Intro to Algorithms, COMP-160, Homework #1

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2. **Same question as 1, different functions.**

   (a) $\log_a n$ vs $\log_b n \rightarrow \log_a n = \Theta(\log_b n)$
   $c_1 \cdot \log_b n \leq \log_a n \leq c_2 \cdot \log_b n$
   $c_1 \leq \frac{\log_a n}{\log_b n} = \frac{\log a}{\log n} \cdot \frac{\log n}{\log b} = \frac{\log a}{\log b} \leq c_2$
   Therefore, $\exists c_1, c_2$ such that $c_1 \cdot \log_b n \leq \log_a n \leq c_2 \cdot \log_b n$, so $\log_a n = \Theta(\log_b n)$

   (b) $\log(\Theta(1) \cdot n)$ vs $\log n \rightarrow \log(\Theta(1) \cdot n) = \Theta(\log n)$
   $c_1 \cdot \log n \leq \log(\Theta(1) \cdot n) = \log n + \log(\Theta(1)) = \log n + c_n \leq c_2 \log n$
   Since $\log(\Theta(1)) = c_n \geq 0$, $c_1 \leq 1$
   Next we want to find a c_2 such that $\log n + \log(\Theta(1)) \leq c_2 \cdot \log n$
   We can assume that $c_2 + \log n \leq c_2 \cdot \log n$, so we can say $\log(\Theta(1)) + \log n \leq c_2 + \log n$
   We now see that c_2 is valid as long as $\log(\Theta(1)) \leq c_2$.
   Therefore, $\exists c_1, c_2$ such that $c_1 \cdot \log n \leq \log(\Theta(1) \cdot n) \leq c_2 \cdot \log n$
   So $\log(\Theta(1) \cdot n) = \Theta(\log n)$

   (c) $\log n^{\Theta(1)}$ vs $\log n \rightarrow \log n^{\Theta(1)} = \Theta(\log n)$
   $c_1 \cdot \log n \leq \log n^{\Theta(1)} = \Theta(1) \log(n) \leq c_2 \cdot \log n$
   $c_1 \leq \Theta(1) \cdot \frac{\log n}{\log n} = \Theta(1) \leq c_2$
   Therefore, $\exists c_1, c_2$ such that $c_1 \cdot \log n \leq \log n^{\Theta(1)} \leq c_2 \cdot \log n$, so $\log n^{\Theta(1)} = \Theta(\log n)$