Intro to Algorithms, COMP-160, Homework #2
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1. **Use the master method for the following.**

   (a) $T(n) = 16 \cdot T(\frac{n}{4}) + n^2$
   $a = 16$, $b = 4$, $f(n) = n^2$, leaf-level $= n^{\log_4 16} = n^2$
   $f(n) = \Theta(n^{\log_4 16}) = \Theta(n^2)$
   Therefore, case #2: $T(n) = \Theta(n^2 \cdot \log_4 n)$

   (b) $T(n) = 256 \cdot T(\frac{n}{4}) + \Theta(n^4 \log^4 n)$
   $a = 256$, $b = 4$, $f(n) = \Theta(n^4 \log^4 n)$, leaf-level $= n^{\log_4 256} = n^4$
   $f(n) \neq \Theta(n^4)$ but $f(n)$ doesn't dominate n^4 because $f(n) = \Theta(n^4 \cdot \log^4 n)$
   $\exists k$ such that $f(n) = \Theta(n^4 \cdot \log^k n) \rightarrow$ special case #2
   Therefore, case #2: $T(n) = \Theta(n^4 \cdot \log^4 n \cdot \log n) = \Theta(n^4 \cdot \log^5 n)$

   (c) $T(n) = 157 \cdot T(\frac{n}{157}) + n^2$
   $a = 157$, $b = 157$, $f(n) = n^2$, leaf-level $= n^{\log_{157} 157} = n$
   $f(n) = \Omega(n^{1+\epsilon})$ where $\epsilon \leq 1$ so root-level dominates
   Therefore, case #3: $T(n) = \Theta(f(n)) = \Theta(n^2)$

   (d) $T(n) = T(\frac{159n}{732}) + \Theta(n^2)$
   $a = 1$, $b = \frac{732}{159}$, $f(n) = \Theta(n^2)$, leaf-level $= n^{\log_{\frac{732}{159}} 1} = n^0 = 1$
   $f(n) = \Omega(n^{0+\epsilon})$ where $\epsilon \leq 2$ so root-level dominates
   Therefore, case #3: $T(n) = \Theta(f(n)) = \Theta(n^2)$