Decoherence versus Dynamical Casimir Effect

Ralf Schützhold* **and Markus Tiersch**+

Institut für Theoretische Physik, Technische Universität Dresden,
01062 Dresden, Germany

Abstract. By means of two simple examples: phase and amplitude damping, the impact of decoherence on the dynamical Casimir effect is investigated. Even without dissipating energy (i.e., pure phase damping), the amount of created particles can be diminished significantly via the coupling to the environment (reservoir theory) inducing decoherence. For a simple microscopic model, it is demonstrated that spontaneous decays within the medium generate those problems – Rabi oscillations are far more advantageous in that respect. These findings are particularly relevant in view of a recently proposed experimental verification of the dynamical Casimir effect.

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1. Introduction

Nearly a quarter of a century after the discovery of the static Casimir effect [1], it has been realised that the (non-inertial) motion of one of the mirrors should induce the creation of real particles out of the virtual quantum vacuum fluctuations [2]. Unfortunately, this striking prediction (the dynamical Casimir effect) has not yet been verified experimentally.

To this end, it should be advantageous to exploit the phenomenon of parametric resonance through a periodic perturbation of the discrete eigen-frequencies of a finite cavity with the external (perturbation) frequency matching twice the unperturbed eigen-frequency of one of the cavity modes, for example. In the exact resonance case, the effective (time-averaged) Hamiltonian $\hat{H}_{\rm eff}$ can be derived by means of the rotating wave approximation. If the aspect ratio of the cavity and the resonant mode is chosen such that there is no resonant inter-mode coupling (see, e.g., [3]), the effective Hamiltonian is just the generator of a squeezing operator ($\hbar = c = 1$ throughout)

$$\hat{H}_{\rm eff} = i\,\xi\left[(\hat{a}^\dagger)^2 - \hat{a}^2\right]\,, \tag{1}$$

with ξ depending on the strength of the perturbation etc., and $\hat{a}^\dagger$, $\hat{a}$ denoting the usual bosonic creation and annihilation operators for the resonant mode with the well-known commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. For example, if the perturbation is induced by an oscillation of the dielectric permittivity of a thin dielectric slab with thickness a at one

* corresponding author: schuetz@theory.phy.tu-dresden.de
+ markus@theory.phy.tu-dresden.de

of the walls of the cavity with length $L \gg a$, the squeezing parameter ξ depends on the wave-number $k_\|$ parallel to the thin slab and the external (perturbation) frequency ω in the following way [3]

$$\xi = \frac{1}{2}\frac{k_\|^2}{\omega}\frac{a}{L}\chi\,, \tag{2}$$

with χ denoting the amplitude of oscillation of the inverse dielectric permittivity. The dynamically squeezed vacuum state contains an exponentially growing particle number

$$\langle\hat{n}\rangle = \langle 0|\exp\{+i\hat{H}_{\rm eff}t\}\,\hat{n}\,\exp\{-i\hat{H}_{\rm eff}t\}\,|0\rangle = \sinh^2(2\xi t)\,, \tag{3}$$

which should facilitate the measurement (in the ideal case).

Of course, aiming at an experimental verification of the dynamical Casimir effect (cf. [4]), deviations from the ideal behaviour described above have to be taken into account as well. Previous studies have been devoted to the effects of detuning (i.e., a deviation of the external frequency from the exact resonance) and losses (i.e., a finite quality factor of the cavity), see, e.g., [5]. However, squeezed states such as $\exp\{-i\hat{H}_{\rm eff}t\}\,|0\rangle$ are highly non-classical states (see, e.g., [6, 7, 8]) and therefore particularly vulnerable to decoherence in a more general form (see, e.g., [9]).

The impact of decoherence on the dynamical Casimir effect will be studied in this Article by means of some representative examples for decoherence channels as well as an explicit microscopic model. It should be mentioned here that the following investigations are devoted to the decoherence of the photon field due to the coupling to the environment – not to be confused with the decoherence of the quantised position of a mirror due to the photon field (i.e, the dynamical Casimir effect) as discussed in [10].

