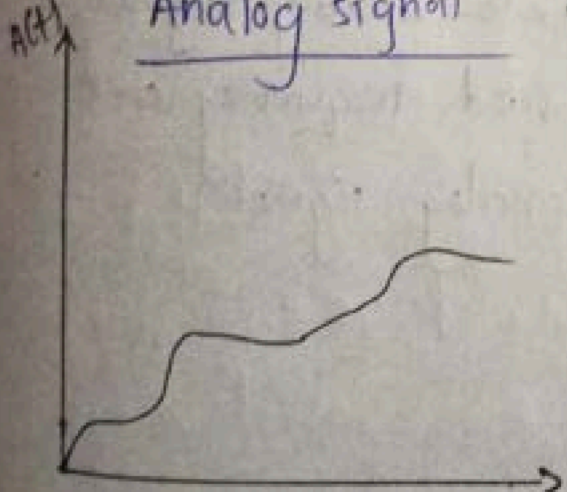
DSP unit-1

Introduction to digital signal processing

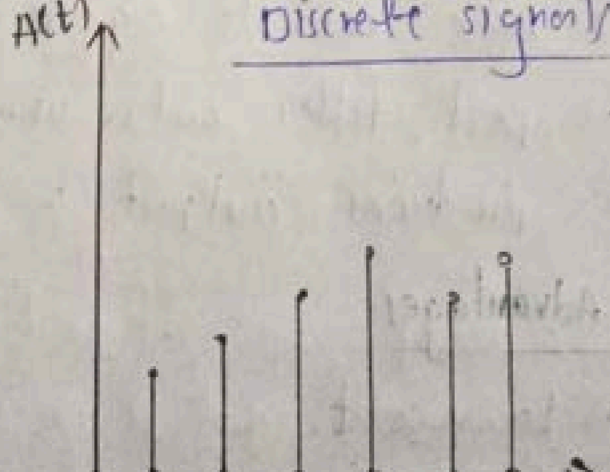
→ **Digital signal processing** :- Digital signal processing is defined as representation of signals by sequence of numbers (or) symbols. and processing these sequences is called DSP

- **signal** : A signal is a function that conveys the information, about the behaviour of physical system.

Analog signal



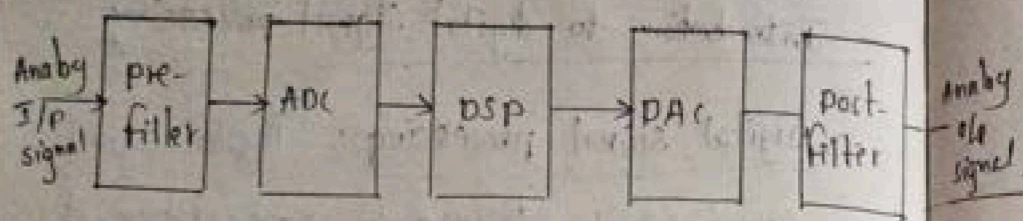
Discrete signals



→ In analog signals both time and amplitude are continuous

→ In discrete signals, signals is defined at discrete units of time.

-> Block diagram of DSP



-> Here input is analog signal, given for prefilter

-> prefilter forwards analog signal to ADC, which converts to digital signal, and sends to DSP.

-> DSP processes it, sends to DAC.

-> DAC converts again into analog signal.

-> post filter cuts unwanted frequency and produces output as analog signal.

Advantages

-> Low cost

-> flexibility

-> error detection and correction.

Disadvantages

-> High power consumption.

Applications:

-> Speech and image processing

-> audio and video ,,

Review of discrete time signals and systems

Discrete time signal: The signals which are defined at particular intervals of time is called discrete time signal.

-> classification of discrete time signals

1. Deterministic and Random signal
2. periodic and aperiodic signal
3. Energy and power signal
4. causal and Non-causal signal
5. Even and odd signal.

1. Deterministic signal: A deterministic signal can be completely represented by a mathematical equation.

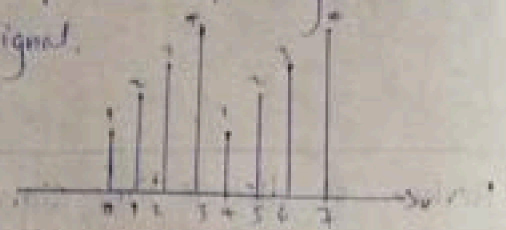
Ex: sine & cos signals

Non deterministic signal: A Non-deterministic signal cannot be represented by a mathematical equation.

Ex: Noise

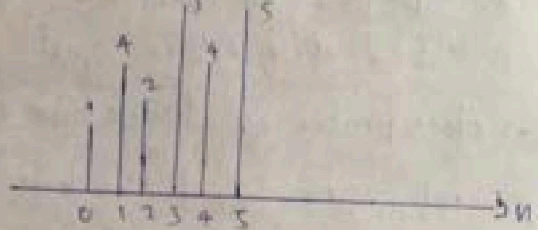
② Periodic signal : A signal which repeats its ~~pattern~~ pattern repeatedly is called periodic signal.

Ex



Aperiodic signal A signal which does not repeat it pattern itself is called aperiodic signal

Ex



③. Energy signal : A sy signal is said to be energy signal if and only if it total energy is finite and power is zero is called energy signal

Ex Energy = 1
power = 0.

Power signal : A signal is said to be power signal if and only it's power is finite and energy is infinite is called power signals

Ex : power = 1
Energy = ∞

④. Causal signal : A signal is said to be causal if $x(n) = 0$ for $n < 0$ is called causal signal

Non-causal signal : A signal is said to be Non-causal if $x(n) = 0$ for $n \geq 0$ is called non-causal signal

⑤. Even signal : A signal $x(n)$ is said to be even signal if it satisfies the condition

$$\boxed{x(-n) = x(n)}$$

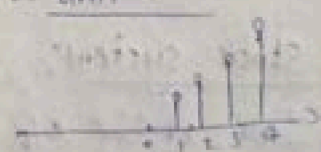
ODD signal A signal $x(n)$ is said to be odd signal if it satisfies the condition

$$\boxed{x(-n) = -x(n)}$$

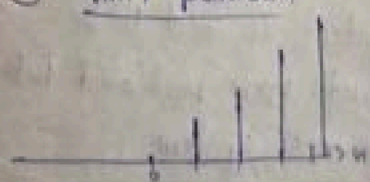
→ Standard discrete time signals

1 unit step

2. unit Ramp



③. unit parabola



④. unit impulse

Basic operations on discrete time signal

-> Time shifting
-> Time Reversal
-> Time scaling
-> Amplitude scaling
-> signal addition
-> signal multiplication

Discrete time system: A system that takes a discrete time signal as input and generates discrete time signal as output is called discrete time system.

-> classification of discrete time systems

1. static and dynamic system
2. Time invariant and Time variant system
3. Linear and Non-linear system
4. stable and unstable system
5. FIR and IIR systems

1. Static system: A system is said to be static, if the o/p response depends upon present I/p only is called static system

Dynamic system: A system is said to be dynamic if the o/p depends upon past and future I/ps only is called dynamic system.

2. Time Invariant system: A system is said to be time invariant if of I/p and o/p characteristics does not change with time

Time variant system: A system is said to be time variant if I/p and o/p characteristics change with time is called time variant system.

3. Linear system: A system which satisfies the superposition theorem is called linear system

$$T[ax_1(n) + bx_2(n)] = ay_1(n) + by_2(n)$$

Non-linear system: A system which does not satisfies superposition theorem is called Non linear system.

4. stable system and unstable: A system is said to be stable if it is bounded input and bounded outputs otherwise unstable

5. FIR: if the impulse response of the system is finite is called FIR system.

IIR: if the impulse respons of the system is infinite is called IIR system

pole-zero plot and system stability

→ when we identify roots of the numerator called zeros

→ when we identify roots of the denominator called poles

(P). Find pole zero plot for given transfer function:

$$H(z) = \frac{z^{-1} - 0.5z^{-2}}{1+1.2z^{-1}+0.45z^{-2}}$$

Sol: given transfer function:

$$H(z) = \frac{z^{-1} - 0.5z^{-2}}{1+1.2z^{-1}+0.45z^{-2}}$$

→ Multiply numerator and denominator with "z^2"

$$H(z) = \frac{(z^{-1} - 0.5z^{-2})z^2}{(1+1.2z^{-1}+0.45z^{-2})z^2}$$

$$= \frac{z^{-3} - 0.5z^{-2}}{z^2+1.2z^{-3}+0.45z^{-4}}$$

$$= \frac{z - 0.5}{z^2+1.2z+0.45}$$

$$H(z) = \frac{z-0.5}{(z+0.6-j0.3)(z+0.6+j0.3)}$$

→ To find zeros

$z - 0.5 = 0$

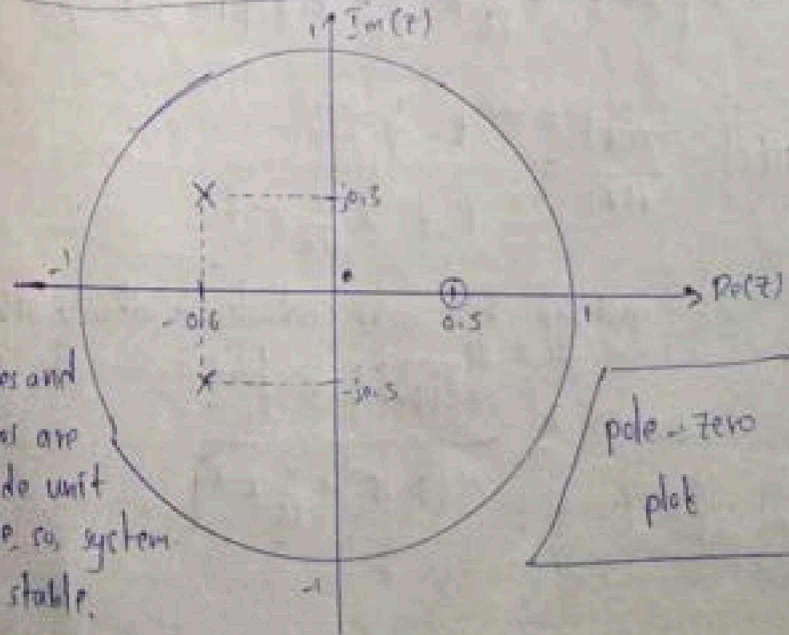
$\boxed{z = 0.5}$

→ To find poles

$z+0.6-j0.3=0$ $\quad$ $z+0.6+j0.3$

$z = -0.6+j0.3$ $\quad$ $z = -0.6-j0.3$

$P_1 = -0.6+j0.3$ $\quad$ $P_2 = -0.6-j0.3$



→ poles and zeros are inside unit circle so system is stable.

