n')
limit((-7*n+10)/sqrt(9*n**2+10*n),n,oo)
```

Out[65]: $-\frac{7}{3}$

Вычислите предел последовательности

$\lim\limits_{n\to\infty}\left(\frac{\sqrt{5n^{14}-3n^8+2n-1}}{2n^7-8n^4+1}\right)$.

In [66]:
```
import sympy
n=sympy.symbols('n')
limit(sqrt(5*n**14-3*n**8+2*n-1)/(2*n**7-8*n**4+1),n,oo)
```

Out[66]: $\frac{\sqrt{5}}{2}$

Вычислите предел $\lim\limits_{x\to\infty}\frac{-5x^2-8x-4}{2x^2-x-2}$.

In [67]:
```
import sympy
x=sympy.symbols('x')
limit((-5*x**2-8*x-4)/(2*x**2-x-2),x,oo)
```

Out[67]: $-\frac{5}{2}$

Вычислите предел $\lim\limits_{x\to-\infty} x(\sqrt{5x^2+6}-\sqrt{5x^2-6})$.

In [68]:
```
import sympy
x=sympy.symbols('x')
limit(x*(sqrt(5*x**2+6)-sqrt(5*x**2-6)),x,-oo)
```

Out[68]: $-\frac{6\sqrt{5}}{5}$

Вычислите предел $\lim\limits_{x\to-\infty}\left(\frac{1}{\sqrt{2x^2+2x-3}-\sqrt{2x^2-5x-5}}\right)$.

In [70]:
```
import sympy
x=sympy.symbols('x')
limit(1/(sqrt(2*x**2+2*x-3)-sqrt(2*x**2-5*x-5)),x,-oo)
```

Out[70]: $-\frac{2\sqrt{2}}{7}$

Вычислите предел $\lim\limits_{x\to1}\left(\frac{9(1-x^{\frac{1}{7}})}{x^{\frac{1}{8}}-1}\right)$.

In [71]:
```
import sympy
x=sympy.symbols('x')
limit(9*(1-x**(1/7))/(x**(1/8)-1), x,1)
```

Out[71]: $\frac{9-9\mathrm{x}^{0.142857142857143}}{\mathrm{x}^{0.125}-1}$

Задачи для самостоятельного решения

Вычислить предел:

$\lim\limits_{n\to\infty}\frac{4n^6+8n^5-6n^3+5n-1}{-8n^6+6n^5-5n^2+n}$.

Ответ: $-\frac{1}{2}$.

In [78]:
```
import sympy
n=sympy.symbols('n')
```

```
limit((4*n**6+8*n**5-6*n**3+5*n-1)/(-8*n**6+6*n**5-5*n**2+n),n,oo)
```

Out[78]: $-\frac{1}{2}$

Вычислить предел:

$$\lim_{n\to\infty}\frac{9n+4\sin^3 4n+8}{8n+6\cos^6 4n-4}.$$

Ответ: $-\frac{9}{8}$.

In [16]:
```
import sympy
n=sympy.symbols('n')
limit((9*n+(4*sin(4*n)**3)+8)/(8*n+(6*cos(4*n)**6)-4),n,oo)
```

Out[16]: $\frac{9}{8}$

Вычислить предел:

$$\lim_{n\to\infty}\left(\frac{n^3}{n^2+3n+1}-n\right).$$

Ответ: -3.

In [117…
```
import sympy
n=sympy.symbols('n')
limit((n**3/(n**2+3*n+1)-n),n,oo)
```

Out[117… -3

Вычислить предел:

$$\lim_{n\to\infty} \frac{2-3n}{\sqrt{n^2-n+1}-\sqrt{25n^2+2n+1}}.$$

Ответ: $\frac{3}{4}$.

In [9]:
```
import sympy
n=sympy.symbols('n')
limit((2-3*n) / (sqrt(n**2-n+1)-sqrt(25*n**2+2*n+1)),n,oo)
```

Out[9]: $\frac{3}{4}$

Вычислить предел:

$$\lim_{x\to-\infty} \frac{7\cdot 6^{x+4}+9}{5-7\cdot 6^{x+7}}.$$

Ответ: $\frac{9}{5}$.

In [139…
```
import sympy
n=sympy.symbols('x')
limit((7*(6**(x+4))+9)/(5-7*6**(x+7)),x,-oo)
```

Out[139… $\frac{9}{5}$

Вычислите предел $\lim_{x\to+\infty} \left(\frac{4x+4}{5-2\ln 2x}\right)$.

Ответ: $-\infty$.

In [19]:
```
import sympy
n=sympy.symbols('x')
```

