Quantitative text analysis: Supervised Scaling Methods

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MY 459: Quantitative Text Analysis

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Course website: lse-my459.github.io

Overview of text as data methods

Outline

From Classification to Scaling

- Machine learning focuses on identifying classes (classification), while social science is typically interested in locating things on latent traits (scaling), e.g.:
  - Policy positions on economic vs social dimension
  - Inter- and intra-party differences
  - Soft news vs hard news
  - Petitioner vs respondent in legal briefs
  - ...and any other continuous scale
- But the two methods overlap and can be adapted – will demonstrate later using the Naive Bayes classifier
- In fact, the class predictions for a collection of words from NB can be adapted to scaling

Outline

Supervised scaling methods

Wordscores method (Laver, Benoit & Garry, 2003):

- ▶ Two sets of texts
  - ▶ Reference texts: texts about which we know something (a scalar dimensional score)
  - ▶ Virgin texts: texts about which we know nothing (but whose dimensional score we'd like to know)
- ▶ These are analogous to a "training set" and a "test set" in classification
- ▶ Basic procedure:
  1. Analyze reference texts to obtain word scores
  2. Use word scores to score virgin texts

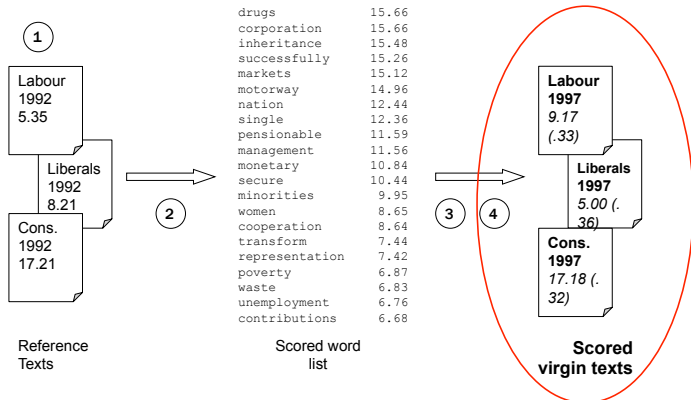# Wordscores Procedure

The Wordscore Procedure
(Using the UK 1997-2001 Example)

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corporation	15.66
inheritance	15.48
successfully	15.26
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motorway	14.96
nation	12.44
single	12.36
pensionable	11.59
management	11.56
monetary	10.84
secure	10.44
minorities	9.95
women	8.65
cooperation	8.64
transform	7.44
representation	7.42
poverty	6.87
waste	6.83
unemployment	6.76
contributions	6.68

① **Labour 1992 5.35**

Liberals 1992 8.21

Cons. 1992 17.21

Reference Texts

②

Scored word list

③ ④

Labour 1997 *9.17 (.33)*

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Scored virgin texts

Step 1: Obtain reference texts with a priori known positions (`setref`)
Step 2: Generate word scores from reference texts (`wordscore`)
Step 3: Score each virgin text using word scores (`textscore`)
Step 4: (optional) Transform virgin text scores to original metric

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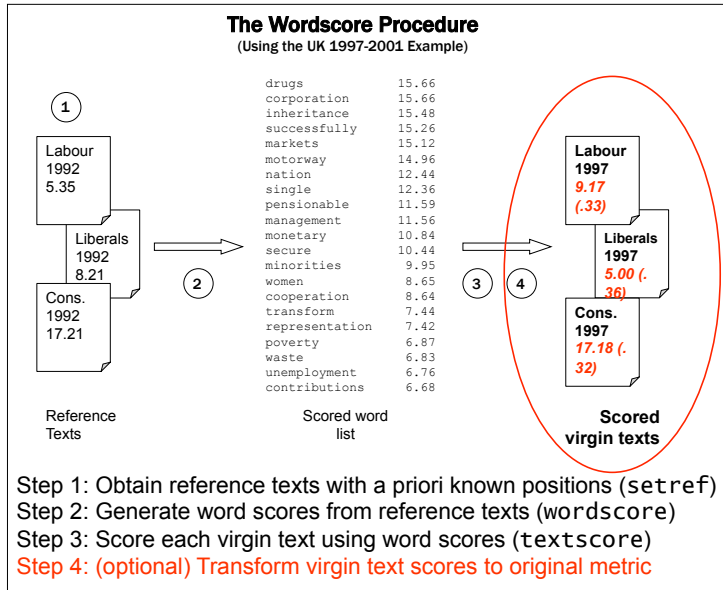- Liberals 1997 5.00 (.36)
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1 — 2 — Scored word list — 3 — 4