pole-zero plot

(P). Find pole-zero, stability from difference equation given below.

$$y(n) = y(n-1) + \frac{3}{16}y(n-2) = x(n) - \frac{1}{2}x(n-1)$$

Sol

given that

$$y(n) - y(n-1) + \frac{3}{16}y(n-2) = x(n) - \frac{1}{2}x(n-1)$$

Step1 Apply z-Transform.

$$Y(z) - z^{-1}Y(z) + \frac{3}{16}z^{-2}Y(z) = X(z) - \frac{1}{2}z^{-1}X(z).$$

Step2 Take common $X(z)$ and $Y(z)$.

$$Y(z)\left[1 - z^{-1} + \frac{3}{16}z^{-2}\right] = X(z)\left[1 - \frac{1}{2}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2}z^{-1}}{1 - z^{-1} + \frac{3}{16}z^{-2}}$$

- multiply numerator and denominator with z^2

$$H(z) = \frac{z^2\left[1 - \frac{1}{2}z^{-1}\right]}{z^2\left[1 - z^{-1} + \frac{3}{16}z^{-2}\right]}$$

$$H(z) = \frac{z^2 - \frac{1}{2}z}{z^2 - z + \frac{3}{16}}$$

$$H(z) = \frac{z(z - \frac{1}{2})}{z^2 - z + \frac{3}{16}}$$ find roots for this

$$= \frac{z(z - \frac{1}{2})}{(z - \frac{1}{4})(z - \frac{3}{4})}$$

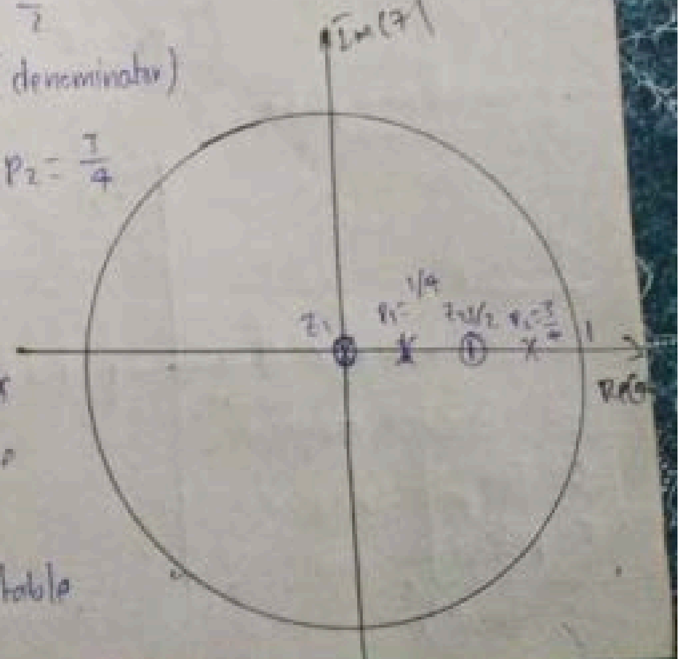
$$H(z) = \frac{z(z - \frac{1}{2})}{(z - \frac{1}{4})(z - \frac{3}{4})}$$

→ To find zeros (numerator).

$z_1 = 0$, $z_2 = \frac{1}{2}$

→ To find poles (denominator)

$p_1 = \frac{1}{4}$, $p_2 = \frac{3}{4}$



poles and zeros are inside the unit.

So, system is stable

Representation of poles and zeros in s-plane

Q. plot poles and zeros for given transfer function

$$H(s) = \frac{s+2}{(s+3)(s+4)} \ ?$$

Sol given that

$$H(s) = \frac{s+2}{(s+3)(s+4)}$$

zeros (z) = $s+2=0$

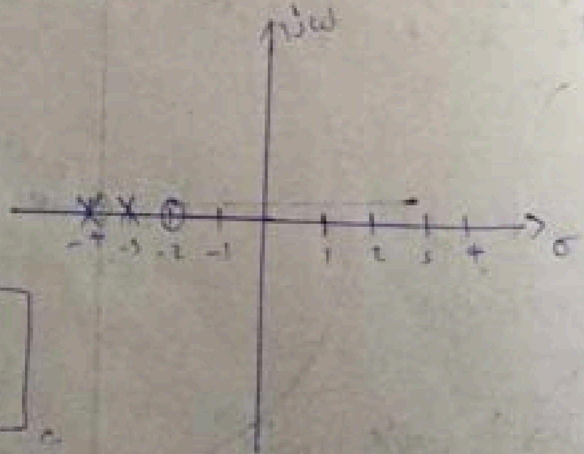
$\boxed{z_1 \quad s=-2}$

poles (p) = $s+3=0$ $\quad$ $s+4=0$

$s=-3$ $\quad$ $s=-4$

$\boxed{P_1=-3 \qquad P_2=-4}$

-> consider s-plane



O - zeros
X - poles

Q. plot poles and zeros given T.F

$$H(s) = \frac{s^2+1}{s(s^2+2s+5)} \ ?$$

Sol

zeros

$s^2+1=0$

$s^2=-1$

$s=\sqrt{-1}$

$\boxed{s=+j1, -j1}$

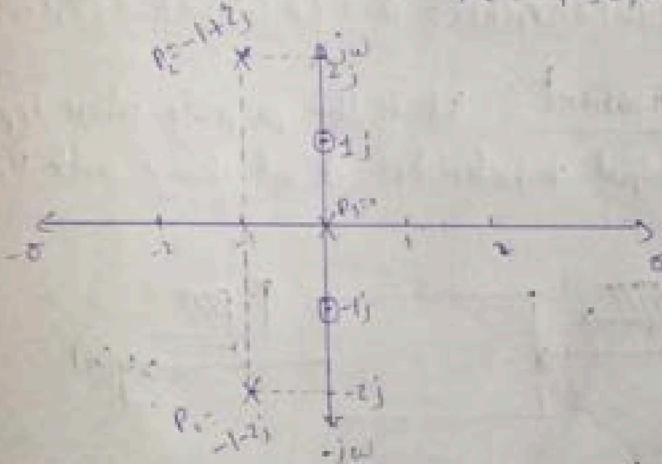
poles

$s(s^2+2s+5)=0$

$\boxed{P_1: s=0}$

$P_2 = s^2+2s+5=0$

$P_2 = -1-j2, -1+j2$



Analysis of LTI - DT Systems

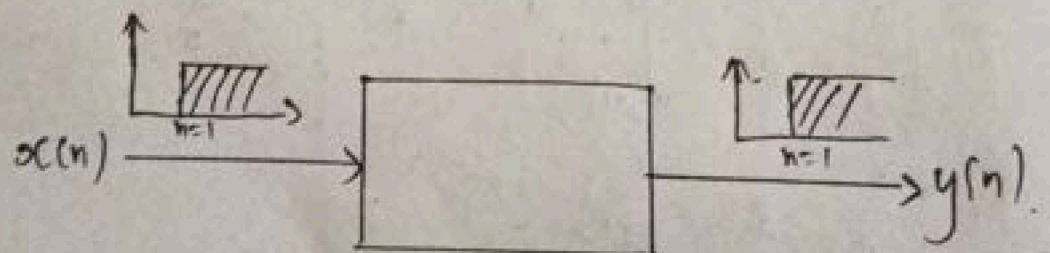
-> Def of LTI - DT system

-> It is a system which satisfies both linearity and Time invariant properties is called LTI-DT system.

Linearity : The transform of weighted signals in time domain is equal to the weighted sum of their spectra.

$$T[ax_1(n) + bx_2(n)] = a \cdot T[x_1(n)] + b \cdot T\{x_2(n)\}$$

Time invariant : It is the property where input and output characteristics do not change with time.



Techniques :-

- -> solution of difference equations
- -> Transfer function
- -> Impulse Response
- -> convolution sum
- -> stability

unit - II

Discrete fourier series (DFS) :-

→ The term discrete fourier series is any periodic discrete time signal comprising hormonically - related discrete real sinusoids combined by weighted summation

→ It is expressed as

$$x(n) = \sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N} \quad \text{--- (1)} \qquad n = 0, 1, \ldots N-1$$

$$\sum_{n=0}^{N-1} e^{j2\pi nk/N} = \begin{cases} N; & k = 0, \pm N, \pm 2N \\ 0; & \text{otherwise} \end{cases}$$

Multiply equation (1).

In above eq $X(k)$ is fourier series coefficient.

$$k = 0, 1, \ldots N-1.$$

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

replace k to $k+N$

$$X(k+N) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi n(k+N)/N}$$

$$X(k+N) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \cdot \underbrace{e^{-j2\pi nN/N}}_{=1}$$

$$x(k+N) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

$$\boxed{x(k+N) = X(k)}$$

Discrete fourier series is a periodic signal..

