COMP-170: Homework #10

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Problem 2

For every string $w = w_1w_2\dots w_n$, the string written in reverse, denoted w^r, is the string $w_nw_{n-1}\dots w_1$. For any language L, let $L^R = \{w^r \mid w \in L\}$. Show that if L has a DFA then L^R has a DFA.

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To prove L^R has a DFA if L has a DFA, we can use a proof by construction. Specifically, we can construct a DFA d^r for L^R using a DFA d for L.

Suppose the language L has a DFA d. This means, given any $w \in L$, d accepts w. In a similar way that we visualize DFAs, we can also represent any DFA (d included) as a graph where the states of d are vertices and the transitions of d are direct edges. Given the graphical representation of d, we can see that if d accepts some w (where $w \in L$), there is a path from the start state of d to the accepting state of d, determined by the characters of w. If d doesn't accept w, there exists no such path.

Consider any $w \in L$. Because $w \in L$, we know that d accepts w, and thus that there is a path p (from the start state to the accepting state) in the graphical representation of d for w. Now consider the reverse path p^r, where p^r is simply the steps of p backwards. Since p represents a path that traverses d as we consume the characters of w from front to back (left to right), we can see p^r represents a path that would consume the characters of w from back to front (or the characters of w^r from front to back). Thus we can see that if we were to flip the transition directions in d (i.e. if there is a transition from q_i to q_j for some character c, we replace it with a transition from q_j to q_i for the character c) and made d's start state an accepting state (and vice versa), we would form an alternate DFA d^r thats graphical representation would contain p^r. Thus, we can see that d^r would accept w^r, for any $w \in L$. Thus, we can see that d^r would be a valid DFA for any $w^r \in L^R$, so d^r is a valid DFA for L^R. Therefore, we can see that if L has a DFA, so does L^R. $\boxtimes$