COMP-170: Homework #2

Ben Tanen - February 5, 2017

Problem 2

Let x be a binary string of length n, and let x_i, $0 \leq i \leq n-1$, be the character (bit) located at position i. A string y is x bitwise doubled if for all y_i, $y_i = x_{\lfloor \frac{i}{2} \rfloor}$. Note that the empty string, λ, bitwise doubled is the empty string, λ.

For example, 010 bitwise doubled is 001100 and 00110 bitwise doubled is 0000111100.

Write a Turing machine (high level pseudo code) to decide the following language:

$$L = \{y01x \mid x \in \{0,1\}^* \text{ and } y \text{ is } x \text{ bitwise doubled } \}$$

* * *

M on input s

1. If $s[0] = B$ (the first cell of s is B), *REJECT* // Reject the empty string
2. If $s[0] = 0$, $s[1] = 1$, and $s[2] = B$, *ACCEPT* // x and y are empty strings
3. Repeat loop to find separating 01 between x and y:
   (a) Mark current cell with X (0 becomes $0X$, 1 becomes $1X$, etc.)
   (b) Walk right one cell
      i. If current cell is B, *REJECT*
      ii. Otherwise, mark cell with X
   (c) Walk right two cells
      i. If current cell is B or marked, *REJECT*
      ii. Otherwise, continue
   (d) Walk right until we hit first B or marked cell
   (e) Walk left one cell and mark with X
   (f) Walk left two cells
      i. If current cell is marked, *REJECT*
      ii. Otherwise, continue
   (g) Walk left one cell
      i. If current cell is marked, go to step 4
      ii. Otherwise, continue

(h) Walk left until we hit first marked cell

(i) Walk right one cell

4. Walk right one cell
   (a) If current cell is an unmarked 0, mark with #
   (b) Otherwise, *REJECT*
5. Walk right one cell
   (a) If current cell is an unmarked 1, mark with #
   (b) Otherwise, *REJECT*
6. From beginning of tape, walk through tape, removing X markings, until we hit first B
7. Walk back to beginning of tape
8. Repeat loop to check for bitwise doubling:
   (a) Mark current cell with X
   (b) Walk right one cell
      i. If current cell is the same as previous cell's unmarked value, mark with X (ex: if previous cell was 0 and current cell is 0, add X)
      ii. Otherwise, *REJECT*
   (c) Walk right until we hit second cell with #
   (d) Walk right until we hit first 0, 1, or B
      i. If current cell is the same as cell the unmarked value from 6(a), add additional X (ex: if 6(a) cell was 0, mark current cell if it is also 0)
      ii. Otherwise, *REJECT*
   (e) Walk left until we hit second cell with #
   (f) Walk left one cell
      i. If cell is marked, go to step 9 // Done marking for bitwise doubles
   (g) Walk left until we hit first X marked cell
   (h) Walk right one cell
9. From beginning of tape, walk through tape until we hit first B or unmarked cell
   (a) If we find an unmarked cell (cell without # or X), *REJECT*
   (b) If we find B, *ACCEPT*