2. Reservoir

In the following, we discuss a very simplified model for the reservoir – which nevertheless yields some insight into the basic mechanism and illustrates the main idea. Let us consider a weak interaction of the mode $\hat{a}^\dagger, \hat{a}$ under investigation to a reservoir containing many degrees of freedom, which will be approximated by independent two-level systems. Without any loss of generality, the undisturbed Hamiltonian of the two-level systems can be cast (after a suitable internal rotation) into the form $\sum_I \omega_I \sigma_z^I$ with $2\omega_I$ denoting their intrinsic energy gap and $\vec{\sigma}^I$ being the usual Pauli spin matrices. The undisturbed Hamiltonian of the mode $\hat{a}^\dagger, \hat{a}$ without squeezing is just $\Omega\hat{a}^\dagger\hat{a}$. In this simple model for the reservoir, the most general first-order interaction Hamiltonian reads

$$\hat{H}_{\rm int}^{(1)} = \sum_I \left(\hat{a}^\dagger\,\vec{\lambda}_I\cdot\vec{\sigma}_I + \hat{a}\,\vec{\lambda}_I^*\cdot\vec{\sigma}_I\right)\,, \tag{4}$$

where $\vec{\lambda}_I$ denote the small $|\vec{\lambda}_I| \ll \Omega$ first-order coupling constants. Since the coupling is supposed to be small, we may apply the rotating wave approximation after which only the resonant terms ($\omega_I = \Omega$) survive (see, e.g., [8, 7])

$$\hat{H}_{\rm int}^{(1)} \stackrel{\rm RWA}{=} \sum_{\omega_I=\Omega} \lambda_I\left(\hat{a}^\dagger\sigma_-^I + \hat{a}\sigma_+^I\right)\,. \tag{5}$$

Consequently, this effective Hamiltonian consists of hopping operators and describes the decay channel. Now let us consider the most general second-order interaction Hamiltonian

$$\hat{H}_{\text{int}}^{(2)} = \sum_I \left((\hat{a}^\dagger)^2 \vec{\zeta}_I \cdot \vec{\sigma}_I + \hat{a}^\dagger \hat{a}\, \vec{\Lambda}_I \cdot \vec{\sigma}_I + \hat{a}^2 \vec{\zeta}_I^{\,*} \cdot \vec{\sigma}_I \right) , \tag{6}$$

containing the second-order coupling constants $\vec{\zeta}_I$ and $\vec{\Lambda}_I$ which are usually much smaller than the first-order couplings λ_I. Again applying the rotating wave approximation

$$\hat{H}_{\text{int}}^{(2)} \overset{\text{RWA}}{=} \sum_I \hat{a}^\dagger \hat{a}\, \Lambda_I \sigma_z + \sum_{\omega_I = 2\Omega} \zeta_I \left((\hat{a}^\dagger)^2 \sigma_-^I + \hat{a}^2 \sigma_+^I \right) , \tag{7}$$

we observe that the second term describes resonant two-particle hopping and is usually small compared to one-particle hopping $\hat{H}_{\text{int}}^{(1)}$ since the second-order perturbation ζ_I is supposed to be smaller than the first order λ_I.

However, the first term contains a sum over all reservoir modes – without restrictions due to resonance conditions – and thus may well be of comparable magnitude or even larger than $\hat{H}_{\text{int}}^{(1)}$. This term corresponds to weak measurements (see, e.g., [9]) of the particle number $\hat{a}^\dagger \hat{a}$ without dissipating energy (as $[\hat{a}^\dagger \hat{a}\, \Lambda_I \sigma_z, \hat{a}^\dagger \hat{a}] = 0$) and is called phase damping. In summary, we obtain the two leading contributions

$$\hat{H}_{\text{int}}^{\text{RWA}} = \sum_{\omega_I = \Omega} \lambda_I \left(\hat{a}^\dagger \sigma_-^I + \hat{a} \sigma_+^I \right) + \sum_I \hat{a}^\dagger \hat{a}\, \Lambda_I \sigma_z \,, \tag{8}$$

where the first (hopping) term describes amplitude damping (decay channel) and the second contribution corresponds to phase damping.

