A goal programming model for advertising investment strategy

1. Introduction

With people spending more and more time online, online advertising has grown to a vital portion of the advertising market. Online advertising, distinct from traditional advertising, targets customers more precisely based on user data (Evans, 2009). This provides a unique opportunity for businesses to improve the target audience's willingness to buy the product by pushing the advertisements to them online.

When customers research the products, the role of key opinion leaders (KOLs) is crucial because they impact buying tendencies by presenting products to their followers (AsiaKOL, 2020). In consequence, utilizing the influence of KOLs for promoting products is an effective advertising strategy. However, it is difficult to effectively allocate the advertising budget to a large number of KOLs on different platforms to achieve the best promotion. This paper aims to provide decision-makers of a company with recommendations for optimizing online advertising strategies by employing Goal Programming to allocate the limited budget reasonably to KOLs on different online platforms and taking advantage of their influence to improve the engagement of the target audience for the product.

2.Model Formulation

Goal programming is a mathematical approach for dealing with multi-objective optimization problems by and minimizes deviations from aspiration levels. Our research aims to optimize the display of advertising on three social media platforms, with the main goal of reaching the desired total number of ad engagements, while taking budget constraints for specific platform into account.

2.1 Variables

As shown in Table 1, our model includes 9 decision variables and the other associated input variables.

Table 1

Notation	Description	Type

p	Three different social media platforms (1 = YouTube, 2 = TikTok, 3= Instagram)	Index
i	Three different types of KOL/influencers 1=Micro influencer (with 10-100k followers) 2=Mid-tier influencer (with 100k-500k followers) 3=Macro influencer (with 500k-1m followers)	Index
x_{ip}	Number of ads placed to i type KOL on platform p	Integer (decision variable)
c_{ip}	Ad quotations for i type KOL on platform p	Integer
u	three types of users' metrics (1=age, 2= income, 3=education level)	Index
w_u	The relative importance of the three user indicators	continuous
r_{ip}	The engagement rate for different KOLs on each platform	continuous
N_{up}	Percentage of users of indicator u on platform p	continuous
I	Ad engagement due to advertising placement	Integer
M	Total budget	integer
a_i	The average number of followers for each type of KOL 1=Micro influencer： 50,005 2=Mid-tier influencer：300,000 3=Macro influencer ： 750,000	integer

2.2 Quantify engagement

Choose three types of users' metrics and set w_u as their relative importance, as present in Table 2.

Table 2

	Descriptions	Weight
Age	From 18 to 24	w_1
Income	Medium income	w_2
Education level	Bachelor and above	w_3

Note: The exact value of w_u has been showing in case study.

The matrix shown in Table 3 is the percentage of target audience in each platform and let these be N_{up}.

Table 3

	YouTube	Tik Tok	Instagram
Age	N_{11}	N_{12}	N_{13}
Income	N_{21}	N_{22}	N_{23}
Education	N_{31}	N_{32}	N_{33}

Then, the engagements of active users for one advertisement on per platform are:

YouTube: $k_{i1} = (W_1N_{11} + W_2N_{21} + W_3N_{31})r_{i1} \times a_i$

Tik Tok: $k_{i2} = (W_1N_{12} + W_2N_{22} + W_3N_{32})r_{i2} \times a_i$

Instagram: $k_{i3} = (W_1N_{13} + W_2N_{23} + W_3N_{33})r_{i3} \times a_i$

Therefore, the total engagement function would be:

$$I = \sum_{i=1}^{3} (k_{i1}x_{i1} + k_{i2}x_{i2} + k_{i3}x_{i3})$$

2.3 Ad Cost

Table 4 shows the cost of each type of KOL across platforms.

Table 4

	YouTube	Tik Tok	Instagram
Micro influencer	$c_{11}x_{11}$	$c_{12}x_{12}$	$c_{13}x_{13}$

Mid-tier influencer	$c_{21}x_{21}$	$c_{22}x_{22}$	$c_{23}x_{23}$
Macro influencer	$c_{31}x_{31}$	$c_{32}x_{32}$	$c_{33}x_{33}$

Thus, the amount of total cost is:

$$C = \sum c_{ip}\, x_{ip} \ \ (for\ i = 1,2,3;\ p = 1,2,3), \quad x_{ip} \geq 0$$

2.4 Assumption

In this study, we have made the following assumptions:

1. The same user may access the same advertisement on different platforms, which produces a simple and direct cumulative effect.