Properties of DFS

1 Linearity property

Statement: If $x_1(n) \xleftrightarrow{DFS} C_{1k}$

and $x_2(n) \longleftrightarrow C_{2k}$

then $ax_1(n)+bx_2(n) \xleftrightarrow{DFS} aC_{1k}+bC_{2k}$.

Proof By the defi..

$$DFS[x(n)] = C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}.$$

$$DFS[ax_1(n)+bx_2(n)] = \frac{1}{N}\sum_{n=\langle N\rangle}[ax_1(n)+bx_2(n)]e^{-j2\pi nk/N}$$

$$\boxed{ax_1(n)+bx_2(n) \Longleftrightarrow aC_{1k}+bC_{2k}.}$$

② shifting property (Time):

$$x[n] \xleftrightarrow{DFS} C_k.$$

$$x[n \pm n_0] \xleftrightarrow{DFS} e^{-\frac{jk2\pi n_0}{N}} C_k.$$

Proof: By the defi..

$$DFS[x(n)] = C_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$$

$$DFS[n-n_0] = \frac{1}{N}x[n-n_0]\,e^{-j2\pi nk/N}$$

let $n-n_0 = r$, $n = r+n_0$

$$= \frac{1}{N}x[r] - e^{-j2\pi(r+n_0)k/N}$$

$$\boxed{DFS[n-n_0] = e^{-j2\pi n_0 k/N}C_k.}$$

③ frequency shifting property

Statement: If $x[n] \xleftrightarrow{DFS} C_k$

then $e^{\pm jk2\pi n_0/N} \xleftrightarrow{DFS} C_{k\pm k_0}$

Proof By the defination

$$DFS[x(n)] = C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N} \text{ —①}$$

$$\cancel{DFS[x(n)] = \frac{1}{N}\sum_{n=0}^{N-1}}$$

$$DFS[e^{jk2\pi n_0/N}x(n)] = \frac{1}{N}\sum_{n=0}^{N-1}[e^{jk2\pi n_0/N}x(n)]\; e^{-j2\pi nk/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\,e^{-jk2\pi(k-k_0)n} \text{ —①.}$$

$$\boxed{e^{\pm jk_0 2\pi n_0/N} \xleftrightarrow{DFS} C_{k-k_0}.}$$

④ Conjugate property

Statement If $x(n) \xleftrightarrow{DFS} C_k$

then $x^*(n) \xleftrightarrow{DFS} C^*_{-k}$

Proof By the defination

$$DFS[x(n)] = C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/n} \quad (1)$$

Apply conjugate on B.S

$$C_k^* = \left[\frac{1}{N}\sum_{n=<N>} x(n)\, e^{-\frac{j2\pi nk}{N}}\right]^*$$

$$C^*_k = \frac{1}{N}\sum_{n=<N>} x^*(n)\, e^{j2\pi nk/N}$$

replace k by -k

$$C^*_{-k} = \frac{1}{N}\sum_{n=<N>} x^*N\, e^{-j2\pi nk/N}$$

$$x^*[n] \xleftrightarrow{DFS} C^*_{-k}$$

⑤ Time Reversal property

Statement If $x(n) \xleftrightarrow{DFS} C_k$

$x(-n) \longleftrightarrow C_{-k}$

proof By the defination

$$DFS[x(n)] = C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$$DFS[x(-n)] = \frac{1}{N}\sum_{n=0}^{N-1} x(-n)\, e^{-j2\pi nk/N}$$

$n = r$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[r]\, e^{j2\pi rk}$$

$$\boxed{DFS[x(-n)] = C_{-k}}$$

Discrete fourier Transform

-> Discrete fourier Transform is more important in all areas of digital signal processing.

-> DFT is used derive frequency domain representation of the signal.

-> DFT does not act on signal that exist all the time.

-> DFT -> $x(n)$; $0 \le n \le N-1$

$$\boxed{DFT\{x(n)\} = X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}KN}} \quad (1)$$

$$\boxed{e^{-j\frac{2\pi}{N}KN} = W_N} \qquad 0 \le K \le N-1.$$

$$\boxed{DFT\{x(n)\} = x(K) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{kn}} \quad (2).$$

$0 \le K \le N-1$

Here, W_N = Twiddle factor.

$x(n)$ = Discrete time signal.

$x(K)$ = sequence in frequency domain.

Inverse Discrete fourier Transform

-> The Inverse discrete fourier transform [IDFT] is the reverse process of DFT.

-> IDFT is used to transform a signal from frequency domain to time domain.

$$\boxed{\text{-> } IDFT\{x(K)\} = x(n) = \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}kn}}$$

$0 \le K \le N-1.$

$e^{-j\frac{2\pi}{N}KN} = W_N.$

$$\boxed{IDFT\{x(K)\} = x(n) = \sum_{K=0}^{N-1} x(K)\, \omega_N^{-KN}}.$$

$0 \le n \le N-1.$

where,

W_N = Twiddle factor.

$x(n)$ = sequence in time domain.

$x(K)$ = Discrete time signal.

Problems on DFT

① Find DFT of sequence $x(n) = \{1,1,1,1\}$

sol $N = 4$

$k = 0, 1, \dots, 4-1$

$k = 0, 1, 2, 3$

→ $k = 0$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}$$

$$x(0) = \sum_{n=0}^{3} x(n)\, e^{-0}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$x(0) = 4$

→ $k = 1$

$$x(1) = \sum_{n=0}^{3} x(1)\, e^{-j\frac{2\pi n(1)}{4_2}}$$

$$= \sum_{n=0}^{3} x(1)\, e^{-jn\pi/2}$$

$$= x(0)e^{0} + x(0)e^{-j\pi/2} + x(1)e^{-j\pi} + x(2)e^{-j3\pi/2}$$

$= 0$

= $k = 2$

$$x(2) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi n(2)/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)e^{-j\pi} + x(1)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$x(2) = 0$

$k = 3$

$$x(3) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi n \cdot 3/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn3\pi/2}$$

$$= x(0)e^{0} + x(1)e^{-3\pi/2} + x(1)e^{-j3\pi/2} + x(3)e^{-j9\pi/2}$$

$x(3) = 0$

$x(k) = \{4, 0, 0, 0\}$

(P). compute the 4-point DFT of $x(n) = \{0,1,2,3\}$.

sol

$$\text{DFT}\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$$\text{IDFT}\{X(k)\} = x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k)\, e^{-j2\pi nk/N}$$

given $N = 4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j2\pi nk}{4}} \quad ; k = 0,1,2,3$$

$k=0 \Rightarrow X(0) = x(0)+x(1)+x(2)+x(3)$

$k=1 \Rightarrow X(1) = x(0)+x(1)e^{-j\pi/2}+x(2)e^{-j\pi}+x(3)e^{-j3\pi/2}$

$k=2 \Rightarrow X(2) = x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}$

$k=3 \Rightarrow X(3) = x(0)+x(1)e^{-j3\pi/2}+x(2)e^{-j3\pi}+x(4)e^{-j9\pi/2}$

$X(0) = 0+1+2+3 = 6.$

$X(1) = 0+1\left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + 2\left(\cos\pi + j\sin\pi\right) +$
$3\left(\cos 3\pi/2 - j\sin 3\pi/2\right).$

$X(1) = -2+2j$

$X(2) = -2.$

$X(3) = -2-2j.$

$x(n) = \{0,1,2,3\} \xleftrightarrow{DFT} X(k) = \{6, -2+2j, -2, -2-2j\}$

Difference b/w DFT and FFT.

DFT.	FFT
-> DFT stands for Discrete fourier transform.	-> FFT stands for fast fourier transform
-> speed is slower.	-> speed is faster
-> It is the discrete version of fourier transform.	-> It is the fast version of DFT.
-> It is an implementation of DFT.	-> It is the relationship between time domain and frequency domain representation
-> Application:	Application
- cross correlation	-> Radar communication.
-> matched filtering	-> sensor data signals.
- system Identification.	-> Diagnostics.
->	

properties of DFT.

① Linearity property

If $DFT\{x_1(n)\} = X_1(k)$

$DFT\{x_2(n)\} = X_2(k)$

$$\boxed{DFT\{ax_1(n) + bx_2(n)\} = aX_1(k) + bX_2(k)} \quad ①$$

Proof : $DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} = X(k)$

Take L.H.S of eq ①

$$DFT\{ax_1(n) + bx_2(n)\} = \sum_{n=0}^{N-1} [ax_1(n) + bx_2(n)] e^{-j2\pi nk/N}$$

$$= \sum_{n=0}^{N-1} ax_1(n) e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} bx_2(n) e^{-j2\pi kn/N}$$

$$= a \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi kn/N} + b \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi kn/N}$$

$$= aX_1(k) + bX_2(k) \quad R.H.S.$$

②. periodicity property

$$\boxed{\begin{array}{l} x(n+N) = x(n) \text{ for all } n \\ X(k+N) = X(k) \text{ for all } k \end{array}}$$

Proof : The IDFT of the X(k) is.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$$

$$x(n+N) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi k(n+N)/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N} \cdot e^{j2\pi kN/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N} \cdot e^{j2\pi k}$$

$$x(n+N) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N} \quad \boxed{\because e^{j2\pi k} = 1}$$

$$\boxed{x(n+N) = x(n)}$$

-> Hence periodicity property is proved.