After averaging over the degrees of freedom of the reservoir by means of the usual (e.g., Born-Markov [7, 8]) approximations, we obtain the master equation in the Lindblad form (see, e.g., [9])

$$\frac{d\hat{\varrho}}{dt} = -i \left[\hat{H}_{\text{eff}}, \hat{\varrho} \right] + \sum_\alpha \left(2\hat{L}_\alpha\, \hat{\varrho}\, \hat{L}_\alpha^\dagger - \left\{ \hat{L}_\alpha^\dagger \hat{L}_\alpha, \hat{\varrho} \right\} \right) , \tag{9}$$

with two different Lindblad operators $\hat{L}_\alpha$. The first one $\hat{L}_1$ arises from the first term in $\hat{H}_{\text{int}}^{\text{RWA}}$ and describes amplitude damping $\hat{L}_1 = \sqrt{\gamma}\, \hat{a}$, whereas the second contribution in $\hat{H}_{\text{int}}^{\text{RWA}}$ generates phase damping with the Lindblad operator $\hat{L}_2 = \sqrt{\Gamma}\, \hat{a}^\dagger \hat{a}$. The two damping coefficients γ and Γ are determined by the coupling constants λ_I and Λ_I as well as the number of involved degrees of freedom of the reservoir. Since this number is much larger for phase damping due to the absence of resonance conditions, the phase damping rate Γ can be as large as or even larger than the amplitude damping rate γ in spite of the fact that the second-order coupling constants Λ_I are usually much smaller than the first order λ_I.

3. Master Equation and Particle Creation

For the two decoherence channels discussed above, phase and amplitude damping, we shall now investigate the impact of decoherence on the amount of created particles

$\hat{n} = \hat{a}^\dagger\hat{a}$. The mode under consideration $\hat{a}^\dagger, \hat{a}$ is described by the effective density matrix $\hat{\varrho}$ whose evolution is governed by the master equation (9)

$$\frac{d}{dt}\hat{\varrho} = \xi[(\hat{a}^\dagger)^2 - \hat{a}^2, \hat{\varrho}] + \frac{\Gamma}{2}\left(2\hat{n}\hat{\varrho}\hat{n} - \{\hat{n}^2, \hat{\varrho}\}\right) + \frac{\gamma}{2}\left(2\hat{a}\hat{\varrho}\hat{a}^\dagger - \{\hat{n}, \hat{\varrho}\}\right), \quad (10)$$

with the squeezing parameter ξ and the two damping coefficients γ and Γ representing the amplitude damping rate and the phase damping rate, respectively.

Instead of deriving the complete solution $\hat{\varrho}(t)$ of the above equation, we just focus on the most interesting quantity, the time-dependent expectation value of the particle number (in the interaction picture with $d\hat{n}/dt = 0$)

$$\frac{d}{dt}\langle\hat{n}\rangle = \frac{d}{dt}\,\mathrm{Tr}\,\{\hat{\varrho}(t)\,\hat{n}\} = \mathrm{Tr}\left\{\left(\frac{d}{dt}\hat{\varrho}(t)\right)\hat{n}\right\}. \quad (11)$$

After inserting the master equation (10), only the first term (ξ) and the amplitude damping term (γ) contribute (as phase damping does not dissipate energy), and, after some algebra, one obtains

$$\frac{d}{dt}\langle\hat{n}\rangle = 2\xi\langle\hat{a}^{\dagger 2} + \hat{a}^2\rangle - \gamma\langle\hat{n}\rangle\,. \quad (12)$$

In view of the squeezing generated by $\hat{H}_{\rm eff}$, the expectation value $\langle\hat{a}^{\dagger 2} + \hat{a}^2\rangle$ will be non-zero in general. By using the same method as above, one can derive the time-dependence of this quantity

$$\frac{d}{dt}\langle\hat{a}^{\dagger 2} + \hat{a}^2\rangle = 8\xi\langle\hat{n}\rangle - (2\Gamma + \gamma)\langle\hat{a}^{\dagger 2} + \hat{a}^2\rangle + 4\xi \quad (13)$$

which fortunately closes the system of equations. This system of two first-order differential equations may be solved easily and gives the general solution

$$\begin{aligned}\langle\hat{n}\rangle(t) = {} & C_+\frac{\Gamma + \sqrt{\Gamma^2 + 16\xi^2}}{8\xi}\,\exp\left\{-(\gamma+\Gamma) + \sqrt{\Gamma^2 + 16\xi^2}\right\} \\ & + C_-\frac{\Gamma - \sqrt{\Gamma^2 + 16\xi^2}}{8\xi}\,\exp\left\{-(\gamma+\Gamma) - \sqrt{\Gamma^2 + 16\xi^2}\right\} \\ & + \frac{8\xi^2}{2\Gamma\gamma + \gamma^2 - 16\xi^2}\,, \end{aligned} \quad (14)$$

where $C_\pm$ are integration constants. After incorporating the initial conditions $\langle\hat{n}\rangle(t=0) = n_0$ and $\langle(\hat{a}^\dagger)^2 + \hat{a}^2\rangle(t=0) = 0$ (which holds for any initial density matrix that is diagonal in the particle number basis such as a thermal ensemble) one arrives at

