Destination index :- It is a 16 bit ...

16.25

0.25 X 16

7 7 7 7.

open loop control system :- A system in which output is not fed to the input. So the control action is independent of the desired o/p.

closed loop control system :- A system in which, output is fed to the Input, so, the control action is dependend on the desired o/p.

Unit-1

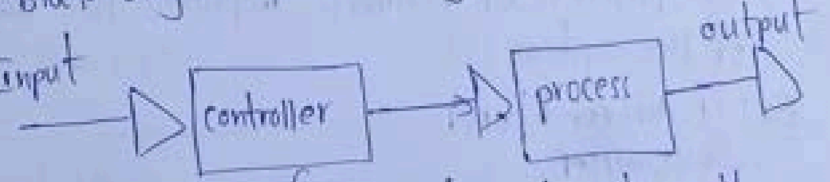
1 Define control system? Explain types of control systems with one example?

(A) Control System: A control system is a system that is used to control the behaviour of device (or) process.

-> It is made up of three main components sensor, controller, actuator.

Types :- i open loop control system: A control system in which action is totally independent of the output of the system then it is called open-loop control system.

-> Black diagram of open loop control system shown below.



-> The accuracy of the system depends on the experience of the user.

Ex : 1 Immersion water heater.
2. Toaster.

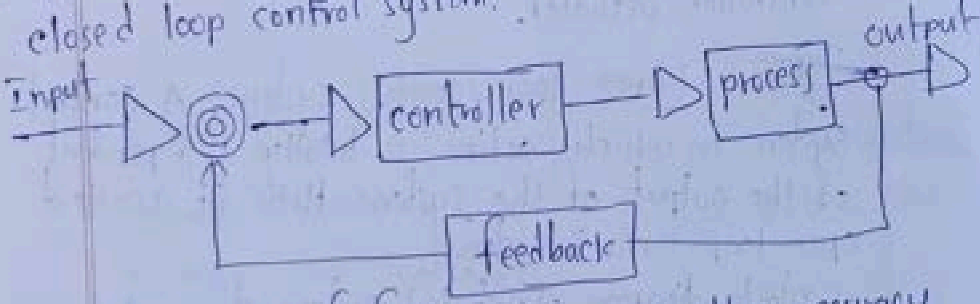
Advantage
-> simple to construct and design.
-> Easy to maintain

Disadvantage
-> inaccurate
-> unreliable.

ii closed loop control system: A control system in which the output has an effect on the input quantity is called closed loop control system.

-> A ~~closed~~ open loop control system can be converted into a closed loop control system by providing feedback.

-> The below figure shows the block diagram of closed loop control system.



-> The presence of feedback improves the accuracy of the system.

Ex: 1. Air conditioner.
2. Geyser.

Advantages:-
-> More accurate
-> Bandwidth is large.
-> less affected by noise.

Disadvantagg
-> It is constlier.
-> Difficult to design.
-> Requires more maintenance.

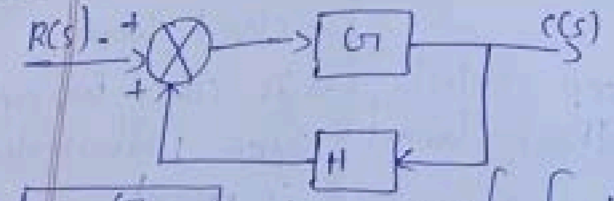
open loop	closed loop
-> In open loop control system feedback element is absent.	-> In closed loop control system feedback element is present
-> Error detector is absent	-> Error detector is present
-> Easy to construct.	-> Difficult to construct.
-> It is economical	-> It is costlier.
-> Having small bandwidth	-> Having large bandwidth
-> Requires less maintenance.	-> Requires more maintenance
-> It is inaccurate.	-> It is accurate.
-> It is unreliable.	-> It is reliable.
-> It is stable.	-> It is unstable.
Ex: Tea maker.	Ex: servo voltage stabilizer.

Feedback:- Feedback is the output of the system is used as input back into the system is called feedback.

Types:-

- positive feedback: The +ve feedback adds the reference point R(s) and feedback output. is called ±

-> Block diagram:

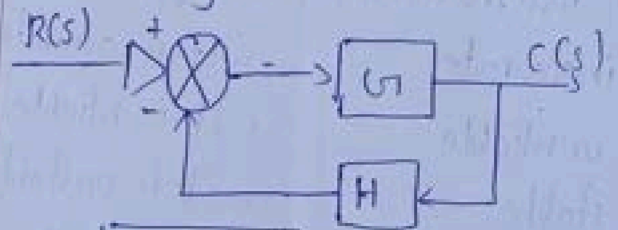


→ $\boxed{T = \frac{G}{1-GH}}$

T = Transfer function
G = open loop gain
H = gain of feedback path.

Negative feedback : The -ve feedback reduces the error between reference input R(s) and system o/p.

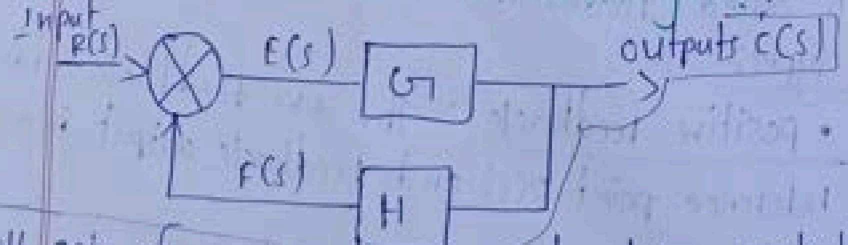
→ Block diagram.



→ $\boxed{T = \frac{G}{1+GH}}$

Effects of feedback

1. Effect of feedback of overall gain :



Overall gain of negative feedback closed loop control system is the ratio of G and 1+GH.

→ Transfer function with feedback is

$\boxed{T(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH}}$

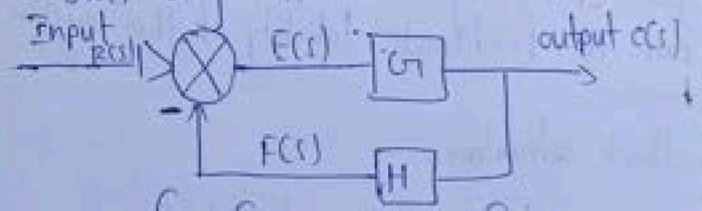
→ Effects studied from Transfer function

- If H is increases, then 1+GH will get increases, and transfer function will decreases

→ If H is decreases, then 1+GH will get decreases and transfer function will increases.

2. Effect of feedback on sensitivity

→ Block diagram:



→ Transfer function with feedback.

$\boxed{T(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH}}$

→ Sensitivity is defined by $S = \frac{\%\ \text{change in } T(s)}{\%\ \text{change in } G(s)}$

$$S = \frac{dT/T}{dG/G} \quad \frac{(dT/T \times 100)}{(dG/G \times 100)}$$

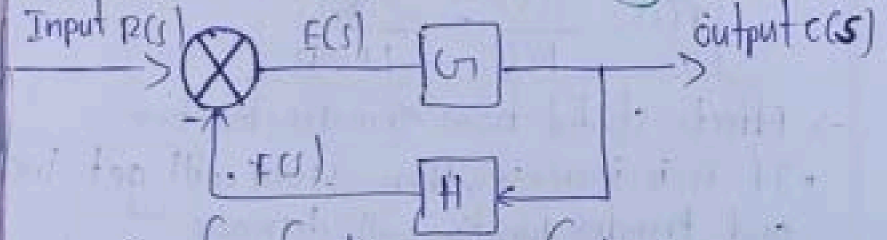
$$S = \frac{dT}{dG} \times \frac{G}{T} \qquad \left(\frac{G}{T} = 1+GH\right)$$

$$S = \frac{dT}{dG}(1+GH)$$

$$\frac{dT}{dG} = \frac{1}{(1+GH)^2}$$

$$\boxed{S = \frac{dT}{dG} \times \frac{G}{T} = \frac{1}{1+GH}}$$

3. Effect of feedback on stability



-> So, Transfer function with feedback is

$$\boxed{T(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH}}$$

-> gain of the system without feedback is G.
-> Due to feedback stability of the system decreases

Feedback advantages

-> Reduction in noise and disturbance
-> " of non-linear effects
-> Improved efficiency, accuracy, quality
-> " system stability and process certainty.

Feedback of characteristics :-

-> Increased accuracy
-> feedback decreas/ Reduced sensitivity.
-> More stability
-> More reliable.
-> Increase the Bandwidth.
-> Reduced effect of noise.
-> " " non-linearities

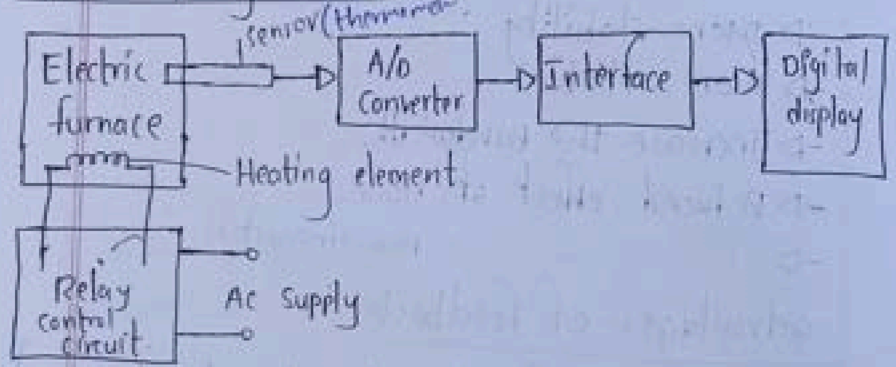
advantages of feedback :

-> Reduction of ~~external~~ external noise and disturbance.
-> Reduction of non-linear effects.
-> Improved efficiency, accuracy and quality.
-> Improved gain and bandwidth.
-> The transient response can be easily controlled

Examples of control system

1 Temperature control system:

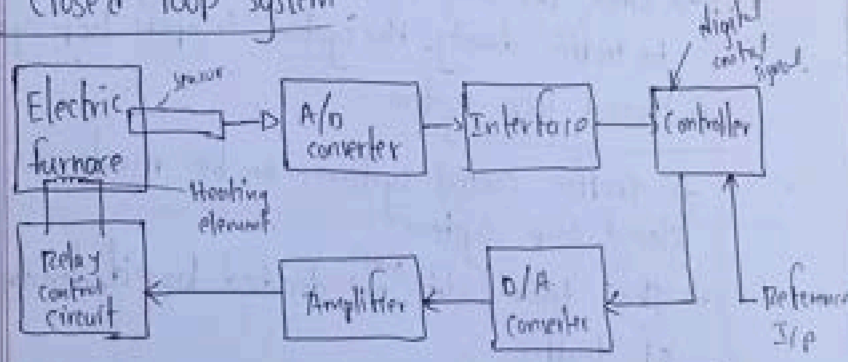
i open loop system:



-> The above block diagram shows open loop temperature control system.
-> The ON and OFF of the supply is governed by the relay control circuit.
-> The temperature is measured by a sensor (thermometer), which gives analog signal corresponding to furnace.
-> This analog signal is converted to digital signal by A/D converter.
-> The temperature of the system is raised by heating element.
-> This digital signal given to the digital display to display the temperature.

-> The o/p temperature depends upon heater remains ON.

ii closed loop system:



-> The above block diagram shows the closed loop temperature control system.
-> The o/p of the system is desired temperature, it depends on heater remains on.
-> The switching on and off of the relay is controlled by controller.

2. Traffic control system

i open loop system:
-> Traffic control means, traffic signal operated on time basis.
-> The sequence of the control signals are based on time slot given for each signal.

→ The time slots are decided based on traffic study.

→ Since, the time slot does not changes according to traffic density the system is open loop system.

ii. closed loop system :

→ Traffic control system can be made as closed loop system.

→ The time slots signals are decided based on density of traffic.

→ The timings of control signals are decided by the computer.

→ The flow of vehicles is better than open loop system.

Masons gain formula

Mason's gain formula is used to determine transfer function

$$T(s) = \frac{C(s)}{R(s)}$$

→ It is used to find overall transfer function from signal flow graph.

$$\therefore \text{ over all Transfer function} = \boxed{\frac{\sum T_k \Delta_k}{\Delta}}$$

where,

k = No. of forward path

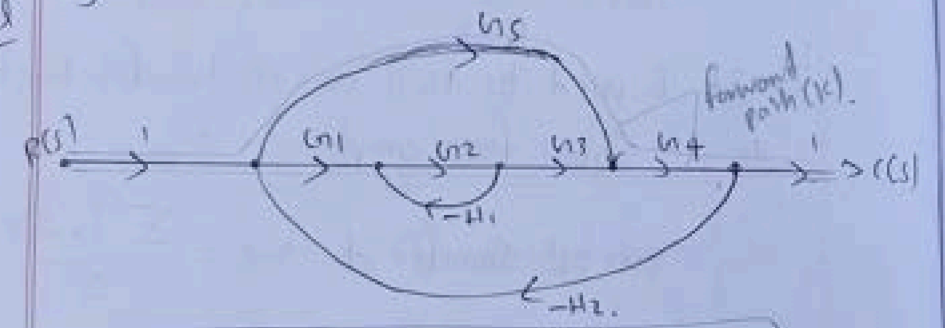
T_k = gain of k^{th} forward path

$\Delta = 1 -$ (Sum of individual loop gain) + (Sum of product of non touching loop gains taken two at a time) − (sum of product of non-touching loop gains taken three at a time t.) + ...

Δ_k = (1 − (loop gain which does not touch forward path).

(P). Determine closed loop transfer function for the given signal flow graph using masons gain formula?

sol



$$\boxed{\text{Masons gain formula} = \frac{\sum T_K \Delta_K}{\Delta}}$$

i) No. of forward path (K):

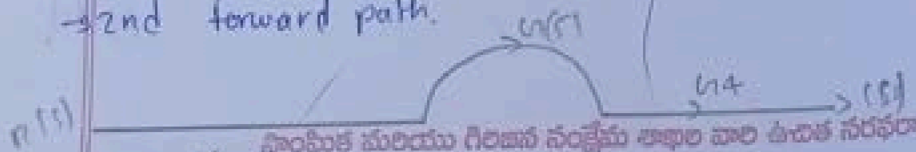
-> No. of forward path $K = 2$.

$$T.F = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

-> 1st forward path.

$T_1 = G_1 G_2 G_3 G_4$.

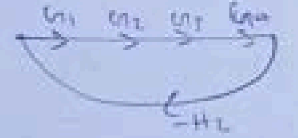
-> 2nd forward path.



$T_2 = G_5 G_4$.

ii) Individual loop gain

$L_1 = -G_2H_1$, $L_2 = -G_1G_2G_3G_4H_2$



$L_3 = -G_5G_4H_2$

iii) Non-touching loop pairs

Two Non-touching loop pairs are there.

$L_1 \times L_3 = -G_2H_1 \times -G_5G_4H_2 = G_2G_4G_5H_1H_2$

iv) Find Δ

$\Delta = 1 - [\text{Sum of individual loop gain}] + [\text{Sum of gain product of two non-touching loop}]$

$= 1 - [-G_2H_1 - G_1G_2G_3G_4H_2 - G_5G_4H_2] + [G_2G_4G_5H_1H_2]$.

$$\boxed{\Delta = 1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_5G_4H_2 + G_2G_4G_5H_1H_2}$$

$\Delta_k = \Delta_1, \Delta_2$

$\Delta_k = 1 -$ [Non touching part of 1st forward path)

$\Delta_1 = [1-0]$, $\Delta_2 = 1-[-G_2H_1]$

$\boxed{\Delta_1 = 1.}$ $\boxed{\Delta_2 = 1+G_2H_1}$

$$\boxed{\text{Transfer function (TF)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_5G_4[1+G_2H_1]}{1+G_2H_2+G_1G_2G_3G_4H_1H_2+G_5G_4H_2+G_2G_4G_5H_1H_2}}$$

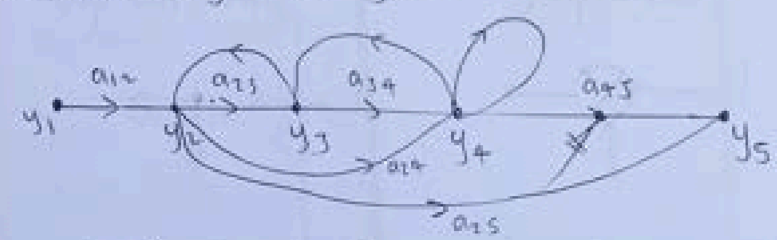
Signal flow graph (Introduction):

-> Signal flow graph (SFG) is a graphical representation of block diagram.

