COMP-170: Homework #5

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Problem 1

Let $D5 = \{$The set of integers divisble by 5$\}$. Prove every decidable set many one reduces to $D5$.

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To show every decidable set many one reduces to $D5$, we will take any decidable set L and prove $L \leq_m D5$. In order to do this, we will construct a function $f : \Sigma_L^* \to \Sigma_{D5}^*$ such that $x \in L \Leftrightarrow f(x) \in D5$. Once we show that $L \leq_m D5$, because L could have been any decidable set, we will have shown that every decidable set many one reduces to $D5$.

First, let L be any decidable set. With this, we will claim $L \leq_m D5$. By definition, because L is decidable, we know there exists some machine D_L that decides L, such that if $x \in L$, D_L accepts x and if $y \notin L$, D_L rejects y. Given this information, we can show how L and $D5$ relate to each other (using the "Tony Square").

	L	$D5$
IN	$x \in L$	y is divisible by 5
OUT	$x \notin L$	y is not divisible by 5

Now that we've built our "Tony Square", we can define our function. Let $f : \Sigma_L^* \to \Sigma_{D5}^*$ be defined as follows:

f on input x outputs y, where y is defined by:

Run D_L on x

If D_L accepts x, $y = 5$

If D_L rejects x, $y = 1$

Given our definition of f, we claim that f is computable. Because D_L is a valid Turing machine and f is a finite function, we can see that f is indeed computable.

Now, we can go through two cases to show that f correctly satisfies $x \in L \Leftrightarrow f(x) \in D5$. Consider the following two cases:

1. Suppose $x \in L$ such that D_L accepts x. Since D_L accepts x, we can see that y will be set as 5, which is divisible by 5. Thus, if $x \in L$, $y \in D5$.
2. Suppose $x \notin L$ such that D_L rejects x. Since D_L rejects x, we can see that y will be set as 1, which is not divisible by 5. Thus, if $x \notin L$, $y \notin D5$.

Given these two cases, we can thus see that f does indeed satisfy the claim $x \in L \Leftrightarrow f(x) \in D5$. Therefore, we can see that $L \leq_m D5$. Because we chose L to be any decidable set, we can see that every decidable set many one reduces to $D5$. $\boxtimes$