Intro to Algorithms, COMP-160, Homework #1

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1. **Compare each of the following pairs of functions asymptotically. To compare $f(n)$ and $g(n)$, you should prove that $f(n)$ is $O(g(n))$ or $\Omega(g(n))$, or both.**

   (a) 2^{n+1} vs $2^n \rightarrow 2^{n+1} = \Theta(2^n)$
   $c_1 \cdot 2^n \leq 2^{n+1} = 2 \cdot 2^n \leq c_2 \cdot 2^n$
   $c_1 \leq 2\frac{2^n}{2^n} \leq c_2$
   $c_1 \leq 2 \leq c_2$ and $n_0 \geq 1$
   Therefore $\exists c_1, c_2$ such that $c_1 \cdot 2^n \leq 2^{n+1} \leq c_2 \cdot 2^n$, so $2^{n+1} = \Theta(2^n)$

   (b) 2^{2n} vs $2^n \rightarrow 2^{2n} = \Omega(2^n)$
   $c \cdot 2^n \leq 2^{2n} = 4^n$
   $c \leq \frac{4^n}{2^n} = 2^n$
   Since $n_0 \geq 1$, $c \leq 2$
   Therefore, $\exists c$ such that $c \cdot 2^n \leq 2^{2n}$, so $2^{2n} = \Omega(2^n)$

   (c) 4^n vs $2^{2n} \rightarrow 4^n = \Theta(2^{2n})$
   $c_1 \cdot 2^{2n} \leq 4^n \leq c_2 \cdot 2^{2n}$
   $c_1 \cdot 4^n \leq 4^n \leq c_2 \cdot 4^n$
   $c_1 \leq \frac{4^n}{4^n} \leq c_2$
   $c_1 \leq 1 \leq c_2$ and $n_0 \geq 0$
   Therefore $\exists c_1, c_2$ such that $c_1 \cdot 2^{2n} \leq 4^n \leq c_2 \cdot 2^{2n}$, so $4^n = \Theta(2^{2n})$

   (d) 2^n vs $4^n \rightarrow 2^n = O(4^n)$
   $2^n \leq c \cdot 4^n$
   $\frac{1}{2^n} \leq c$
   Since $n_0 \geq 1$, $c \geq \frac{1}{2}$
   Therefore, $\exists c$ such that $2^n \leq c \cdot 4^n$, so $2^n = O(4^n)$