-> Signal flow graph is used to determine over all transfer function of a control system.

-> Signal flow graph is an easier method to determine transfer function as compared to block diagram reduction.

-> Element of signal flow graph



output Node: The node which has only incoming branches.

path: The traversal of connected brances in the direction of branch arrows.

Forward path: It is a path from input node to o/p node.

Forward path gain: It is the product of branch gains.

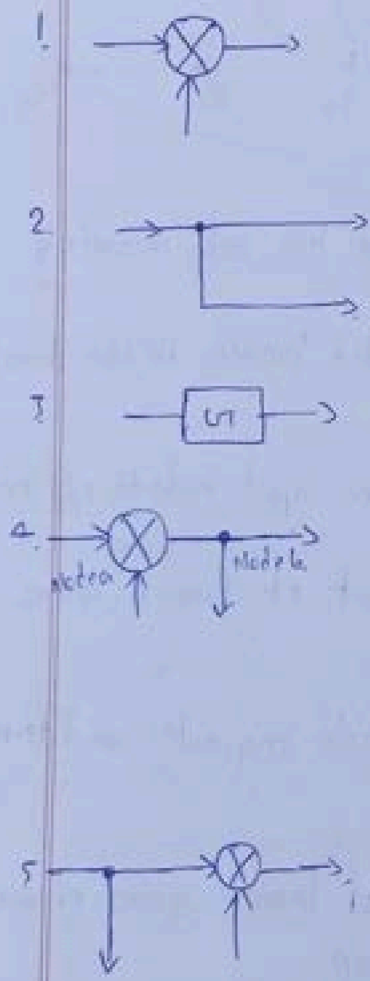
loop: It is a path which originates and terminates at the same node.

loop gain: It is product of branch gains encounted in traversing a loop.

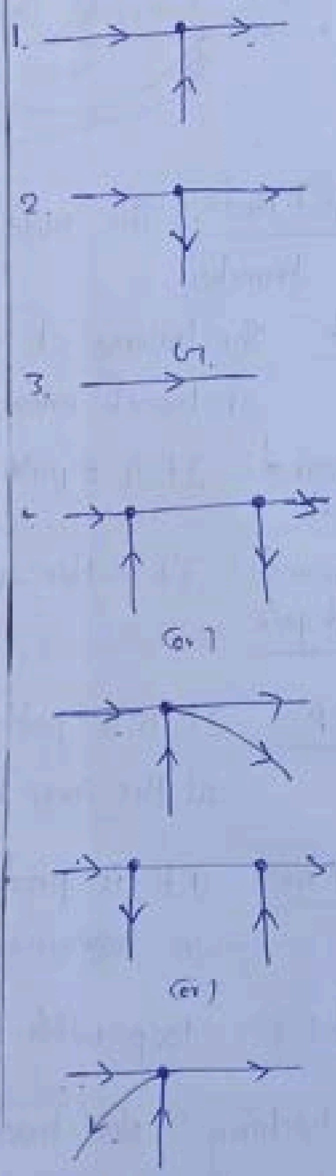
Self loop: loop with one branch is called self loop.

Non-touching loop: Non-touching loops if they do not passes any common node.

Block diagram



Signal flow graph



(P). obtain signal flow graph from following algebraic equation

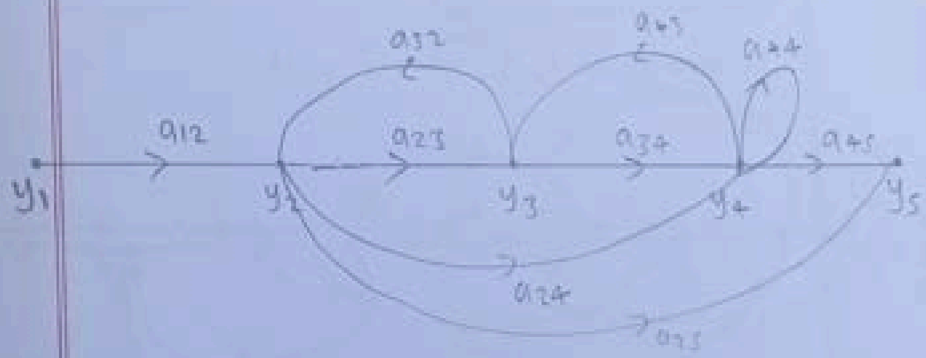
$y_2 = a_{12}y_1 + a_{32}y_3$

$y_3 = a_{23}y_2 + a_{43}y_4$

$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$

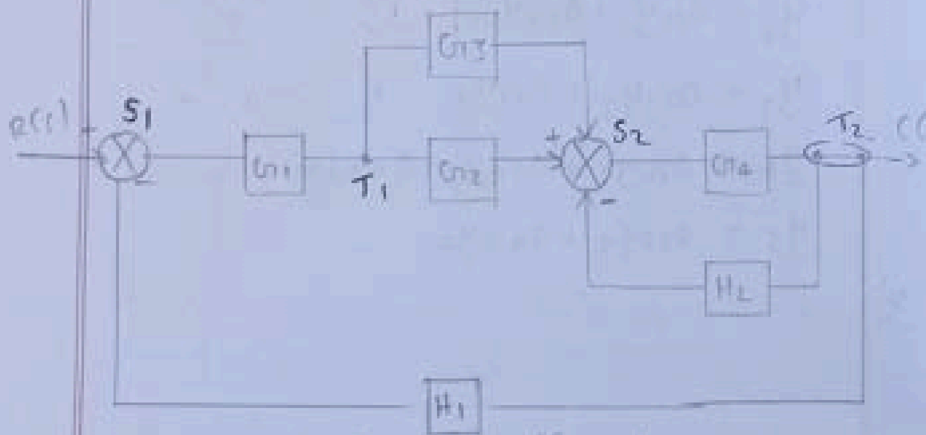
$y_5 = a_{25}y_2 + a_{45}y_4$

Sol



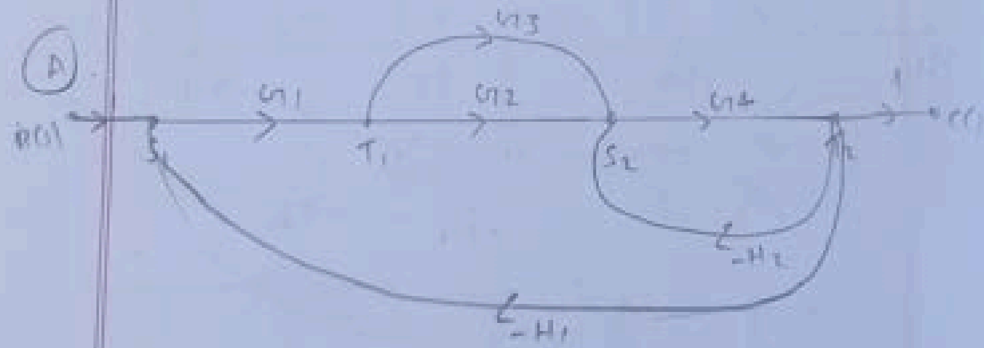
సాంఘిక మరియు గిరిజన సంక్షేమ శాఖల వారి ఉచిత సరఫరా

(P) obtain signal flow graph from block diagram.

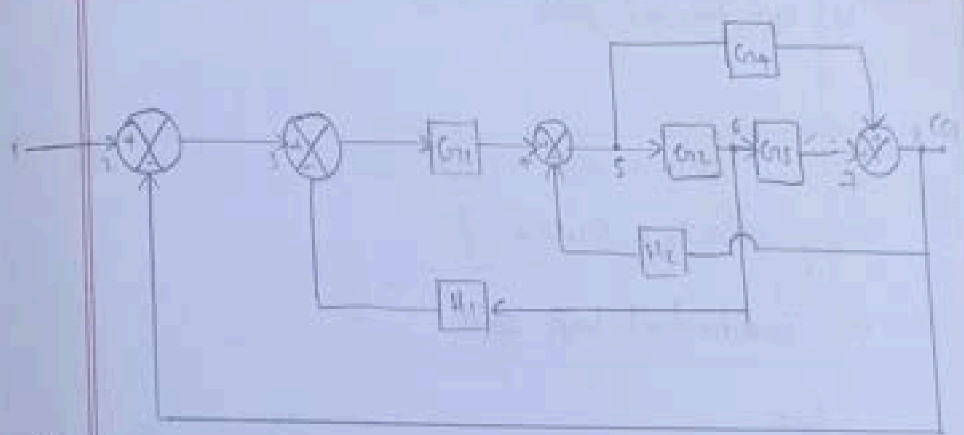


S= summing point, T= Takeoff point

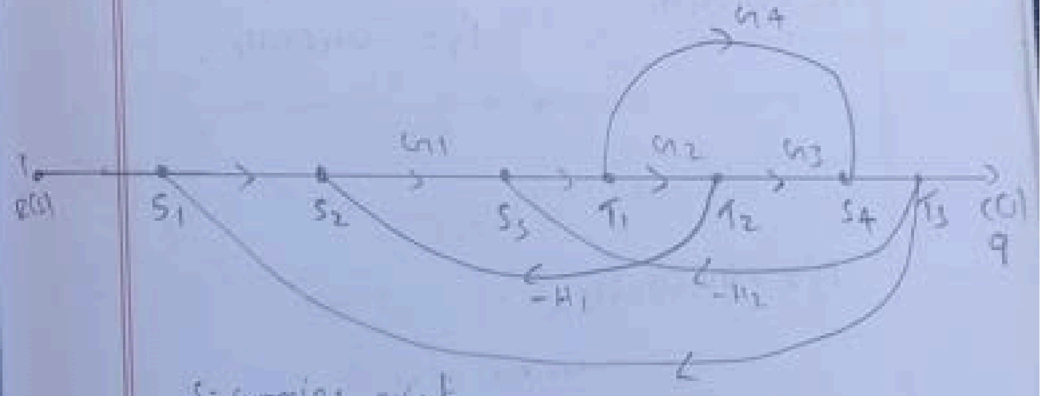
(A).



(P) convert the block diagram into signal flow graph and find Transfer function?



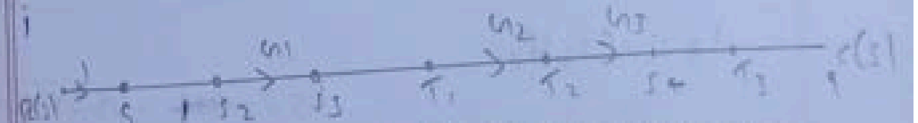
(A).



S: summing point

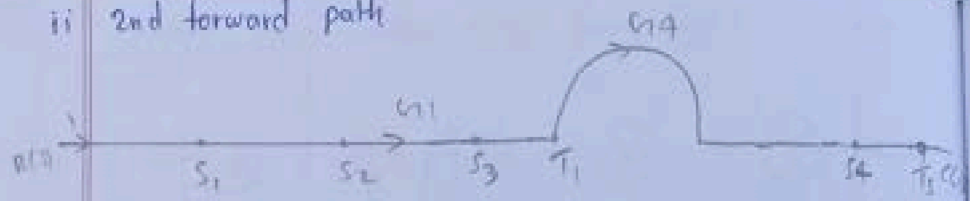
T= Take off point.

i No. of forward path (K) and gain :- K = 2



gain $T_1 = G_1 G_2 G_3$

ii 2nd forward path



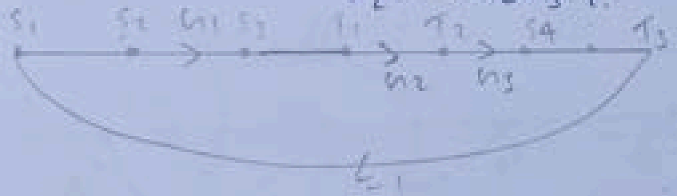
$\boxed{\text{gain } T_2 = G_1G_4}$

ii Individual loop gain

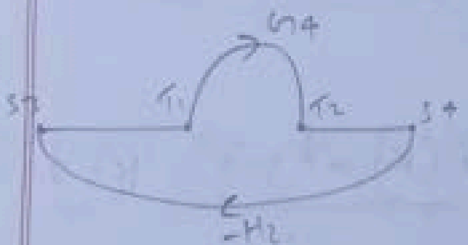
$L_1 =$

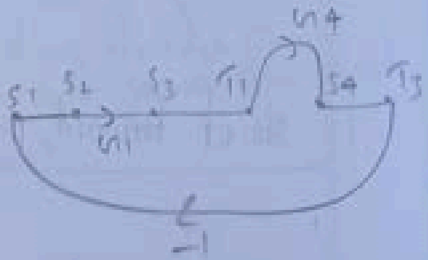
$L_1 = -G_1G_2H_1$

$L_2 = -G_2G_3H_2$



$L_3 = -G_1G_2G_3$





$L_4 = -G_4H_2$

$L_5 = -G_1G_4$

iii product of two Non-touching loops = 0

iv calculate Δ and Δ_k.

$\Delta = 1 - [\text{sum of individual loop gain}] + [\text{sum of two non touching loop pair gains}]$

$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_4H_2 + G_1G_4$

$\Delta_1 = \Delta_2 = 1$

$T.F = \dfrac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$

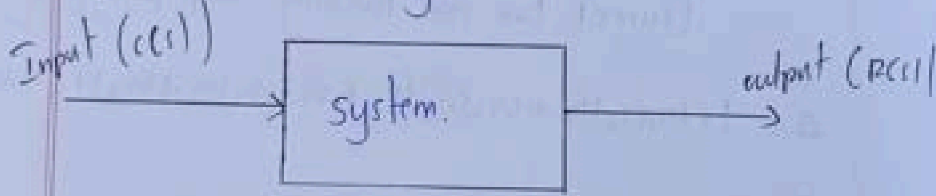
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_4H_2 + G_1G_4}$$

సాంఘిక మరియు గిరిజన సంక్షేమ శాఖల వారి ఉచిత సరఫరా

Transfer functions

-> The relationship between input and output is called Transfer function.

-> It describes the system.



-> Transfer function = $\frac{C(s)}{R(s)} = \frac{N(s)}{D(s)}$.

Here N(s) order gives total zeros
D(s) order gives total poles.

-> Types of transfer function

i Proper Transfer function :

-> If poles are greater than zeros [P>Z] then it is called proper transfer function.

ii Improper transfer function:

-> If zeros are greater than poles [Z>P] then it is called improper transfer function.

properties :

-> The ratio of laplace transform output to laplace transform input, assuming all initial conditions to be zero.

-> The transfer function of a system does not depend on inputs.

-> The systems poles and zeros can be determined from it's transfer function

-> stability can be found from Transfer function.

-> Transfer function is applicable for LTI systems only.

Advantages

-> poles and zeros can be Identified.

-> It is depend on system, not on input

-> Integral and differential equations are converted to simple algebraic eq.

Disadvantages

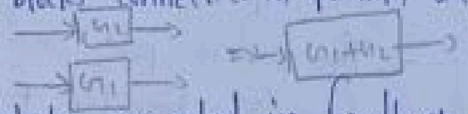
-> It is applicable only for LTI systems.

Block diagram Reduction

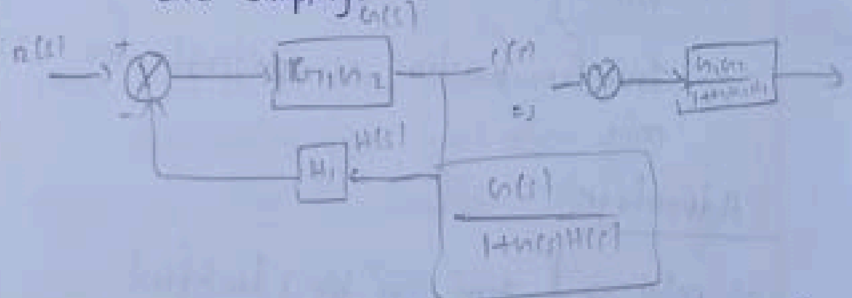
→ Block diagram reduction rules

Rule 1 : check for blocks connected in series and simplify

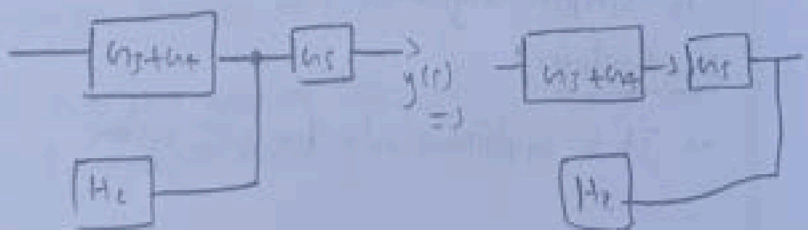
Rule 2 ; check for blocks connected in parallel and simplify.



