COMP-170: Homework #3

Ben Tanen - February 12, 2017

Problem 4

Prove that $L = \{\langle M \rangle \mid M \text{ accepts input } 1011\}$ is recognizable.

* * *

We would like to show that the language $L = \{\langle M \rangle \mid M \text{ accepts input } 1011\}$ is recognizable. In order to do this, we will use proof by construction. More specifically, we will construct a machine R_L that will accept any $x \in L$.

Let's define R_L as the following:

> **R_L on input $\langle M \rangle$:**
>
> Run M on 1011
>
> If M accepts 1011, *ACCEPT*
>
> If M reject 1011, *REJECT*
>
> END

We will now claim that R_L recognizes L. To show this, consider the following cases:

1. Suppose $x \in L$ such that $x = \langle M \rangle$ for some M and M accepts the input 1011. In this case, when we run M on input 1011, we know M will accept the input, which will cause R_L to accept x correctly.
2. Suppose $y \notin L$ such that $y = \langle M \rangle$ for some M and M rejects the input 1011. In this case, when we run M on input 1011, we know M will reject the input, which will cause R_L to reject y correctly.
3. Suppose $z \notin L$ such that $z = \langle M \rangle$ for some M and M loops on the input 1011. In this case, when we run M on input 1011, M will loop, which means R_L will loop. However, because $z \notin L$, R_L can do anything other than accept z, which it does. Thus, we can see R_L functions correctly on z.

Thus, since R_L accepts all $x \in L$ and doesn't accept all $x \notin L$, we can see that R_L recognizes L. This shows that L is indeed recognizable. ⊠