

# Quantitative text analysis: Supervised Scaling Methods

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MY 459: Quantitative Text Analysis

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Course website: [lse-my459.github.io](https://lse-my459.github.io)

1. Overview and Fundamentals
2. Descriptive Statistical Methods for Text Analysis
3. Automated Dictionary Methods
4. Machine Learning for Texts
5. Supervised Scaling Models for Texts
6. *Reading Week*
7. Unsupervised Models for Scaling Texts
8. Similarity and Clustering Methods
9. Topic models
10. Word embeddings
11. Working with Social Media

# Overview of text as data methods



# Outline

- ▶ From classification to scaling
  - ▶ Basics of supervised scaling methods
  - ▶ Wordscores
  - ▶ Practical aspects
  - ▶ Examples

# From Classification to Scaling

- ▶ Machine learning focuses on identifying classes (**classification**), while social science is typically interested in locating things on latent traits (**scaling**), e.g.:
  - ▶ Policy positions on economic vs social dimension
  - ▶ Inter- and intra-party differences
  - ▶ Soft news vs hard news
  - ▶ Petitioner vs respondent in legal briefs
  - ▶ ...and any other continuous scale
- ▶ But the two methods overlap and can be adapted – will demonstrate later using the Naive Bayes classifier
- ▶ In fact, the class predictions for a collection of words from NB can be adapted to scaling

# Outline

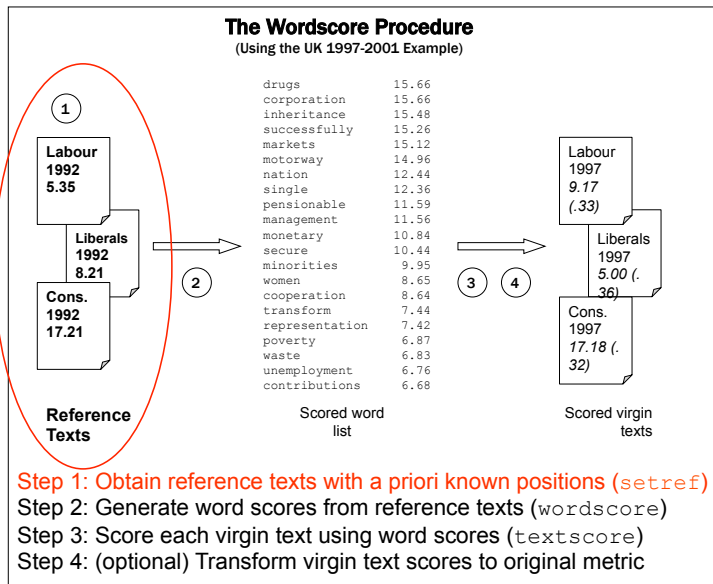
- ▶ From classification to scaling
  - ▶ Basics of supervised scaling methods
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  - ▶ Practical aspects
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# Supervised scaling methods

Wordscores method (Laver, Benoit & Garry, 2003):

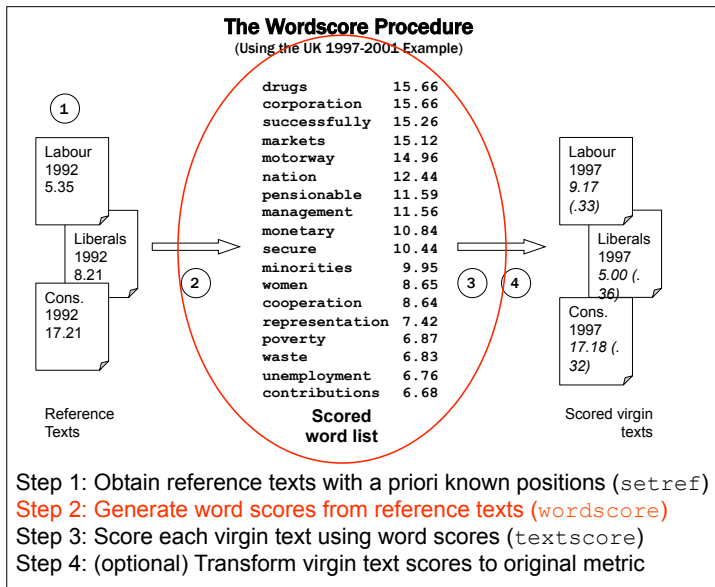
- ▶ Two sets of texts
  - ▶ **Reference texts**: texts about which we know something (a scalar dimensional score)
  - ▶ **Virgin texts**: texts about which we know nothing (but whose dimensional score we'd like to know)
- ▶ These are analogous to a “training set” and a “test set” in classification
- ▶ Basic procedure:
  1. Analyze reference texts to obtain word scores
  2. Use word scores to score virgin texts