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▶ Start with a set of I *reference* texts, represented by an $I \times J$ document-feature matrix C_{ij}, where i indexes the document and j indexes the J total word types

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  - This can be on a scale metric, such as 1–20
  - Can use arbitrary endpoints, such as -1, 1
- We *normalize* the document-feature matrix within each document by converting C_{ij} into a *relative* document-feature matrix (within document), by dividing C_{ij} by its word total marginals:

$$F_{ij} = \frac{C_{ij}}{C_{i\cdot}} \tag{1}$$

where $C_{i\cdot} = \sum_{j=1}^{J} C_{ij}$

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▶ This tells us the probability that given the observation of a specific word j, that we are reading a text of a certain reference document i

- ▶ Assume we have two reference texts, A and B

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- ▶ Assume we have two reference texts, A and B
- ▶ The word "choice" is used 10 times per 1,000 words in Text A and 30 times per 1,000 words in Text B
- ▶ So F_i "choice" $= \{.010, .030\}$
- ▶ If we know only that we are reading the word choice in one of the two reference texts, then probability is 0.25 that we are reading Text A, and 0.75 that we are reading Text B

$$P_{A \text{ "choice"}} = \frac{.010}{(.010 + .030)} = 0.25 \tag{3}$$

$$P_{B \text{ "choice"}} = \frac{.030}{(.010 + .030)} = 0.75 \tag{4}$$

Wordscores mathematically: Word scores

► Compute a J-length "score" vector S for each word j as the average of each document i's scores a_i, weighted by each word's P_{ij}:

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- This procedure will yield a single "score" for every word that reflects the balance of the scores of the reference documents, weighted by the relative document frequency of its normalized term frequency

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- Continuing with our example:
    - We "know" (from independent sources) that Reference Text A has a position of -1.0, and Reference Text B has a position of $+1.0$
    - The score of the word "choice" is then $0.25(-1.0) + 0.75(1.0) = -0.25 + 0.75 = +0.50$

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- ▶ Note that new words outside of the set J may appear in the K virgin documents — these are simply ignored (because we have no information on their scores)

- ▶ Note also that nothing prohibits reference documents from also being scored as virgin documents

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- ▶ Martin and Vanberg (2008) have proposed alternatives to the LBG (2003) rescaling

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- An alternative would be to bootstrap the textual data prior to constructing C_{ij} and C_{kj} — see Lowe and Benoit (2012)

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- ▶ Language-blind: all we need to know are reference scores
- ▶ Could potentially work on texts like this:

(See http://www.kli.org)

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- Need to be from the same lexical universe as virgin texts
- Should contain lots of words

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- ▶ With three or more reference values, the mid-point is mapped onto a multi-dimensional simplex. The values now matter but only in relative terms (we are still investigating this fully)

Multinomial Bayes model of Class given a Word
Class posterior probabilities

$$P(c_k|w_j) = \frac{P(w_j|c_k)P(c_k)}{P(w_j|c_k)P(c_k) + P(w_j|c_{\neg k})P(c_{\neg k})}$$

▶ This represents the posterior probability of membership in class k for word j

▶ Under *certain conditions*, this is identical to what LBG (2003) called P_{wr}

▶ Under those conditions, the LBG "wordscore" is the linear difference between $P(c_k|w_j)$ and $P(c_{\neg k}|w_j)$

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- Consider two "reference" scores s_1 and s_2 attached to two classes $k = 1$ and $k = 2$. Taking P_1 as the posterior $P(k = 1|w = j)$ and P_2 as $P(k = 2|w = j)$, A generalised score s_j^* for the word j is then

$$
\begin{aligned}
s_j^* &= s_1 P_1 + s_2 P_2 \\
&= s_1 P_1 + s_2(1 - P_1) \\
&= s_1 P_1 + s_2 - s_2 P_1 \\
&= P_1(s_1 - s_2) + s_2
\end{aligned}
$$

▶ For more than two reference classes, if the reference scores are ordered such that $s_1 < s_2 < \cdots < s_K$, then

$$
\begin{aligned}
s_j^* &= s_1 P_1 + s_2 P_2 + \cdots + s_K P_K \\
&= s_1 P_1 + s_2 P_2 + \cdots + s_K \left(1 - \sum_{k=1}^{K-1} P_k\right) \\
&= \sum_{k=1}^{K-1} P_i (s_k - s_K) + s_I
\end{aligned}
$$