$$\begin{aligned}\langle\hat{n}\rangle(t) = {} & \frac{\sqrt{\Gamma^2 + 16\xi^2} + \Gamma}{4\sqrt{\Gamma^2 + 16\xi^2}}\left(2n_0 + \frac{\sqrt{\Gamma^2 + 16\xi^2} - \Gamma}{\sqrt{\Gamma^2 + 16\xi^2} - \Gamma - \gamma}\right)e^{+(\sqrt{\Gamma^2+16\xi^2}-\Gamma-\gamma)t} \\ & + \frac{\sqrt{\Gamma^2 + 16\xi^2} - \Gamma}{4\sqrt{\Gamma^2 + 16\xi^2}}\left(2n_0 + \frac{\sqrt{\Gamma^2 + 16\xi^2} + \Gamma}{\sqrt{\Gamma^2 + 16\xi^2} + \Gamma + \gamma}\right)e^{-(\sqrt{\Gamma^2+16\xi^2}+\Gamma+\gamma)t} \\ & - \frac{1}{2}\frac{16\xi^2}{\Gamma^2 + 16\xi^2 - (\Gamma+\gamma)^2}\,. \end{aligned} \quad (15)$$

Note that the apparent divergence at $\sqrt{\Gamma^2 + 16\xi^2} = \Gamma + \gamma$ disappears since the singularities in the first and last terms cancel each other.

As a consistency check, we reproduce Eq. (3) for the initial vacuum state $n_0 = 0$ and no decoherence $\Gamma = \gamma = 0$. Furthermore, for negligible phase damping $\Gamma \ll \xi$, the characteristic exponent is given by $4\xi - \gamma$ in accordance with [5]. I.e., one only obtains exponential growth if the particle creation process (i.e., squeezing) is faster than the decay $4\xi > \gamma$ (as one might expect, cf. [5]). In the presence of additional phase damping, this threshold is shifted to $4\xi > \sqrt{\gamma^2 + 2\gamma\Gamma}$ according to Eq. (15).

Let us study the behaviour at very short and late times, respectively. For a positive amplitude damping rate $\gamma > 0$, the particle number $\langle\hat{n}\rangle(t)$ first decreases linearly in time (unless $n_0 = 0$, of course) as the squeezing mechanism sets in at quadratic order t^2 only. At late times, the particle number either increases exponentially (above the threshold $4\xi > \sqrt{\gamma^2 + 2\gamma\Gamma}$) or settles down to a constant (and for $\xi > 0$ positive) value (below the threshold $4\xi < \sqrt{\gamma^2 + 2\gamma\Gamma}$).

4. Decoherence without Dissipation – Phase Damping

Without amplitude damping $\gamma = 0$ (high quality factor), the situation is qualitatively different from the scenario with dissipation. For short times, the number of particles increases quadratically and for late times, the particle number is always growing exponentially, but the characteristic exponent is reduced. For fast decoherence rates $\Gamma \gg \xi$ (and a high quality factor), the leading term behaves as

$$\langle\hat{n}\rangle(t) \approx \left(n_0 + \frac{1}{2}\right) \exp\left\{\frac{8\xi^2}{\Gamma}\,t\right\} - \frac{1}{2}\,, \tag{16}$$

i.e., the particle creation rate is strongly reduced by a factor of order $\Gamma/\xi \gg 1$. (Note that, for fast decoherence $\Gamma \gg \xi$ and intermediate times $1/\Gamma \ll t \ll \Gamma/\xi^2$, one obtains quasi-linear behaviour according to the equation above.)

The reduction of the number of created particles can be understood in the following way: The permanent weak measurement of the particle number implies a suppression of the off-diagonal elements of the density matrix (i.e., the interference terms) in the particle number basis (see, e.g., [9]). E.g., without squeezing $\xi = 0$, we would have

$$\varrho_{mn}(t) = \langle n|\,\hat{\varrho}(t)\,|m\rangle = \varrho_{mn}(0)\exp\left\{-\frac{\Gamma}{2}(m-n)^2 t\right\}\,. \tag{17}$$

A strong – i.e., decisive – measurement of the particle number would set all off-diagonal elements to zero. On the other hand, squeezing is a highly non-classical effect (see, e.g., [6, 7, 8]) and the related mechanism for particle creation involves the transfer of occupation from the diagonal to the off-diagonal elements and back

$$\varrho_{mn} = \begin{pmatrix} \varrho_{00} & \rightarrow & \varrho_{02} & \cdots \\ \downarrow & \varrho_{11} & \downarrow & \cdots \\ \varrho_{20} & \rightarrow & \varrho_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}\,. \tag{18}$$

Therefore, the suppression of the off-diagonal elements diminishes this effect. Roughly speaking, we lose the phase information/phase control which is necessary for constructive interference and resonance.

