Digital Electronics and Microprocessors

Class 5

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Karnaugh Map Method

- A graphical method of simplifying logic equations or truth tables. Also called a K map.
- Theoretically can be used for any number of input variables, but practically limited to 5 or 6 variables.
- K Map shows the relationship between inputs & outputs

Karnaugh Map Method

- The truth table values are placed in the K map as shown in the next page.
- Adjacent K map square differ in only one variable both horizontally and vertically. $\overline{A}\,\overline{B}, \overline{A}B, AB, A\overline{B}$
- A SOP expression can be obtained by ORing all squares that contain a 1.

A	B	X
0	0	1 → $\overline{A}\,\overline{B}$
0	1	0
1	0	0
1	1	1 → AB

$$\left\{ X = \overline{A}\,\overline{B} + AB \right\}$$

	$\overline{B}$	B
$\overline{A}$	1	0
A	0	1

(a)

Karnaugh maps and truth tables for three variables.

A	B	C	X	
0	0	0	1	$\rightarrow \bar{A}\bar{B}\bar{C}$
0	0	1	1	$\rightarrow \bar{A}\bar{B}C$
0	1	0	1	$\rightarrow \bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$\rightarrow AB\bar{C}$
1	1	1	0	

$$\left\{ \begin{aligned} X = {} & \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C \\ & + \bar{A}B\bar{C} + AB\bar{C} \end{aligned} \right\}$$

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	0
AB	1	0
$A\bar{B}$	0	0

(b)

Karnaugh maps and truth tables for four variables.

A	B	C	D	X	
0	0	0	0	0	
0	0	0	1	1	$\rightarrow \bar{A}\bar{B}\bar{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\rightarrow \bar{A}B\bar{C}D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$\rightarrow AB\bar{C}D$
1	1	1	0	0	
1	1	1	1	1	$\rightarrow ABCD$

$$\left\{ \begin{aligned} X = \bar{A}\bar{B}\bar{C}D &+ \bar{A}B\bar{C}D \\ + AB\bar{C}D &+ ABCD \end{aligned} \right\}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	0	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

(c)

Karnaugh Map Method

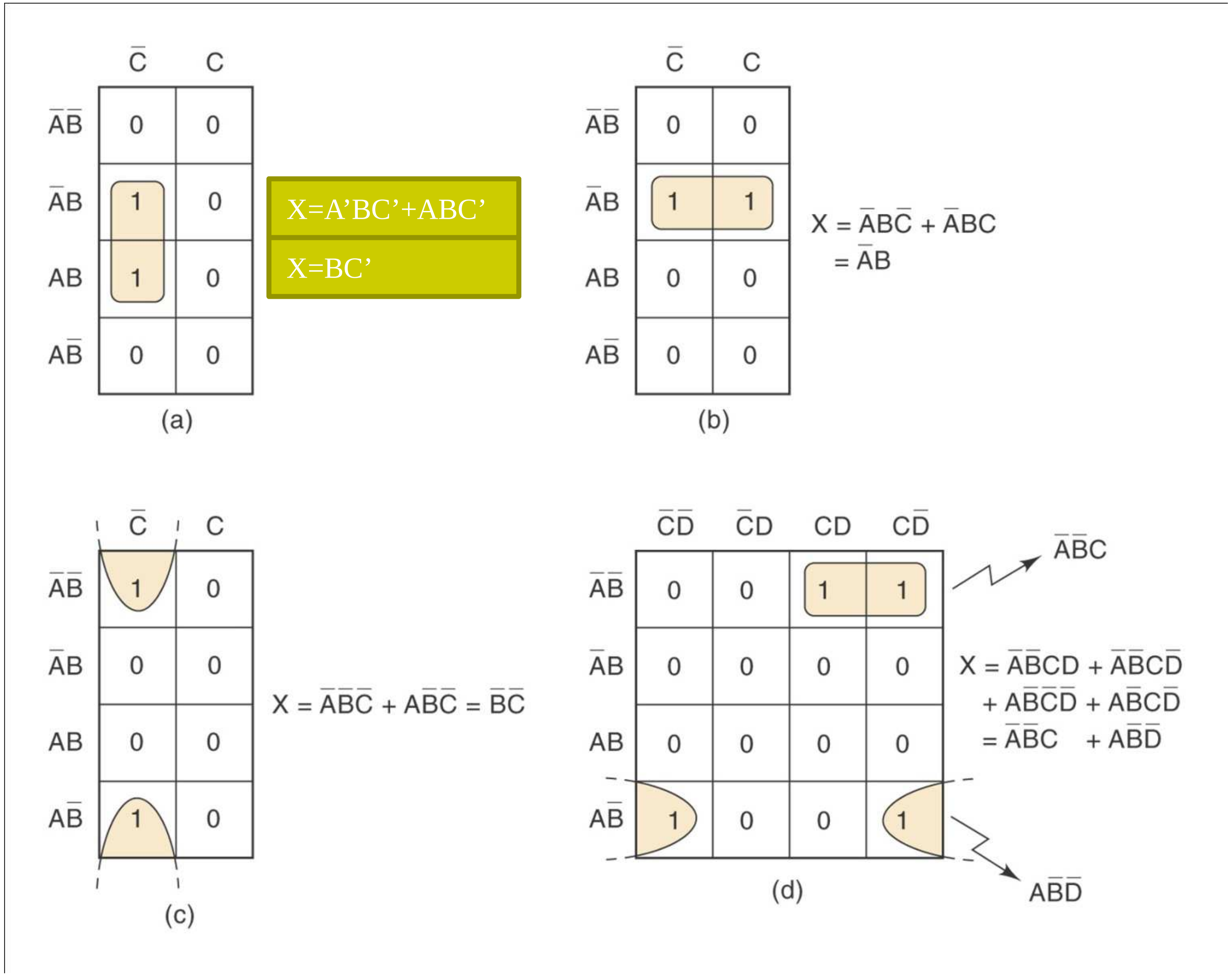
Looping is a process combining the squares which contain 1s. The output expression can be simplified by looping.

