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MATH- III Tutorial - 5

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1 Using the generating function of the Legendre polynomials prove the following:

$$(a)\ P_n(1) = 1 \qquad (b)\ P_n(-1) = (-1)^n$$
$$(c)\ P_{2n+1}(0) = 0 \qquad (c)\ P_{2n}(0) = (-1)^n \frac{1.3.5\cdots 2n-1}{2^n n!}.$$

2 Establish the following recurrence relations for $P_n(x)$

$$(a)\ (n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$
$$(b)\ \int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2-1}$$
$$(c)\ P'_{n+1}(x) - xP'_n(x) - (n+1)P_n(x) = 0.$$

3 Express $f(x) = 3x^4 + 5x - 2$ as a linear combination of $P_n(x),\ n = 0,1,2,3$ and 4, where $P_n(x)$ are Legendre polynomials.

4 If $p(x)$ is a polynomial of degree $n \geq 1$ such that

$$\int_{-1}^{1} x^k p(x)dx = 0, \qquad \text{for } k = 0,1,\cdots,n-1,$$

show that $p(x) = cP_n(x)$ for some constant c.

5 Find the first three terms of the Legendre series of the function $f(x) = \sin x$.

6 Show that

$$(a)\ J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x \qquad (b)\ J_{-1/2}(x)\sqrt{\frac{2}{\pi x}}\cos x$$
$$(c)\ \left(n+\frac{1}{2}\right)! = \frac{(2n+1)!}{2^{2n+1}n!}\sqrt{\pi} \qquad (d)\ \Gamma(1/2) = \sqrt{\pi}.$$

7 Prove that the positive zeros of $J_0(x)$ and $J_1(x)$ occur alternatively.

8 When n is an integer show that

(a) J_n is an even function if n is even

(b) J_n is an odd function if n is odd

(c) $J_{-n} = (-1)^n J_n$.

9 Show that $J_3 + 3J'_0 + 4J'''_0 = 0$.