Rule 3 : check for blocks connected in feedback loop and simplify.



Rule 4 If there is a difficulty with take-off point while simplifying, shift it ~~twow~~ towards right



Rule 5 If there is difficulty with summing point. shift it towards left.

Rule 6 Repeat the about steps upto you get simplified form, i.e. single block.

Unit - II

Servomotors : The motors that are used in automatic control systems are called servomotors.

(or)

The control systes which are used to control the position, velocity and acceleration are called servomotors.

-> Depending upon the supply required to run the motor, they are broadly classified into two types

1. DC servomotor.   2. AC servomotor.

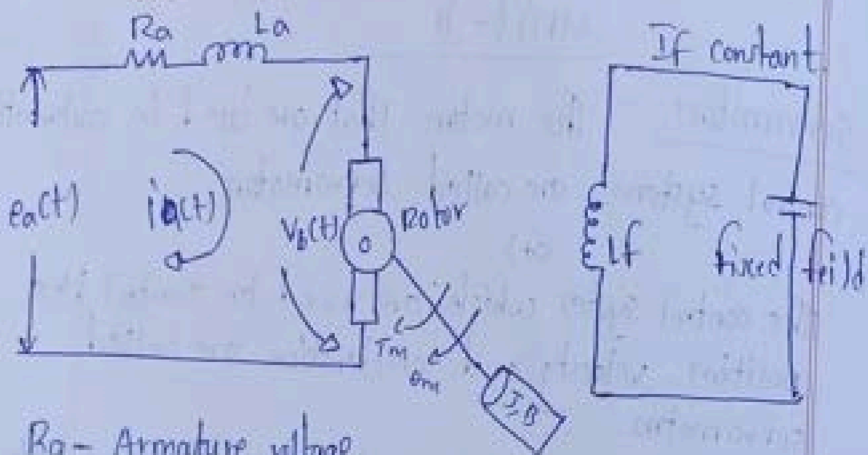
1. DC servomotor : A servomotor that uses dc electrical input and produces mechanical output like position, velocity and acceleration is called dc servomotor.

-> It is of two types.

I Armature controlled dc servomotor:

-> **Transfer function of armature controlled dc servo motor:**

-> Armature controlled dc servometer have controlled armature and fixed feild magnet.



Ra - Armature voltage
La - Armature Inductance
Lf - feild "
ea(t) - Armature voltage
ia(t) - Armature current

-> Transfer function of armature controlled dc servometer is $\boxed{T(s) = \frac{\theta m(s)}{Ea(s)}}$

-> In this case output is $\theta m(t)$ and input is $ea(t)$

-> Apply kvl in Armature.

$$ea(t) = Ra . ia(t) + La \frac{d\, ia(t)}{dt} + V_b(t)$$

-> Apply laplace Transform

$$\boxed{Ea(s) = Ra\, Ia(s) + La\, s\, Ia(s) + V_b(s)} \text{ — } 1.$$

-> The back emf $v_b(t)$ is proportional to the velocity

$$V_b(t) \propto \frac{d\theta m(t)}{dt}$$

$$V_b(t) = k_b \frac{d\theta m(t)}{dt}$$

-> Apply L.T

$$\boxed{V_b(s) = k_b\, s\, \theta m(s)} \text{ — } 2.$$

Sub eq 2 in eq 1.

$$Ea(s) = Ra\, Ia(s) + La\, s\, Ia(s) + k_b\, s\, \theta m(s)$$

$$\boxed{Ea(s) = Ia(s)[Ra + La\, s] + k_b\, s\, \theta m(s)} \text{ — } 3.$$

-> The torque of dc motor depends on armature current

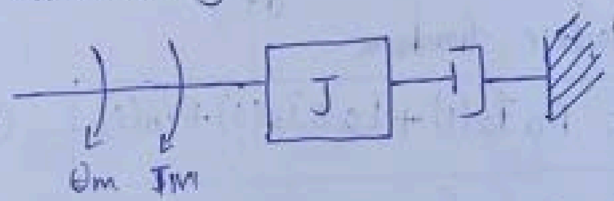
$$Tm \propto ia(t)$$

$$Tm = k_t\, ia(t)$$

-> Apply laplace Transform

$$Tm(s) = k_t\, s\, Ia(s) \text{ — } 4.$$

→ The mechanical system of the motor is given by



→ The differential equation equation representing the above system can be written as:

$$J\frac{d^2\theta_m}{dt^2} + B\frac{d\theta_m}{dt} = T_m$$

→ Apply laplace Transform

$$\boxed{Js^2\theta_m(s) + Bs\,\theta_m(s) = T_m(s)} \quad (5)$$

sub eq (4) in eq (5)

$$Js^2\theta_m(s) + Bs\,\theta_m(s) = K_t\, I_a(s)$$

$$\theta_m(s)\left[Js^2 + Bs\right] = K_t\, I_a(s)$$

$$\boxed{I_a(s) = \frac{\theta_m(s)\left[Js^2+Bs\right]}{K_t}} \quad (6)$$

sub eq (3) in eq (6)

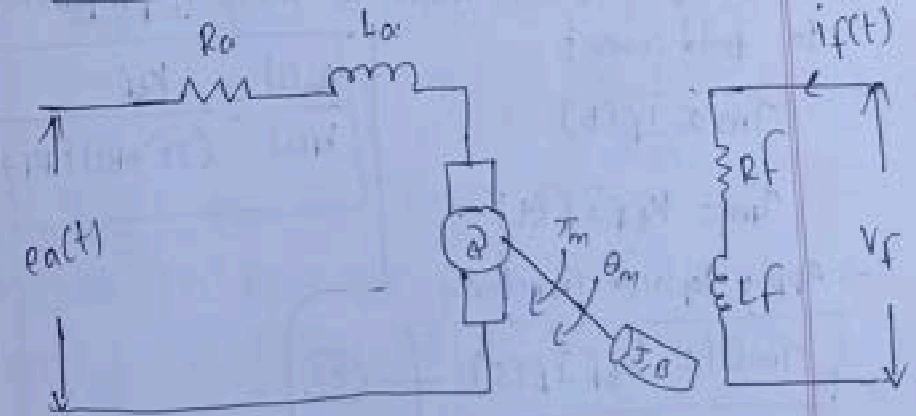
$$E_a(s) = \frac{\theta_m(s)\left[Js^2+Bs\right]}{K_t}\left[R_a + L_a s\right] + K_b s\,\theta_m(s)$$

$$E_a(s) = \theta_m(s)\left\{\frac{\left[Js^2+Bs\right]\left[R_a+L_a s\right] + \left(K_t K_b s\right)}{K_t}\right\}$$

$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t}{(Js^2+Bs)(R_a+L_a s) + K_t K_b s}}$$

2. Feild controlled DC servometer

Transfer function of feild controlled dc servometer



where, R_f - feild resistance.
L_f - " inductance.
i_f - " current
V_f - " voltage.
J = moment of inertia.
B - frictional coefficient

-> Apply KVL in feild,

$$V_f(t) = i_f(t) R_f + L_f \frac{di_f(t)}{dt}$$

-> Apply laplace transform

$$\boxed{V_f(s) = i_f(s) R_f + L_f s\, i_f(s)} \text{ — } ①.$$

-> The torque of the motor is directly proportional to feild current

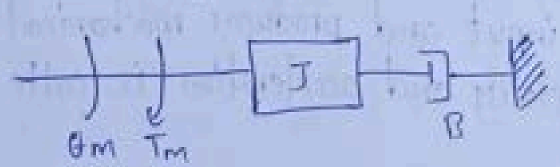
$$T_m \propto I_f(t)$$

$$T_m = K_{tf} I_f(t)$$

-> Apply laplace Transform.

$$\boxed{T_m(s) = K_{tf} I_f(s)} \text{ — } ②.$$

-> The mechanical system of dc motor is given by.



-> Differential equation representing above equation.

$$T_m = J \frac{d^2\theta_m}{dt^2} + B \frac{d\theta_m}{dt}$$

-> Apply L.T

$$T_m(s) = J s^2 \theta_m(s) + Bs\, \theta_m(s)$$

$$\boxed{T_m(s) = \theta_m(s)\left[Js^2 + Bs\right]} \text{ — } ③.$$

sub eq ② in eq ③.

$$\cancel{K_{tf}(s) I_f(s)} \quad K_{tf} I_f(s) = \theta_m(s)\left[Js^2 + Bs\right]$$

$$\boxed{I_f(s) = \frac{\theta_m(s)}{K_{tf}}\left[Js^2 + Bs\right]} \text{ — } ④.$$

sub ④ in ①.

$$V_f(s) = I_f(s) R_f + L_f s\, I_f(s)$$

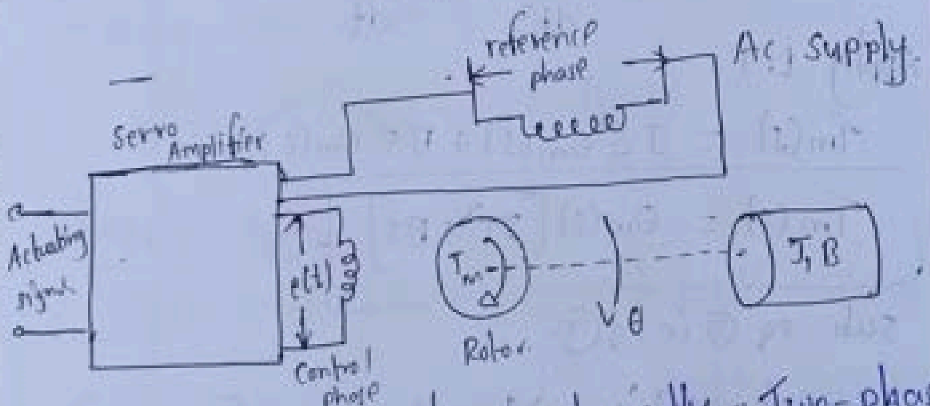
$$V_f(s) = I_f(s)\left[R_f + L_f s\right]$$

$$V_f(s) = \frac{\theta_m(s)}{K_{tf}}\left[Js^2 + Bs\right]\left[R_f + L_f s\right]$$

$$\boxed{\frac{\theta_m(s)}{V_f(s)} = \frac{K_{tf}}{(Js^2 + Bs)\left[R_f + L_f s\right]}}$$

Ac Servometer :- A servometer that uses Ac electrical input and produces mechanical output like position, velocity and acceleration is called Ac servometer.

Transfer function of Ac servometer :-



→ An Ac servometer is basically Two-phase induction motor. A two phase motor consist of two stator windings oriented 90°.

→ It has two phases control phase and reference phase.

→ The control phase is energized by voltage is 90° with respect to reference phase.

→ The control phase voltage is supplied from a servo Amplifier.

where, T_m = Torque developed by servometer.

θ = Angle displacement of rotor.

J = Moment of inertia.

T_l = Torque required by the load

e_{car} = carrier signal

e_c = control signal

e_{cm} = Modulated control signal

→ Torque developed by the motor.

$$T_m = k_1 e_c - k_2 \frac{d\theta}{dt}$$

→ Torque developed by the load.

$$T_l = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

→ Motor torque = load torque.

$$k_1 e_c - k_2\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

→ Apply laplace transform:

$$k_1 E_c(s) - k_2 s\,\theta(s) = Js^2\theta(s) + Bs\,\theta(s)$$

$$k_1 E_c(s) = \theta(s)\left[Js^2 + Bs + k_2 s\right]$$

$$\boxed{\frac{\theta(s)}{E_c(s)} = \frac{k_1}{s\,\theta(s)\left[Js + B + k_2\right]}}$$

Time Response Analysis

-> The ~~response~~ time response of the system means the o/p of the system is expressed in terms of time.

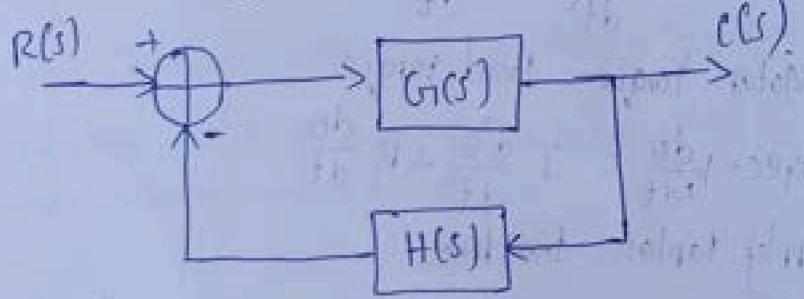
-> The o/p in s-domain is $C(s)$, which is product of Transfer function and Input of the system.

-> There are two types of time response analysis.

i Transient response.

ii steady state response.

-> Block diagram



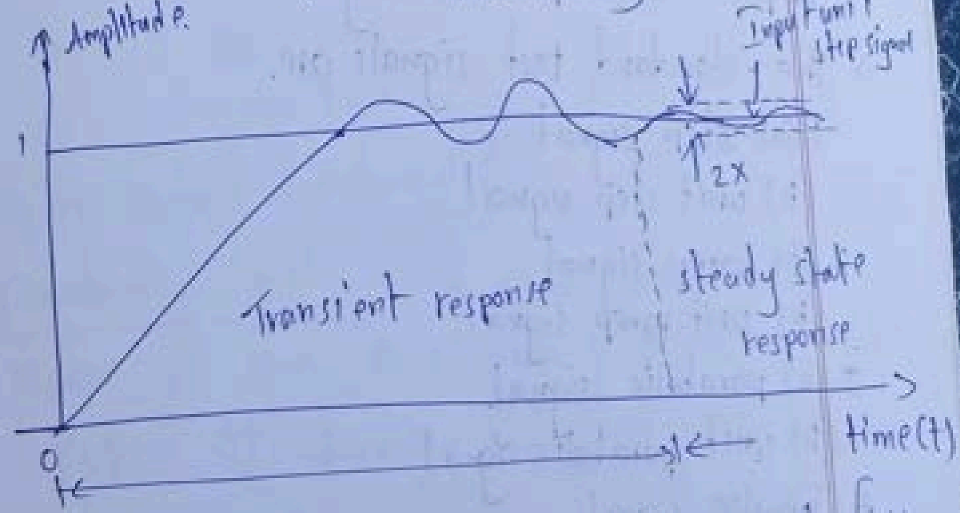
-> $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$C(s) = R(s)\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

-> For time domain output

$$c(t) = L^{-1} C(s).$$

$$c(t) = L^{-1}\left[R(s)\left(\frac{G(s)}{1+G(s)H(s)}\right)\right]$$



-> In Transient response we study speed of the system, and accuracy of the system.

-> In steady state response we study steady state error of the system.

Standard Test signals

-> The commonly used test signals are step, Ramp Impulse and parabolic signals.

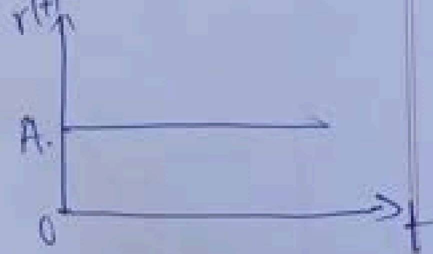
-> The standard test signals are.

1. (a) step signal.
   (b) unit step signal.
2. (a) Ramp signal
   (b) unit ramp signal.
3. (a) parabolic signal.
   (b) unit parabolic signal.
4. Impulse signal
5. sinusoidal signal

Step signal : step signal is a signal whose value varies from 0 to A at $t=0$.

$x(t) = Au(t)$.

