LINEAR ALGEBRA PRECOURSE PROGRAM MASTER DEGREE IN DCSC (UNITS) - 2018/2019

Ornela Mulita

September 2018

(18/09/18) Definition of vector spaces and vector subspaces. Examples. Linearly dependence/independence. System of generators. Basis. Dimension of a vector space and dimension of a vector subspace. Properties and examples of basis and of dimensions of vector spaces. Exercises: prove linearly dependence/independence of vectors in $\mathbf{R}^2$. Exercise: possible vector subspaces in $\mathbf{R}^2$ and $\mathbf{R}^3$ and respective dimensions.

(19/09/18) Matrices. Basic definitions and examples. Basic matrix operations and properties. Matrix multiplication and properties, example of the lack of the commutative property. The set of $m \times n$ matrices forms a vector space; its dimension and a possible basis.

(20/09/18) Determinants. Determinants of order 2: definition, properties and examples. Determinants of order 3: definition, properties and examples. Generalization for determinants of order n: definition, properties and examples (Laplace Rule, Binet's Theorem, etc). The cofactor matrix rule for computing the inverse of a matrix.

(21/09/18) Linear Mappings. Definition and examples of linear mappings: the linear map associated with a matrix, the zero map and the identity map. Properties of linear mappings. Definition of the sum operation and scalar multiplication between linear mappings such that the set of all linear mappings forms a vector space. Kernel and Image of linear mappings, examples of $Im(L)$ and $Ker(L)$ for particular mappings, e.g. for the projection mapping, etc. Injective and surjective linear mappings. Matrix associated to a linear map: one to one correspondence between the vector space of all matrices and the set of linear mappings. Examples: given a linear map, find the associated matrix.

(24/09/18) Change of basis/change of coordinates in vector spaces. General derivation of the change of coordinates formula for vectors and different examples. Example: rotation of the coordinate $x-$ and $y-$ axes with an angle θ; its transition matrix as an example of an orthogonal matrix.

(25/09/18) Definition of scalar products on a vector space V. Basic terminology for matrices with complex entries: adjoint matrix, hermitian/skewhermitian matrix, unitary matrix. Example: Classical Euclidean scalar product. Property: orthogonal (resp. unitary) matrices preserve the scalar product. Definition of norms/ seminorms in V. Example of a normed vector space: ($\mathbf{R}^n$, $p-$ norms). Cauchy Schwarz and Holder inequalities. Definition of orthogonality between vectors, orthogonal space of a vector subspace. Example: The Pythagoras theorem. Exercise: find an orthonormal basis of $\mathbf{R}^2$ among four given vectors. Definition of norm equivalence. Theorem: If $dim(V) < \infty$, then all norms are equivalent; illustrative example: table of equivalence constants for the main norms in $\mathbf{R}^n$.

(26/09/18) Definition of matrix norm. Consistent and submultiplicative matrix norms. Example of a non submultiplicative matrix norm. Examples: the Frobenius norm, the induced matrix norm from a vector norm, in particular matrix p-norms. Exercises for computing different matrix norms. Definition of eigenvalues and eigenvectors. Spectrum and spectral radius of a matrix. Relation between norms and spectral radius of a matrix. Symmetric positive definite matrices. Examples and properties. Sylvester Criterion. Exercises.

(28/09/18) Eigenvalues and eigenvectors. Computation of eigenvalues: Rayleigh quotient and the characteristic polynomial. Examples: computation of the eigenpairs of a matrix. Kernel, image and rank of a matrix. Equivalent properties for a nonsingular matrix. Particular types of matrices.