# Wordscores Procedure





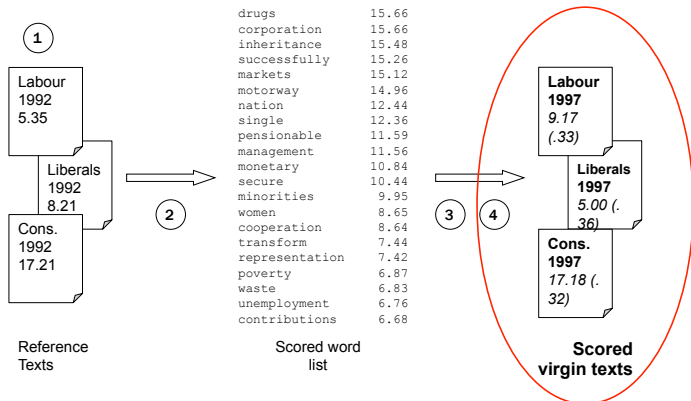
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## The Wordscore Procedure

(Using the UK 1997-2001 Example)



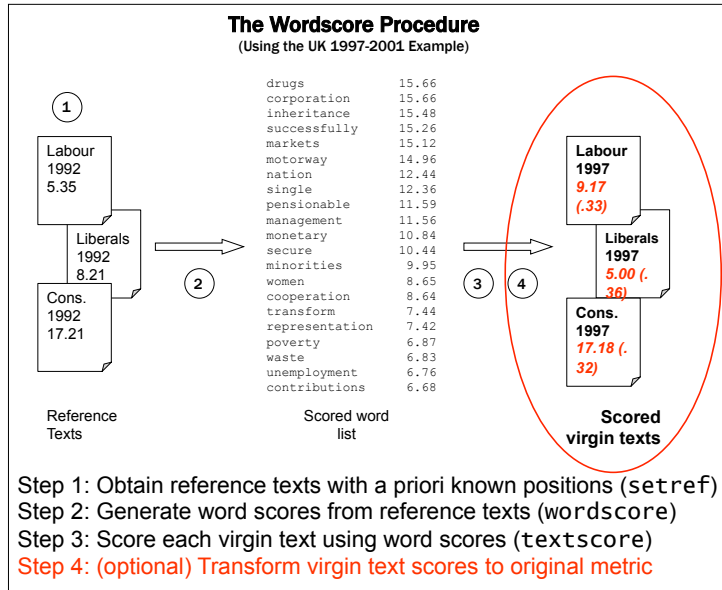
Step 1: Obtain reference texts with a priori known positions (`setref`)

Step 2: Generate word scores from reference texts (`wordscore`)

Step 3: Score each virgin text using word scores (`textscore`)

Step 4: (optional) Transform virgin text scores to original metric

# Wordscores Procedure



## Wordscores mathematically: Reference texts

- ▶ Start with a set of  $I$  *reference* texts, represented by an  $I \times J$  document-feature matrix  $C_{ij}$ , where  $i$  indexes the document and  $j$  indexes the  $J$  total word types

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  - ▶ This can be on a scale metric, such as 1–20
  - ▶ Can use arbitrary endpoints, such as -1, 1
- ▶ We *normalize* the document-feature matrix within each document by converting  $C_{ij}$  into a *relative* document-feature matrix (within document), by dividing  $C_{ij}$  by its word total marginals:

$$F_{ij} = \frac{C_{ij}}{C_{i.}} \quad (1)$$

where  $C_{i.} = \sum_{j=1}^J C_{ij}$



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- This tells us the probability that given the observation of a specific word  $j$ , that we are reading a text of a certain reference document  $i$