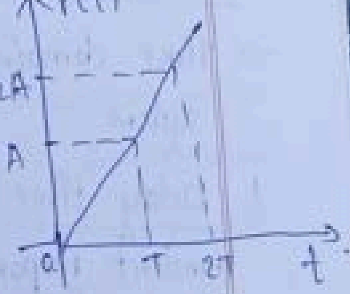
$u(t) = 1 ; t \geq 0$.
$\quad = 0 ; t < 0$.



-> In Laplace Transform $= \frac{1}{s}$.

Ramp signal : A Ramp signal is a signal whose values vary linearly with time from initial value of "0" at $t=0$.

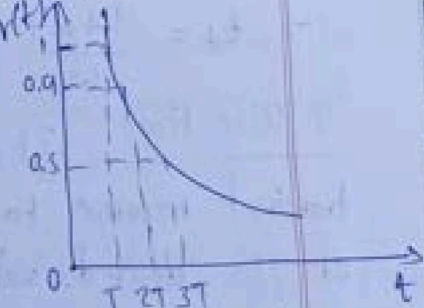
$r(t) = At ; t \geq 0$
$\quad 0 ; t < 0$.



-> In laplace transform - $\frac{1}{s^2}$.

parabolic signal : A signal varies with square of the time from initial value 0 at $t=0$.

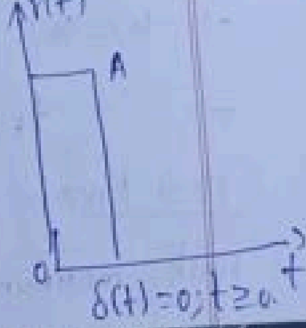
$r(t) = \frac{At^2}{2} ; t \geq 0$
$\quad 0 ; t < 0$



-> In laplace transform - $\frac{1}{s^3}$.

Impulse signal : Impulse signal happens in very short duration time.

$\lim_{t_1 \to 0} \int_{t_1}^{t_2} \delta(t)\, dt = 1$.



-> In L.T - 1

Design (or) Time specifications of second order system :

→ Time domain specifications of II^{nd} order system.

1. **Delay time (t_d)** : It is required for transient response to reach 50% of it's final value in first attempt.

→ final value = 1

→ $t_d = \frac{1+0.7}{\omega_n}$

2. **Rise time** : It is the time required for transient response to reach 10% to 90% (or) 0 to 100% of it's desired value in first attempt.

→ 0–100% – under damped systems

→ 10%–90% – over damped systems.

→ $t_r = \frac{\pi - \theta}{\omega_d}$

3. **peak time** : It is the time required to reach maximum overshoot / peak overshoot

→ At $t = t_p$ the first derivative of time response is zero.

$$t_p = \frac{\pi}{\omega_d}$$

4. **Maximum overshoot (M_p)** : The maximum positive derivative output with respect to desired value.

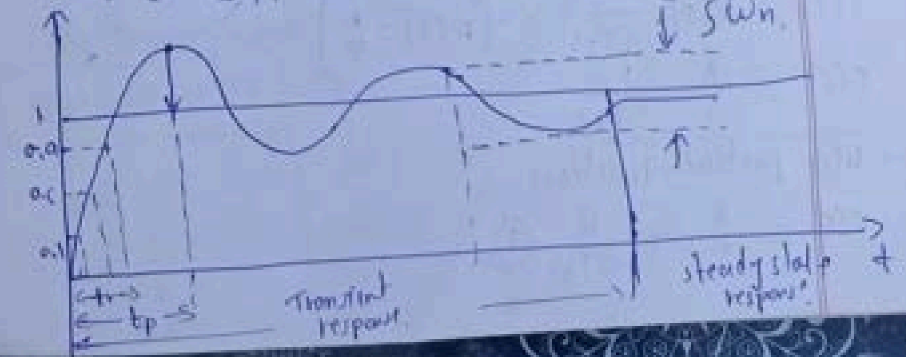
→ It occurs when time response curve after input signal becomes zero (when $t = t_p$).

→ $M_p = \frac{C(t)_{max} - 1}{1}$

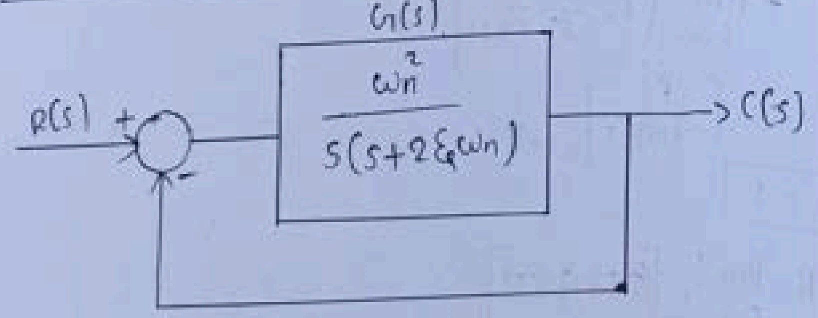
(5) **Setting Time (t_s)** : It is the time needed to settle down the oscillation within tolerance band of 2% to 5% of desired value of o/p

→ on 2% tolerance band's $t_s = 4 \frac{1}{\zeta\omega_n}$

→ on 5% tolerance bas $t_s = 3 \frac{1}{\zeta\omega_n}$



Time Response of second order system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}\cdot 1} \qquad \boxed{H(s)=1}$$

$$\boxed{\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} \quad (1)$$

ω_n = Natural frequency

ξ = Damping ratio.

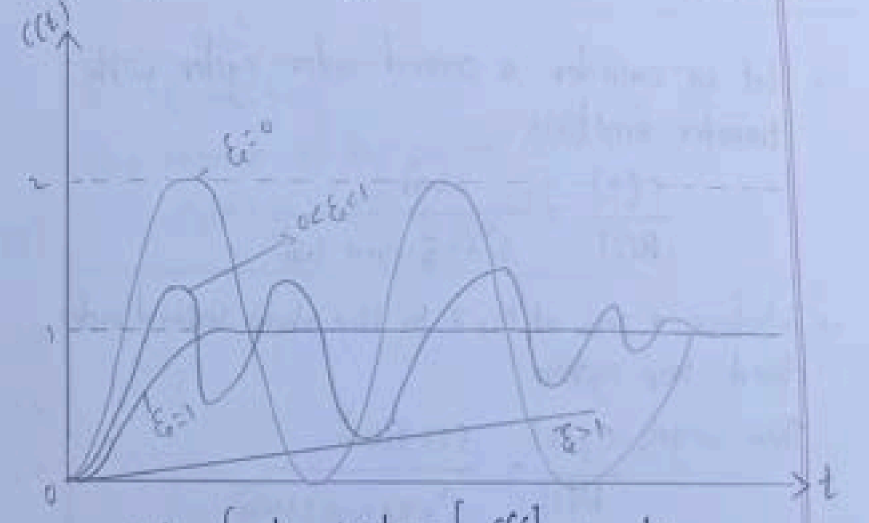
If $\xi = 0$ → undamped s/m

$0<\xi<1$ → under damped s/m

$\xi > 1$ → over damped s/m

$\xi = 1$ → critically damped s/m.

→ Response of the system is shown in below figure:



→ Roots of denominator of $\frac{C(s)}{R(s)}$ → poles.

→ Roots of Numerator of $\frac{C(s)}{R(s)}$ → zeros.

→ The time response of the system is characterized by poles of the Transfer function.

→ $1+G(s).H(s)$ - characteristic equation.

$1+G(s).H(s)=0.$

$s^2+2\xi\omega_n+\omega_n^2=0$

→ find roots for above equation using $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get

$$\boxed{s=-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}}$$

i. $\xi=0$; $s=\pm\omega_n\sqrt{-1}$ - roots are imaginary

ii. $\xi=1$; $s=-\omega_n$, real -ve and equal roots

iii. $0<\xi<1$; poles → complex conjugate, iv. $\xi>1$ poles → real -ve and unequal

Effect of adding a zero to a system

-> Let us consider a second order system with transfer function

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n + \omega_n^2}$$

-> Introduce a zero at $s=-z$ to the above second order closed loop system.

Then we get $$\frac{C_z(s)}{R(s)} = \frac{(s+z)\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

-> divide the numerator by z,

$$\frac{C_z(s)}{R(s)} = \frac{(s+z)\omega_n^2/z}{s^2+2\xi\omega_n s+\omega_n^2}$$

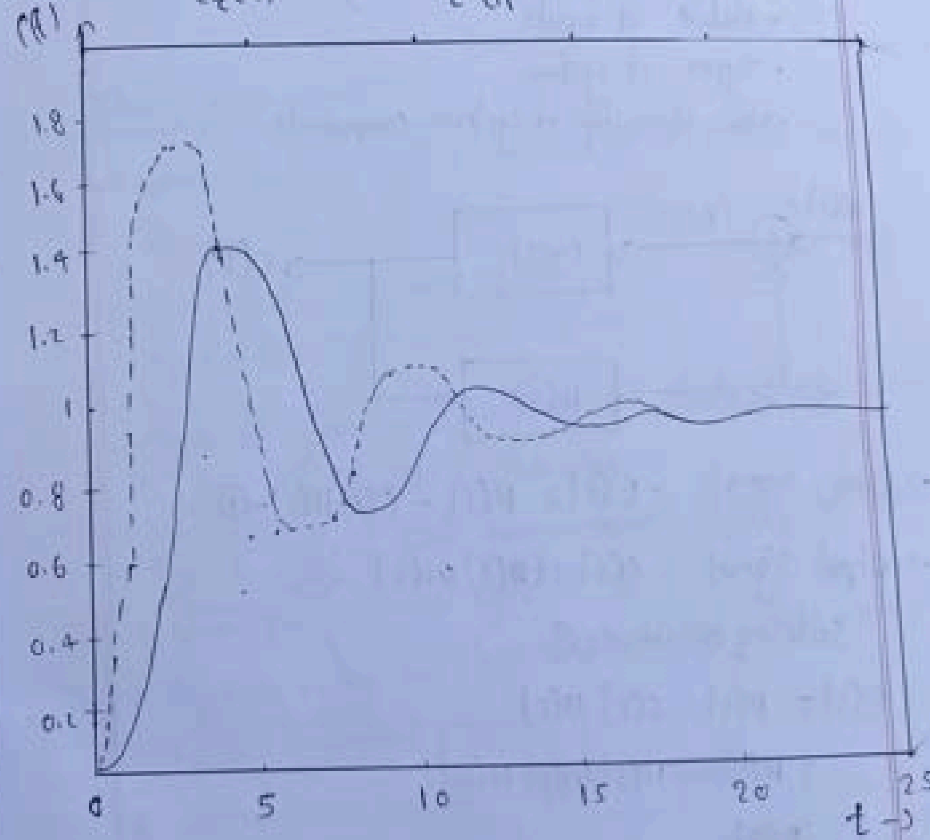
-> To analyze the effect of adding a zero to the system, we can rewrite the above eq as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} + \frac{s}{z}\left(\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right)$$

-> Let $c(t)$ is the step response of the system $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$.

-> step response of the system with added zero.

$$c_z(t) = c(t) + \frac{1}{z}\frac{d}{dt}c(t)$$



Effect of adding zero to a system

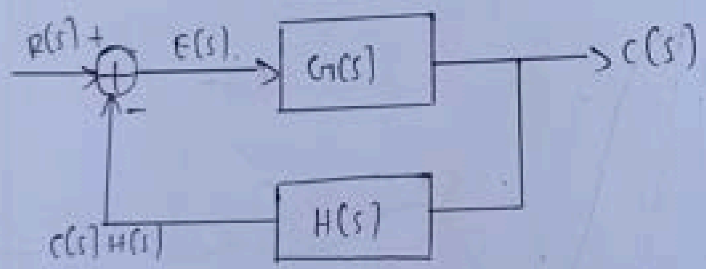
Steady state errors

-> It is the error at time 'T' goes to ∞.

-> It measures the accuracy of the system.

-> Errors may arises from following reasons

- Nature of inputs
- Types of systems
- Non-linearity of system components.



-> Error signal - $E(s) = R(s) - C(s)H(s)$ —①

-> output signal - $C(s) = E(s)G(s)$ —②

Sub eq ② in eq ①

$E(s) = R(s) - C(s)H(s)$

$\cancel{= R(s) - R(s)G(s)H(s)}$

$E(s) = R(s) - E(s)G(s)H(s)$

$E(s)[1 + G(s)H(s)] = R(s)$

$$\boxed{E(s) = \frac{R(s)}{1+G(s)H(s)}}$$

-> steady state error

$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

$= \lim_{s\to 0} s\left[\frac{R(s)}{1+G(s)H(s)}\right]$

Static error constants

-> when control system excited to standard signal, it may be zero, constant, infinity.

-> The value of steady state error depends on the type of the system and I/p signal.

-> Type '0' system will have constant steady state error with when I/p signal is unit step.

-> Type '1' system will have constant steady state error. when I/p is Ramp signal.

-> Type '2' system will have constant steady state error when I/p is parabolic signal.

* position error constant, $k_p = \lim_{s\to 0} G(s)H(s)$

* velocity " " , $k_v = \lim_{s\to 0} sG(s)H(s)$

* Acceleration " " , $k_a = \lim_{s\to 0} s^2G(s)H(s)$

Ac Tachometer

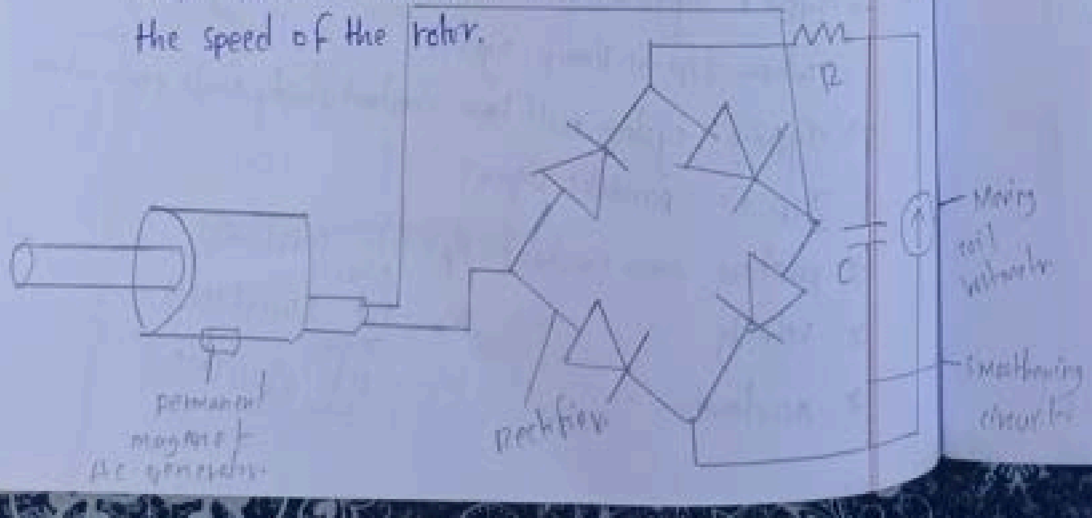
Tachometer : A tachometer is an ~~revolution~~ counter instrument, that measures the rotation speed of a shaft (or) disk. in a motor is called tachometer.

Ac Tachometer : The Ac tachometer has stationary armature and rotating magnetic feild.

-> The commutator and brushes are absent in Ac tachometer

-> The rotating magnetic feild induces the EMF in the stationary coil of the stator.

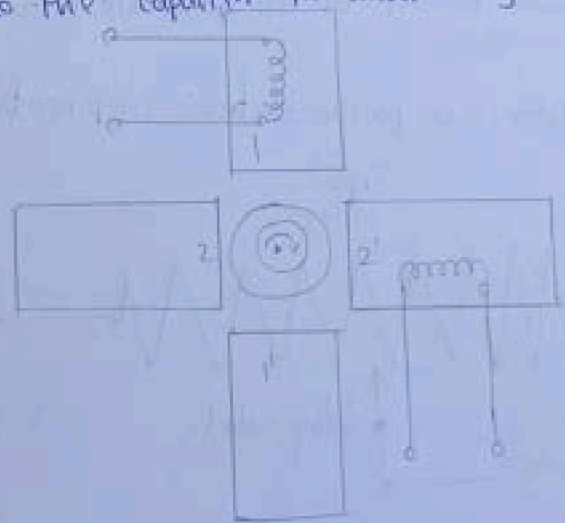
-> The below mentioned circuit used for measuring the speed of the rotor.