③ Time Reversal property :

If $DFT\{x(n)\} = X(k)$, then

$$DFT\{x((-n))_N\} = X((-k))_N$$

$$DFT\{x(N-n)\} = X(N-k)$$

Proof: $DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

For time reversal

$$DFT\{x(N-n)\} = \sum_{n=0}^{N-1} x(N-n)\, e^{-j2\pi kn/N}$$

$$\boxed{DFT[x(N-n)] = X(N-k)}$$

Hence proved

Circular convolution property :

Statement

If $DFT\{x(n)\} = X(k)$, then

$$DFT\{x_1(n) \circledast x_2(n)\} = X_1(k) \cdot X_2(k)$$

-> Circular convolution in time domain = multiplication in frequency domain

Proof : $x_1(n) \circledast x_2(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2(n-m)$ — ①

-> $DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

-> DFT of circular convolution

$$DFT\{x_1(n) \circledast x_2(n)\} = \sum_{n=0}^{N-1} [x_1(n) \circledast x_2(n)]\, e^{-j2\pi kn/N} \quad -②$$

sub eq ① in eq ②

$$= \sum_{n=0}^{N-1}\left[\sum_{m=0}^{N-1} x_1(m) \cdot x_2(n-m)\right] e^{-j2\pi kn/N} \quad -③$$

let $\left.\begin{array}{l} n-m = p \\ n = p+m \end{array}\right\}$ sub in eq ③

$$= \sum_{m=0}^{N-1}\sum_{p=0}^{N-1} x_1(m)\, x_2(p)\, e^{-j2\pi(p+m)k/N}$$

$$= \sum_{m=0}^{N-1}\sum_{p=0}^{N-1} x_1(m)\, x_2(p)\, e^{-j2\pi pk/N} \cdot e^{-j2\pi mk/N}$$

$$\Rightarrow \sum_{m=0}^{N-1} x_1(m) e^{-j2\pi mk/N} \cdot \sum_{p=0}^{N-1} x_2(p) e^{-j2\pi pk/N}$$

$$\Rightarrow x_1(k) \cdot x_2(k)$$

$$DFT\{x_1(n) \; (N) \; x_2(n)\} = X_1(k) \cdot X_2(k)$$

-> Hence circular convolution is proved

6. Multiplication property

-> Multiplication property states that

$$x_1(n) \xleftrightarrow{DFT} X_1(k) \text{ and}$$

$$x_2(n) \xleftrightarrow{DFT} X_2(k) \text{ then}$$

$$\boxed{x_1(n) \cdot x_2(n) \xleftrightarrow{DFT} \cancel{\tfrac{1}{N} X_1(k)} \cdot \frac{1}{N} X_1(k) \; (N) \; X_2(k)}$$

-> Hence multiplication property is proved.

7. Parseval's Theorem

-> The parseval's theorem states that

$$\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k) Y^*(k).$$

Linear convolution

-> Multiplication of two sequences in time domain is called linear convolution.

-> Two signal sequences
- i Input signal $x(n)$
- ii Impulse response $h(n)$
- iii output $y(n)$

-> Finally linear convolution is given by the

$$y(n) = x(n) * h(n) \text{ and is calculated as}$$

$$y(n) = \sum_{0}^{\infty} x(k) \, h(n-k)$$

Linear convolution	circular convolution
-> Multiplication of two sequences in time domain is called Linear convolution	-> circular convolution is also known as cyclic convolution
-> less complexity	-> More complexity.
-> It is linear shifting	-> It is circular shifting
-> No. of samples N_1+N_2-1	-> $Max(N_1, N_2)$.
-> Formula in Matlab conv(a,b).	-> Formula in matlab cconv(a,b).

Fast fourier Transform

-> The fast fourier tranform [FFT] is an efficient algorithm.

-> FFT is used to converting time domain signal into a equivalent frequency domain signal is called ~~fast fou~~ based on DFT

-> It is the fast version of DFT.

-> FFT is based on the principle of decomposing.

-> FFT used two properties of twiddle factor.

i, periodicity.

$$W_N^{k+N} = W_N^{k}$$

ii symmetry.

$$W_N^{k+\frac{N}{2}} = -W_N^{k}$$

N. points	No. of complex multiplication: Direct DFT	FFT	Speed fact = $\frac{\text{Direct DFT}}{\text{FFT}}$
4	16	4	4
16	256	32	8
64	4096	192	~~20.89~~ 21.33

Advantages

-> FFT is used to converting time domain sequence into frequency domain sequence.

-> It reduces N^2 to $\frac{N}{2}\log_2 N$

Application

-> Digital spectral analysis
-> pattern recognition
-> Fault analysis

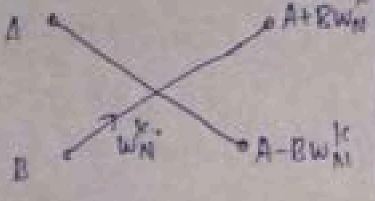
Types of FFT

i Decimation - in Time [DIT-FFT]
ii Decimation in frequency [DIF-FFT]

Decimation-in Time DIT-FFT Algorithm	Decimation in frequency DIF-FFT Algorithm
-> DIT algorithm is based on the decomposition of input sequence into smaller subsequences.	-> DIF algorithm is based on the decomposition of output sequence into smaller subsequences.
$x(n) = X_0(n), X_e(n)$.	$X(k) = X_e(k), X_0(k)$.
-> Decomposition is done in time domain.	-> Decomposition is done in frequency domain.

-> In DIT, input is bit reversed and output is in natural order.

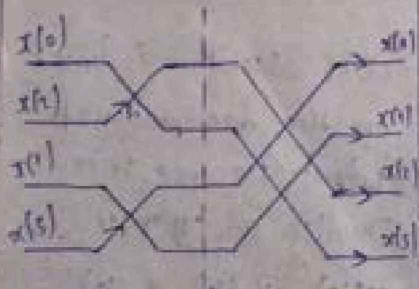
-> computational diagram



-> The complex multiplication is before add-subtract operation.

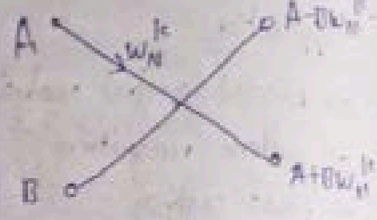
-> Butterfly diagram:

N = 4



-> In DIF, input is in natural order and output is bit reversed.

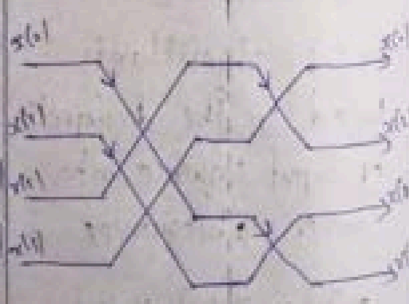
-> computational diagram



-> The complex multiplication is after add-subtract operation.

-> Butterfly diagram

N = 4



problems on DIT-FFT

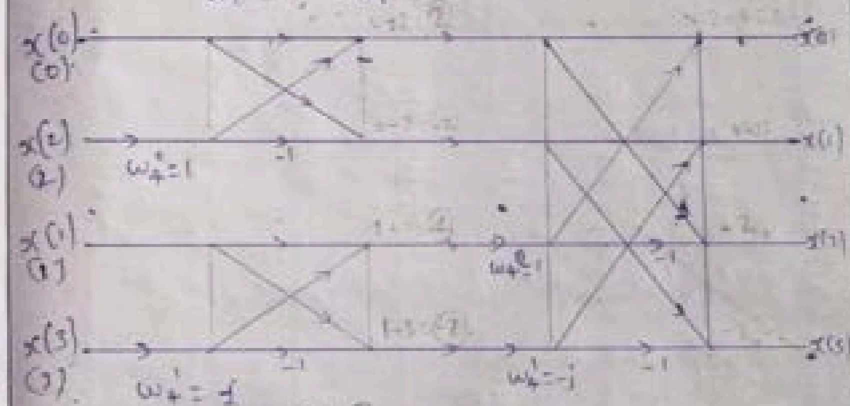
4-point DIT-FFT

(P). given $x(n) = \{0, 1, 2, 3\}$ find $X(k)$ using DIT-FFT Algorithm.

sol $x(n)$ = 4 samples.

$\therefore N = 4$

$W_4^0 = 1, \quad W_4^1 = -j$

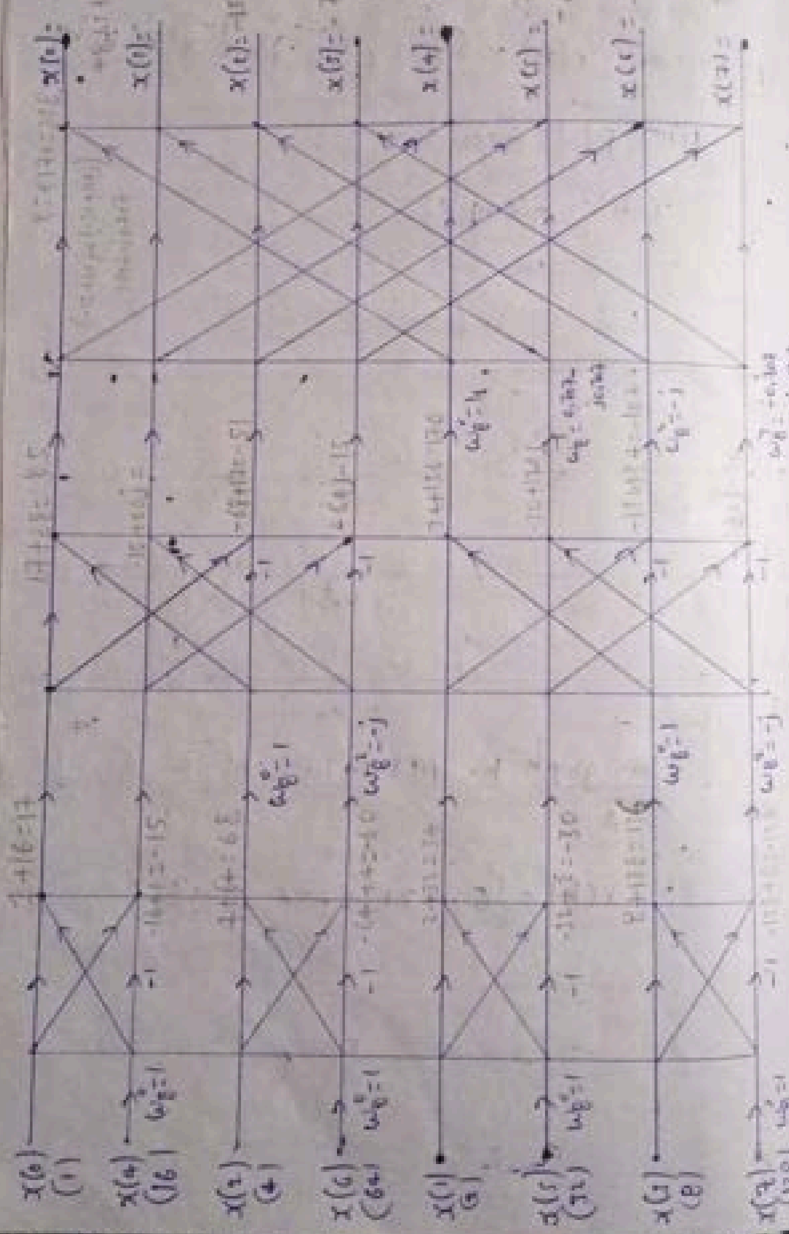


flow graph for DIT-FFT : N = 4

$\therefore X(k) = \{6, -2+2j, -2, -2-2j\}$

(P). given $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$ find $X(k)$ using DIT-FFT?