- Looping adjacent groups of 2, 4, or 8 1s will result in further simplification.
- When the largest possible groups have been looped, only the common terms are placed in the final expression.
- Looping may also be wrapped between top, bottom, and sides.

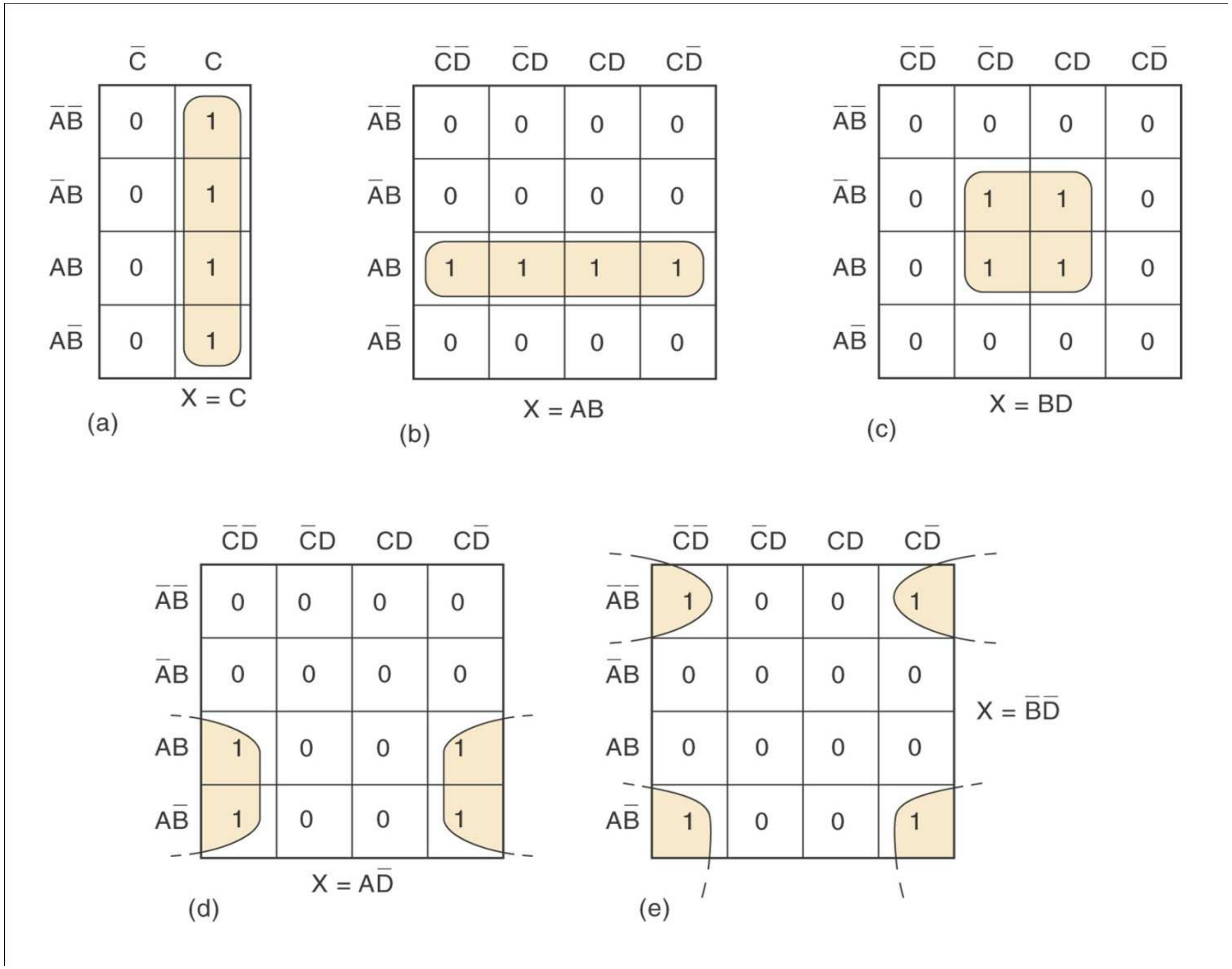
Rule for loops of any size