-> The block diagram shows Ac tachometer generator.

-> Ac tachometer circuit consist of permanent magnet, moving coil voltmeter, resistor, capacitor and Rectifier.

-> The induced voltage are rectified and then passes to the capacitor for smoothening.



Drag cup Ac tachometer

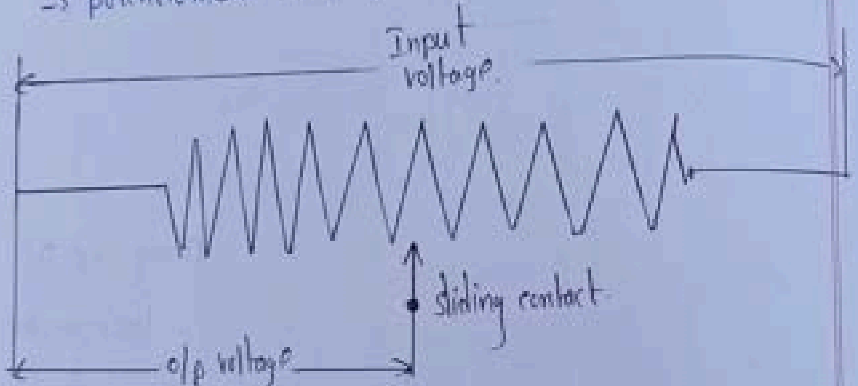
Advantages	Disadvantages
-> cost is very less	-> o/p signal depends on supply voltage.
-> Drag cup Tachometer generate ripple free o/p voltage.	-> limited accuracy.

potentiometer. potentiometer is a instrument used to measure unknown voltage by comparing it with the known voltage.

-> potentiometer is also known as potmeter.

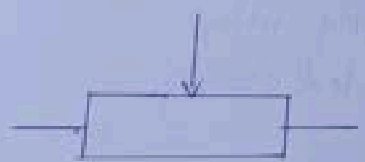
-> potentiometer is act as an adjustable voltage divider.

-> potentiometer is a passive electronic component.



-> potentiometer work by varying the position of the sliding contact.

-> symbol of potentiometer.



i potentiometer symbol (IEC)

ii potentiometer symbol (ANSI)

Types of potentiometer

-> There are three types of potentiometers are there.

i Rotary potentiometer :-

-> These are most common type of potentiometer, where the wiper moves along a circular path.

-> The best example of rotary potentiometer is radio transistor volume controller.

-> These potentiometers are used where voltage control is required.

ii Linear potentiometer

-> In this type of potentiometer, wiper moves along a linear path.

-> This is similar to rotary potentiometer, but work done linearly.

-> These potentiometers are used to calculate voltage in circuits.

iii Digital potentiometer :-

-> Digital potentiometers are also called as Digipots.

-> used to control analog signals

-> These are also called as RDAC's (resistive - digital to analog converter).

Advantages

- High realiability
- High precision.
- Low power consumption.

Applications

- In steering systems
- In audio applications
- Mountain bikes.

Unit - III

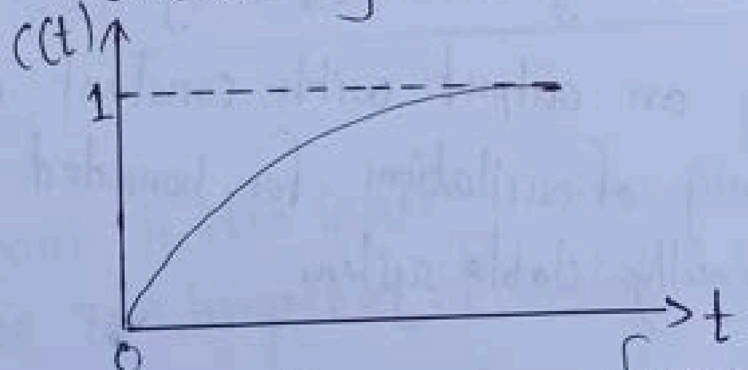
Concepts of Stability

The concept of stability :-

Stability : A system is said to be stable, if it's output is under control, otherwise, it is said to be unstable.

-> A stable system produces bounded output for a given bounded input.

-> The following figure shows the response of a stable system.



-> This is the response of the first order control system for unit step input, the response is b/w 0 and 1, so it is bounded output.

-> Types of systems based on stability

→ classification of systems based on stability.

- Absolutely stable system.
- conditionally stable system.
- Marginally stable system.

Absolutely stable system : If a system output is stable for all variations of it's parameters is called Absolutely stable system.

conditionally stable system : If a system output is stable for limited range of variations of it's parameters is called conditionally stable system.

Marginally stable system : If a system is stable by producing an output with constant amplitude and frequency of oscillations for bounded input is called marginally stable system.

Necessary conditions for stability

→ The essential requirement for the control system is that it should be stable.

→ conditions for stability

i BIBO (Bounded input bounded output) :

→ It means the system responds bounded output for the intial bounded input is called BIBO.

→ This system is said to be BIBO.

→ It is commonly used for signal processing systems.

ii Linear input and output irrespective of initial conditions :

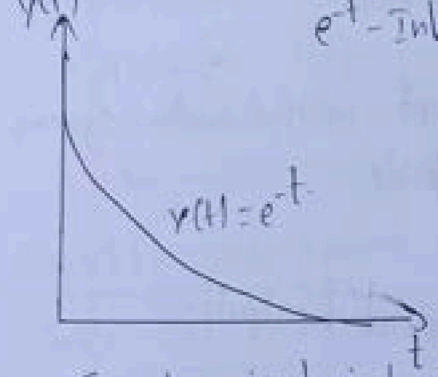
→ It means if the input is zero, the output should also be zero, irrespective of initial conditions.

→ let us consider two systems

- $y(t) = x(t)$.
- $y(t) = \{x(t)\}^{-1}$

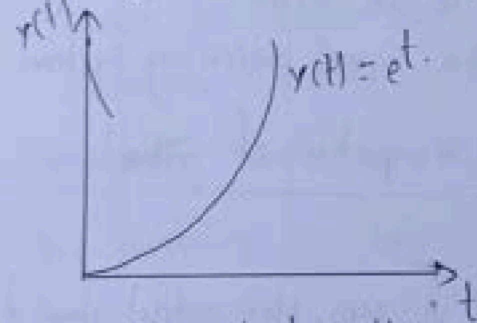
where $x(t) = e^{-t}$

e^{-t} - Inverse exponential graph.



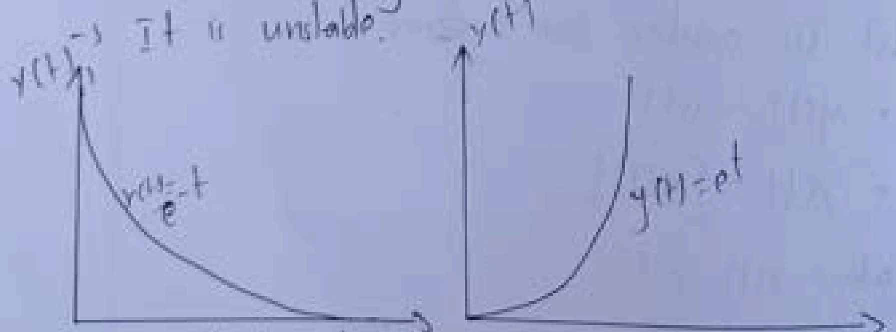
-> The above input is bounded input, the output also needs to be bounded.

-> let us consider +ve exponential graph.



-> The output in this case is increases with time, it tends to infinity.

-> It is unstable.



(a) stable system. t (b) unstable system. t

iii Stability with respect to location of poles and zeros

-> In terms of poles and zeros, we can represent transfer function as:

$$G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\dots(s+z_m)}{s^n(s+p_1)(s+p_2)(s+p_3)\dots(s+p_n)}$$

-> Here 'z' represent zeros and 'p' represent poles.

-> For zeros, the numerator of the transfer function equate to zero.

-> For poles, the denominator of the transfer function equate to zero.

-> The roots of numerator can be represented as $z_1, -z_2, z_3 \dots -z_m$.

-> The roots of denominator can be represented as $p_1, -p_2, p_3 \dots -p_n$

Necessary conditions of stability

-> The closed loop poles are roots of the characteristic equation.

-> But if the characteristic equation is higher order, then it is not possible to calculate roots.

-> The characteristic equation is $D(s) = 0$.

$$a_0 s^n + a_1 s^{n-1} + \dots\dots + a_n = 0$$

-> All the coefficients of the polynomial must have some sign.

-> None of the coefficients vanishes, all the powers of 's' must be present in the characteristic equation.

-> If any polynomial satisfies above two conditions it is called as Hurwitz polynomial.

Routh Hurwitz criterion :-

-> It is an analytical procedure for determining whether all the roots of a polynomial have -ve real parts or not.

-> Routh Hurwitz criterion states that any system can be stable if and only if all the roots of first column have the same sign. or there is a sign change.

-> Necessary but not sufficient conditions

-> There are some necessary conditions to make system stable

-> consider a system with characteristic equation.

$$a_0 s^m + a_0 s^{m-1} + \dots + a_m = 0.$$

i All the coefficients of the equation should have the same sign.

ii There should be no missing term.

-> If above two conditions are satisfied, we didn't say system is stable. for this we use Routh Hurwitz criterion, to check stability.

Advantages

- → we can find the stability of the system without solving equation.
- → Easly determine the relative stability.

Limitation

- → This criterion is applicable for linear system only.
- → It does not provide exact position of poles on the s-plane.

Example: consider this characteristic polynomial

$$a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0.$$

sol coefficients of a_0, a_1 have same sign

step1 Arrange of coefficients of above equation in two rows.

Row 1	a_0	a_2	$a_4 \dots$
Row 2	a_1	a_3	a_5

step2 from this two rows we will form row 3

$$b_1 = -\frac{1}{a_1}\begin{bmatrix} a_0 & a_2 \\ a_1 & a_3 \end{bmatrix} = -\frac{(a_0 a_3 - a_1 a_2)}{a_1}$$

$$b_3 = -\frac{1}{a_1}\begin{bmatrix} a_0 & a_4 \\ a_1 & a_5 \end{bmatrix} = -\frac{(a_0 a_5 - a_1 a_4)}{a_1}$$

Row 1	a_0	a_2	$a_4 \dots$
Row 2	a_1	a_3	a_5
Row 3	b_1	b_3	b_5

step3 from above Row 2 and Row we will form Row 4

$$c_1 = -\frac{1}{b_1}\begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} = -\frac{(a_1 a_3 - b_1 a_3)}{c_1}$$

$$c_3 = -\frac{1}{b_1}\begin{bmatrix} a_1 & a_5 \\ b_1 & b_5 \end{bmatrix} = -\frac{(a_1 b_5 - b_1 a_5)}{c_3}$$

step4 continue this procedure.

(P) check the stability of the system whose characteristic equation is

$$s^4+2s^3+6s^2+4s+1=0$$

Sol obtain array of coefficients.

s^4	1	6	1
s^3	2	4	
s^2	4	1	
s^1	3.5		
s^0			

a_1	a_2	b_3
b_1	b_2	b_3

for $s^2 = -\frac{1}{2}\begin{bmatrix}1 & 6\\ 2 & 4\end{bmatrix} = -\frac{(4-12)}{2} = -\frac{(-8)}{2} = 4$

$\boxed{b_1 = 4}$

for $s^2 = -\frac{1}{2}\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix} = -\frac{(0-2)}{2} = \frac{2}{2} = 1$

$\boxed{b_2 = 1}$

for s^1

$c_1 = -\frac{1}{4}\begin{bmatrix}2 & 4\\ 4 & 1\end{bmatrix} = -\frac{(2-16)}{4} = \frac{14}{4} = 3.5$

$c_1 = 3.5$

→ Since all coefficients in the first column are of the same sign i.e +ve,

→ The system is said to be stable.

(2). Find the stability of given equation using Routh's method

I. $s^3+6s^2+11s+6=0$

Sol $s^3+6s^2+11s+6=0$.

$a_0=1,\ a_1=6,\ a_2=11,\ a_3=6$

s^3	1	11
s^2	6	6
s^1	10	
s^0	6	

For s^1 =

$-\frac{1}{6}\begin{bmatrix}1 & 11\\ 6 & 6\end{bmatrix} = -\frac{(6\times1-11\times6)}{6} = -\frac{(6-66)}{6} = \frac{60}{6} = 10$

For $s^1 = 0$.

For $s^0 = -\frac{1}{10}\begin{bmatrix}6 & 6\\ 10 & 0\end{bmatrix} = -\frac{(6\times0-6\times10)}{10} = -\frac{(-60)}{10} = 6$.

$\boxed{\text{System is stable}}$

ii s^3+4s^2+s+16

Sol s^3+4s^2+s+16

$a_0=1,\ a_1=4,\ a_2=1,\ a_3=16$

s^3	1	1
s^2	4	16
s^1	-3	
s^0	16	

For $s^1 = -\frac{1}{4}\begin{bmatrix}1 & 1\\ 4 & 16\end{bmatrix} = -\frac{(1\times16-1\times4)}{4} = -\frac{(16-4)}{4}$

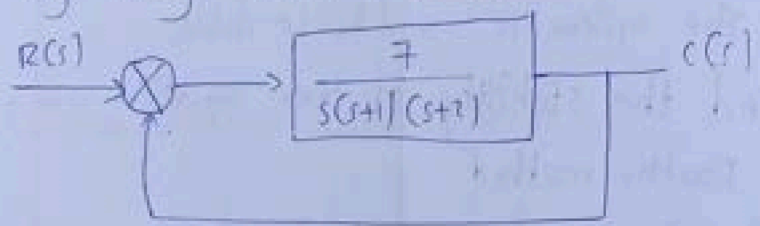
$s^1 = -3$

For $s^1 = 0$

For s^0 $-\frac{1}{(-3)}\begin{bmatrix}4 & 16\\ -3 & 0\end{bmatrix} = +\frac{(0-48)}{-3} = 16$

system is unstable.

(P). check whether the system is stable (or) not by using Routh criterion.



sol The characteristic equation is

$1 + G(s)H(s) = 0$

$1 + \frac{7}{s(s+1)(s+2)} = 0$

→ $s(s+1)(s+2)$

→ $(s^2+s)(s+2)$

$s^3+2s^2+s^2+2s$

s^3+3s^2+2s+7.

Equation is s^3+3s^2+2s+7.

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & 7 \\ s^1 & -\frac{1}{3} & 0 \\ s^0 & 7 & \end{array}$$

$s^1 \rightarrow \frac{3\times2-7\times1}{3} = \frac{6-7}{3} = -\frac{1}{3}$

$s^0 \rightarrow \frac{\left(-\frac{1}{3}\right)7 - 3\times0}{-1/3} = 7$

→ First column have different signs for coefficients.

→ ∴ system is unstable.

Routh-Criterion special case-i

(P). Find the stability of the system for

$s^5+s^4+2s^3+2s^2+3s+5=0$.

Routhing column

$$\begin{array}{c|c|ccc} 1 & s^5 & 1 & 2 & 3 \\ 1 & s^4 & 1 & 2 & 5 \\ \varepsilon & s^3 & \boxed{0}\ \varepsilon & -2 & \\ +ve & s^2 & \frac{2\varepsilon+2}{\varepsilon} & 5 & \\ -2 & s^1 & \frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2} & 0 & \\ 5 & s^0 & 5 & & \end{array}$$

special case

For s^3 - $\frac{(1\times2-2\times1)}{1} = 0$

$\frac{(1\times5-3\times1)}{1} = \frac{5-3}{1} = -2$

For s^2 - $\frac{\varepsilon\times2-(-2)(1)}{\varepsilon} = \frac{2\varepsilon+2}{\varepsilon}$

For s^1 $\frac{\frac{2\varepsilon+2}{\varepsilon}(-2)-5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}}$

$= \frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2}$

For s^0 -

→ Sub $\varepsilon=0$ in first colum.

$\lim_{\varepsilon\to0} \frac{2\varepsilon+2}{\varepsilon} = \lim_{\varepsilon\to0} \frac{2+\frac{2}{\varepsilon}}{} = 2+\frac{2}{0} = 2+\infty = (+\infty)$

$\lim_{\varepsilon\to0} \frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2} = \lim_{\varepsilon\to0} -\frac{4}{2} = -2$.

