COMP-170: Homework #1

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Problem 4

Prove, by the contrapositive method, that if c is an odd integer, then the equation $n^2+n-c=0$ has no integer solution.

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We want to show that if c is an odd integer, then the equation $n^2+n-c=0$ has no integer solutions. In order to prove this, we will use the contrapositive of the statement. More specifically, we will show that if $n^2+n-c=0$ has an integer solution, then c is an even integer.

Let c be some integer such that the equation $n^2+n-c=0$ has at least one integer solution. Let x be any solution to the equation such that $x^2+x-c=0$. From here, we can slightly transform the equation to see that $x^2+x=(x)(x+1)=c$. We know that either x is even or $x+1$ is even, which means $(x)(x+1)$ is divisible by 2. Thus, we know c is also divisible by 2 which implies c is an even integer. This shows that if $n^2+n-c=0$ has an integer solution for some $c \in \mathbb{Z}$ then c is an even integer.

We can use the above statement as the contrapositive to also show that if c is odd, $n^2+n-c=0$ has no integer solutions. ⊠