Problem set 6

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1. Ex. a) gcd(532, 15435) = 7

2. Ex. c) lcm(532, 1083) = 30324

3. Ex. e) 72 and 42

$72 \Rightarrow 1 \cdot 42 + 30$
$42 \Rightarrow 1 \cdot 30 + 12$
$30 \Rightarrow 2 \cdot 12 + 6$
$12 \Rightarrow 2 \cdot 6$

$\gcd(72, 42) = \boxed{6}$

Expressed as a linear combination:

$\gcd(72,42) \Rightarrow 72 \cdot 3 + 42 \cdot (-5) \Rightarrow 6$

$\gcd(72, 42) = 6$

y	x	r
72	42	30
42	30	12
30	12	6
12	6	0

3(cont.) Ex g) 630 and 147

$630 \Rightarrow 4 \cdot 147 + 42$
$147 \Rightarrow 3 \cdot 42 + 21$
$42 \Rightarrow 2 \cdot 21$

$gcd(630, 147) = \boxed{21}$

Expressed as a linear combination:

$gcd(630, 147) \Rightarrow 630 \cdot (-3) + 147 \cdot 13 \Rightarrow \boxed{21}$

$gcd(630, 147) \Rightarrow 21$

y	x	r
630	147	42
147	42	21
42	21	0

4. Ex. g) $x = 34$, $n = 55$

$gcd(34, 55) = \boxed{1}$

$55 \Rightarrow 34 \cdot 1 + 21$	$1 \Rightarrow 3 - 2$
$34 \Rightarrow 21 \cdot 1 + 13$	$1 \Rightarrow 3 - (5 - 3)$
$21 \Rightarrow 13 \cdot 1 + 8$	$1 \Rightarrow 3 \cdot 2 - 5$
$13 \Rightarrow 8 \cdot 1 + 5$	$1 \Rightarrow (8 - 5) \cdot 2 - 5$
$8 \Rightarrow 5 \cdot 1 + 3$	$1 \Rightarrow (8 \cdot 2) - (5 \cdot 3)$
$5 \Rightarrow 3 \cdot 1 + 2$	$1 \Rightarrow (8 \cdot 2) - (13 - 8) \cdot 3$
$3 \Rightarrow 2 \cdot 1 + 1$	$1 \Rightarrow (8 \cdot 5) - (13 \cdot 3)$

4 (cont).

$1 \Rightarrow (21-13)\cdot 5 - (13\cdot 3)$

$1 \Rightarrow (21\cdot 5) - (13\cdot 8)$

$1 \Rightarrow (21\cdot 5) - (34-21)\cdot 8$

$1 \Rightarrow (21\cdot 13) - (34\cdot 8)$

$1 \Rightarrow (55-34)\cdot 13 - (34\cdot 8)$

$1 \Rightarrow (55\cdot 13) - (34\cdot 21)$

$1 = (55\cdot 13) - (34\cdot 21)$ or $= ((-21)\cdot 34) \% 55$

$-21 \% 55$ is the inverse of 34 or $55 - 21 = 34$

$\boxed{\text{34 is the inverse of 34 given modulo 55}}$

$sx \bmod m = 1$ check:

$s = 34,\ x = 34,\ m = 55$

$$\boxed{\begin{array}{l} 34 \cdot 34 \% 55 = 1 \\ 1156 \% 55 = 1 \end{array}}$$

5. a) total characters = 40 (digits + letters + special)

$\boxed{40^6}$

b) $40^7 + 40^8 + 40^9$

c) 14 = digits + special : (first char can't be letter)

$14\cdot 40^6 + 14\cdot 40^7 + 14\cdot 40^8$

6. a) Each page can be printed by any each of the four printers, so for 100 pages, there is $\boxed{4^{100}}$ ways to print.

b) 1st and 100th must be printed on Either of the two color printers, the remaining 98 can be printed on any of the printers

1st page + last page = 4 ways
other pages = 4^{98} ways

So the number of total ways is $\boxed{4^{99} \text{ ways}}$

C) Each consecutive group can be printed four ways. For ~~100 pages~~, you get 4 groups.
So there are $\boxed{4^{4} \text{ ways}}$ to print

7. a) 40 total characters which means the total ways are expressable by

$r = 6,\ n = 40$

$n \cdot n-1 \cdot n-2 \cdot n-3 \cdot n-4 \cdot n-5$ or
$40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35$ or $\boxed{2,763,633,600}$

b) Similar to a), but our first n-term is 36 since 40 (total characters) - 4 (special) = 36