→ Here two sign changes.
→ Here two roots are there.
→ ∴ System is unstable

(P). Determine stability by Routh Criterion

$$s^5+2s^4+3s^3+6s^2+2s+1=0$$

sol

$$\begin{array}{cc|ccc}
1 & s^5 & 1 & 3 & 2 \\
2 & s^4 & 2 & 6 & 1 \\
\varepsilon & s^3 & \boxed{0}\ \varepsilon & 1.5 & \\
-ve & s^2 & \boxed{\infty}\ \frac{6\varepsilon-3}{\varepsilon} & 1 & \\
1.5 & s^1 & \frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3} & & \\
1 & s^0 & 1 & &
\end{array}$$

→ For s^3

$$= \frac{1\times6-2\times3}{2}, \quad (2\times2-1\times1)/2 \quad \cancel{\frac{1\times1-2\times2}{2}}$$

$$= \frac{6-6}{0} \qquad = \frac{\cancel{1-4}}{2} = \frac{4-1}{2}$$

$$= 0 \qquad = -3/2 = 3/2$$

$$= 1.5$$

For s^2

$$= \frac{6\times\varepsilon-2\times1.5}{\varepsilon} \quad 1$$

$$= \frac{6\varepsilon-3}{\varepsilon}$$

For s^1

$$= \frac{\frac{6\varepsilon-3}{\varepsilon}\times1.5-\varepsilon}{\frac{6\varepsilon-3}{\varepsilon}}$$

$$= \frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3}$$

→ for checking stability sub $\varepsilon=0$ in column 1.

→ $\lim\limits_{\varepsilon\to0} \frac{6\varepsilon-3}{\varepsilon} = \lim\limits_{\varepsilon\to0} 6-\frac{3}{\varepsilon} = 6-\infty = -\infty$ -ve sign.

→ $\lim\limits_{\varepsilon\to0} \frac{9\varepsilon-4.5-\varepsilon^2}{6\varepsilon-3} = \lim\limits_{\varepsilon\to0} \frac{-4.5}{-3} = 1.5$

→ Here two sign changes are there.

→ it means two roots are there

→ so system is unstable.

Routh Hurwitz criterion special case-ii

Special case ii All elements of any row array are zero.

Example problem

(P). Determine the stability of the given characteristic equation by Routh Hurwitz criterion.

$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$.

Sol given Eq. $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	
→ s^4	2	12	16	
s^3	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	
	8	24	0	
s^2	6	16		
s^1	2.67	0		
s^0	16			

special case complete row zero

For s^4 i $\frac{(2\times8-12\times1)}{2}$ ii $\frac{(20\times2-16\times1)}{2}$

$=\frac{16-12}{2}$ $=\frac{40-16}{2}$

$=\frac{4}{2}$ $=\frac{24}{2}$

$=2$ $=12$

For s^3 i $\frac{(2\times12-2\times12)}{2}$ ii $\frac{(2\times16-2\times16)}{2}$

$=0$ $=0$

→ s^3- entire row becomes zero, to overcome is this considering auxilary equation, above the zero row.

→ Auxilary equation,

$A(s) = 2s^4+12s^2+16$.

differentiate the above equation with respect to s

$\frac{dA(s)}{d(s)} = 8s^3+24s$ $\boxed{\frac{d}{ds}(16)=0}$

For s^2 i $\frac{(8\times12-24\times2)}{8}$ ii $\frac{(8\times16-2\times0)}{8}$

$=\frac{96-48}{8}$ $=\frac{128}{8}$

$=\frac{48}{8}=6$ $=16$

For s^1

$$= \frac{(8\times16 - 24\times6)}{6} = \frac{(6\times24 - 8\times16)}{6}$$

$$128 - 144 \quad = \frac{144 - 128}{6}$$

$$= \frac{16}{6}$$

$$= 2.66$$

For s^0 $\frac{(16\times2.66 - 6\times0)}{2.66}$

$$= \frac{42.56}{2.66}$$

$$= 16.$$

-> There is no sign changes here.

-> It means no roots are there.

-> So, this system is stable.

Relative Stability analysis

-> The relative stability is "the measure of how close the system is to instability".

(P). consider a 3rd order system with characteristic equation. $s^3 + 10.1s^2 + 21s + 2 = 0$. is this system stable?

If we shift the imaginary axis to the left by 0.2 analyze the relative stability!

Sol characteristic equation.

$$s^3 + 10.1s^2 + 21s + 2 = 0$$

Routh's array

s^3	1	21
s^2	10.1	2.
s^1	20.802	0.
s_0	2.	

For s^1 : $\frac{(21\times10.1 - 2\times1)}{10.1}$

$$= \frac{212.1 - 2}{10.1}$$

$$= \frac{210.1}{10.1}$$

$$= 20.802.$$

$$\therefore \frac{(20.802\times2 - 10.1\times0)}{20.802}$$

$$= \frac{41.604}{20.802}$$

$$= 2$$

-> There is no sign change.

For s^1

$= \frac{(8\times16 - 24\times6)}{6} = \frac{(6\times24 - 8\times16)}{6}$

$128 - 144 \quad = \frac{144-128}{6}$

$= \frac{16}{6}$

$= 2.66$

For s^0 $\frac{(16\times2.66 - 6\times0)}{2.66}$

$= \frac{42.56}{2.66}$

$= 16.$

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s^0	2.	

For s^1 ; $\frac{(21\times10.1 - 2\times1)}{10.1}$

$= \frac{212.1 - 2}{10.1}$

$= \frac{210.1}{10.1}$

$= 20.802.$

s^0 ; $\frac{(20.802\times2 - 10.1\times0)}{20.802}$

$= \frac{41.604}{20.802}$

$= 2$

- There is no sign change.

→ There no sign change here.
→ It means no roots are there.
→ ∴ system is stable.

∴ Relative stability

To investigate relative stability about ~~s=0~~, $s=0.2$,
we replace 's' by $s'-0.2$.
→ characteristic equation is

$$s^3 + 10.1s^2 + 21s + 2 = 0.$$

put $s = s'-0.2$

$$(s'-0.2)^3 + 10.1(s'-0.2)^2 + 21(s'-0.2) + 2 = 0.$$

$$CE_2 : s'^3 + 9.5s'^2 + 17.08s' - 1.804 = 0.$$

s'^3	1	17.08
s'^2	9.5	-1.804
s'	17.269	0
s^0	-1.804	

For s'

$$\frac{(9.5 \times 17.08) + 1.804}{9.5}$$

$$= \frac{162.26 + 1.804}{9.5}$$

$$= \frac{164.064}{9.5}$$

$$= 17.269$$

→ Sign change is there, roots are there.
→ ∴ system is unstable.

Relative 'k' stability

Q. Find the value of 'k' for the system to be stable.

(a). $$G(s)H(s) = \frac{k(1-s)}{s(s^2+5s+9)}$$

Sol

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{k(1-s)}{s(s^2+5s+9)} = 0$$

$$\frac{s(s^2+5s+9) + k(1-s)}{s(s^2+5s+9)} = 0$$

$CE: s^3+5s^2+9s+k-ks=0.$

$a_0=1 \quad a_1=5, \; a_2=9-k \quad a_3=k.$

s^3	1	$9-k$
s^2	5	$k.$
s^1	$\frac{45-5k-k}{5}$	0
s^0	k	

For s^1

$$\frac{5(9-k)-k}{5}$$

$$=\frac{45-5k-k}{5}$$

$k>0.$

$\frac{45-6k}{5}>0 \Rightarrow 45-6k>0.$

$45>6k \Rightarrow k<\frac{45}{6}.$

$k<7.5$

The range of 'k' for stability is $0<k<7.5$.

(P). $s^3+3ks^2+(k+2)s+4=0$

sol $CE: s^3+3ks^2+(k+2)s+4=0.$

s^3	1	$k+2$
s^2	$3k$	4
s^1	$\frac{3k^2+6k-4}{3k}$	$0.$
s^0	4	

For $s^1 = \frac{3k(k+2)-4}{3k.}$

$= \frac{3k^2+6k-4}{3k.}$

$\therefore 3k>0 \Rightarrow k>0$

$\frac{3k^2+6k-4}{3k}>0 \Rightarrow 3k^2+6k-4>0$

$k>0.5275 \; \& \; k>-2.5275$

$0.5275<k<\infty.$

Root Locus Technique :- (Introduction)

The root locus method is a graphical technique for determination of the zeros of $m(s)+n(s)$ from the zeros of $m(s)$ and $n(s)$

-> Root locus method is a fundamental tool.

-> It is used in control systems engineering to analyze the behaviour and stability of feedback control systems.

-> purpose of root locus

-> Find the stability of closed loop system.

-> Find the range of 'k' for system to be stable

-> Find 'ω' for marginally stable system

-> Find 'k' for system to be overdamped, underdamped, critically damped, undamped condition.

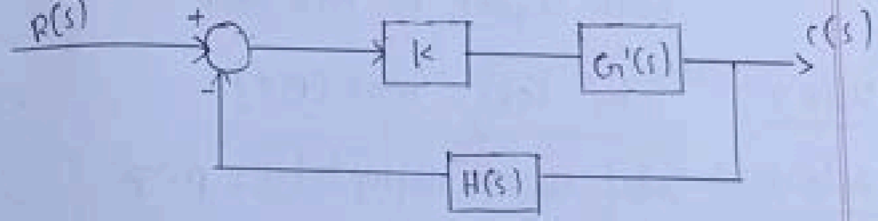
-: The root locus concept:

-> The characteristic equation of root locus closed loop system is

$$1+G(s)H(s)=0 \quad (1)$$

-> In root locus 'k' is assumed to be variable parameter.

-> The block diagram of root locus



-> Here $G(s) = K.G'(s)$ $\boxed{K = \text{system gain}}$

Sub $G(s)$ value in eq (1)

$$\boxed{1+K.G'(s)H(s)=0.}$$

-> from above equation

-> K = variable parameter.

-> The roots of above equations depends upon 'k'

-> If 'k' values is varied from $-\infty$ to ∞, for each value of 'k', we will get seperate set of roots.

-> All the roots are joined on s-plane and resulting locus root locus

Root locus: The locus of closed loop poles obtained when system gain 'k' is varied from $-\infty$ to ∞

$k = 0$ to ∞ => Direct root locus.

$k = -\infty$ to 0 => Inverse root locus.

Construction Rules of root Locus :

1 Rule-1 : The root locus is always symmetric with respect to real axis

Rule 2 : Total Loci = max (P, X)

Rule-3 : Total no. of Asymptotes = P - Z.

Rule-4 : Angle of Asymptotes.

$$\theta = \frac{(2X+1)}{P-Z} 180^\circ ; X = 0, 1, 2 \ldots$$

Rule-5 Centroid of Asymptotes.

$$C_a = \frac{\Sigma_{Real} P - \Sigma Real Z}{P-Z}$$

Rule-6 : Break away point.

Identify...

- Characteristic equation $(1+G(s)H(s) = 0)$
- compute k = polynomial
- compute $\frac{dk}{ds} = 0$ [s = Break away point]

Rule 7 Angle of departure

Rule-8 : Intersection to Imaginary axis.

Identify...

- characteristic equation $(1+G(s)H(s) = 0)$
- Construct Routh array.
- find k (for marginary stable)
- place k in Auxilary eqn.
- s = intersection to imaginary axis

(P). A unity feedback control system has an open loop transfer function of $\cancel{G(s) = \frac{k(s+4/s)}{s^2(s+12)}}$

$$G(s)H(s) = \frac{k(s+2)(s+3)}{(s+1)(s-1)}$$

Draw root locus of given system? k varies from 0 to ∞.

sol Finding zeros and poles, here numerator indicates zeros and denominator indicates poles

i zeros = -2, -3

ii poles = -1, 1.

Step 1 : No. of Loci = max (P, Z)
= 2

Step 2 No. of Asymptotes $= P - Z$

$= 2 - 0$

$= 0.$

Step 3 Angle of asymptotes

$$\theta = \frac{(2x+1)}{P-Z}180^\circ \quad x = 0, 1.$$

$$0; \quad \theta = \frac{2(0)+1}{0}180^\circ$$

$$1, \quad \theta = \frac{2(1)+1}{0}180^\circ$$

Not defined.

Step 3 ; Break away point.

- characteristic equation.

$= 1 + G(s)H(s) = 0.$

$$= 1 + \frac{k(s+2)(s+3)}{(s+1)(s-1)} = 0$$

~~$= (s+1)(s-1) + k(s+2)(s+3) = 0$~~

$$k = -\frac{(s+1)(s-1)}{(s+2)(s+3)}$$

$$k = -\frac{s^2-1}{s^2+5s+6}$$

- Find $\left(\frac{dk}{ds} = 0\right)$.

$$k = -\frac{s^2-1}{(s^2+5s+6)}$$

$$\boxed{\frac{d(A/B)}{dx} = \frac{B\frac{dA}{dx} - A\frac{dB}{dx}}{B^2}}$$

$$\frac{(s^2+5s+6)(2s) - (s^2-1)(2s+5)}{(s^2+5s+6)^2} = 0$$

A = Numerator
B = denominator

$\cancel{2s^3} + 10s^2 + 12s - \cancel{2s^3} - 5s^2 + 2s + 5 = 0.$

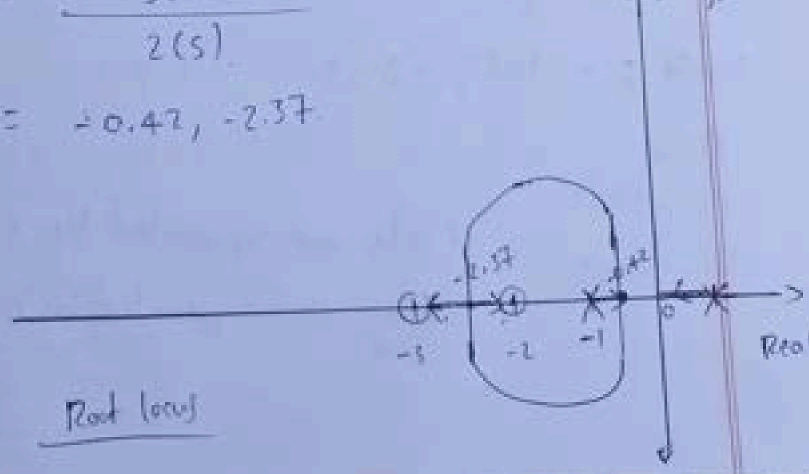
$\Rightarrow 5s^2 + 14s + 5 = 0$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = \frac{-14 \pm \sqrt{(14)^2 - 4(5)(5)}}{2(5)}$$

$= -0.42, -2.37$

x - poles
O - zeros
• Break away point



Root locus

Q. A unity feedback control system has open loop transfer function $G(s) = \frac{k}{s(s^2+4s+13)}$, sketch the root locus?

Sol: To locate poles and zeros

$$s(s^2+4s+13)=0$$

$s=0$

$$s^2+4s+13=0$$

$a=1, b=4, c=13$

$$s = \frac{-4 \pm \sqrt{4^2-4(1)(13)}}{2.1}$$

$$s = \frac{-4 \pm \sqrt{16-52}}{2}$$

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$s = 0, -2+j3, -2-j3$

$P_1 = 0$

$P_2 = -2+j3$

$P_3 = -2-j3$

* poles are represented by x

* zeros are represented by 'o'

Step 2: No. of Asymptotes $= P-Z$

$= 3-0$

$= 3$

Step 3: Angle of asymptotes

$$\theta = \frac{(2x+1)}{P-Z} 180 \quad x=0,1$$

$$\theta = \frac{2(0)+1}{3-0} 180^\circ = \frac{1}{3} 180^\circ = 60^\circ$$

$$\theta = \frac{2(1)+1}{3-0} 180^\circ = \frac{3}{3} 180 = 180^\circ$$

$\theta = 60^\circ, 180^\circ$

Step 4: Centroid of Asymptotes

$$= \frac{\Sigma \text{Real } P - \Sigma \text{Real } Z}{P-Z}$$

$$= \frac{0-2+j3-2-j3-0}{3-0}$$

$$= -\frac{4}{3}$$

$\boxed{\text{Centroid} = -1.33}$

Step 5 Break ere away point.