sol: $N = 8$



$X(k) = \{256,\ 48.68+166.05j,\ -51+j102,\ -78.63+j48.05,\ -85,\ -78.63-j48.05,\ -51-j102,\ 48.63-j166.05\}$

---

problem on DIF-FFT.

(P). compute the DFT of $x(n) = \cos\frac{n\pi}{2}$ where $N = 4$ using DIF-FFT?

sol: $x(n) = \cos\frac{n\pi}{2}$

given $N = 4 - (0, 1, 2, 3)$

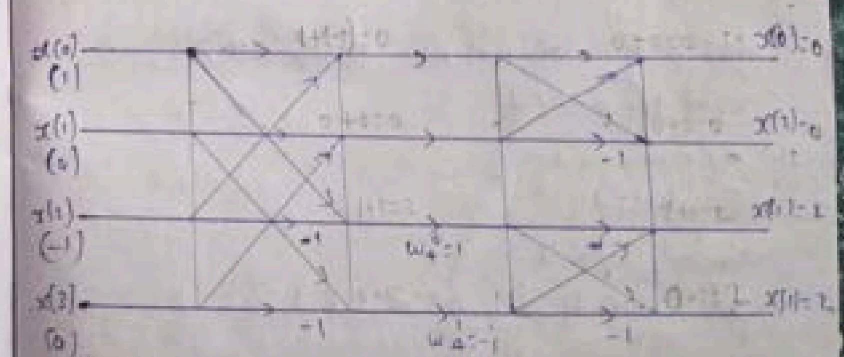
i $n = 0 - \cos\frac{0\pi}{2} = 1$

ii $n = 1 - \cos\frac{\pi}{2} = 0$

iii $n = 2 - \cos\frac{2\pi}{2} = -1$

iv $n = 3 - \cos\frac{3\pi}{2} = 0$

So, $x(n) = \{1, 0, -1, 0\}$



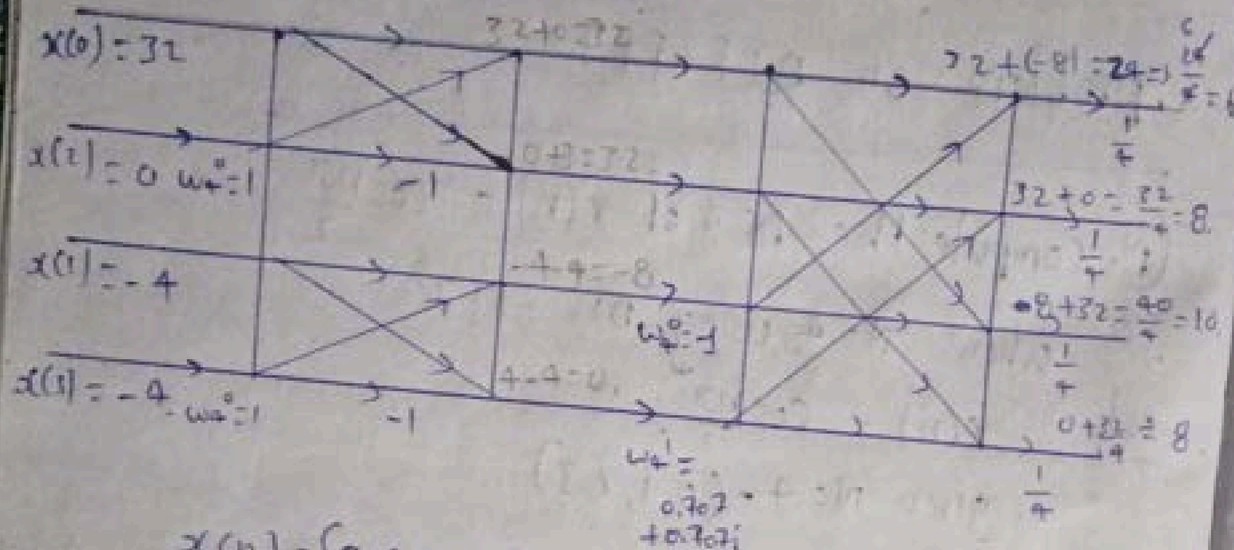
$X(k) = \{0, 0, 2, 2\}$

Q. compute $x(n)$ for $x(k) = \{32, -4, 0, -4\}$ using DIF-FFT?

Sol

given $N = 4$

$x(k) = \{32, -4, 0, -4\}$



$x(n) = \{6, 8, 10, 8\}$

Q. given $x(n) = n+1$ for $0 \le n \le 7$. find $x(k)$ using DIF-fft?

Sol

$N = 8$. $x(n) = n+1$ — (1)

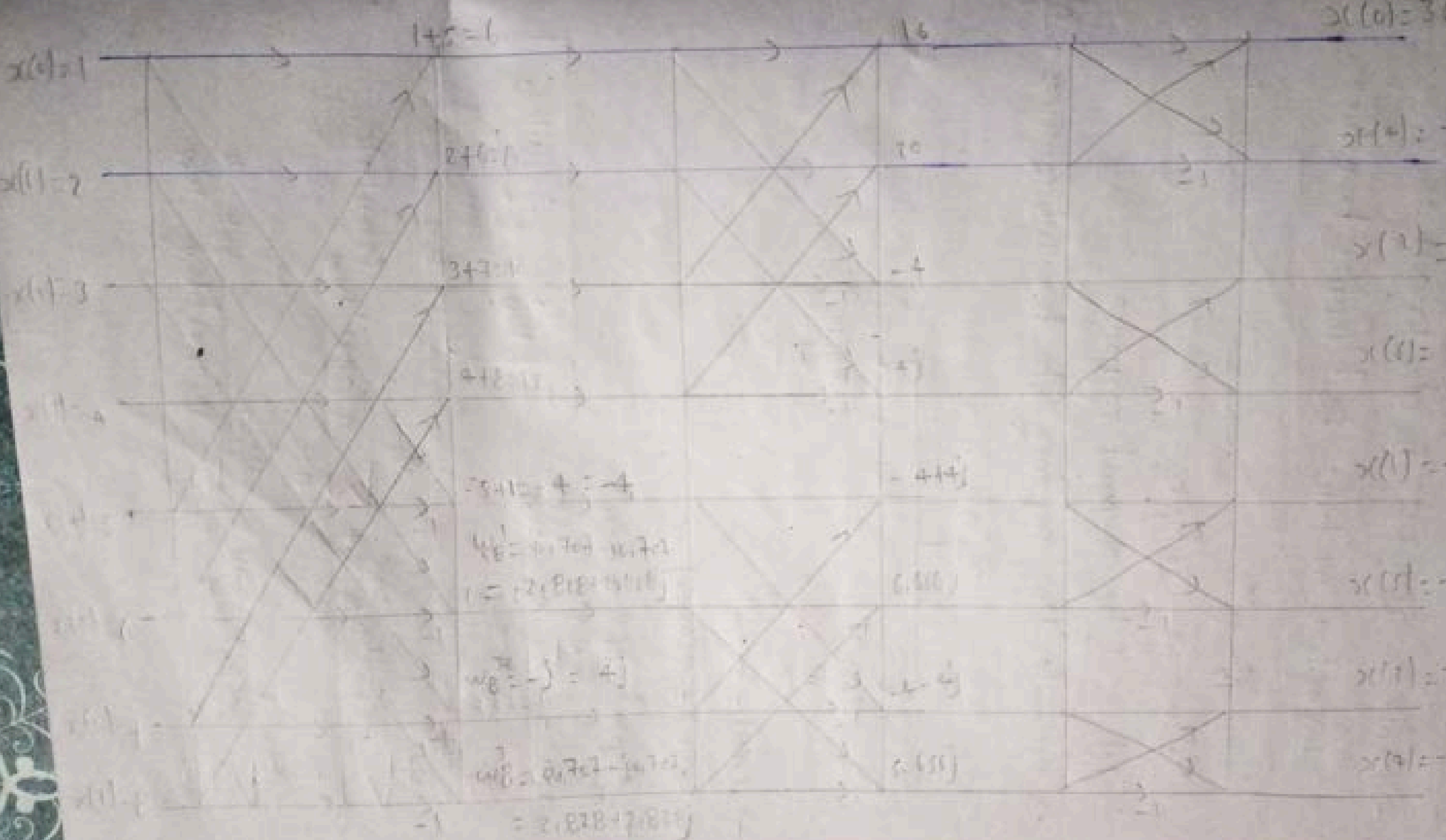
$x(n)$ for $0 \le n \le 7$.

put $n = 0$ in eq(1).