- When a variable appears in both complemented & uncomplemented form within a loop, that variable is eliminated from the expression.
- Variables that are the same for all squares of the loop must appear in the final expression.

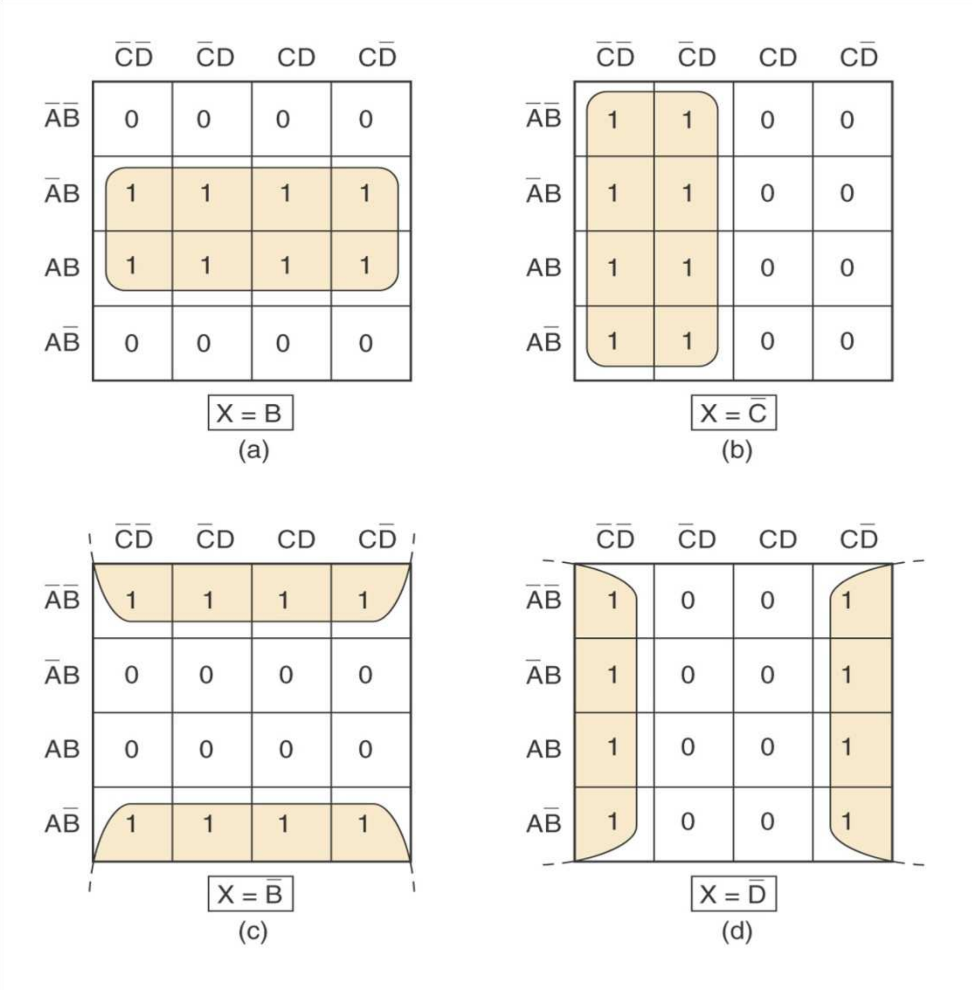
Examples of looping pairs of adjacent 1s.