- characteristic eqn

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{k}{s(s^2+4s+13)}}{1+\frac{k}{s(s^2+4s+13)}}$$

$$= \frac{k}{s(s^2+4s+13)+k}$$

$$s(s^2+4s+13)+k = 0$$

$$s^3+4s^2+13s+k = 0$$

$$\boxed{k = -(s^3+4s^2+13s)}$$

- Compute $\frac{dk}{ds} = \frac{d}{ds}\left[s^3+4s^2+13s\right]$

$$= 3s^2+8s+13 = 0$$

$$a=3,\ b=8\quad c=13$$

$$= \frac{-8 \pm \sqrt{(8)^2-4(3)(13)}}{2\cdot 3}$$

$$= \frac{-8 \pm \sqrt{64-156}}{6}$$

$$= \frac{-8 \pm \sqrt{-96}}{6} \Rightarrow \frac{-8}{6} \pm \frac{\sqrt{-96}}{6}$$

$$\boxed{s = -1.33 \pm j1.6}$$

$k \neq$ +ve and real

graph in pdf

(P) $\frac{k(s+9)}{s(s^2+4s+11)}$ sketch the root locus of the system?

sol given that

$$\frac{k(s+9)}{s(s^2+4s+11)}$$

Numerator = zeros.

$$s+9 = 0$$

$\boxed{s = -9}$ — zero.

Denominator = poles.

$$s(s^2+4s+11) = 0$$

$$s^3+4s^2+11 = 0$$

poles = $\cancel{-1+\sqrt{38}}$, $\cancel{-1-\sqrt{38}}$, $0, -2+j\sqrt{7}, -2-j\sqrt{7}$.

poles = 3, zeros = 1.

Step 1 Total loci = max (P, Z)

$= 3.$

Step 2 Total no. of Asymptotes $= P - Z$

$= 3 - 1$

$= 2.$

Step 3 $\theta = \left(\frac{2x+1}{P-Z}\right) 180^\circ \quad x = 0, 1.$

$\theta = \left(\frac{2\times 0+1}{2}\right) 180^\circ = 90^\circ$

$\theta = \left(\frac{2+1}{2}\right) 180^\circ = 270^\circ.$

Step 4 centroid of Asymptotes :

$C_a = \frac{\Sigma \text{real } p - \Sigma \text{real } z}{P - Z}.$

$= \frac{0 - 2 - 2 - (-9)}{2}.$

Centroid $= \frac{5}{2} = 2.5,$

Step-5 Break away point.

→ characteristic eqn

$1 + G(s)H(s) = 0.$

$1 + \frac{k(s+9)}{s(s^2+4s+11)} = 0.$

$s^3 + 4s^2 + 11s + ks + 9k = 0.$

$s^3 + 4s^2 + 11s + k(s+9) = 0.$

$k(s+9) = \frac{-s^3 - 4s^2 - 11s}{s+9}$ — (1)

Unit-4

Frequency Response analysis

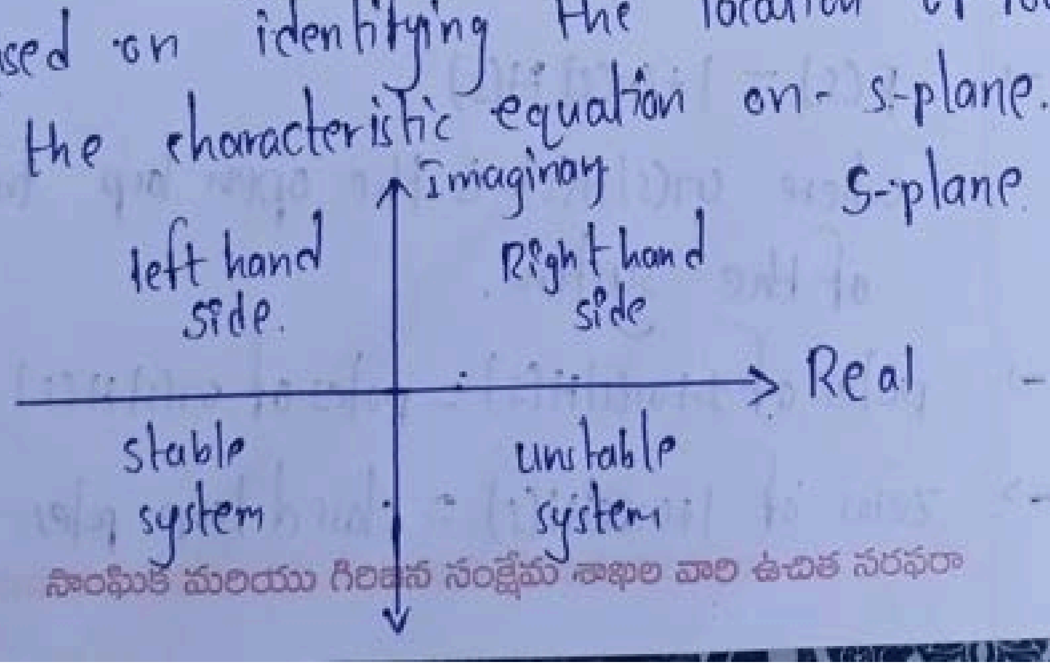
Nyquist stability criterior : ~~Nquis~~

-> ~~Nq~~ Nyquist criterion (or) Nyquist stability criterion is a graphical method which is used for finding the stability of closed loop control system with a feedback loop.

-> This criterion plays crucial role for design and analysis purpose of the system with feedback

-> The principle of ~~Nq~~ Nyquist criterion was independently proposed by Felix strecker in 1930.

-> The stability of a feedback control system is based on identifying the location of roots of the characteristic equation on s-plane.



- Nyquist stability criterior is applicable for linear systems, not for non-linear systems.
- IN Nyquist stability criterion both absolute and relative stability can be found, based on frequency response
- No need to calculate the roots of the characteristic equations
- It is based on polar plots
- Nyquist plots are continuation of polar plots for finding the stability of closed loop control system by varying ω from $-\infty$ to ∞.
- Nyquist plots are used to draw the complete frequency response of the open loop transfer function.
- $F(s) = 1+G(s)H(s)$
  where $G(s)H(s)$ is the open loop transfer function of the system.
- poles of $1+G(s)H(s)$ = poles of $G(s)H(s)$ = open loop poles
- zeros of $1+G(s)H(s)$ = closed loop poles.

- Suppose P donates the number of poles of $1+G(s)H(s)$ located in the right half of s-plane, then number on zeros on right half of s-plane is

$$\boxed{Z = N+P}$$

- In stability there is no zeros of $1+G(s)H(s)$ in the right half,
- This means for stable system Z must be 0.

$$\boxed{N = -P}$$

- This is called Nyquist stability criterion.

Relation between Time and frequency Response

-> The calculation of system parameters for higher order systems is difficult in time domain analysis.

-> This can be overcome by frequency domain analysis.

-> frequency response; can be determining by calculating the phase and amplitude of the given system transfer function.

Time domain specifications

i Delay time : $t_r = \frac{\pi - \theta}{\omega_d}$ sec

ii peak time : $t_p = \frac{\pi}{\omega_d}$ sec.

iii Maximum overshoot : $\%M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$

iv setting time : $t_s = \frac{4}{\zeta\omega_n}$ sec (for 2% error)

$t_s = \frac{3}{\zeta\omega_n}$ sec (for 5% error).

ω_d = damping frequency

ω_n = natural "

ζ = damping ratio.

frequency domain specifications

i Resonant Peak $(M_r) = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

ii Resonant frequency $(\omega_r) = \omega_n\sqrt{1-2\zeta^2}$

iii cut-off rate.

Second order system closed loop response $\rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

ζ = damping ratio.

ω_n = Natural frequency.

Sub $s = j\omega$ in above equation

$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} \quad \boxed{j^2 = -1}$$

$$T(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2\zeta\omega_n j\omega + \omega_n^2}$$

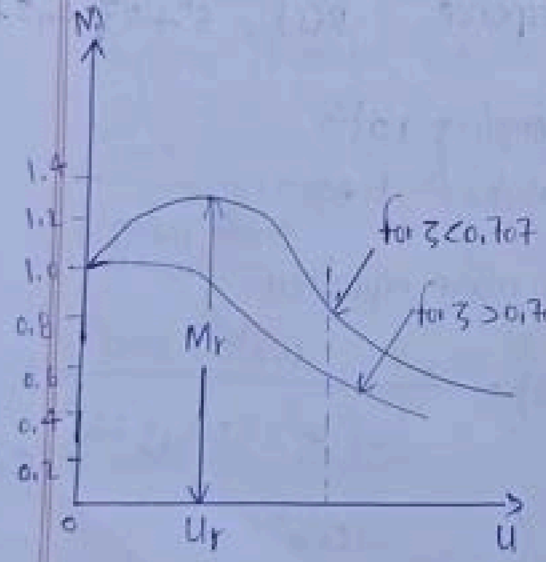
$$T(j\omega) = \frac{\omega_n^2}{\omega_n^2\left(-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta j\frac{\omega}{\omega_n} + 1\right)}$$

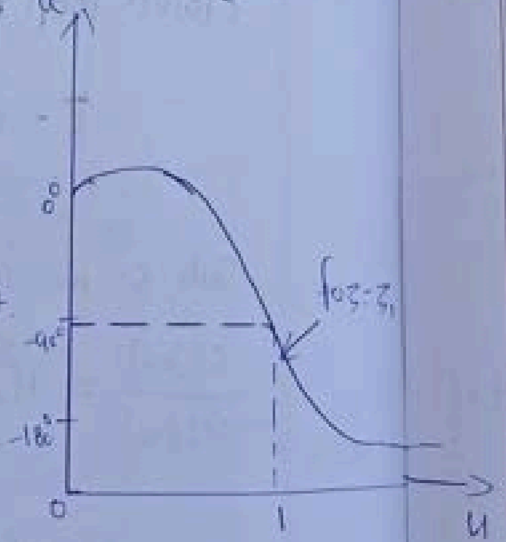
$$T(j\omega) = \frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2+2\zeta j\frac{\omega}{\omega_n}\right)}$$

$\boxed{\text{Replace } \frac{\omega}{\omega_n} = u}$

$$\boxed{T(j\omega) = \frac{1}{(1-u^2+2\zeta j u)}}$$

$$M = \frac{1}{\sqrt{(1-u^2)^2+(2\zeta u)^2}}, \quad \alpha = -\tan^{-1}\left(\frac{2\zeta u}{(1-u^2)}\right)$$



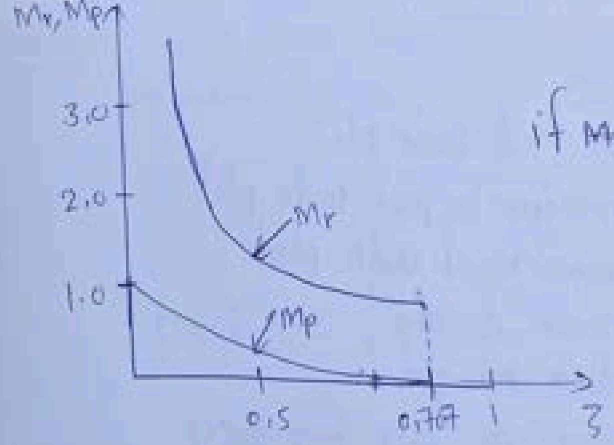


Resonant peak (M_r) $= \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ Resonant frequency (ω_r) $= \omega_n\sqrt{1-2\zeta^2}$

if $\zeta = 0, M_r = \infty$

If $\zeta = 0, \omega_r = \omega_n$

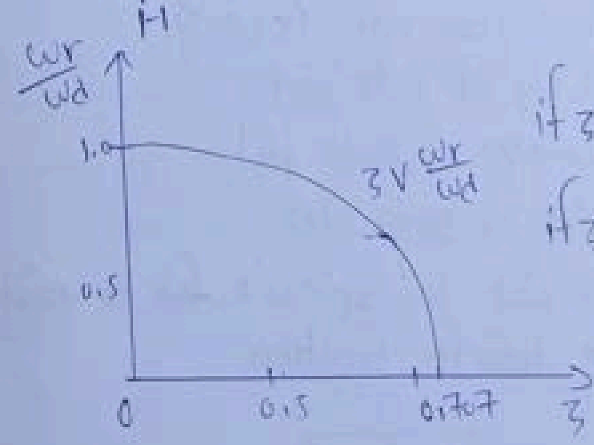
→ curve b/w M_r, M_p and ζ



if M_r $\zeta=0, M_r=1$

$M_p = \infty$

ii)



if $\zeta = 0, \frac{\omega_r}{\omega_d} = max$

if $\zeta = 0.707, \frac{\omega_r}{\omega_d} = min$

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BODE plots

outlines

- → Basics of Bode plot.
- → procedure to plot Bode plot
- → parameters of Bode plot.
  - * Gain margin
  - * phase margin
  - * Gain cross over frequency
  - * phase cross over frequency.
- → stability of Bode plot
- → Advantages of Bode plot.
- → Basics of Bode plot

• Bode plot is applicable ~~for~~ to minimum phase transfer function.

$$y_{in} = A\sin\omega t \longrightarrow \boxed{\text{System}} \longrightarrow y_{out} = \underset{\substack{\uparrow \\ \text{gain}}}{A'}\sin(\omega t + \underset{\substack{\uparrow \\ \text{phase}}}{\phi})$$

→ Here we have two plots

i gain plot $[\,|G(j\omega)| \rightarrow \omega\,]$

ii phase plot $[\,\angle G(j\omega) \rightarrow \omega\,]$

procedure

Step 1 : write given transfer function in standard form

$$G(s) = \frac{(s+a)(s+b)}{(s+p)(s+q)}$$

$$G(s) = \frac{ab}{pq} \times \frac{\left(1+\frac{s}{a}\right)\left(1+\frac{s}{b}\right)}{\left(1+\frac{s}{p}\right)\left(1+\frac{s}{q}\right)}$$

Take H as constant 'k'

Step 2 Identify slope of 1st line for bode plot.

→ slope of 1st line is based on poles and zeros of origin.

Step 3 : gain of 1st line at $\omega = 1$ rad/sec.

$$\text{gain}\big|_{1\,rad/sec} = 20\log k$$

Step 4 ; write all corner frequencies in ascending order and define slope for each line

$$a > p > b < q$$

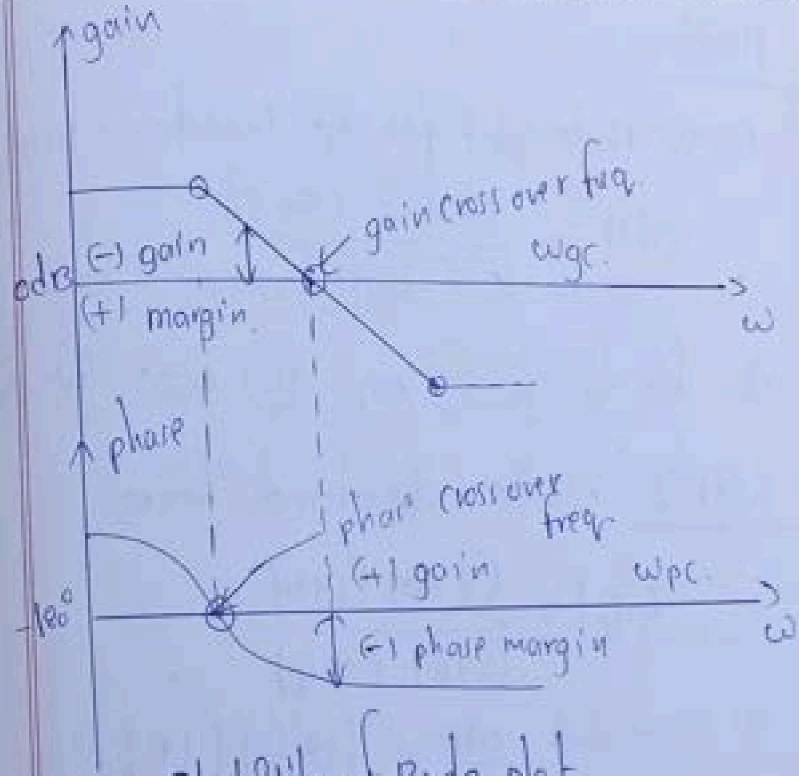
ω	pole/zero	slope	Resultant slope
a	zero	+20dB/dec	= 20 dB/dec
p	pole	-20dB/dec	= 0 dB/dec
b	zero	+20dB/dec	= 20dB/dec
q	pole	-20dB/dec	= 0 dB/dec

Step 5; write phase equation and make a table of $\phi \to \omega$

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{\omega}{b}\right) - \tan^{-1}\left(\frac{\omega}{p}\right) - \tan^{-1}\left(\frac{\omega}{q}\right)$$

$\phi \to \omega$.

parameters of Bode plot



Stability of Bode plot

- $\omega_{pc} > \omega_{gc} \to$ stable
- $\omega_{pc} < \omega_{gc} \to$ unstable
- $\omega_{pc} = \omega_{gc} \to$ marginal stable.