2. The same level KOI produces the same advertising effect, ignoring the effect of small differences in the number of followers.

3. KOIs with accounts on multiple platforms will not be double-selected.

2.5 Case Description

The purpose of this report is to help a company to promote the product online. The company providing career planning services aims to promote career planning products by improving the advertisement engagements of the target audience through the influence of KOLs on YouTube, TikTok and Instagram under the limited budget.

Holmes (2023) investigated the conversion rate for social media of professional services is 2.3% in 2023. The company expects to attract 5,000 new customers through this program. According to the conversion rate, the total engagement should be 217,391. So setting the target value of total engagement as 220,000.

Currently, this company is willing to pay $500 for every 1,000 engagements. The total advertising budget is $110,000. Geyser (2023) examined that TikTok has become the most common online influencer marketing platform, while Instagram and YouTube rank second and fourth. As a result, TikTok, Instagram, and YouTube are respectively allocated 50%, 30%, and 20% of the total budget.

The goals are as follows:

Goal 1: Create at least 220,000 total engagements.

Goal 2: Invest no more than $33,000 in Instagram.

Goal 3: Invest no more than $55,000 in TikTok.

Goal 4: Invest no more than $22,000 in YouTube.

The target audience of career planning products is users who are between 18 and 24 years old, who with middle income, and who have bachelor's degree or higher level of education. Referring to the user profile data of different platforms provided by Statista (2022) and Statista (2023) (see *Appendix 2*), the proportion of target audience to the total users is displayed in Table 5.

Table 5

	YouTube	TikTok	Instagram
Customer Age	15.5%	36.20%	30.80%
Income level	34%	31%	32%
Education level	37%	36%	44%

Assuming that the importance of target audience in each group is consistent, assign the weight of target audience of each group to 0.33 according to *Appendix 1*. (w_i = 0.33)

Based on the different influences of KOLs, promoting the product by three levels of them: Micro influencer, Mid-tier influencer and Macro influencer. The engagement rates of different KOLs on the three online platforms provided by Statista (2023) are displayed in Table 6.

Table 6

	YouTube	TikTok	Instagram
Micro influencer	1.9%	12.4%	1.06%
Mid-tier influencer	3.47%	10.9%	0.91%
Macro influencer	3.4%	10.8%	0.86%

Referring to the report by Dogtiev (2023), taking the median of the price range or the fixed

value of price as the advertising quotation for KOLs with different influence, as shown in Table 7.

Table 7

	YouTube ($)	TikTok ($)	Instagram ($)
Micro influencer	600	75	219
Mid-tier influencer	5500	688	775
Macro influencer	15000	1875	3138

This company expects to have a wide range of ways to advertise rather than a single, so the number of advertisements per platform is set to no more than 25 advertisements for each type of KOLs.

The goal programming model is written as follows:

$$\text{Min} \quad P_1(d_1^-) + P_2(d_2^+) + P_3(d_3^+) + P_4(d_4^+)$$

s.t.

$$x_{ip} \leq 25$$

$$I - d_1^+ + d_1^- = 220{,}000 \qquad \text{(Goal 1)}$$

$$\sum c_{i3}\, x_{i3} - d_2^+ + d_2^- = 33{,}000 \quad \text{(Goal 2)}$$

$$\sum c_{i2}\, x_{i2} - d_3^+ + d_3^- = 55{,}000 \quad \text{(Goal 3)}$$

$$\sum c_{i1}\, x_{i1} - d_4^+ + d_4^- = 22{,}000 \quad \text{(Goal 4)}$$

$$x_{ip}, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, d_4^+, d_4^- \geq 0$$

Using goal programming, the optimal value for advertisement allocation on each platform is obtained by python (see Appendix 3) as shown in Table 8.

Table 8

	YouTube	TikTok	Instagram
Micro influencer	1	24	0
Mid-tier influencer	0	0	25

Macro influencer	1	5	0

According to the company's planning for advertising, the optimal solution would be to place $15,600 in ads on YouTube, $11,175 in ads on TikTok, and $19,375 in ads on Instagram, which would give the company 220,211 ads engagements.