$n-4 \cdot n-1 \cdot n-2 \cdot n-3 \cdot n-4 \cdot n-5$ or
$36 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35$ or
$\boxed{2,487,270,240}$

8. $r = \{a, b, c\}$

1. The first letter can be any letter in r, making 3 options for the first letter.

2. the next 9 letters can be chosen from 2 options in r, since only 2 letters are available (no consecutive chars)

which means there are $3 \cdot 2^9$ possibilities or 1536 possibilities

9. a) $\{0, 1\}^7$ is the set of all len(x) = 7 binary strings, Therefore, the total number of different functions is 2^7 or $\boxed{128}$

c) $\{0, 1\}^5$ has $\boxed{32}$ elements (2^5)
$\{0, 1\}^7$ has $\boxed{128}$ elements (2^7)

So the number of different functions are:
$2^7 \cdot 2^7 - 1 \cdot 2^7 - 2 \ldots, 2^7 - 2^5 + 1$

or:

$\frac{(2^7)!}{(2^5)!} = \boxed{\frac{128!}{32!}}$

10. b)

d)

11. a) $\boxed{82}\boxed{X} - \boxed{XXXX}$

Constant

any digits 0-9

10^4 ways

Either 4 or 5

2 ways

1 Way

1 Way

1st digit 3rd digit digits 4-7

$1 \times 1 \times 2 \times 10^4 = \boxed{20,000}$

3rd digit

b) Similar to a, the first 3 digit possibilities can be expressed $1 \times 1 \times 2$, but the last four are now all required to be unique. so, $10 \times 9 \times 8 \times 7 \times 1 \times 1 \times 2 = \boxed{10,080}$

12.

a) $10!$ ways or $3{,}628{,}800$ ways

b) $\underset{\text{groom}}{1} \cdot \overset{\text{bride}}{9} \cdot \underset{\text{remaining people}}{8!}$ ways or $362{,}880$ ways

c) $9! \cdot 2!$ ways or $725{,}760$ ways

13. a) We have $\boxed{\binom{12}{3}}$ strings, since the remaining characters of the total twelve must be 'b'

b) Similar to a, we have $\binom{12}{5}$ strings but we have two letters to choose from to fill the remaining characters. Thus, we have $\binom{12}{5} \cdot 2^7$ total combinations.

14. a) $\binom{52}{5}$

b) $\binom{39}{3} \times \binom{13}{2}$

c) $\binom{13}{5} \times \binom{13}{0} + \binom{13}{4} \times \binom{13}{1} + \binom{13}{3} \times \binom{13}{2} +$
$\binom{13}{2} \times \binom{13}{3} + \binom{13}{1}\binom{13}{4} + \binom{13}{0} \times \binom{13}{5}$

d) $\binom{13}{1} \times \binom{39}{1}$

e) $\binom{13}{1} \times \binom{4}{3}\binom{12}{1}\binom{4}{2}$

f) $\binom{13}{5} \times 4^5$

15. a) $\binom{20}{5}$ orders

b) 20^5 or 3,200,000 orders

c) 1,860,480 orders

16 a) we have 10 digits and 26 letters so $10 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36$
or 604,661,760 possible passwords

b) $10 \cdot 26 \cdot 36 \cdot 36 \cdot 36 \cdot 36$ or 436,700,160 possible passwords
digit letter any $\longrightarrow$

17 a) In a deck of 52 cards, 13 are clubs

So there are $\binom{39}{5}$ hands with no clubs therefore

there are $\binom{52}{5} - \binom{39}{5}$ hands that have atleast one club

b) $\binom{52}{5} - \binom{13}{5} \times 9^5$ hands

18 c) We have 7 total letters, with 's' repeating 3 times. Therefore we have $\frac{7!}{3!}$ or 840 permutations.

19 a) 10^{20} ways

b) $\frac{20!}{2^{10}}$ ways

20. a) $\frac{28!}{3!25!}$ ways

b) $\frac{23!}{3!20!}$ ways

c) $\frac{28!}{3!25!} - \frac{17!}{3!14!}$ ways

21 a) $(2^{10})-1$ strings

b) 2^9 strings

c $\frac{10!}{5!\,5!} + 2^9$ strings