# BITS, PILANI - K. K. BIRLA GOA CAMPUS

**MATH- III Tutorial - 4**

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1 Determine the nature of the point $x = 0$ for each of the following equations

$(a)$ $y'' + (\sin x)y = 0$
$(b)$ $x^2y'' + (\sin x)y = 0$
$(c)$ $x^2y'' + (2 - x)y' + y = 0$

2 Classify the singular points of the following differential equation

$$(x^2 + x - 2)^2y'' + 3(x + 2)y' + (x - 1)y = 0$$

3 Find the general solution of the following differential equations near the singular point $x = 0$

$(a)$ $x^2y'' - x(2 - x)y' + (2 + x^2)y = 0,$
$(b)$ $x^2y'' + xy' + (x^2 - 1/4)y = 0$
$(c)$ $xy'' - y\prime + 4x^3y = 0$

4 Show that the differential equation $x^2y'' + xy' + (x^2 - 1)y = 0$ has only one Frobenius series solution. Then find it.

5 Find the general solution of each of the following differential equations, near the indicated singular point, in terms of Hypergeometric functions

$(a)$ $x(1 - x)y'' + (3/2 - 2x)y' + 2y = 0,\ x = 0$
$(b)$ $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0,\ x = -1$
$(c)$ $(1 - x^2)y'' - xy\prime + p^2y = 0,\ x = 1$

6 Verify each of the following

$(a)$ $(1 + x)^p = F(-p, b, b, -x)$
$(b)$ $e^x = \lim_{b\to\infty} F(a, b, a, x/b)$
$(c)$ $\log(1 + x) = xF(1, 1, 2, -x).$