For fast decoherence $\Gamma \gg \xi$, one could visualise the major effect via a *gedanken* experiment combining the time-evolution governed by $\hat{H}_{\rm eff}$ with permanently repeated strong measurements of the particle number at equidistant time intervals $\Delta t = \mathcal{O}(1/\Gamma)$. In the Heisenberg picture, the unitary evolution between two measurements can be described by a Bogoliubov transformation

$$\hat{a}(t + \Delta t) = \alpha(\Delta t)\hat{a}(t) + \beta(\Delta t)\hat{a}^\dagger(t)\,, \tag{19}$$

with the Bogoliubov coefficients obeying the bosonic unitarity relation $|\alpha|^2 - |\beta|^2 = 1$. For an initial state $\hat{\varrho}$ which is diagonal in particle number basis (such as after a strong measurement of $\hat{n}$), the expectation value evolves as

$$\langle \hat{n}(t + \Delta t)\rangle = \langle \hat{n}(t)\rangle + |\beta(\Delta t)|^2[1 + 2\langle \hat{n}(t)\rangle]\,. \tag{20}$$

In the second term on the right-hand side, one may distinguish the pure quantum vacuum effect $|\beta(\Delta t)|^2$ and the contribution of classical resonance $2|\beta(\Delta t)|^2\langle \hat{n}(t)\rangle$. For fast decoherence $\Gamma \gg \xi$, the time intervals are small $\xi\Delta t \ll 1$ facilitating the introduction of an approximate differential equation $\Delta t \to dt$

$$\frac{d}{dt}\,\langle \hat{n}(t)\rangle \propto \frac{\xi^2}{\Gamma}[1 + 2\langle \hat{n}(t)\rangle]\,, \tag{21}$$

where we have used $|\beta(\Delta t)|^2 = \sinh^2(2\xi\Delta t) \approx 4\xi^2\Delta t^2$ and $\Delta t = \mathcal{O}(1/\Gamma)$. We observe that the pure quantum vacuum effect $d\langle \hat{n}(t)\rangle/dt \propto \xi^2/\Gamma$ would only yield a linear increase $\propto t\xi^2/\Gamma$ (permanent reshuffling from diagonal to off-diagonal elements of ϱ_{mn} and accumulating higher diagonal terms), but the classical resonance contributions $d\langle \hat{n}(t)\rangle/dt \propto 2\langle \hat{n}(t)\rangle\xi^2/\Gamma$ amplify all particles present and thereby lead to an exponential growth with a reduced exponent $\propto t\xi^2/\Gamma$ as in Eq. (16).

5. Microscopic Model

In order to apply the above results to an explicit scenario, one has to estimate the magnitude of the involved quantities – such as the phase and amplitude damping rates Γ and γ, respectively. This can be achieved either by experimental means, i.e., measuring the quantum coherence time ($\propto 1/\Gamma$) and the quality factor ($\propto 1/\gamma$) of the used cavity etc., or via theoretical calculations – based on a microscopic model, for example. In the following, we shall present a very simple microscopic model for a material with a time-dependent index of refraction – which, nevertheless, yields some interesting conclusions.

Let us consider a specifically designed semi-conductor at low temperatures, for example, whose localised single valence electrons (far below the conducting band) can occupy three levels described by the amplitudes $\psi_{a,b,c}(t)$. All the other levels are supposed to be unimportant and are not occupied at all. Illuminating the semi-conductor with a strong external Laser beam tuned to the frequency of transition from the lowest-lying electronic state a to the first excited state b, we can manipulate the amplitudes of the electronic states $\psi_{a,b}(t)$. In addition, a small and slowly varying electric test field $\mathcal{E}$ (e.g., micro-waves) is acting on the three-level system. The spatial distribution of the ground state a is supposed to be very compact such that it is basically unaffected by the electric test field $\mathcal{E}(t)$; but the excited states b and c are more spread out and hence couple to $\mathcal{E}(t)$ with the dipole moment κ (dipole approximation). In the rotating wave approximation, the Lagrangian of the described three-level system reads