A simpler formulation:
Use reference scores such that $s_1 = -1.0, s_K = 1.0$

- From above equations, it should be clear that any set of reference scores can be linearly rescaled to endpoints of $-1.0, 1.0$
- This simplifies the "simple word score"

$$s_j^* = (1 - 2P_1) + \sum_{k=2}^{K-1} P_k(s_k - 1)$$

- which simplifies with just two reference classes to:

$$s_j^* = 1 - 2P_1$$

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- There is a special role for reference classes in between $-1.0, 1.0$, as they balance between "pure" classes — more in a moment

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▶ There is a special role for reference classes in between $-1.0, 1.0$, as they balance between "pure" classes — more in a moment

▶ There are alternative scaling models, such that used in Beauchamp's (2012) "Bayesscore", which is simply the difference in logged class posteriors at the word level. For $s_1 = -1.0$, $s_2 = 1.0$,

$$
\begin{aligned}
s_j^B &= -\log P_1 + \log P_2 \\
&= \log \frac{1 - P_1}{P_1}
\end{aligned}
$$

Moving to the document level

▶ The "Naive" Bayes model of a joint document-level class posterior assumes conditional independence, to multiply the word likelihoods from a "test" document, to produce:

$$P(c|d) = P(c) \, \frac{\prod_j P(w_j|c)}{P(w_j)}$$

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▶ So we *could* consider a document-level relative score, e.g. $1 - 2P(c_1|d)$ (for a two-class problem)

▶ But this turns out to be *useless*, since the predictions of class are highly separated

Moving to the document level

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▶ This is exactly the solution proposed by LBG (2003)

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- ▶ A better solution is to score a test document as the arithmetic mean of the scores of its words
- ▶ This is exactly the solution proposed by LBG (2003)
- ▶ Beauchamp (2012) proposes a "Bayesscore" which is the arithmetic mean of the log difference word scores in a document – which yields extremely similar results

And now for some demonstrations with data…

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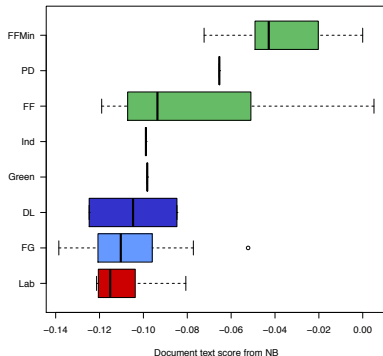
Application 1: Dail speeches from LBG (2003)
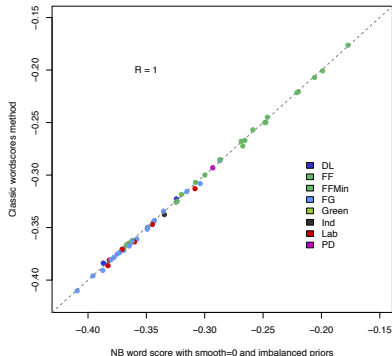
- three reference classes (Opposition, Opposition, Government) at {-1, -1, 1}
- no smoothing

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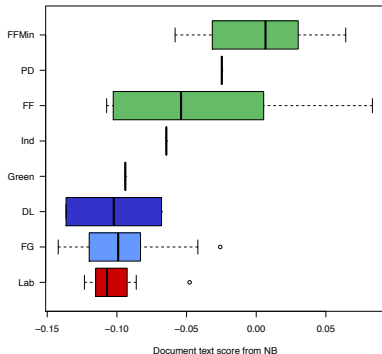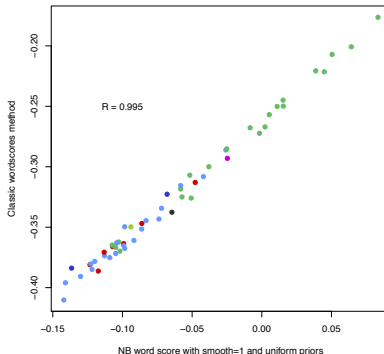
- two reference classes (Opposition+Opposition, Government) at {-1, 1}
- Laplace smoothing