```
limit((4*x+4) / (5-2*ln(2*x)),x,+oo)
```

Out[19]: $-\infty$

Индивидуальное задание

Численное интегрирование

In [43]:

```
from numpy import ones,copy,cos,tan,pi,linspace

def gaussxw(N):

    # Начальное приближение к корням многочлена Лежандра
    a = linspace(3,4*N-1,N)/(4*N+2)
    x = cos(pi*a+1/(8*N*N*tan(a)))

    # Найдите корни, используя метод Ньютона
    epsilon = 1e-15
    delta = 1.0
    while delta>epsilon:
        p0 = ones(N,float)
        p1 = copy(x)
        for k in range(1,N):
            p0,p1 = p1,((2*k+1)*x*p1-k*p0)/(k+1)
        dp = (N+1)*(p0-x*p1)/(1-x*x)
        dx = p1/dp
        x -= dx
        delta = max(abs(dx))

    # Рассчитайте веса
    w = 2*(N+1)*(N+1)/(N*N*(1-x*x)*dp*dp)

    return x,w

def gaussxwab(N,a,b):
    x,w = gaussxw(N)
    return 0.5*(b-a)*x+0.5*(b+a),0.5*(b-a)*w
```

In [44]:

```
def f(x):
    return x**4 - 2*x + 1

N = 10
a = 0.0
b = 2.0
h = (b-a)/N

s = 0.5*f(a) + 0.5*f(b)
for k in range(1,N):
    s += f(a+k*h)

print(h*s)
```

```
4.50656
```

In [45]:

```
def f(x):
    return x**4 - 2*x + 1

N = 3
a = 0.0
b = 2.0

# Вычислите точки выборки и веса, затем сопоставьте их
# к требуемому домену интеграции
x,w = gaussxw(N)
xp = 0.5*(b-a)*x + 0.5*(b+a)
wp = 0.5*(b-a)*w

# Выполните интеграцию
s = 0.0
for k in range(N):
    s += wp[k]*f(xp[k])

print(s)
```

```
4.4000000000000075
```

In [40]:

```
# Методы интеграции
from scipy.integrate import quad
from pylab import loglog, show, xlabel, ylabel, legend
```

```
from sympy import integrate, symbols
from math import sin, cos
from numpy import sum, linspace

def trapezoid(a,b,N):
    h = (b-a)/N
    sum = 0.5*(f(a)+f(b))
    x = a
    for i in range(1,N):
        x += h
        sum += f(x)
    return sum*h

def simpson(a,b,f,N):
    h = (b-a)/N
    x = linspace(a,b,N+1)
    y = f(x)
    return h/3.*sum(y[0:-1:2] + 4*y[1::2] + y[2::2])

def f(x):
    return x**4 - 2*x + 1
a=0.
b=2.
TrapezoidResiduals = []
SimpsonResiduals = []
Nsteps = []
x = symbols('x')
IntAnalytic = integrate(x**4 - 2*x + 1, (x, a, b))
integr = integrate(x**4 - 2*x + 1)
print(integr.subs(x,b))
IntScipy = quad(f,a,b)[0]
for i in range(10):
    N = 2**i
    Nsteps.append(N)
    TrapezoidResiduals.append(abs(trapezoid(a,b,N)-IntScipy))
    SimpsonResiduals.append(abs(simpson(a,b,f,N)-IntScipy))
print (SimpsonResiduals)

loglog(Nsteps,TrapezoidResiduals, '-r', label='trapeziod')
loglog(Nsteps,SimpsonResiduals, '-b', label='simpson')
xlabel('N')
ylabel('error')
legend()
show()
```

```
4.40000000000000
[4.3999999999999995, 0.2666666666666666, 0.016666666666666607, 0.0010416666666666075, 6.510416666660745e-05, 4.069010416607455e-06, 2.5431315098245477e-07, 1.5894571880892272e-08, 9.934106870446158e-10, 6.208811242913725e-11]
```