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- ▶ The word “choice” is used 10 times per 1,000 words in Text A and 30 times per 1,000 words in Text B
- ▶ So  $F_i$  “choice” = { .010, .030 }
- ▶ If we know only that we are reading the word choice in one of the two reference texts, then probability is 0.25 that we are reading Text A, and 0.75 that we are reading Text B

$$P_A \text{ “choice”} = \frac{.010}{(.010 + .030)} = 0.25 \quad (3)$$

$$P_B \text{ “choice”} = \frac{.030}{(.010 + .030)} = 0.75 \quad (4)$$

## Wordscores mathematically: Word scores

- Compute a  $J$ -length “score” vector  $S$  for each word  $j$  as the average of each document  $i$ ’s scores  $a_i$ , weighted by each word’s  $P_{ij}$ :

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- ▶ In matrix algebra,  $S = \underset{1 \times J}{a} \cdot \underset{1 \times I}{a} \cdot \underset{I \times J}{P}$
- ▶ This procedure will yield a single “score” for every word that reflects the balance of the scores of the reference documents, weighted by the relative document frequency of its normalized term frequency

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  - ▶ We “know” (from independent sources) that Reference Text A has a position of  $-1.0$ , and Reference Text B has a position of  $+1.0$
  - ▶ The score of the word “choice” is then
$$0.25(-1.0) + 0.75(1.0) = -0.25 + 0.75 = +0.50$$

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$$v_k = \sum_j (F_{kj} \cdot s_j) \quad (6)$$

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- ▶ Note also that nothing prohibits reference documents from also being scored as virgin documents

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- ▶ Martin and Vanberg (2008) have proposed alternatives to the LBG (2003) rescaling

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- ▶ An alternative would be to bootstrap the textual data prior to constructing  $C_{ij}$  and  $C_{kj}$  — see Lowe and Benoit (2012)

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- ▶ Should contain lots of words

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- ▶ With three or more reference values, the mid-point is mapped onto a multi-dimensional simplex. The values now matter but only in relative terms (we are still investigating this fully)

## Multinomial Bayes model of Class given a Word

### Class posterior probabilities

$$P(c_k|w_j) = \frac{P(w_j|c_k)P(c_k)}{P(w_j|c_k)P(c_k) + P(w_j|c_{\neg k})P(c_{\neg k})}$$

- ▶ This represents the **posterior probability of membership in class  $k$**  for word  $j$
- ▶ Under *certain conditions*, this is identical to what LBG (2003) called  $P_{wr}$
- ▶ Under those conditions, **the LBG “wordscore” is the linear difference between  $P(c_k|w_j)$  and  $P(c_{\neg k}|w_j)$**



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- ▶ Consider two “reference” scores  $s_1$  and  $s_2$  attached to two classes  $k = 1$  and  $k = 2$ . Taking  $P_1$  as the posterior  $P(k = 1|w = j)$  and  $P_2$  as  $P(k = 2|w = j)$ , A generalised score  $s_j^*$  for the word  $j$  is then

$$\begin{aligned}s_j^* &= s_1 P_1 + s_2 P_2 \\&= s_1 P_1 + s_2 (1 - P_1) \\&= s_1 P_1 + s_2 - s_2 P_1 \\&= P_1 (s_1 - s_2) + s_2\end{aligned}$$

## “Certain conditions”: More than two reference classes

- For more than two reference classes, if the reference scores are ordered such that  $s_1 < s_2 < \cdots < s_K$ , then

$$\begin{aligned}s_j^* &= s_1 P_1 + s_2 P_2 + \cdots + s_K P_K \\&= s_1 P_1 + s_2 P_2 + \cdots + s_K \left(1 - \sum_{k=1}^{K-1} P_k\right) \\&= \sum_{k=1}^{K-1} P_i (s_k - s_K) + s_I\end{aligned}$$

A simpler formulation:

Use reference scores such that  $s_1 = -1.0, s_K = 1.0$

- ▶ From above equations, it should be clear that any set of reference scores can be linearly rescaled to endpoints of  $-1.0, 1.0$
- ▶ This simplifies the “simple word score”

$$s_j^* = (1 - 2P_1) + \sum_{k=2}^{K-1} P_k(s_k - 1)$$

- ▶ which simplifies with just two reference classes to:

$$s_j^* = 1 - 2P_1$$

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- ▶ We might as well always anchor reference scores at  $-1.0, 1.0$
- ▶ There is a special role for reference classes in between  $-1.0, 1.0$ , as they balance between “pure” classes — more in a moment
- ▶ There are alternative scaling models, such that used in Beauchamp's (2012) “Bayesscore”, which is simply the difference in logged class posteriors at the word level. For  $s_1 = -1.0, s_2 = 1.0$ ,

$$\begin{aligned}s_j^B &= -\log P_1 + \log P_2 \\ &= \log \frac{1 - P_1}{P_1}\end{aligned}$$