# Application 2: Classifying legal briefs (Evans et al 2007)
Wordscores v. Bayesscore
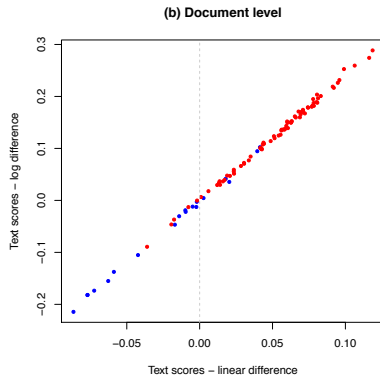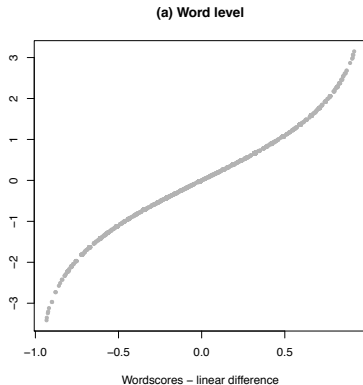
- Training set: Petitioner and Respondent litigant briefs from *Grutter/Gratz v. Bollinger* (a U.S. Supreme Court case)
- Test set: 98 amicus curiae briefs (whose P or R class is known)

Application 2: Classifying legal briefs (Evans et al 2007)
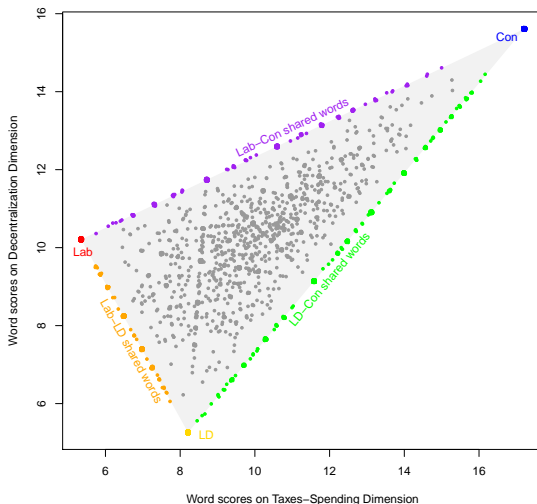## Posterior class prediction from NB versus log wordscores
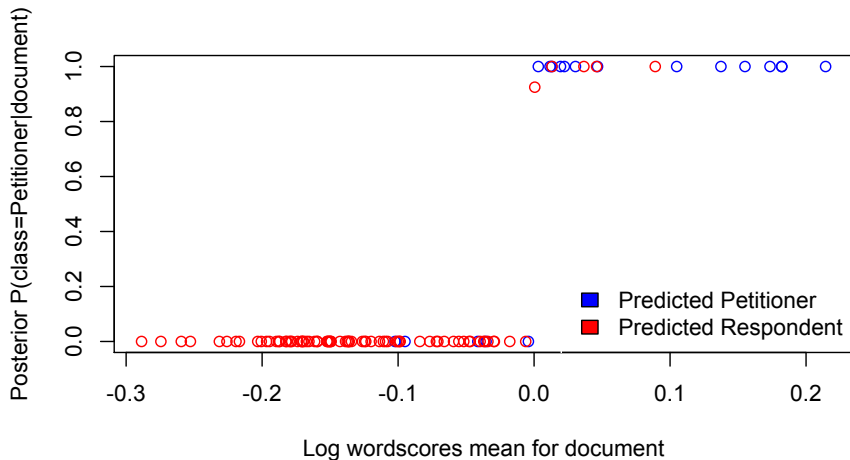
Application 3: Scaling environmental interest groups (Klüver 2009)
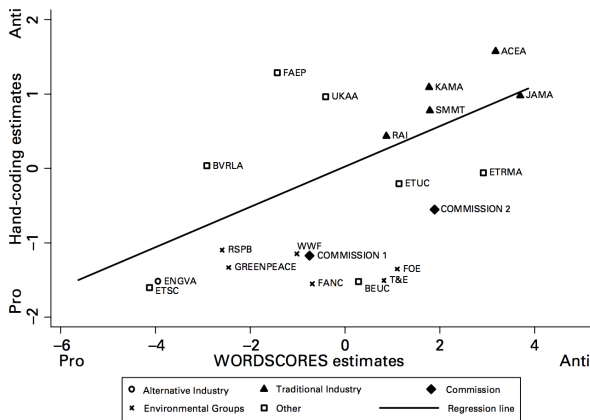
- Dataset: text of online consultation on EU environmental regulations
- Reference texts: most extreme pro- and anti-regulation groups

Application 4: LBG's British manifestos
More than two reference classes

- *x*-axis: Reference scores of {5.35, 8.21, 17.21} for Lab, LD, Conservatives
- *y*-axis: Reference scores of {10.21, 5.26, 15.61}