COMP-170: Homework #1

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Problem 3

Prove, by the contrapositive method, that if n^2 is even then n is even.

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We want to show that if n^2 is even, then n is also even. In order to prove this, we will use the contrapositive of the statement. More specifically, we will show that if n is odd, then n^2 is also odd.

Let n be odd, a.k.a not even. Since n is odd, we know that $n = 2m+1$ for some $m \in \mathbb{Z}$. Thus, we know that $n^2 = (2m+1)^2 = 4m^2 + 4m + 1$. Since $m \in \mathbb{Z}$, we know that $4m^2 + 4m \in \mathbb{Z}$ and that $2m^2 + 2m \in \mathbb{Z}$. Thus, we can see $n^2 = 4m^2 + 4m + 1 = 2q + 1$ for $q = 2m^2 + 2m \in \mathbb{Z}$. Since n^2 can be represented as $2q + 1$ where $q \in \mathbb{Z}$, we can see that n^2 is odd, just like n. Thus, we can see that if n is odd, n^2 is odd.

We can use the above statement as the contrapositive to also show that if n^2 is even, n is even. $\boxtimes$