1.a

a. $f = \Theta(n^8)$

b. $f = \Theta(3^n)$

c. $f = \Theta(3^n)$

d. $f = \Theta(n)$

e. $f = \Theta(n \log n)$

~~g~~f. $f = \Theta(n \log n)$

g. $n > 1: f = \Theta(n^{2})$, $n \leq 1: f = \Theta(1.1^n)$

h. $\Theta(n^3)$

1b. c) $\sum_{j=1}^{n} j(j-1) = \frac{n(n^2-1)}{3}$

Base case: $P(1): 1(1-1) = \frac{1(1^2-1)}{3}$

$0 = 0$

I. H.: $\sum_{j=1}^{K} j(j-1) = \frac{K(K^2-1)}{3}$

Induction:

For $n = K+1$

$$\sum_{j=1}^{K+1} j(j-1) = \frac{(K+1)((K+1)^2-1)}{3}$$

$$= (K+1)\left(\frac{(K+1)^2-1}{3}\right)$$

1c.

Prove $C_n = 5^{2^K}$

$P(n): C_n = 5^{2^K}$

For $n = 1$

$$C_1 = 5^{2^1}$$

$$\Rightarrow (C_{1-1})^2 = 5^2$$

$$\Rightarrow (C_0)^2 = 25$$

$$\Rightarrow (5)^2 = 25$$

$$\Rightarrow 25 = 25$$

For $n = K+1$

$$C_K = 5^{2^K}$$

$$\Rightarrow C_{K+1}$$

$$\Rightarrow (C_{K+1})^2$$

$$\Rightarrow (C_{K+1-1})^2$$

$$\Rightarrow (C_K)^2$$

$$\Rightarrow (5^{2^K})^2$$

$$\Rightarrow 5^{2K+2K}$$

$$\Rightarrow 5^{2^{K+1}}$$ ✓

1d.

If $P(n)$, $P(n+1)$, $P(n+2)$ ... $P(\cancel{k}K)$ is true then $P(K+1)$ is true for $K > n$

$P(K) = a \cdot 5 + b \cdot 3$, $a, b \geq 0$

Base Step: $P(8) = 1 \cdot 5 + 1 \cdot 3 = 8$

$P(9) = 3 + 3 + 3 = 9$

$P(10) = 5 + 5 = 10$

$P(k+1) = 5 \cdot a + 3 \cdot b + 1$

$b \geq 2$: $P(k+1) = 5(a+2) + 3\cancel{b}(b-3)$

$b = 1$: $P(k+1) = 5(a-1) + 9$

$b = 0$: $P(k+1) = 5(a-1) + 3(2)$

As long as $K > 8$, $P(K) = 5a + 3b$ = true

1e

c. let $n = k$;

$f(0) = 2^0 + 3^0 + 7^{0+2}$

$= 1 + 1 + 49$

$= 51$ ✓

$f(1) = 2^1 + 3^1 + 7^{1+2}$

$= 2 + 3 + 343$

$= 348$ ✓

Let $n = k+1$:

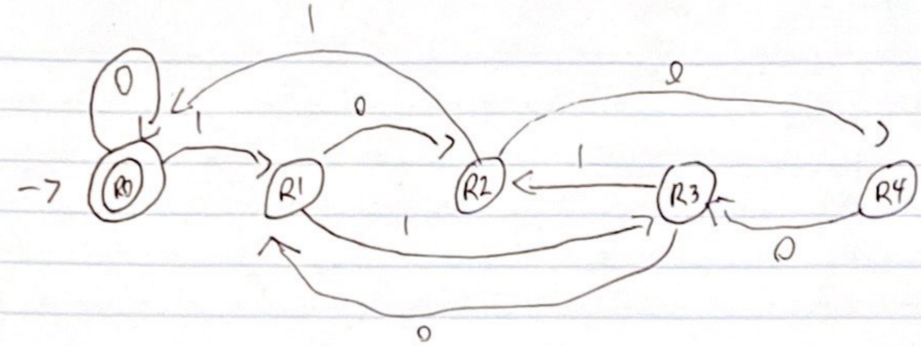
$f(k+1) = 2^{k+1} + 3^{k+1} + 7^{k+3}$

because

$5g_{k+1-1} - 6g_{k+1-2} + 20 \cdot 7^{k+1}$

which is true for all $n \geq 0$

2.



States R0 – R4 correspond to the remainder after dividing by 5. If the input so far is a multiple of 5, you are in state R0. A '0' bit would multiply the number by 4, which would still result in a number divisible by 5 with no remainder, therefore you are still in state R0. A '1' bit would multiply it by 4 then add 1, moving it to state R1. Then, an input of '0' doubles the number, moving it to R2. Or, an input of '1' doubles the number, putting it in state R3. In R2, an input of '1' moves you to state R0. In R2, an input of '0' moves you ~~[illegible]~~ to R4.