4.1.10

(page 124)

Determine all triplets of integers (x, y, z) satisfying the equation.

$$x^3 + y^3 + z^3 = (x + y + z)^3$$

proof)

$$x^3 + y^3 + z^3 = (x + y + z)^3$$

$$x^3 + y^3 + z^3 - (x + y + z)^3 = 0$$

$$(x^3 + y^3) + (z^3 - (x + y + z)^3) = 0$$

$$(x + y)(x^2 - xy + y^2) + (z - (x + y + z))(z^2 + z(x + y + z) + (x + y + z)^2) = 0$$

$$(x+y)(x^2-xy+y^2)-(x+y)(z^2+zx+zy+z^2+(x^2+y^2+z^2+2xy+2yz+2zx)) = 0$$

$$(x + y)((x^2 - xy + y^2) - (x^2 + y^2 + 3z^2 + 2xy + 3yz + 3zx)) = 0$$

$$(x + y)(-3xy - 3z^2 - 3yz - 3zx) = 0$$

$$-3(x + y)((xy + yz) + (zx + z^2)) = 0$$

$$-3(x + y)(y(x + z) + z(x + z)) = 0$$

$$-3(x + y)(y + z)(z + x) = 0$$

$$(x + y)(y + z)(z + x) = 0$$

Thus, either $x + y = 0$, $y + z = 0$, or $z + x = 0$ then we can conclude that $x = -y$, $y = -z$, or $z = -x$