COMP-170: Homework #4

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Problem 3

Let $L = \{\langle M\rangle \mid M \text{ only accepts odd inputs }\}$. Prove $\overline{A_{TM}} \leq_m L$.

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We want to prove $\overline{A_{TM}} \leq_m L$. To do this, we will construct a function $f : \Sigma^*_{\overline{ATM}} \to \Sigma^*_L$ such that $x \in \overline{A_{TM}} \Leftrightarrow f(x) \in L$. Once we show f satisfies this criteria, we can see that $\overline{A_{TM}} \leq_m L$.

First, we will claim $\overline{A_{TM}} \leq_m L$. To prove this claim, we will first show how $\overline{A_{TM}}$ and L relate to each other (using the "Tony Square").

	$\overline{A_{TM}}$	L
IN	M doesn't accept w	M' only accepts odd inputs
OUT	M accepts w	M' does not just accepts odd inputs

Now that we've built our "Tony Square", we can define our function. Let $f : \Sigma^*_{\overline{ATM}} \to \Sigma^*_L$ be defined as follows:

f on input $\langle M, w\rangle$ outputs $\langle M'\rangle$, where M' is defined as:

> **M' on input x:**
>
> > If $x = 1$, $ACCEPT$
> >
> > If $x = 2$, run M on w
> >
> > > If M accepts w, $ACCEPT$
> > >
> > > Else, $LOOP$
> >
> > Else, $LOOP$

Given our definition of f, we claim that f is computable. Because M' is a valid Turing machine and f is a finite function, we can see that f is indeed computable.

Now, we can go through two cases to show that f correctly satisfies $x \in \overline{A_{TM}} \Leftrightarrow f(x) \in L$. Consider the following two cases:

1. Suppose $\langle M, w\rangle \in \overline{A_{TM}}$ such that M does not accept w. We can see that regardless of $\langle M, w\rangle$, M' accepts 1. Because M doesn't accept w, we can see that M' will loop on every other input (including 2). Thus, $L(M') = \{1\}$ so we can see that M' only accepts odd inputs. Thus $\langle M'\rangle \in L$.

2. Suppose $\langle M, w\rangle \notin \overline{A_{TM}}$ such that M accepts w. Again, we see that M' accepts 1. Additionally, because M accepts w, we can see that M' also accepts 2. M' will loop on every other input. Thus, $L(M') = \{1, 2\}$ so we can see that M' accepts odd and even inputs. Thus $\langle M'\rangle \notin L$.

Given these two cases, we can thus see that f does indeed satisfy the claim $x \in \overline{A_{TM}} \Leftrightarrow f(x) \in L$. Therefore, we can see that $\overline{A_{TM}} \leq_m L$. $\boxtimes$