COMP-170: Homework #4

Ben Tanen - February 26, 2017

Problem 4

Let $L = \{\langle M \rangle \mid M \text{ only accepts odd inputs } \}$. Prove that $\overline{L}$ is recognizable.

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To show $\overline{L}$ is recognizable, we will use a proof by construction. Specifically, we will construct a recognizer R for the language $\overline{L}$ such that for any $x \in \overline{L}$, R will accept x and for any $y \notin \overline{L}$, R doesn't accept y.

Given this, let's define R as follows:

R **on input** $\langle M \rangle$**:**

For $i = 0 \to \infty$

For $j = 0 \to i$

Run M on input s_j for i steps

If M accepts s_j in i steps and s_j is an even input, $ACCEPT$

We will now claim that R is a recognizer of $\overline{L}$. To show this, consider the following cases:

1. Let $\langle M \rangle \in \overline{L}$, where M does not just except odd inputs. Given this, we know M accepts at least one even input. Thus, we know there exists an input string s that is even and that M accepts in fewer (or equal) steps than any other even input string. Suppose M accepts s in k steps (where k is a finite number). With our loop structure, we know that we will eventually run M on s for k steps, which will cause M to accept s. This will cause R to correctly accept $\langle M \rangle$.
2. Let $\langle M \rangle \notin \overline{L}$, where M only accepts odd inputs. Because $L(M)$ only contains odd inputs, we can see that M will never accept an even input, no matter how many steps M is allowed. Thus, because M will never accept an even input, R will correctly never accept $\langle M \rangle$.

Based on our construction and these cases, we can see that R accepts all $x \in \overline{L}$ and that R doesn't accept all $y \notin \overline{L}$. Therefore, we can see that R is a recognizer of $\overline{L}$, and since there exists a machine that recognizes $\overline{L}$, we know that $\overline{L}$ is recognizable. $\boxtimes$