3. Discussion

According to the results, surprisingly, TikTok, which has the largest ad budget, yet spends the least. This could be attributed to the extremely low single ad offer for micro-influencers and macro-influencers in TikTok, which is even 12.5 percent of YouTube's and the engagement rate exceed expectation.

In contrast, Instagram spends the most on advertising, and all of it is used to buy ads for Mid-tier influencers. By examining the data, the cost of ads for Mid- tier influencer's advertisements are the most cost-effective. However, when developing the final advertising strategy, the choice of KOL should be adjusted appropriately by taking into account the diversity of advertisements.

On YouTube, the company chooses only one Micro influencer and one Macro influencer to post advertisements, mainly due to the high cost of advertisements quoted by the KOL of YouTube. But its large user base can still be a reason for the company to choose it.

4. Limitation & Future

However, there are still several limitations in our study which has been listed below:

1. This study simply gives equal weight to the four goals in model construction.

2. This study makes some simple assumptions that distinguish it from the real world.

3. This study doesn’t take the trend of user disruption into consideration.

Meanwhile, in the future research, it is vital that for us to

1.Consider using more decision variables. For example, exploring the possibility of replacing adverts but at the same time promoting the product, e.g. by introducing reviews, sharing

coupons, etc.

2. Consider the impact of national policies on purchasing behavior.

3. The investor and popular users can be considered as two levels, and two-level multi-objective linear programming can be introduced to optimize the objective function based on the considerations of both.

5. Conclusion

This study builds a model based on goal programming model for optimizing ad costs and ad engagement, this model considers user characteristics, KOL's advertisement offer and other factors to develop an optimal strategy for companies to place KOL advertisements. Real data are used for simulation in the case study, but the study still has some limitations, such as the gap between assumptions and complex reality. Based on this study, companies should consider the comparison of the relevance of users and target customers on different platforms as well as the ratio of advertisement consumption to revenue. In addition, more realistic factors, such as the characteristics of fans of different KOL, etc., should be considered when formulating advertising strategies, so as to develop more effective marketing strategies with limited funds.

Reference

AsiaKOL. (2020). Zero Moment of Truth And KOLs. [Online]. Available at: https://en.asiakol.com/article/97

Dogtiev, A. (2023). Influencer Marketing Costs (2023). *Business of Apps.* [Online]. Available at: https://www.businessofapps.com/marketplace/influencer-marketing/research/influencer-marketing-costs/

Evans, D.S. (2009). 'The Online Advertising Industry: Economics, Evolution, and Privacy', *Journal of Economic Perspectives*, 23(3), pp. 37–60. Available at: https://doi.org/10.1257/jep.23.3.37.

Geyser, W. (2023). The State of Influencer Marketing 2023: Benchmark Report. [Online]. Available at: https://influencermarketinghub.com/influencer-marketing-benchmark-report/

Holmes, K. (2023). Updated 2023: Average Conversion Rate by Industry and Marketing Source. [Online]. Available at: https://www.ruleranalytics.com/blog/insight/conversion-rate-by-industry/

Appendix 1

Comparison matrix is as follow:

	age	income	education level
age	1	1	1
income	1	1	1
education level	1	1	1

Normalize the matrix is as follow:

	age	income	education level
age	0.33	0.33	0.33
income	0.33	0.33	0.33
education level	0.33	0.33	0.33

Consistency check

Consistency check is one of the most key steps in AHP, it can help to judge the rationality of decisions. It is determined by the consistency ratio (CR). When the CR is lower than or equal to 0.1, the consistency of pairwise comparisons is considered acceptable. The formula is as follows:

$$CR = \frac{CI}{RI},\ CI = \frac{\lambda_{max} - n}{n-1}$$

CI is the consistency index and since we set all user metrics to have the same weight on each platform, our matrix has full consistency. As Brunelli and Matteo (2015) describe, '' λ_{max}, is equal to n if and only if the matrix is consistent…''. So, the value of CI=0, so that we can get CR=0 and can accept the comparison result.