$$\mathcal{L} = i\psi_a^*\dot{\psi}_a + i\psi_b^*\dot{\psi}_b + i\psi_c^*\dot{\psi}_c - \Delta\omega\,\psi_c^*\psi_c + (\kappa\,\mathcal{E}(t)\psi_b^*\psi_c + \Omega_{\rm R}(t)\psi_a^*\psi_b + \text{h.c.})\,, \qquad (22)$$

with $\Delta\omega$ denoting the energy gap between states b and c, and $\Omega_{\rm R}(t)$ the Rabi frequency of the strong external Laser field (see, e.g., [7, 8]). Since $\mathcal{E}(t)$ is small, we may use linear response theory ($\psi_c \ll \psi_a, \psi_b$), and employing the adiabatic approximation*

$$\kappa\,\mathcal{E}(t)\psi_b = \Delta\omega\,\psi_c - i\dot{\psi}_c \approx \Delta\omega\,\psi_c\,, \qquad (23)$$

because $\mathcal{E}(t)$ is supposed to be slowly varying, we finally (after eliminating ψ_c) arrive at the contribution to the effective Lagrangian for the test field $\mathcal{E}$

$$\mathcal{L}_{\rm eff} = |\psi_b(t)|^2\frac{\kappa^2}{\Delta\omega}\,\mathcal{E}^2(t) = \frac{\varepsilon_{\rm eff}(t)-1}{2}\,\mathcal{E}^2(t)\,. \qquad (24)$$

Consequently, by means of a Laser-induced excitation of the polarisable b-level $\psi_b(t)$, we may generate a time-dependent effective dielectric permittivity $\varepsilon_{\rm eff}$. This, in turn, can serve as a perturbation for the dynamical Casimir effect [3, 4].

For a rough estimate of the maximum possible order of magnitude of $\varepsilon_{\rm eff}$, we assume one three-level system per lattice site with a dipole coupling κ corresponding to the lattice spacing of a few Ångströms (i.e., the electronic states b and c are spread out over a few Ångströms). The Rabi frequency $\Omega_{\rm R}$ determines the time-scale for the dynamics of the micro-wave test field $\mathcal{E}$ and should be around a few GHz in order to achieve resonance [4]. The adiabatic approximation employed above requires the energy gap $\Delta\omega$ to be much larger than that frequency. If we tune the three-level system in order to make this energy gap as small as possible within the region of validity of the adiabatic approximation, say $\mathcal{O}(100\text{ GHz})$, the effective dielectric permittivity $\varepsilon_{\rm eff}$ can be as large as $\varepsilon_{\rm eff} \leq \mathcal{O}(10^5)$, which is more than sufficient.

6. Spontaneous decay → noise/decoherence

So far, we only considered stimulated transitions $a \leftrightarrow b$ induced by the (external) Laser field. Under this assumption, one obtains an effective Lagrangian (24) leading to a unitary evolution, i.e., no decoherence. As it will become evident below, this result changes drastically for a spontaneous decay back to the ground state a instead of a stimulated transition.

Assuming that the spontaneous decay (involving optical frequencies) occurs much faster than the (slow) dynamics (e.g., micro-wave frequencies) of the test field $\mathcal{E}(t)$, we may adopt the sudden approximation and omit the time-dependence of $\mathcal{E}(t)$ during that process. The influence of the field $\mathcal{E}$ perturbs the polarisable b-state inducing a mixing

* Outside the region of validity of the adiabatic approximation, $\Delta\omega$ should be replaced by the detuning between the frequency of the test field $\mathcal{E}$ and the $b \leftrightarrow c$ transition frequency $\Delta\omega$.

with the c-state and thus an energy-shift, which can be calculated using second-order stationary perturbation theory

$$E_b^{(2)} = E_b^{(0)} + \langle b|\,\hat{\mathcal{E}}\,|b\rangle + \frac{|\,\langle b|\,\hat{\mathcal{E}}\,|c\rangle\,|^2}{E_b^{(0)} - E_c^{(0)}}\,. \tag{25}$$