Examples of looping groups of four 1s (quads).



Examples of looping groups of eight 1s (octets).



Karnaugh Map Method

- Complete K map simplification process:
  - Construct the K map, place 1s as indicated in the truth table.
  - Loop 1s that are not adjacent to any other 1s.
  - Loop 1s that are in pairs
  - Loop 1s in octets even if they have already been looped.
  - Loop quads that have one or more 1s not already looped.
  - Loop any pairs necessary to include 1 not already looped.
  - Form the OR sum of terms generated by each loop.

Refer Examples 4-10 to 4-13

Example:- use K-map to simplify $y=C'(A'B'D'+D)+AB'C+D'$

	C'D'	C'D	CD	CD'
A'B'	1	1	0	1
A'B	1	1	0	1
AB	1	1	0	1
AB'	1	1	1	1

$Y=AB'+C'+D'$

Examples

	C'D'	C'D	CD	CD'
A'B'	0 0	0 1	0 3	1 2
A'B	4	1 5	1 7	6
AB	12	1 13	1 15	14
AB'	8	9	1 11	10

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	0 $_{1}$	0 $_{2}$	0 $_{3}$	1 $_{4}$
$\bar A B$	0 $_{5}$	1 $_{6}$	1 $_{7}$	0 $_{8}$
AB	0 $_{9}$	1 $_{10}$	1 $_{11}$	0 $_{12}$
$A\bar B$	0 $_{13}$	0 $_{14}$	1 $_{15}$	0 $_{16}$

$$X = \underbrace{\bar A\bar B C\bar D}_{\text{loop 4}} + \underbrace{ACD}_{\text{loop 11, 15}} + \underbrace{BD}_{\text{loop 6, 7, 10, 11}}$$

(a)

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	0 $_{1}$	0 $_{2}$	1 $_{3}$	0 $_{4}$
$\bar A B$	1 $_{5}$	1 $_{6}$	1 $_{7}$	1 $_{8}$
AB	1 $_{9}$	1 $_{10}$	0 $_{11}$	0 $_{12}$
$A\bar B$	0 $_{13}$	0 $_{14}$	0 $_{15}$	0 $_{16}$

$$X = \underbrace{\bar A B}_{\text{loop 5, 6, 7, 8}} + \underbrace{B\bar C}_{\text{loop 5, 6, 9, 10}} + \underbrace{\bar A C D}_{\text{loop 3,7}}$$

(b)

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	0 $_{1}$	1 $_{2}$	0 $_{3}$	0 $_{4}$
$\bar A B$	0 $_{5}$	1 $_{6}$	1 $_{7}$	1 $_{8}$
AB	1 $_{9}$	1 $_{10}$	1 $_{11}$	0 $_{12}$
$A\bar B$	0 $_{13}$	0 $_{14}$	1 $_{15}$	0 $_{16}$

$$X = \underbrace{AB\bar C}_{9,\ 10} + \underbrace{\bar A\bar C D}_{2,\ 6} + \underbrace{\bar A B C}_{7,\ 8} + \underbrace{ACD}_{11,\ 15}$$

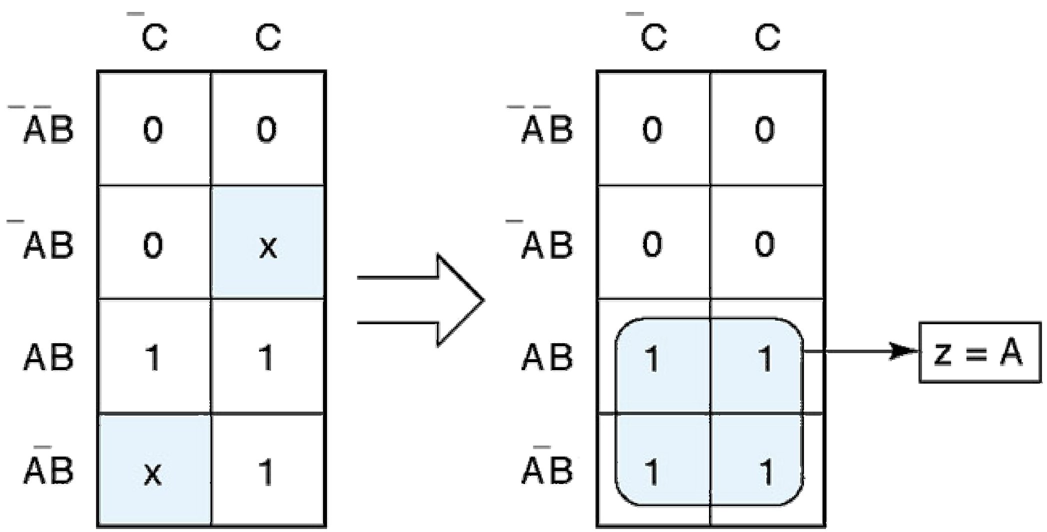
(c)