## Moving to the document level

- ▶ The “Naive” Bayes model of a joint document-level class posterior assumes conditional independence, to multiply the word likelihoods from a “test” document, to produce:

$$P(c|d) = P(c) \frac{\prod_j P(w_j|c)}{P(w_j)}$$

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- ▶ So we *could* consider a document-level relative score, e.g.  $1 - 2P(c_1|d)$  (for a two-class problem)
- ▶ But this turns out to be *useless*, since the predictions of class are **highly separated**

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- ▶ A better solution is to score a test document as the **arithmetic mean** of the **scores of its words**
- ▶ This is exactly the solution proposed by LBG (2003)
- ▶ Beauchamp (2012) proposes a “Bayesscore” which is the arithmetic mean of the log difference word scores in a document – which yields extremely similar results

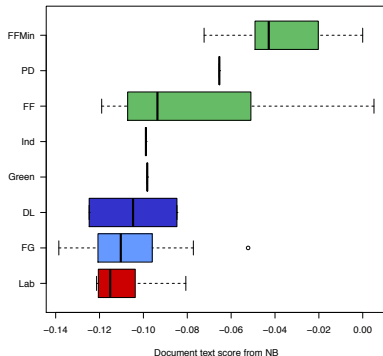
And now for some demonstrations with data...

# Outline

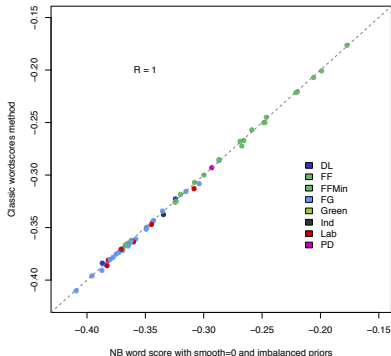
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# Application 1: Daily speeches from LBG (2003)

(a) NB Speech scores by party, smooth=0, imbalanced priors



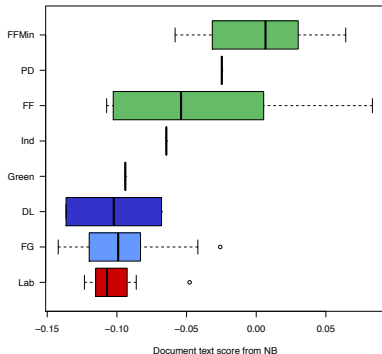
(b) Document scores from NB v. Classic Wordscores



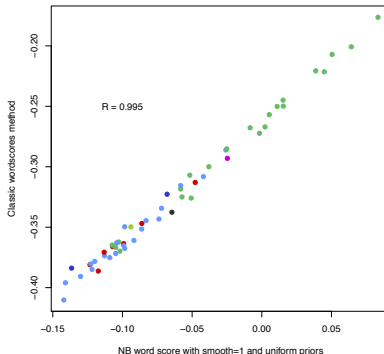
- ▶ three reference classes (Opposition, Opposition, Government) at  $\{-1, -1, 1\}$
- ▶ no smoothing

# Application 1: Daily speeches from LBG (2003)

(c) NB Speech scores by party, smooth=1, uniform class priors



(d) Document scores from NB v. Classic Wordscores

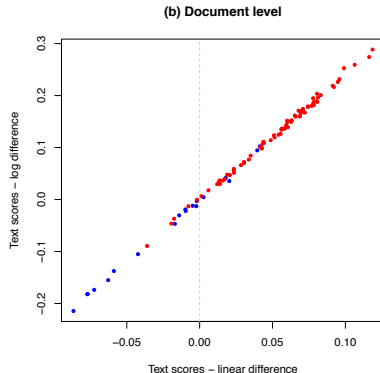
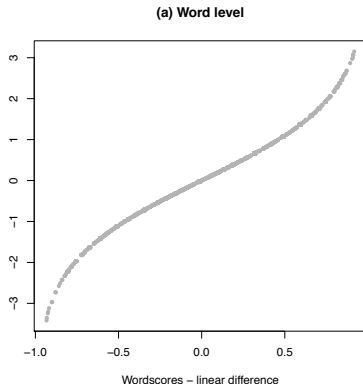


