# Properties of Logs, Exponents, and Series

## Conventions

- $\lg n = \log_2 n$
- $\ln n = \log_e n$
- $\log_a n$ specifies that the base of the log is $a$
- $\log n$ indicates that the base of the log doesn't matter

## Logs

- $\lg^k n \; = \; (\lg n)^k$
- $\lg \lg n \; = \; \lg (\lg n)$
- $a^{\log_b c} = c^{\log_b a}$
- $a^{\log_a b} = \; b$
- $\log b^n \; = n \; \log b$
- $\log_a \; n \; = \; \frac{\log_b n}{\log_b a}$
- $\log xy \; = \; \log x \; + \; \log y$
- $\log_a b \; = \; \frac{1}{\log_b a}$

## Exponents

- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

## Series

- Geometric

  if $\alpha > 1$: $\sum_{i=0}^{n} \alpha^i < \frac{\alpha}{\alpha - 1} \alpha^n$

  if $\alpha < 1$: $\sum_{i=0}^{\infty} \alpha^i < \frac{1}{1 - \alpha}$

- Harmonic

  $\sum_{i=1}^{n} \frac{1}{i} < \ln(1 + n)$

- Arithmetic

  $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

- Factoring

  $\sum_{i=0}^{n} x \cdot i = x \sum_{i=0}^{n} i$