The first-order term vanishes in the dipole approximation (cf. the selection rules) $\langle b|\,\hat{\mathcal{E}}\,|b\rangle = 0$. Depending on the slowly varying electric field $\mathcal{E}(t)$, the spontaneous decay of the excited state – which is a mixture of the states b and c – releases a different energy $E_b^{(2)}[\mathcal{E}(t)]$ than the originally absorbed Laser photon in general. Note that the totalised energy shift $E_b^{(2)} - E_b^{(0)}$ in Eq. (25) exactly corresponds to the polarisation term in Eq. (24), i.e., the polarisation energy $\mathcal{E}\mathcal{P}$ of the medium

$$\varepsilon_{\rm eff}\mathcal{E}^2 = \mathcal{E}\mathcal{D} = \mathcal{E}^2 + \mathcal{E}\mathcal{P}\,. \tag{26}$$

So far, the electric field $\mathcal{E}$ has been treated as a classical perturbation in Eq. (25) which is strictly valid in the classical limit (coherent state with many micro-wave photons) only. For the quantised micro-wave radiation field in a dynamically squeezed state, the situation is more complicated. Expressed in terms of the relevant mode $\hat{a}^\dagger$, $\hat{a}$ with the frequency Ω, the electric field operator can be estimated by

$$\hat{\mathcal{E}}^2 = \mathcal{O}\left(\frac{\Omega}{V}\,[\hat{a}^\dagger + \hat{a}]^2\right)\,, \tag{27}$$

with V denoting the volume of the cavity. In a squeezed state, the expectation value of $\hat{\mathcal{E}}^2$ and its variance are strongly time-dependent in view of $\hat{a}^\dagger(t) = e^{i\Omega t}\hat{a}^\dagger(0)$. Since we may decompose the squeezed state into a linear superposition of (nearly classical) coherent states, the energy of the emitted photons will fluctuate randomly (depending on the moment of decay). Therefore, spontaneous decays inevitably lead to decoherence – first, by generating energy noise (total energy conservation), and, second, by effectively performing a weak measurement, since the emitted photon carries away some information about the electric field (encoded in its energy, for example – see the discussion below). Comparing the energy shift in Eqs. (25) and (26) with the squeezing parameter in Eq. (2), we may estimate the total noise energy per cycle

$$E_{\rm noise} = \mathcal{O}(N\xi)\,, \tag{28}$$

with N being the total number of micro-wave photons inside the cavity. This noise energy is of the same order of magnitude (with the explicit pre-factor depending on the details of the geometry and the dynamics of $\varepsilon_{\rm eff}$ etc.) as the energy gain due to the dynamical Casimir effect in each cycle – which already demonstrates one of the main drawbacks of spontaneous decays.

The estimate of the phase damping rate Γ generated by spontaneous decays is rather complicated. To this end, it is necessary to study the entanglement between the micro-wave mode $\hat{a}^\dagger$, $\hat{a}$ under consideration and the reservoir, cf. [9]. One indicator is the amount of information about the quantum state of the micro-wave field (in particular the particle number) leaking out – which can be used as a lower bound for $\Gamma_{\rm decay}$. Ergo, the question is: how much can we learn about the number of micro-wave photons inside the cavity by looking at the (optical) photons emitted during the spontaneous decays? Since the spontaneous decays have to be repeated with a frequency of order Ω in order to achieve resonance [4], the line-width will also be of order Ω. Let M denote the number of absorbed Laser photons – and hence excited three-level systems. Of course, the same number M of (optical) photons must be emitted by (approximately independent) spontaneous decays in each cycle and hence the variance of the total emitted energy is $\mathcal{O}(\sqrt{M}\Omega)$. This quantity can be contrasted with the accuracy necessary for measuring the total number N of micro-wave photons inside the cavity (by looking at the optical photons emitted). In the presence of N micro-wave photons with an energy of order Ω (resonance condition), the total energy difference in Eqs. (25) and (26) is of order $N\xi$, cf. Eqs. (2) and (27). The ratio of these quantities $N\xi/(\sqrt{M}\Omega)$ determines the information gained in each cycle (i.e., spontaneous decay). As a result, this special type of decoherence becomes important if the number of micro-wave photons N exceeds the threshold set by the number M of absorbed Laser photons

$$N \geq \mathcal{O}(\sqrt{M}\,\Omega/\xi) \;\to\; \Gamma_{\rm decay} \geq \mathcal{O}(\Omega) \gg \xi \,. \tag{29}$$

However, as demonstrated in Section 2, there are far more possibilities for inducing phase damping. For example, if the electrons are pumped (by the Laser) into the conducting band (in contrast to the three-level system discussed above), many more degrees of freedom are available for carrying away energy and information.