$x(n) = 1+1 = 2$.

similarly upto 7.

weget. $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$


x(0)=1
x(1)=2
x(1)=3
1+5=6
2+6=8
3+7=10
4+8=12
16
20
-4
-4+4j
-4-4j
x(0)=36
x(4)=-4
x(2)=-4+4j
x(6)=-4-4j
x(1)=-4+9.656j
x(5)=-4-1.656j
x(3)=-4+1.656j
x(7)=-4-9.656j
-4+4j
1.656j
-4-4j
-1
= 2.828+2.828j

Radix-2 DIT-FFT

-> Radix-2 DIT-FFT is also known as Decimation in time- Fast fourier transform.

$$N = 2^M$$

where $M \rightarrow$ Integer.

-> let $x(n)$ be N-point sequence.

-> Decimate this sequence into two subsequences is $\frac{N}{2}$

$$x(n) \rightarrow x_e(n)\ \tfrac{N}{2}, \quad x_o(n)\ \tfrac{N}{2}$$

-> $x_e(n) = x(2n)$

-> $x_o(n) = x(2n+1)$

-> N-point DFT of $x(n)$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$\boxed{W_N^{nk} = e^{-j2\pi nk/N}}$$

$$\sum_{n=0}^{N/2-1} x(2n) W_N^{2nk} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{(2n+1)nk}$$

$$\sum_{n=0}^{N/2-1} x(2n) W_N^{2nk} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2nk} W_N^{k}$$

$$X(k) = \sum_{n=0}^{N/2-1} x_e(n) W_{\frac{N}{2}}^{nk} + W_N^{k} \sum_{n=0}^{N/2-1} x_o(n) W_{\frac{N}{2}}^{nk}$$

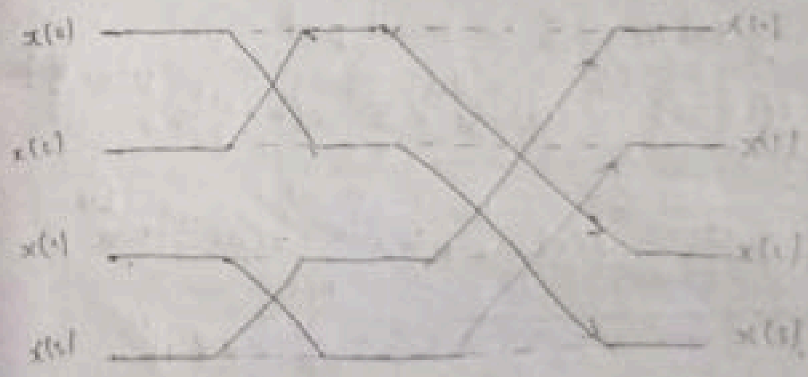
$$X(k) = X_e(k) + W_N^{k} X_o(k) \quad -①$$

sub $k = k - \frac{N}{2}$ in eq①

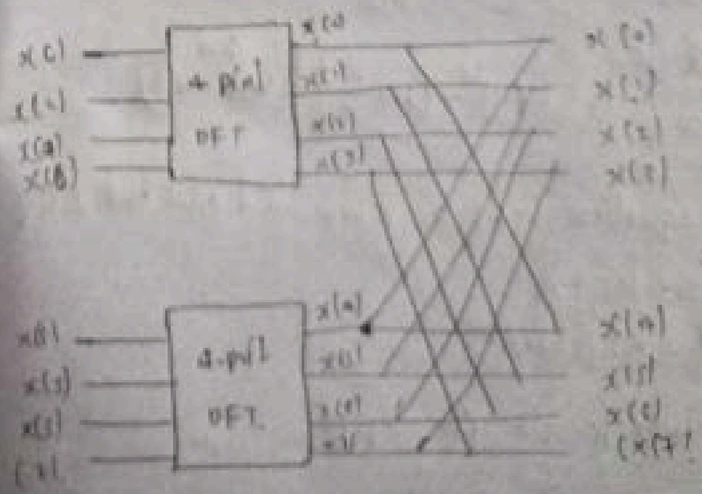
$$X(k) = X_e\left(k-\tfrac{N}{2}\right) + W_N^{k-\frac{N}{2}} X_o\left(k-\tfrac{N}{2}\right) \quad -②$$

-> Butterfly diagram

-> for 4-point DIT-FFT



-> For 8 point



Radix-2 DIF-FFT Algorithm

→ Decimation In time fast fourier transform is also known as Radix-2 DIF-FFT.

→ Decimation in frequency

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \; ; 0 \le k \le N-1 \quad -(1).$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=\frac{N}{2}}^{N-1} x(n) W_N^{kn}$$

→ put $n = n+\frac{N}{2}$ in second part.

$$X(k) = \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) W_N^{k\left(n+\frac{N}{2}\right)}$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + W_N^{k\frac{N}{2}} \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) W_N^{kn}$$

$$\because W_N^{k\frac{N}{2}} = (-1)^k$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + (-1)^k \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) W_N^{kn}$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^k x\left(n+\frac{N}{2}\right)\right] W_N^{kn} \quad -(2).$$

→ Decompose equation $X(k)$ as Even and odd index seq.

- For even - $k = 2r$
- For odd - $k = (2r+1)$

→ For even sub $k = 2r$ in eq (2).

$$X(2r) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + \underbrace{(-1)^{2r}}_{=1} x\left(n+\frac{N}{2}\right)\right] W_N^{2rn}$$

$$X(2r) = \sum_{n=0}^{N/2-1} \underbrace{\left[x(n) + x\left(n+\frac{N}{2}\right)\right]}_{g(n)} W_{N/2}^{rn}$$

$$X(2r) = \sum_{n=0}^{N/2-1} g(n) W_{\frac{N}{2}}^{rn} \quad -(3)$$

→ For odd sub $k = (2r+1)$ in eq (2)

$$X(k) = \sum_{n=0}^{N/2-1} x(n) + \underbrace{(-1)^{2r+1}}_{-1} x\left(n+\frac{N}{2}\right)\Big] W_N^{(2r+1)n}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \underbrace{\left[x(n) - x\left(n+\frac{N}{2}\right)\right]}_{} W_N^{n} \cdot W_{N/2}^{rn}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} h(n) W_N^{n} \cdot W_{N/2}^{rn} \quad -(4)$$

→ $g(n) = x(n) + x\left(n+\frac{N}{2}\right)$ $\quad \frac{N}{2} = 4$

→ $h(n) = x(n) - x\left(n+\frac{N}{2}\right)$

put $n = 0$ to 3 in above $g(n)$ and $h(n)$ equation

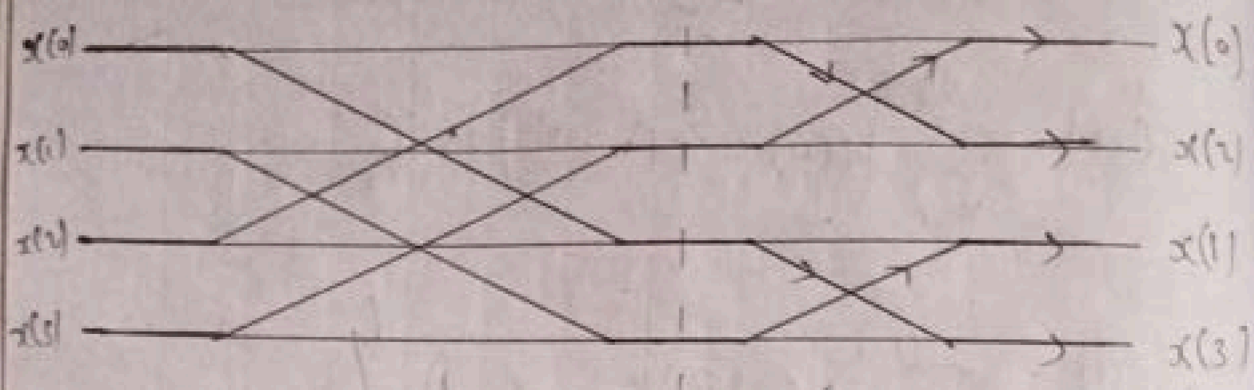
$n=0$, $g(0) = x(0) + x(4)$, $h(0) = x(0) - x(4)$

$n=1$, $g(1) = x(1) + x(5)$, $h(1) = x(1) - x(5)$

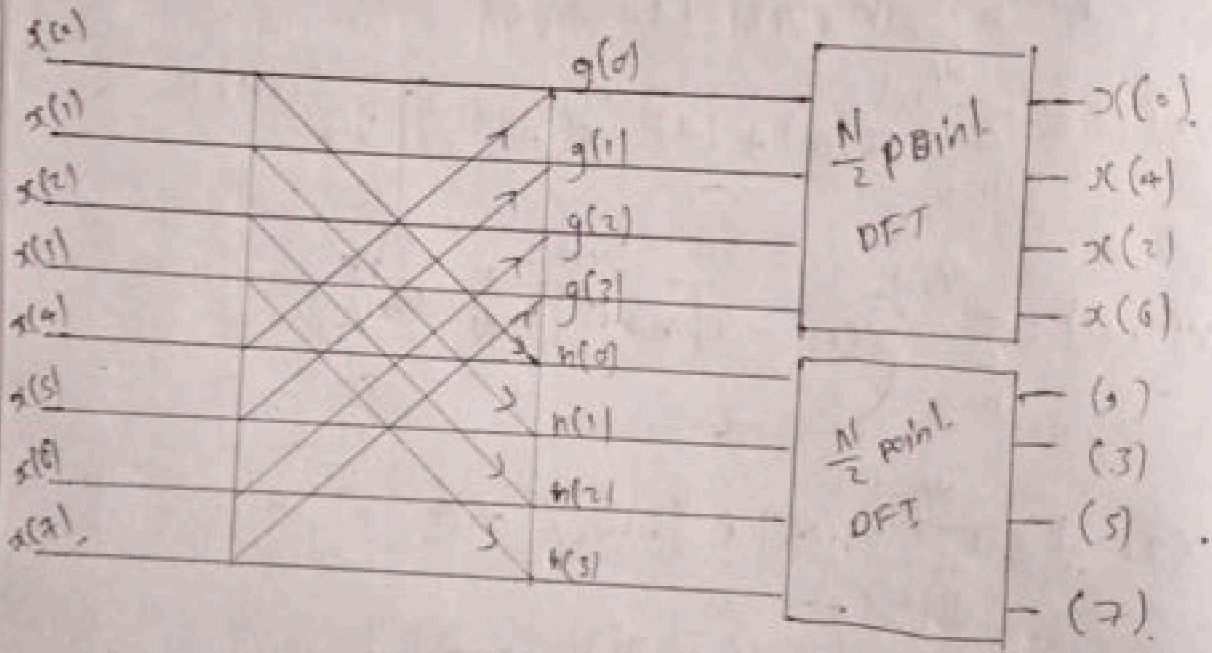
$n=2$, $g(2) = x(2) + x(6)$, $h(2) = x(2) - x(6)$

$n=3$ $g(3) = x(3) + x(7)$ $\quad h(3) = x(3) - x(7)$.

for 4-point DIF-FFT.



