

Intro to Algorithms, COMP-160, Homework #2
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3. **Solve** $T(n) = T(\sqrt{n}) + 1$, **using a change of variables:** $m = \log_2 n$.

$$T(n) = T(n^{\frac{1}{2}}) + 1$$

$$m = \log_2 n \rightarrow 2^m = n$$

From this, we can do change of variable: $T(2^m) = T(2^{\frac{m}{2}}) + 1$

$$T(\log(2^m)) = T(\log(2^{\frac{m}{2}})) + 1$$

$$T(m) = T(\frac{m}{2}) + 1$$

From this, we can use the master-method on $T(m) = T(\frac{m}{2}) + 1$

$$a = 1, b = 2, f(m) = 1, \text{ leaf-level} = m^{\log_2 1} = m^0 = 1$$

$$f(m) = \Theta(m^{\log_2 1}) = \Theta(1)$$

Therefore, case #2: $T(m) = \Theta(1 \cdot \log m) = \Theta(\log m)$

$$T(n) = \Theta(\log(\log_2 n))$$