Intro to Algorithms, COMP-160, Homework #4

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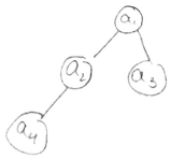
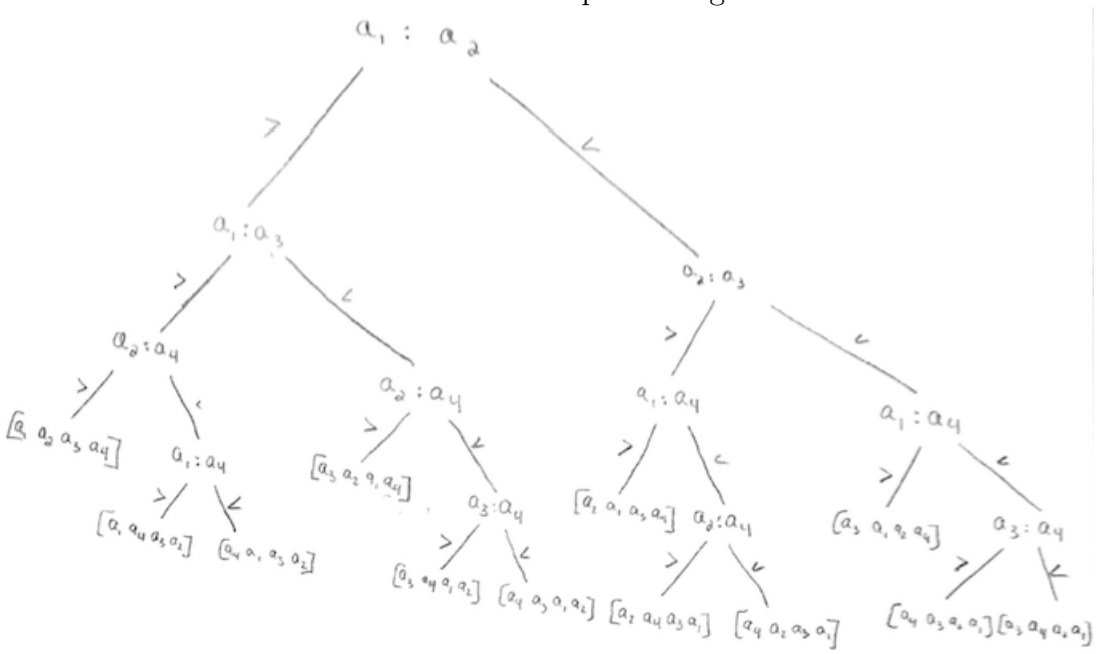
1. (a) For an array $A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$ of distinct numbers, there are two main ways to build a heap, described in the class notes. Show what comparisons each method will make, in the form of a binary decision tree. Every possible output should be represented at the leaf level. The output is simply a permutation of the indices of the elements in A (e.g., 3124) that corresponds to the contents of the output array that represents a heap.

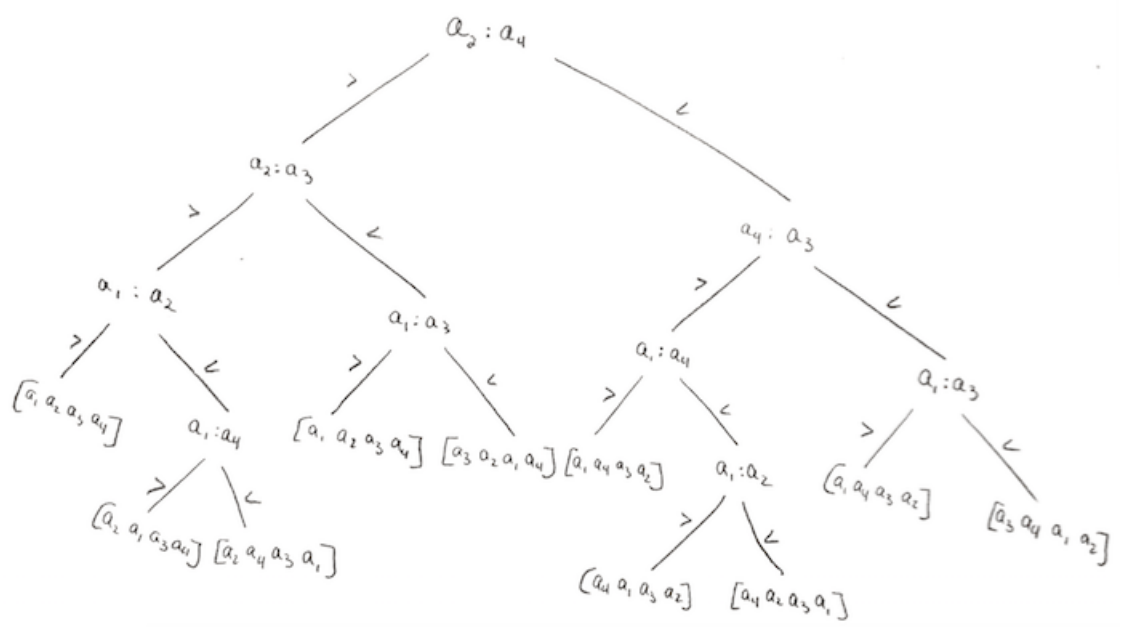
Note: Each output of the format $\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$ forms the heap:



a. Standard Heap Building



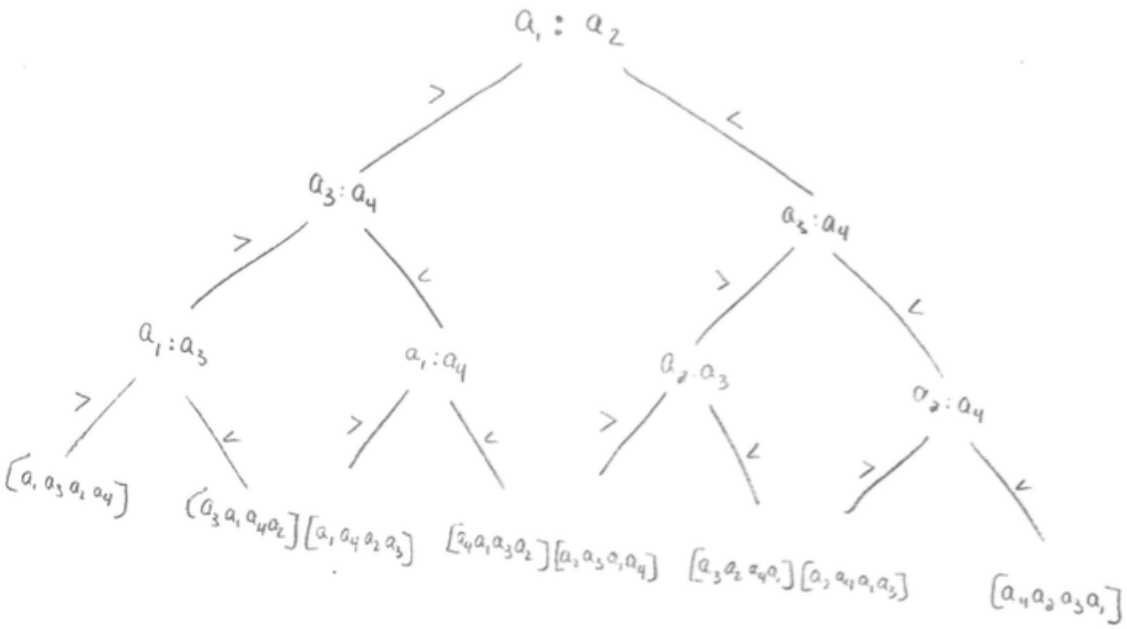
b. In-Place Heap Building



(b) **Give your own heap-building algorithm for inputs of size 4, that uses fewer decisions in the worst-case, compared to the methods in (a).**

An algorithm that we can do for inputs of size 4 works by making two heaps of size 2, and then merging these heaps with a single comparison together to form one heap of size 4.

A decision tree for this algorithm can be seen below.



This algorithm is obviously better (in the worst-case) than the algorithms used

in part (a) because it always takes three comparisons. The worst-case for both algorithms from part (a) have worst-cases of four comparisons.