Reference:

Matteo Brunelli 2015, 'Introduction to the Analytic Hierarchy Process', *SpringerBriefs in Operations Research. P. 83. 978-3-319-12502-2 (electronic),* pp. 29, Available at:

https://core.ac.uk/download/pdf/80714029.pdf

Appendix 2

Data Source:

Statista. (2023). Distribution of Instagram users worldwide as of January 2023, by age and gender. [Online]. Available at: https://www.statista.com/statistics/248769/age-distribution-of-worldwide-instagram-users/ [Accessed 16 Nov 2023].

Statista. (2022). Distribution of leading social media platform users in the United States as of August 2022, by education level. [Online]. Available at: https://www.statista.com/statistics/1337578/us-distribution-leading-social-media-platforms-by-education/ [Accessed 16 Nov 2023].

Statista. (2022). Distribution of leading social media platform users in the United States as of August 2022, by income. [Online]. Available at: https://www.statista.com/statistics/1337616/us-distribution-leading-social-media-platforms-by-income/ [Accessed 16 Nov 2023].

Statista. (2023). Distribution of TikTok users worldwide as of October 2023, by age and gender. [Online]. Available at: https://www.statista.com/statistics/1299771/tiktok-global-user-age-distribution/ [Accessed 16 Nov 2023].

Statista. (2023). Distribution of YouTube users worldwide as of October 2023, by age group and gender. [Online]. Available at: https://www.statista.com/statistics/1287137/youtube-global-users-age-gender-distribution/ [Accessed 16 Nov 2023].

Statista. (2023). Engagement rates among Instagram influencers worldwide in 2022, by number of followers. [Online]. Available at: https://www.statista.com/statistics/992887/growth-engagement-rate-influencers-followers/ [Accessed 16 Nov 2023].

Statista. (2023). TikTok marketing [Online]. Available at: https://www.statista.com/study/102264/tiktok-marketing/ [Accessed 16 Nov 2023].

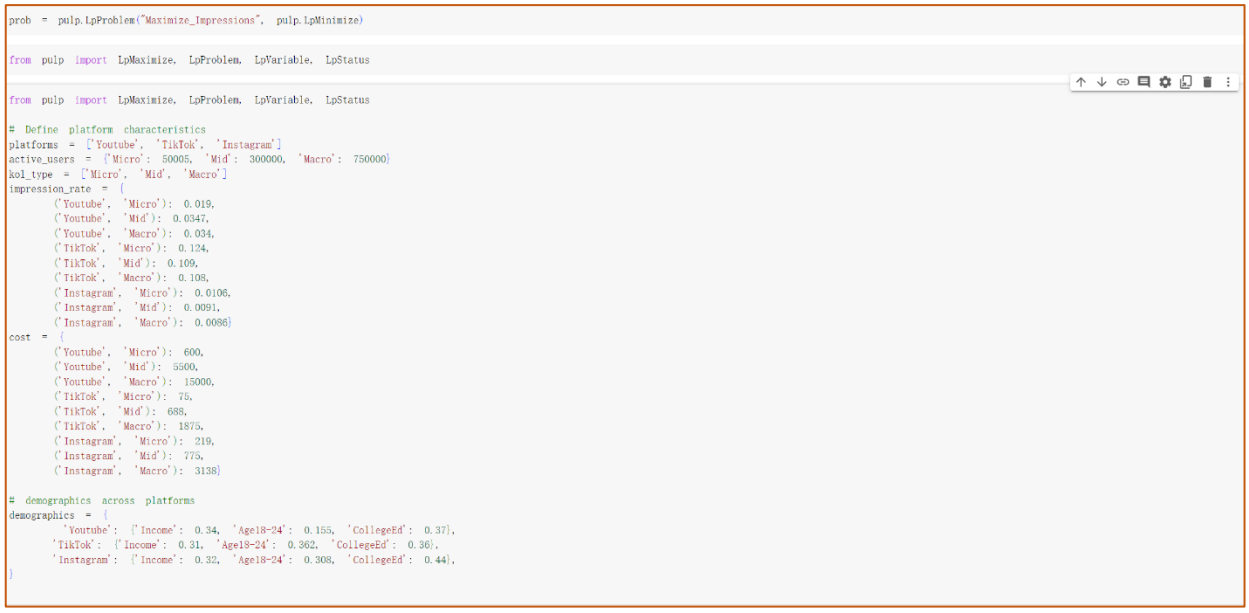
Statista. (2023). YouTube influencer engagement rates worldwide in 2022, by follower count. [Online]. Available at: https://www.statista.com/statistics/1257518/youtube-influencer-engagement-rate/ [Accessed 16 Nov 2023].

