BITS, PILANI - K. K. BIRLA GOA CAMPUS

MATH- III Tutorial - 2

---

1 Find the general solution of the following differential equations, when one of the solution $y_1(x)$ is known

$(i)\ x^2y'' + xy' - 4y = 0,$	$y_1(x) = x^2$
$(ii)\ x^2y'' + xy' + (x^2 - 1/4)y = 0,$	$y_1(x) = x^{-1/2}\sin x$
$(iii)\ x^2y'' - x(x+2)y' + (x+2)y = 0,$	$y_1(x) = x$
$(iv)\ xy'' - (2x+1)y' + (x+1)y = 0,$	$y_1(x) = e^x$

2 Find the general solution of the following differential equations

$(i)\ y'' + y' - 6y = 0,$ $(ii)\ y'' - 9y' + 20y = 0,$
$(iii)\ y'' + y' = 0,$ $(iv)\ y'' + 8y' - 9y = 0,$
$(v)\ y^{(4)} - y = 0,$ $(vi)\ y''' - 3y'' + 4y' - 2y = 0,$

3 Find the general solution of the following equations by reducing them to constant coefficient equation

$(i)\ x^2y'' + pxy' + qy = 0,$ p and q are constants, $x > 0$
$(ii)\ x^2y'' + 2xy' - 12y = 0,$ $x > 0$
$(iii)\ x^2y'' - 3xy' + 4y = 0,$ $x > 0$

4 Consider the general homogeneous equation

$$y'' + P(x)y' + Q(x)y = 0, \tag{1}$$

and change the independent variable from x to $z = z(x)$, where $z(x)$ is an unspecified function of x. Show that equation (1) can be transformed in this way to an equation with constant coefficients if and only if $(Q' + 2PQ)/Q^{3/2}$ is constant.

5 Using the Method of Undetermined Coefficients find particular solution of the following differential equations

$(i)\ y'' - 2y' + 2y = e^x\sin x,$ $(ii)\ y'' - 3y' + 2y = 2xe^x + 3\sin x,$
$(iii)\ y'' + y = \sin^3 x,$ $(iv)\ y^{(4)} - 2y''' + 2y'' - 2y' + y = \sin x$

6 Using the Method of Variation of Parameters find the particular solution of the following differential equations

$(i)\ y'' + y = \tan x,$ $(ii)\ y'' - 3y' + 2y = (1 + e^{-x})^{-1},$
$(iii)\ y'' + y = x\cos x,$

7 Find the general solution of the following equations

$$(i)\ (x^2-1)y'' - 2xy' + 2y = (x^2-1)^2 \qquad (ii)\ x^2y'' - 2xy' + 2y = xe^x$$

8 Using Operator methods (Method 1,2 and 4), and find the particular solution of the following differential equations

$$(i)\ y'' - y = x^2e^{2x}, \qquad (ii)\ y'' - 2y' - 3y = 6e^{5x},$$
$$(iii)\ y'' - 4y = e^{2x}$$

9 Using exponential shift rule to find the general solution of the following differential equations

$$(i)\ (D-2)^3y = e^{2x}, \qquad (ii)\ (D+1)^3y = 12e^{-x},$$

10 Use the exponential shift rule to show that $(D-r)^ky = 0$ has

$$y = (c_1 + c_2x + c_3x^2 + \cdots + c_kx^{k-1})e^{rx}$$

as its general solution.