The undesirable consequences of spontaneous decays can be avoided by inducing stimulated transitions which are much faster than the spontaneous decay rate – for stimulated transitions, the emitted photons have the same quantum numbers as the incident light and thus do not carry away energy or information. E.g., for a Laser beam with a constant intensity $\Omega_{\rm R} = $ const, we obtain the well-known Rabi oscillation

$$\psi_b(t) = \cos(\Omega_{\rm R} t) \,. \tag{30}$$

Rabi oscillations (i.e., stimulated controlled transitions $a \leftrightarrow b$ instead of a spontaneous decay $b, c \to a$) maintain coherence – e.g., they are used to manipulate QuBits [9].

7. Conclusions

The objective was to study the impact of decoherence on the dynamical Casimir effect. For the pure decay channel (amplitude damping), the dissipation of energy results in a subtractive reduction of the characteristic exponent, i.e., the number of particles only grows exponentially if their creation (via squeezing) is faster than their decay (given by the quality factor of the cavity), cf. [5]. However, even without energy loss, e.g., for pure phase damping, the dynamical Casimir effect is diminished by decoherence – but in a different way. For pure phase damping, the number of created particles always grows exponentially, but with a reduced exponent. Furthermore, the pure quantum vacuum contribution (i.e., the original dynamical Casimir effect) would only yield a linear increase of $\langle\hat{n}\rangle$; the (reduced) exponential growth is caused by classical resonance amplifying already present particles. For an experimental verification of the dynamical

Casimir effect, a large enough quality factor of the cavity is necessary, but *not sufficient*, the quantum coherence time is also a very important quantity.

In order to apply the above results to a concrete example, we considered a three-level system as a simple microscopic model allowing the coherent control of $\varepsilon_{\rm eff}(t)$ via Laser illumination (of a semi-conductor, for example). As long as the Laser beam can be treated as a classical external field (stimulated emission only), the evolution is unitary, i.e., without decoherence (under the assumptions made). Spontaneous decays, on the other hand, inevitably generate decoherence: firstly, by dissipating the polarisation energy of the medium in Eq. (26); and, secondly, by allowing information to leak out – which effectively corresponds to weak measurements. Although the explicit decoherence rate depends on the concrete realisation, spontaneous decays are therefore not desirable for a controlled experimental verification of the dynamical Casimir effect.

As one possible solution for this problem, one could use Rabi oscillations which are faster than the rate of spontaneous decays and automatically generate a harmonic dependence of $\varepsilon_{\rm eff}(t)$, cf. Eq. (30), maintaining coherence. Another advantage of Rabi oscillations lies in the fact that they do not deposit energy into the material, i.e., there is (ideally) no heating – avoiding unwanted excitations which generate decoherence.

If one departs from the three-level system consisting of localised electrons and excites electrons in the conducting band, similar difficulties arise. Apart from the aforementioned problems owing to spontaneous decays, a finite conductivity always entails additional noise and decoherence according to the fluctuation-dissipation theorem. Furthermore, one should be very careful not to excite degrees of freedom generating a non-linearity which induces a coupling to higher harmonics. E.g., via parametric down-conversion, such a coupling might overcome the frequency separation between the Laser and the micro-wave photons – and thereby completely swamp the desired signal.

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References

[1] Casimir HBG 1948 *Kon. Ned. Akad. Wetensch. Proc.* **51** 793

[2] Moore GT 1970 *J. Math. Phys.* **11** 2679

[3] Uhlmann M *et al* 2004 *Phys. Rev. Lett.* **93** 193601

[4] Braggio C *et al* 2004 *Rev. Sci. Instrum.* **11** 4967;
(see also `quant-ph/0411085` and `http://www.pd.infn.it/casimir/`)

[5] Dodonov VV 1998 *Phys. Rev.* A **58** 4147

[6] Dodonov VV 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** R1-R33

[7] Mandel L, Wolf E 1995 *Optical Coherence and Quantum Optics*
(Cambridge: Cambridge University Press)

[8] Scully MO, Zubairy MS 1997 *Quantum Optics*
(Cambridge: Cambridge University Press)

[9] Nielsen MA, Chuang IL 2000 *Quantum Computation and Quantum Information*
(Cambridge: Cambridge University Press)

[10] Dalvit DAR, Neto PAM 2000 *Phys. Rev. Lett.* **84** 798