Appendix 3

Link to Google Colaboratory with code:

https://colab.research.google.com/drive/1Fzb431awk3FUHDlWAnZAKUo0bxJPM0_-?usp=sharing

```
# Intall and Import Package/Library
!pip install pulp
import pulp
!apt-get install -y -qq glpk-utils
from pulp import GLPK
from pulp import LpVariable, LpProblem, LpMaximize, LpMinimize, lpSum
```



```
prob = pulp.LpProblem("Maximize_Impressions", pulp.LpMinimize)

from pulp import LpMaximize, LpProblem, LpVariable, LpStatus

from pulp import LpMaximize, LpProblem, LpVariable, LpStatus

# Define platform characteristics
platforms = ['Youtube', 'TikTok', 'Instagram']
active_users = {'Micro': 50005, 'Mid': 300000, 'Macro': 750000}
kol_type = ['Micro', 'Mid', 'Macro']
impression_rate = {
        ('Youtube', 'Micro'): 0.019,
        ('Youtube', 'Mid'): 0.0347,
        ('Youtube', 'Macro'): 0.034,
        ('TikTok', 'Micro'): 0.124,
        ('TikTok', 'Mid'): 0.109,
        ('TikTok', 'Macro'): 0.108,
        ('Instagram', 'Micro'): 0.0106,
        ('Instagram', 'Mid'): 0.0091,
        ('Instagram', 'Macro'): 0.0086}
cost = {
        ('Youtube', 'Micro'): 600,
        ('Youtube', 'Mid'): 5500,
        ('Youtube', 'Macro'): 15000,
        ('TikTok', 'Micro'): 75,
        ('TikTok', 'Mid'): 688,
        ('TikTok', 'Macro'): 1875,
        ('Instagram', 'Micro'): 219,
        ('Instagram', 'Mid'): 775,
        ('Instagram', 'Macro'): 3138}

# demographics across platforms
demographics = {
        'Youtube': {'Income': 0.34, 'Age18-24': 0.155, 'CollegeEd': 0.37},
       'TikTok': {'Income': 0.31, 'Age18-24': 0.362, 'CollegeEd': 0.36},
       'Instagram': {'Income': 0.32, 'Age18-24': 0.308, 'CollegeEd': 0.44},
}
```

```
#the percentage of target users on per platform
demographic_factor_youtube = 0.33 * (demographics['Youtube']['Income'] + demographics['Youtube']['Age18-24'] + demographics['Youtube']['CollegeEd'])
demographic_factor_tiktok = 0.33 * (demographics['TikTok']['Income'] + demographics['TikTok']['Age18-24'] + demographics['TikTok']['CollegeEd'])
demographic_factor_instagram = 0.33 * (demographics['Instagram']['Income'] + demographics['Instagram']['Age18-24'] + demographics['Instagram']['CollegeEd'])

#the reach on per platform
# Youtube
k11 = demographic_factor_youtube * active_users['Micro'] * impression_rate[('Youtube', 'Micro')]
k21 = demographic_factor_youtube * active_users['Mid'] * impression_rate[('Youtube', 'Mid')]
k31 = demographic_factor_youtube * active_users['Macro'] * impression_rate[('Youtube', 'Macro')]

# TikTok
k12 = demographic_factor_tiktok * active_users['Micro'] * impression_rate[('TikTok', 'Micro')]
k22 = demographic_factor_tiktok * active_users['Mid'] * impression_rate[('TikTok', 'Mid')]
k32 = demographic_factor_tiktok * active_users['Macro'] * impression_rate[('TikTok', 'Macro')]

# Instagram
k13 = demographic_factor_instagram * active_users['Micro'] * impression_rate[('Instagram', 'Micro')]
k23 = demographic_factor_instagram * active_users['Mid'] * impression_rate[('Instagram', 'Mid')]
k33 = demographic_factor_instagram * active_users['Macro'] * impression_rate[('Instagram', 'Macro')]

(k11, k21, k31, k12, k22, k32, k13, k23, k33)

