Intro to Algorithms, COMP-160, Homework #2
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2. **Use the master method for the following.**

   (a) $P(n) = 10 \cdot P(\frac{n}{3}) + \Theta(n^2 \log^5 n)$
   $a = 10$, $b = 3$, $f(n) = \Theta(n^2 \log^5 n)$, leaf-level $= n^{\log_3 10} \approx n^{2.096}$
   Since $\exists \epsilon$ such that $f(n) = \Theta(n^{\log_3 10 - \epsilon}) \approx \Theta(n^{2.096-\epsilon})$ $(\epsilon = 0.096)$
   This shows the leaf-level dominates over the root-level.
   Therefore, case #1: $T(n) = \Theta(n^{\log_3 10})$

   (b) $T(n) = 3 \cdot T(\frac{n}{2}) + n^2$
   $a = 3$, $b = 2$, $f(n) = n^2$, leaf-level $= n^{\log_2 3} = n^{1.585}$
   Since $\exists \epsilon$ such that $f(n) = \Theta(n^{\log_2 3 + \epsilon}) \approx \Theta(n^{1.585+\epsilon})$ $(\epsilon = 0.415)$
   This shows the root-level dominates over the leaf-level.
   Therefore, case #3: $T(n) = \Theta(f(n)) = \Theta(n^2)$

   (c) $T(n) = 4 \cdot T(\frac{n}{16}) + \sqrt{n}$
   $a = 4$, $b = 16$, $f(n) = \sqrt{n} = n^{\frac{1}{2}}$, leaf-level $= n^{\log_{16} 4} = n^{\frac{1}{2}}$
   $f(n) = \Theta(n^{\frac{1}{2}}) = \Theta(n^{\log_{16} 4})$
   Therefore, case #2: $T(n) = \Theta(\sqrt{n} \cdot \log_{16} n)$

   (d) $T(n) = 4 \cdot T(\frac{n}{2}) + \Theta(n^2 \log^{-3} n)$
   $a = 4$, $b = 2$, $f(n) = \Theta(n^2 \log^{-3} n)$, leaf-level $= n^{\log_2 4} = n^2$
   $f(n) \neq \Theta(n^2)$ but $f(n)$ doesn't dominate n^2 because $f(n) = \Theta(n^2 \cdot \log^{-3} n)$
   $\exists k$ such that $f(n) = \Theta(n^2 \cdot \log^k n) \to$ special case #2
   Therefore, case #2: $T(n) = \Theta(n^2 \cdot \log^{-3} n \cdot \log n) = \Theta(n^2 \cdot \log^{-2} n)$