Advantage

- we can identify stability of the system.
- we can identify phase margin and gain margin with minimum calculation.

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Problem

Draw Bode plot for the transfer function

$$G(s) = \frac{14400\,(5+s)}{s^2(20+s)(100+s)}$$

also find (i) ω_{gc} (ii) ω_{pc}, (iii) GM (iv) PM

Sol **Step 1** write standard form.

$$G(s) = \frac{(s+a)(s+b)}{(s+p)(s+q)}$$

$$= \frac{ab}{pq} \times \frac{\left(1+\frac{s}{a}\right)\left(1+\frac{s}{b}\right)}{\left(1+\frac{s}{p}\right)\left(1+\frac{s}{q}\right)}$$

$$G(s) = \cancel{1400}\ \frac{14400 \times 5}{20 \times 100} \times \frac{\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{20}\right)\left(1+\frac{s}{100}\right)}$$

$$36$$

$$G(s) = 36\ \frac{\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{20}\right)\left(1+\frac{s}{100}\right)}$$

Step 2 Slope of 1st line.

→ Hence at origin there are two poles (s^2) so slope of 1st line will be -40 dB/dec

Step 3 gain of 1st line at $\omega = 1$ rad/sec

$k = 36$

$$\text{gain}\big|_{\omega=1\ \text{rad/sec}} = 20\log k = 20\log 36 = 31.12\ \text{dB}$$

Step 4 write all corner frequencies in ascending order and define slope for each line

→ For each pole slope = -20 dB

→ For zero slope = $+20$ dB/dec

$0 > 5 > 20 > 100$

ω	pole/zero	slope	Resultant slope
0	pole (2)	-40 dB/dec	$= 40$ dB/dec
5	zero	20 dB/dec	$= 20$ dB/dec
20	pole	-20 dB/dec	$= -40$ dB/dec
100	zero	20 dB/dec	$= -60$ dB/dec

Step 5 write phase eq. and make table of ~~w~~ ϕ. ϕ-$s\omega$

$$\phi = -180 + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

ω	ϕ
1	-172
5	-148
10	-145
50	-191
100	-220

Draw in

Polar plots

Basics

- Polar plot is used for frequency response characteristics of the system.
- Polar plot is a plot of magnitude and phase by varying from 0 to ∞.
- Let we have open loop transfer function of system as $g(s)$

$$G(j\omega) = \underbrace{|G(j\omega)|}_{\text{magnitude}} \underbrace{\angle G(j\omega)}_{\text{phase}}$$

Procedure to plot the polar plot

Step 1 Determine the open loop transfer function

OLTF → $G(s)$.

Step 2: To Indentify standard equation by $s = j\omega$

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

Step 3: To Identify magnitude and phase when $\omega = 0$ and $\omega = \infty$.

-> magnitude $|G(j\omega)|$

-> phase. $\angle G(j\omega)$

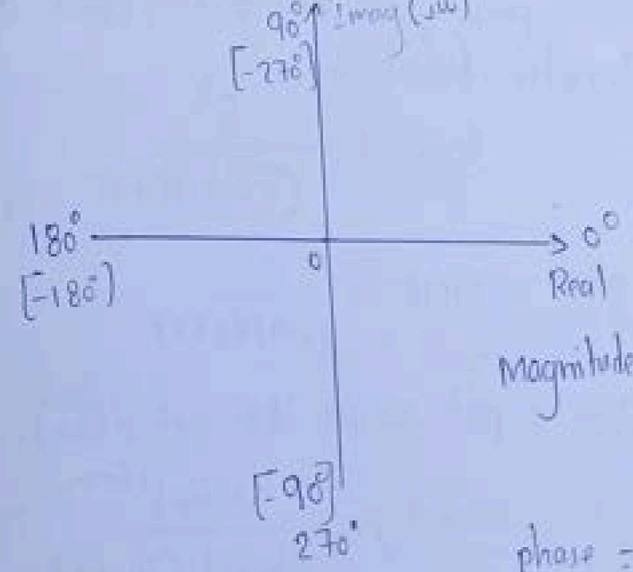
Step-4 Seperate real and Imag parts of $G(j\omega)$

$G(j\omega) = \text{Real}[G(j\omega)] + j\,\text{Imag}[G(j\omega)]$

Step5 If $\text{Real}[G(j\omega)] = 0$, then we will get intersect to imag axis.

if $\text{Imag}[G(j\omega)] = 0$, then we will get inter section to real axis.

Step6 Based on above steps, draw polar plot.



Magnitude = $|G(j\omega)|$ = distance w.r.to center

phase = $\angle G(j\omega)$ = angle w.r.to real axis

phase is +ve in anticlockwise direction and -ve in clockwise direction.

-> $\omega_{pc} > \omega_{gc}$ - stable system.

-> $\omega_{pc} < \omega_{gc}$ - unstable "

-> $\omega_{pc} = \omega_{gc}$ - critically stable

(P) Draw polar plot for given open loop transfer function

$$G(s) = \frac{s^3}{(s+1)(s+2)}$$

Sol

given $G(s) = \dfrac{s^3}{(s+1)(s+2)}$

step 1 put $s = j\omega$ to get $G(j\omega)$

$$G(j\omega) = \frac{G(j\omega)^3}{(j\omega+1)(j\omega+2)}$$

90+90+90

step 2 write equation in polar form

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

$$= \frac{\omega^3}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}} \angle +270^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/2)$$

step 3 find points at $\omega = 0$ & $\omega = \infty$.

at $\omega = 0$
$|G(j\omega)| = 0$.
$\angle G(j\omega) = 270^\circ$

at $\omega = \infty$.
$|G(j\omega)| = \infty$
$\angle G(j\omega) = 90^\circ$

step 4 separate real and Imaginary parts.

$$\cancel{G(j\omega) = \frac{-j\omega^3}{(j\omega+1)(j\omega+2)} \times \frac{(j\omega-1)(j\omega-2)}{(j\omega-1)(j\omega-2)}}$$

$$\cancel{= -j\omega^3(}$$

$$G(j\omega) = \frac{-j\omega^3}{(1+j\omega)(2+j\omega)} \times \frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

$$= \frac{-j\omega^3(2-\omega^2-3j\omega)}{(1+\omega^2)(4+\omega^2)} \quad j^2=-1$$

$$= \frac{-j(\omega^3)(2-\omega^2) + 3(-1)\omega^4}{(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3\omega^4}{(1+\omega^2)(4+\omega^2)} + j\frac{\omega^3(\omega^2-2)}{(1+\omega^2)(4+\omega^2)}$$

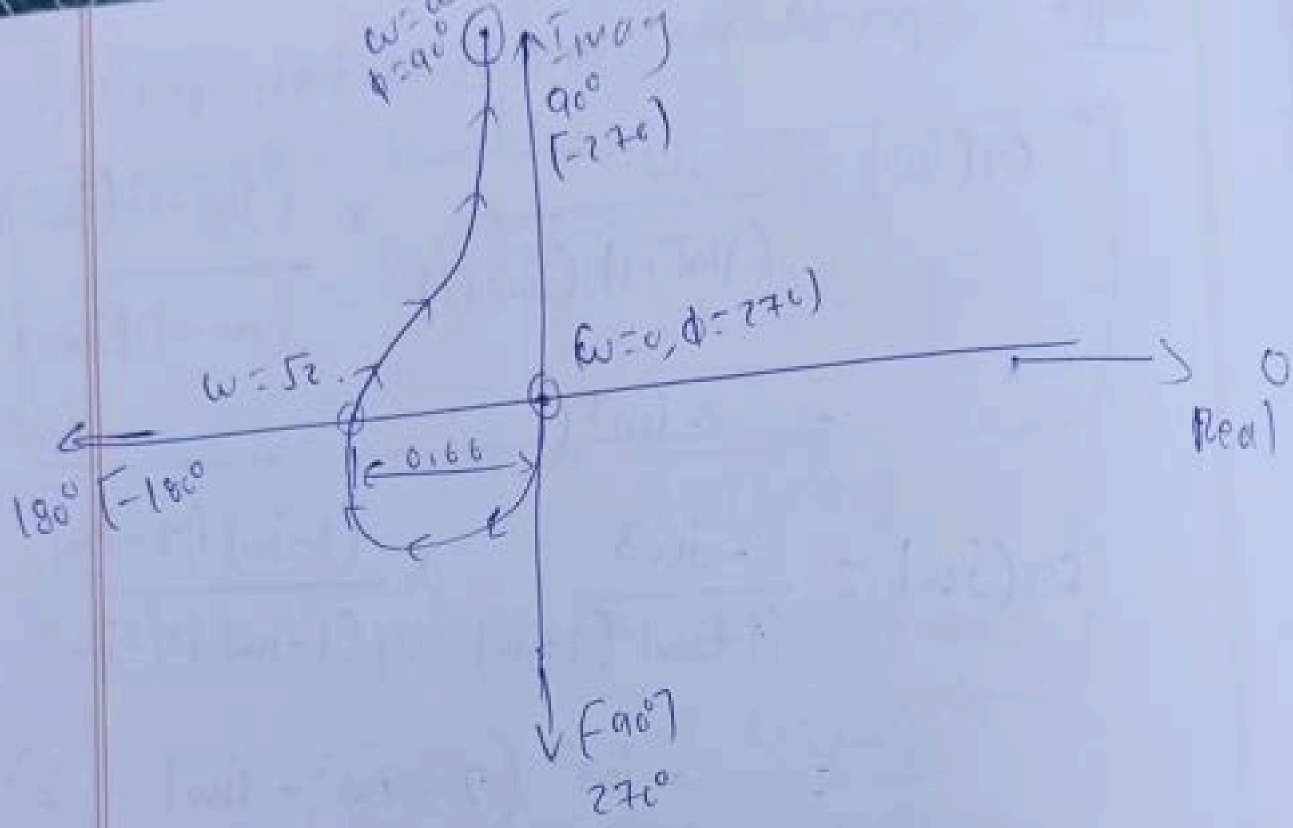
step 5 Intersection to real axis happens at

imag $(G(j\omega)) = 0$.

$\omega = 0$, $\Rightarrow \omega = \sqrt{2}$ rad/sec

step 6 Intersection to Imag axis happens at Real $(G(j\omega)) = 0$.

$\Rightarrow \omega = 0$



$\rightarrow$ Mag $\omega=\sqrt{2}$ in $= \dfrac{\omega^3}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$

$$= \frac{(\sqrt{2})^3}{\sqrt{1+2}\sqrt{4+2}}$$

$$|G(j\omega)| = 0.66$$

$\rightarrow$ phase (ϕ) $= \omega=\sqrt{2}$

$$= 270^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/2)$$

$$= 270^\circ - \tan^{-1}\sqrt{2} - \tan^{-1}(\sqrt{2}/2)$$

$$= 270^\circ - 60 - 30$$

$$= 180^\circ$$

unit-5

concepts of controllability and observability

controllability :

-> controllability verifies the usefulness of the state variables.

-> In controllability test we can find whether the state variable can be controlled to achieve desired output.

Defination of controllability : A system is said to be controllable, if it is possible to transfer the system state from initial state $x(t_0)$ to any other desired state $x(t_d)$ in finite time by control vector $u(t)$.

-> controllability can be tested by kalman's method.

-> kalman's method

-> consider a system with state eq'n $\dot{X} = AX + BU$.

$$Q_c = [B \; AB \; A^2B \ldots\ldots\ldots A^{n-1}B]$$

where,
n = order of the system

-> $|Q_c| \neq 0$ then rank of $Q_c = n$.
then system is completely controllable.

condition: Rank condition is analogous to the Kalman rank condition.

observability

-> In observability test we can find whether the state variable observable (or) measurable.

-> Defination of observability

A system is said to be completely cont observable if every state $X(t)$ can be completely identified by measurements of the output $Y(t)$ over a finite time interval is called observability.

Kalman's method:-

-> consider a system with state model

$$Q_o = [C^T \; A^T C^T \; (A^T)^2 C^T \ldots\ldots (A^T)^{n-1} C^T]$$

where,
n = order of the system

-> $|Q_o| = n$

-> system is completely observable

Solution of state equations

-> The state eq'n for any time invariant system is $\boxed{\dot{X}(t) = AX(t) + BU(t)}$

-> The state equation can classified into two types they are:
* Homogenous state equation
* Non homogenous state equation

-> If 'A' is constant matrix and 'U' is zero vector then equation is called Homogenous equation

-> If 'A' is constant matrix and 'U' is non-zero vector then equation is called Non-homogenous equation

→ **properties**

→ $\phi(t) = e^{At}$

→ $\phi(0) = e^{A(0)} = I$

→ $\phi(t) = e^{At}$

→ $\phi^{-1}(t) = \phi(-t)$

→ $\phi(t_1+t_2) = e^{A(t_1+t_2)} = e^{At_1} \cdot e^{At_2}$

$= \phi(t_1) \cdot \phi(t_2)$

→ $e^{A(t+s)} = e^{At} \cdot e^{As}$

→ $e^{(A+B)t} = e^{At} \cdot e^{Bt}$

→ $[\phi(t)]^n = [e^{At}]^n = e^{Ant}$

(P). obtain complete time response of a system given by

$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ where $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

and $y(t) = [1 \; -1]\, x(t)$

Sol: The given system is homogenous whose solution is

$x(t) = e^{At} x(0) = \phi(t) \cdot x(0)$

$\phi(t) = e^{At} = L^{-1}[SI - A]^{-1}$

$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$

$SI - A = \begin{bmatrix} S & -1 \\ 2 & S \end{bmatrix}$

$[SI - A]^{-1} = \dfrac{Adj(SI-A)}{|SI-A|}$

$Adj[SI-A] = \begin{bmatrix} S & 1 \\ -2 & S \end{bmatrix}$

$|SI - A| = S^2 + 2$

$[SI-A]^{-1} = \begin{bmatrix} \dfrac{S}{S^2+2} & \dfrac{1}{S^2+2} \\ \dfrac{-2}{S^2+2} & \dfrac{S}{S^2+2} \end{bmatrix}$

$\phi(t) = e^{At} = L^{-1}(SI-A)^{-1} = L^{-1}\begin{bmatrix} \dfrac{S}{S^2+2} & \dfrac{1}{S^2+2} \\ \dfrac{-2}{S^2+2} & \dfrac{S}{S^2+2} \end{bmatrix}$

$L^{-1}\left[\frac{s}{s^2+2}\right] = \cos\sqrt{2}t$

$L^{-1}\left[\frac{1}{s^2+2}\right] = \frac{1}{\sqrt{2}}\sin\sqrt{2}t$

$L^{-1}\left[\frac{-2}{s^2+2}\right] = -\sqrt{2}\sin\sqrt{2}t$

$L^{-1}\left[\frac{s}{s^2+2}\right] = \cos\sqrt{2}t$

$$\phi(t) = e^{At} = L^{-1}[sI-A] = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{2}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$= e^{At}x(0) \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

out response $y(t) = [1 \; -1]\, x(t)$

$$= [1 \; -1] \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$= \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t - \cos\sqrt{2}t$$

$y(t) = \frac{3}{\sqrt{2}}\sin\sqrt{2}t$

Concept of state, state variable

- State space mode of Linear Time invariant system

  $\dot{x} = Ax + Bu$ → state equation

  $y = Cx + Du$ → output equation

State: It is a group of variables, which summarizes the history of the system in order predict future values.

State variables: The number of state variables is required is equal to the number of the storage elements.

State vector: It is a vector, which contains state variables as elements.