→ For 8-point DIF FFT



Inverse Fast fourier Transform (Radix-2)

(P).

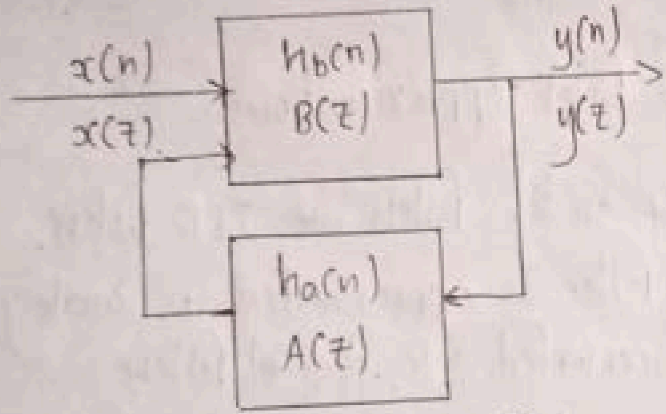
$x(k) = \{6, 2.121+j1.21, -1+5j, -2.121+j3.121,$
$4, -2.121-j3.121, -1-5j, 2.121+j1.2\}$

find IFFT Rad-2

unit - III

IIR filter :

-> IIR stands for infinite impulse response.
-> IIR filter is a recursive filter.
-> IIR filers are digital filters, and have feedback.
-> Block diagram of IIR filter.



-> IIR filters have much better frequency response than FIR filters.

Introduction to digital filters

-> Digital filter is a system that performs mathematical operations on a sampled discrete time signal.
-> There are two types of digital filters. they are:
i FIR - Finite impulse response
ii IIR - Infinite impulse response

-> conceptual representation of a digital filter

$$\xrightarrow[\text{I/p sequence}]{x(n)} \boxed{\begin{array}{c} h(k),\ k=0,1,\cdots \\ \text{impulse response} \end{array}} \xrightarrow[\text{o/p sequence}]{y(n)}$$

Applications

-> Data compression
-> Biomedical signal processing
-> speech and image processing
-> digital audio

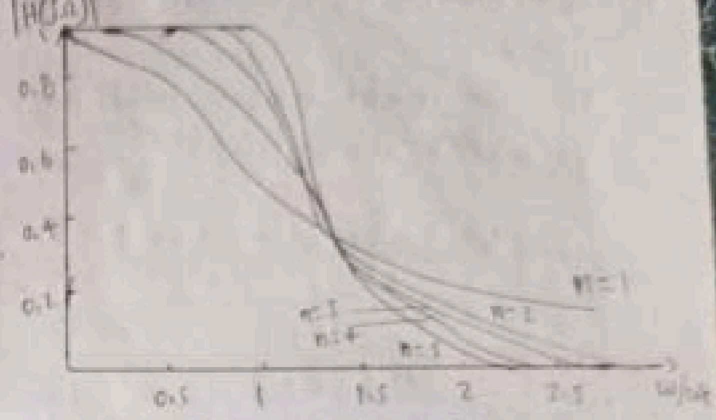
Analog filter approximations

-> usually analog filters are IIR filters.
-> IIR filter is approximated in analog and then converted into digital filters
-> There are two types of analog filter approximations are there, they are
i Butterworth LPF
ii chebyshev LPF

i Analog butterworth of LPF

-> The general transfer function of analog butterworth LPF is given by

$$|H(j\Omega)| = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}} \quad N=1,2,3\ldots$$



-> order of butter low pass filter

$$\log\left(\frac{\Omega_s}{\Omega_p}\right)^{N} = \log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}$$

$$\boxed{N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}}}$$

Steps to design an analog butterworth low-pass filter:

i) Find the order of the filter 'N'

$$N \geq \frac{\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

ii) Round off the order to the next higher integer.

iii) Find the transfer function $H(s)$ for the value of N: $H(s) = \frac{N(s)}{D(s)}$

N	Denominator polynomial of $H(s)$
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.84777s+1)$
5	$(s+1)(s^2+0.618033s+1)(s^2+1.618033s+1)$

iv) Calculate the cut-off frequency

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} \text{ (or) } \frac{\Omega_s}{(10^{0.1\alpha_s}-1)^{1/2N}}$$

v) Find the transfer function $H_a(s)$ of an analog butterworth filter Ω_c

$$\boxed{H_a(s) = H(s)\Big|_{s \to \frac{s}{\Omega_c}}}$$

problem

Design an analog butterworth filter that has -2dB passband attenuation at a frequency of 20 radian/sec and -10dB stopband attenuation at 30 rad/sec

Sol) given data

$\alpha_p = 2dB$; $\alpha_s = 10dB$

$\Omega_p = 20$ rad/sec ; $\Omega_s = 30$ rad/sec

i order of the filter

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(2)}-1}}}{\log\left(\frac{30}{20}\right)}$$

$$N \geq \frac{\log\sqrt{\frac{10-1}{10^{0.2}-1}}}{\log(3/2)} \Rightarrow \frac{\log\sqrt{\frac{9}{0.585}}}{\log(1.5)}$$

$$\geq \frac{\log(3.922)}{\log(1.5)} = \frac{0.5935}{0.1761}$$

$$\boxed{N \geq 3.37}$$

ii Round off 'N' to the next higher integer.

$$N \geq 3.37$$

$$\boxed{N = 4}$$

iii The transfer function for $N = 4$ and $\Omega_c = 1$

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.84775s+1)}$$

iv <u>Calculate cut-off frequency</u>

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} = \frac{\Omega_s}{(10^{0.1\alpha_s}-1)^{1/2N}}$$

$$= \frac{20}{(10^{0.1(20)}-1)^{1/2N}} = \cancel{\frac{20}{}}$$

$$= \frac{20}{(10^{0.2}-1)^{1/2N}} = \frac{20}{(0.585)^{1/2N}}$$

$$\Omega_c = \frac{20}{0.935} = 21.39$$

$$\boxed{\Omega_c = 21.39}$$

v Transfer function can be obtained by substituting,

$$s \to \frac{s}{\Omega_c} \text{ in } H(s)$$

$$s \to \frac{s}{21.39}$$

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.84775s+1)}$$

$$\boxed{H_a(s) = H(s)\Big|_{s \to \frac{s}{\Omega_c}} = \frac{s}{21.39}}$$

$$H_a(s) = \frac{1}{\left[\left(\frac{s^2}{21.39}\right)^2+0.76537\left(\frac{s}{21.39}\right)+1\right]\left[\left(\frac{s}{21.39}\right)^2+1.8477\left(\frac{s}{21.39}\right)+1\right]}$$

$$\boxed{H_a(s) = \frac{209336.63}{(s^2+16.3934s+457.581)(s^2+39.52165s+457.531)}}$$

(P) Find the order of the filter for the given specificance.

$\alpha_p = 1\,dB$, $\alpha_s = 30\,dB$

$\Omega_p = 200$ rd/sec $\Omega_s = 600$ rad/sec.

Sol

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} \quad (\text{or}) \quad \frac{\log(\lambda/\varepsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\sqrt{\dfrac{10^{0.1(30)}-1}{10^{0.1(1)}-1}}}{\log\left(\frac{800}{\text{[illegible]}}\right)}$$

$$N \geq \frac{\sqrt{\dfrac{10^{3}-1}{1.2589-1}}}{\log(3)}$$

$$N \geq \frac{\sqrt{\dfrac{1000-1}{0.2589}}}{\log(3)}$$

$$N \geq \frac{\sqrt{\dfrac{999}{0.2589}}}{\log(3)}$$

$$N \geq \frac{1.7952}{0.477}$$

$$\boxed{N \geq 3.759}$$

Butterworth filter

-> Butterworth filter is a type of signal processing filter.

-> It is also called as maximally flat magnitude filter.

Chebyshev Approximation

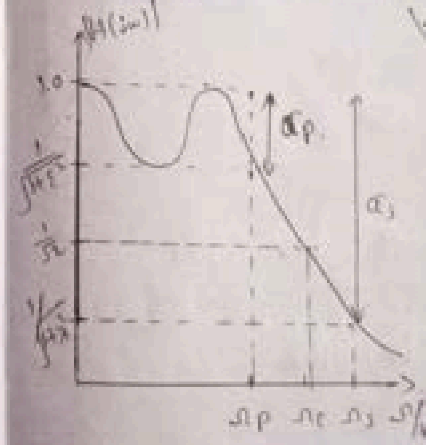
-> Chebyshev filters are used to seperate a signal from another signal.