"Don't-Care" Conditions are certain input conditions for which there are no specified output levels.

"Don't-care" conditions should be changed to either 0 or 1 to produce K-map looping that yields the simplest expression.

A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	"don't
1	0	0	x	care"
1	0	1	1	
1	1	0	1	
1	1	1	1	

(a)



(b) (c)

Application Example:- Elevator Door Controller

Design logic circuit that controls an elevator door in a three storey building. The circuit in fig a has four inputs. M is a logic signal that indicates when the elevator is moving(M=1) or stopped (M=0) F1, F2, and F3 are flour indicator signals that are normally low, and they go HIGH only when the elevator is positioned at the level of that particular floor. For example, when the elevator is lined up level with the second floor, F2=1 and F1=F3=0. The circuit output is the OPEN signal, which is normally LOW and will go HIGH when the elevator door is to be opened.

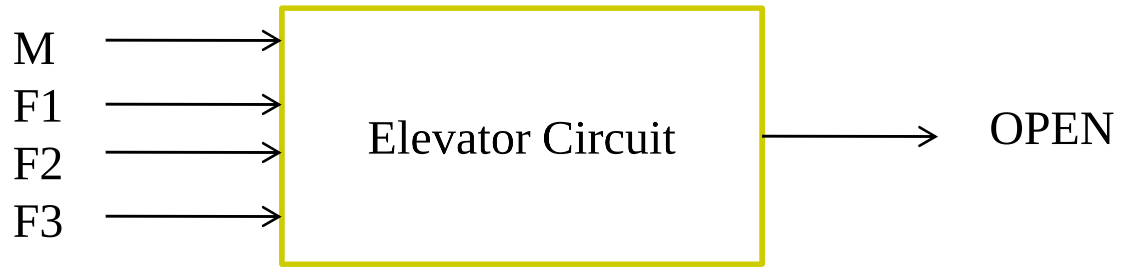


Fig a

We can fill the truth table for open output as follows

1. Since the elevator cannot be lined up with more than one floor at a time, then only one of the floor inputs can be HIGH at any given time. This means that all those cases in the truth table where more than one floor input is a 1 are don't care conditions. We can place an x in the OPEN output column for those 8 cases
2. Looking at other 8 cases, when M=1 the elevator is moving, so OPEN must be 0 since we don't want the elevator door to open. When M=0 (Elevator stopped) we want OPEN =1 provided that one of the floor inputs is 1. when M=0 and all floor inputs are 0, the elevator is stoped but is not properly lined up with any floor, so we want OPEN= 0 to keep the door closed

Truth Table

M	F1	F2	F3	OPEN
0	0	0	0	0
0	**0**	**0**	**1**	**1**
0	**0**	**1**	**0**	**1**
0	0	1	1	X
0	1	0	0	1
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	0
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Fig b

K-map

	$\overline{F2}\,\overline{F3}$	$\overline{F2}\,F3$	$F2\,F3$	$F2\,\overline{F3}$
$\overline{M}\,\overline{F1}$	0	1	X	1
$\overline{M}\,F1$	1	X	X	X
$M\,F1$	0	X	X	X
$M\,\overline{F1}$	0	0	X	0

(c)

	$\overline{F2}\,\overline{F3}$	$\overline{F2}\,F3$	$F2\,F3$	$F2\,\overline{F3}$
$\overline{M}\,\overline{F1}$	0	1	1	1
$\overline{M}\,F1$	1	1	1	1
$M\,F1$	0	0	0	0
$M\,\overline{F1}$	0	0	0	0

$$OPEN = \overline{M}\,(F1 + F2 + F3)$$

(d)

Final Circuit

- Work for you