- ▶ two reference classes (Opposition+Opposition, Government) at  $\{-1, 1\}$
- ▶ Laplace smoothing



## Application 2: Classifying legal briefs (Evans et al 2007)

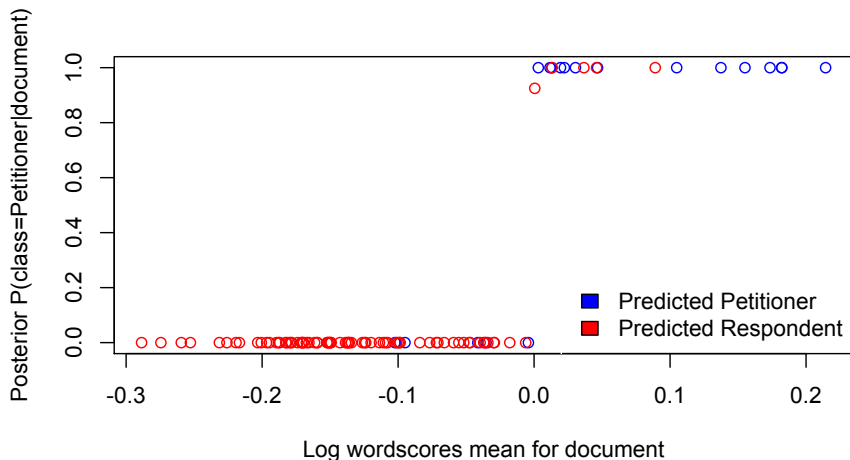
### Wordscores v. Bayesscore



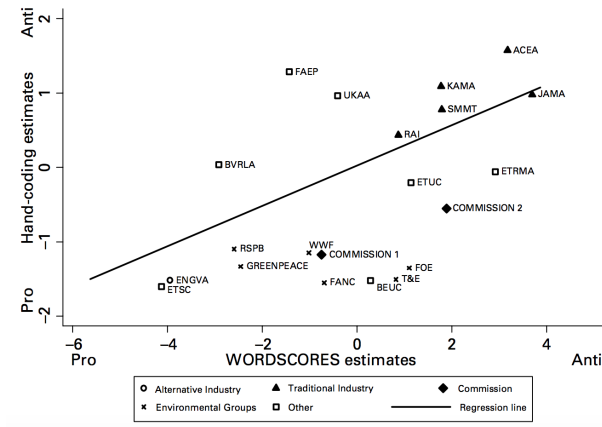
- ▶ Training set: **P**etitioner and **R**espondent litigant briefs from *Grutter/Gratz v. Bollinger* (a U.S. Supreme Court case)
- ▶ Test set: 98 amicus curiae briefs (whose **P** or **R** class is known)

## Application 2: Classifying legal briefs (Evans et al 2007)

### Posterior class prediction from NB versus log wordscores

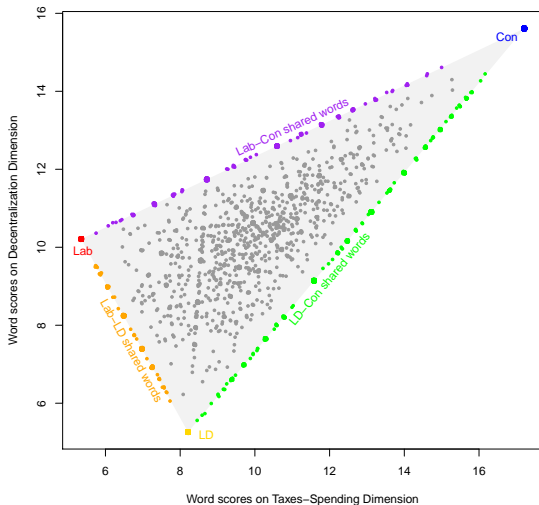


## Application 3: Scaling environmental interest groups (Klüver 2009)



## Application 4: LBG's British manifestos

### More than two reference classes



- ▶ x-axis: Reference scores of  $\{5.35, 8.21, 17.21\}$  for Lab, LD, Conservatives
- ▶ y-axis: Reference scores of  $\{10.21, 5.26, 15.61\}$