(271.20461775000007,
 2971.5345000000007,
 7278.975000000001,
 2111.6831472000003,
 11136.312000000002,
 27585.360000000004,
 186.81187932,
 962.1612000000002,
 2273.238)
```

```
k_value = {
        'Youtube,Micro': k11,
        'Youtube,Mid': k21,
        'Youtube,Macro': k31,
        'TikTok,Micro': k12,
        'TikTok,Mid': k22,
        'TikTok,Macro': k32,
        'Instagram,Micro': k13,
        'Instagram,Mid': k23,
        'Instagram,Macro': k33
}
```

```
#Decision variables
ads = pulp.LpVariable.dicts("Ads", [(platform, kol_type) for platform in platforms for kol_type in ['Micro', 'Mid', 'Macro']], lowBound=0, upBound=25, cat='Integer')


# Deviation variables for each platform-KOL combination
d1_pos = pulp.LpVariable("d1_pos", lowBound=0, upBound=None, cat='Continuous')
d1_neg = pulp.LpVariable("d1_neg", lowBound=0, upBound=None, cat='Continuous')
d2_pos = pulp.LpVariable("d2_pos", lowBound=0, upBound=None, cat='Continuous')
d2_neg = pulp.LpVariable("d2_neg", lowBound=0, upBound=None, cat='Continuous')
d3_pos = pulp.LpVariable("d3_pos", lowBound=0, upBound=None, cat='Continuous')
d3_neg = pulp.LpVariable("d3_neg", lowBound=0, upBound=None, cat='Continuous')
d4_pos = pulp.LpVariable("d4_pos", lowBound=0, upBound=None, cat='Continuous')
d4_neg = pulp.LpVariable("d4_neg", lowBound=0, upBound=None, cat='Continuous')
```

```
# Add Objective Function to the Environment
prob += d4_pos
```

```
# Add Soft Constraints Function to the Environment
# Goal 1 --- Impression
prob += sum([k_value[platform + ',' + kol] * ads[(platform, kol)] for platform in platforms for kol in kol_type]) - d1_pos + d1_neg == 220000


# Goal 2 --- The cost for Instagram
prob += sum(ads['Instagram', kol_type] * cost[('Instagram', kol_type)] for kol_type in ['Micro', 'Mid', 'Macro']) - d2_pos + d2_neg == 33000 #30%

# Goal 3 --- The cost for TikTok
prob += sum(ads['TikTok', kol_type] * cost[('TikTok', kol_type)] for kol_type in ['Micro', 'Mid', 'Macro']) - d3_pos + d3_neg == 55000 #50%

# Goal 4 --- The cost for Youtube
prob += sum(ads['Youtube', kol_type] * cost[('Youtube', kol_type)] for kol_type in ['Micro', 'Mid', 'Macro']) - d4_pos + d4_neg == 22000 #20%

prob += d1_neg == 0
prob += d2_pos == 0
prob += d3_pos == 0
```

```
# Solve the Problem
prob.writeLP("Maximize_Impressions.lp")
prob.solve(GLPK(msg=True, options=['--ranges', 'sensitivity.txt']))

# The Status of the Solution
print("Model Status:{}".format(pulp.LpStatus[prob.status]))

# To Display Optimal Decision Variables & Reduced Cost per Variable
for v in prob.variables():
        print(v.name, "=", v.varValue)

# To Display Optimal Value of Objective Function
print("Objective=", pulp.value(prob.objective))
```

```
Model Status:Optimal
Ads_('Instagram',_'Macro') = 0
Ads_('Instagram',_'Micro') = 0
Ads_('Instagram',_'Mid') = 25
Ads_('TikTok',_'Macro') = 5
Ads_('TikTok',_'Micro') = 24
Ads_('TikTok',_'Mid') = 0
Ads_('Youtube',_'Macro') = 1
Ads_('Youtube',_'Micro') = 1
Ads_('Youtube',_'Mid') = 0
d1_neg = 0.0
d1_pos = 211.408
d2_neg = 13625.0
d2_pos = 0.0
d3_neg = 43825.0
d3_pos = 0.0
d4_neg = 6400.0
d4_pos = 0.0
Objective= 0.0
```