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MATH- III   Tutorial - 1

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1 Verify that the following equations are homogeneous, and solve them:

$$(i)\ (x^2-2y^2)dx+xydy=0 \qquad (ii)\ x\sin(y/x)\frac{dy}{dx}=y\sin(y/x)+x$$
$$(iii)\ x^2y\,'=y^2+2xy.$$

2 If $ae \neq bd$, show that constants h and k can be chosen in such a way that the substitutions $x=z-h$, $y=w-k$ reduce

$$\frac{dy}{dx}=F\left(\frac{ax+by+c}{dx+ey+f}\right)$$

to a homogeneous equation.

3 Solve the following equations:

$$(i)\ \frac{dy}{dx}=\frac{x+y+4}{x-y-6} \qquad (ii)\ (2x+3y-1)dx-4(x+1)dy=0.$$
$$(iii)\ \frac{dy}{dx}=\frac{x+y+4}{x+y-6}$$

4 Determine which of the following equations are exact, and solve the ones that are:

$$(i)\ \left(x+\frac{2}{y}\right)dy+ydx=0 \qquad (ii)\ -\frac{1}{y}\sin(x/y)dx+\frac{x}{y^2}\sin(x/y)dy=0$$
$$(iii)\ dx=\frac{y}{1-x^2y^2}dx+\frac{x}{1-x^2y^2}dy \qquad (iv)\ 2x\sin y\ dx+x^2\cos y\ dy=0.$$

5 Solve

$$\frac{ydx-xdy}{(x+y)^2}+dy=dx$$

as an exact equation in two ways, and reconcile the results.

6 Show that if $(\partial M/\partial y-\partial N/\partial x)/(Ny-Mx)$ is a function $g(z)$ of the product $z=xy$, then

$$\mu=e^{\int g(z)dz}$$

is an integrating factor for the equation $M(x,y)dx+N(x,y)dy$.

7 Solve each of the following equations by finding an integrating factor:

$(a)\ (xy-1)dx+(x^2-xy)dy=0$

$(b)\ ydx+(x-2x^2y^3)dy=0$

$(c)\ (x^3+xy^3)dx+3y^2dy=0$

$(d)\ xdy+ydx+3x^3y^4dy=0.$

8 Under what circumstances will equation $M(x,y)dx + N(x,y)dy$ have an integrating factor that is a function of the sum $z = x + y$?

9 Write the linear equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

in the form $Mdx + Ndy = 0$ and use the idea of exact equations to show that this equation has an integrating factor μ that is a function of x alone. Find μ and obtain the solution.

10 Solve the following linear equations

$(i)\ y' + y = \dfrac{1}{1+e^x}$ $\qquad (ii)\ \dfrac{dx}{dy} + 2yx = e^{-y^2}$

$(iii)\ y' + y = 2xe^{-x} + x^2$ $\qquad (iv)\ L\dfrac{di}{dt} + Ri = E\sin kt$ (Simple Electric Circuit)

11 Solve the following equations as a linear differential equations

$(i)\ xdy + ydx = xy^2dx$ $\qquad (ii)\ y' + xy = \dfrac{x}{y^3}, y \neq 0,$

$(iii)\ (e^y - 2xy)y' = y^2$

12 Solve the following equations (using reduction of order):

$(i)\ yy'' + (y')^2 = 0$ $\qquad (ii)\ xy'' + y' = 4x$

$(iii)\ y'' = 1 + (y')^2$ $\qquad (iv)\ y'' + (y')^2 = 1$