COMP-170: Homework #10

Ben Tanen - April 30, 2017

Problem 5

Prove that for every $k > 1$, a language $A_k \subseteq \{0,1\}^*$ exists that can be recognized by a DFA with k states, but not by one with $(k-1)$ states.

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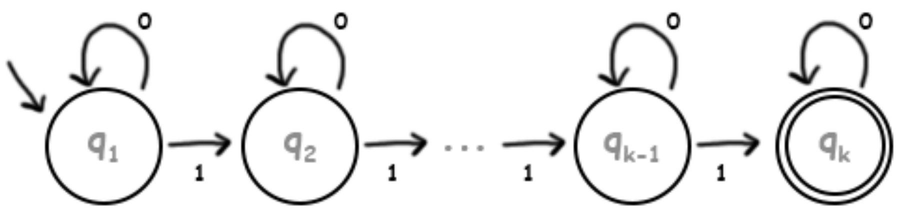
To prove that there exists a language $A_k \subseteq \{0,1\}^*$ that requires k states for every $k > 1$, we can use a proof by construction. Specifically, we can construct such a language A_k for any value of k.

Let k be any integer value greater than 1. Now, let $A_k = \{w \mid w \text{ contains at least } k-1 \text{ 1s }\}$. In order to count these $k-1$ 1s, we know that any DFA for A_k must have at least k states to keep track of how many 1s might have been consumed thus far (a start state, where the count is 0, and a state for all integer values between 1 and k). Thus, since we know that any DFA for A_k requires at least k states, we can see that there exists no DFA for A_k that has $k-1$ states.

Next, to show that a DFA exists that has k states, consider the following DFA.



We can see that this DFA would accept any $w \in A_k$ since it simply transitions as it counts up to k. Thus, since this DFA (with k states) is a valid DFA for A_k, we can thus say such a DFA exists.

Therefore, we can see that, based on our construction of A_k, A_k has a valid DFA with k states, but no valid DFA with any fewer states. Thus, for every $k > 1$, there exists a valid language $A_k \subseteq \{0,1\}^*$. ⊠