-> The analog approximation for the IIR filter using chebyshev LPF as has two types

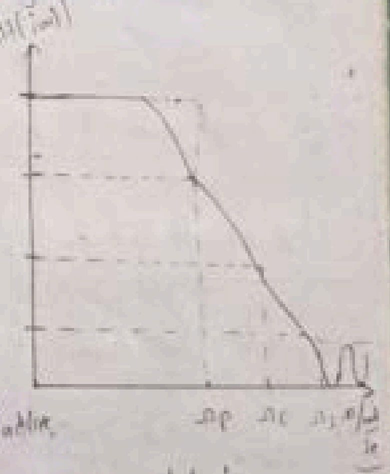
-> Type 1 chebyshev filter.

-> Type 2 chebyshev filter

-> The magnitude plot and frequency response of analog chebyshev Type-I and type-II is shown in below graphs



Typ-I chebyshev



Type 2 chebyshev

-> Type I contains poles only

-> Type II contains both poles and zeros

→ order of the chebyshev filter:

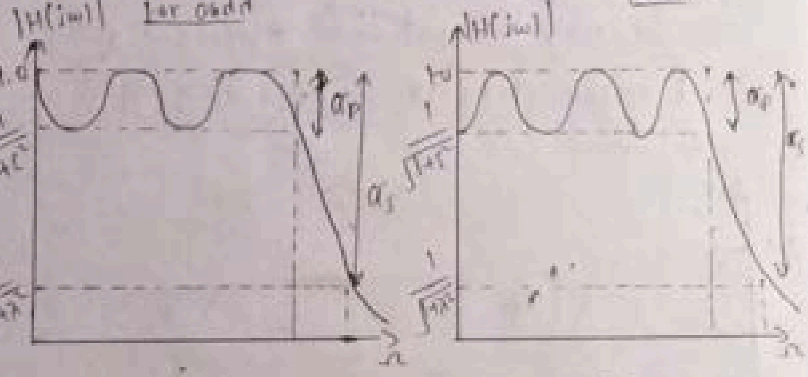
$$N \geq \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

order = odd - curve starts from unity

order = even - " " " $\frac{1}{\sqrt{1+\epsilon^2}}$

For odd

For even



steps to design analog chebyshev's filter

i Find the order of the filter.

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} \quad (or) \quad \frac{\cosh^{-1}(\lambda/\epsilon)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

ii Round-off the 'N' value to the next higher integer.

iii Find the minor axis value 'a' and major axis value 'b' of the ellipse

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where $\mu = \epsilon^{-1} + \sqrt{1+\epsilon^{-2}}$

iv Calculate the poles which lies on ellipse.

$$S_k = a\cos\phi_k + jb\sin\phi_k \qquad \boxed{\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi}$$

v Find the denominator polynomial D(s) using poles.

vi Find numerator polynomial N(s) it depends on value of N.

- For N → odd

$$N(s) = D(s)|_{s=0} \quad [\text{sub } s=0]$$

- For N → even

$$N(s) = D(s)|_{s=0} \quad [\text{divide the result by } \sqrt{1+\epsilon^2}]$$

vii obtain transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

(P) Design a chebyshev filter with a MAX passband attenuation 2.5 dB at $\Omega_p = 20$ rad/sec and stop band attenuation of 30 dB at $\Omega_s = 50$ rad/sec.

sol: given data.

$\alpha_p = 2.5$ dB , $\alpha_s = 30$ dB

$\Omega_p = 20$ rad/sec $\Omega_s = 50$ rad/sec.

* design of chebyshev Type-I LPF

i) order of the system

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]} \quad (\text{or}) \quad N \geq \frac{\cosh^{-1}(\lambda/\varepsilon)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

To find $\lambda = \sqrt{10^{0.1\alpha_s}-1}$

$= \sqrt{10^{0.1(30)}-1}$

$= \sqrt{10^3-1}$

$= \sqrt{999}$

$\boxed{\lambda = 31.60}$

To find $\varepsilon = \sqrt{10^{0.1\alpha_p}-1}$

$= \sqrt{10^{0.1(2.5)}-1}$

$= \sqrt{10^{0.25}-1}$

$= \sqrt{0.778}$

$\boxed{\varepsilon = 0.882}$

$$N \geq \frac{\cosh^{-1}\left(\frac{31.60}{0.882}\right)}{\cosh^{-1}\left(\frac{50}{20}\right)}$$

1.5535

0.5335

$$N \geq \frac{\cosh^{-1}(35.01)}{\cosh^{-1}(2.5)} \Rightarrow \boxed{N \geq 2.73}$$

ii) Round off the value to the next higher integer $\boxed{N = 43}$

iii) Find the values of a and b

$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$

$= (0.882)^{-1} + \sqrt{1+(0.882)^{-2}}$

$= 1.134 + \sqrt{1+1.286}$

$= 1.134 + 1.512$

$\boxed{\mu = 2.65}$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$= 20\left[\frac{(2.65)^{1/N} - (2.65)^{-1/N}}{2}\right]$

$= 20\left[\frac{(2.65)^{1/3} - (2.65)^{-1/3}}{2}\right]$

$= \overset{10}{\cancel{20}}\left[\frac{1.384 - 0.723}{2}\right]$

$\boxed{a = 6.61}$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$= 20\left[\frac{(2.65)^{1/3} + (2.65)^{-1/3}}{2}\right]$

$= \overset{10}{\cancel{20}}\left[\frac{1.384 + 0.723}{2}\right]$

$\boxed{b = \cancel{14.459}\ 21.07}$

iv) calculate the poles

$\boxed{S_k = a\sin\phi_k + j\,b\sin\phi_k}$ $k = 1, 2, \dots N$

2.66

$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \; ; \; k = 1,2,3.$$

$\boxed{k=1,\ N=3}$

$$\phi_1 = \frac{\pi}{2} + \left(\frac{2-1}{6}\right)\pi \Rightarrow \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6}$$

$$\phi_1 = \frac{2\pi}{3}$$

$\boxed{k=2,\ N=3}$

$$\phi_2 = \frac{\pi}{2} + \left(\frac{4-1}{6}\right)\pi \Rightarrow \frac{\pi}{2} + \frac{3\pi}{6} = \frac{6\pi}{6}$$

$$\phi_2 = \pi$$

$k=3,\ N=3$

$$\phi_3 = \frac{\pi}{2} + \left(\frac{6-1}{6}\right)\pi = \frac{\pi}{2} + \frac{5\pi}{6} = \frac{8\pi}{6}$$

$$\phi_3 = \frac{4\pi}{3}$$

$k=1$, $s_k = a\cos\phi_k + j b\sin\phi_k$

sub ϕ_k, a, values in above eq

$$\boxed{s_1 = -3.3 + j18.24}$$

similarly

$$\boxed{s_2 = -6.61}$$

$$\boxed{s_3 = -3.3 - j18.24}$$

V. Denominator poles of H(s)

$s_1 = -3.3 + j18.24$

$s_2 = -6.61$

$s_3 = -3.3 - j18.24$

$$D(s) = (s+6.61)\left\{(s+3.3)^2 - (j18.24)^2\right\}$$

$$= (s+6.61)\left\{s^2 + 10.89 + 6.6s - j^2 332.33\right\}$$

$$= (s+6.61)\left\{s^2 + 6.6s + 10.89 + 332.33\right\}$$

$$\boxed{D(s) = (s+6.61)(s^2 + 6.6s + 343.22)}$$

vi Numerator polynomial N(s) of H(s)

For N = 3 [odd]

$$N(s) = D(s)\big|_{s=0}$$

$$N(s) = (0+6.61)(0+6.61+343.22)$$

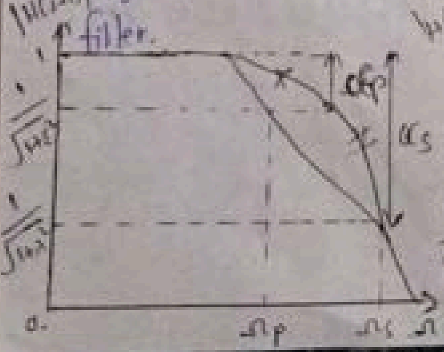
$$N(s) = (6.61)(343.22)$$

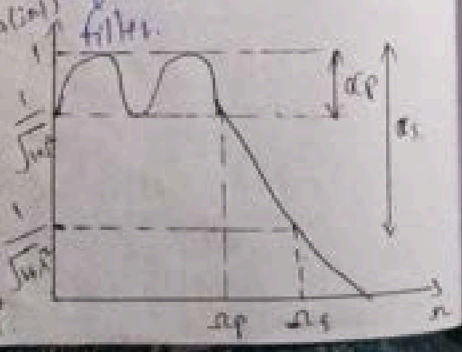
$$\boxed{N(s) = 2268.68}$$

vii obtain H(s)

$$H(s) = \frac{N(s)}{D(s)} \Rightarrow \frac{2268.68}{(s+6.61)(s^2+6.6s+343.22)}$$

Butterworth	Chebyshev
-> Butterworth filter is a type of signal processing filter.	-> Chebyshev filter is used for seperate one signal from another.
-> The magnitude response $\lvert H(j\Omega)\rvert$ decreases monotonically as the frequency 'Ω' increases from $0-\infty$.	-> The magnitude response $\lvert H(j\Omega)\rvert$ exhibits ripples in the passband or stopband
-> The Normalized poles lie on a circle.	-> The Normalized poles lie on an ellipse.
-> The order of the filter $N \geq \dfrac{\log(\lambda/\varepsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$	-> The order of the filter $N \geq \dfrac{\cosh^{-1}(\lambda/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)}$
-> wide transition b/w PBand SB	-> sharper transition b/w PBand SB
-> More transition band.	-> less transition band.
-> More no. of poles.	-> less no. of poles.
-> High order filter.	-> low order filter.
-> More no. of components required to construct filter.	-> less no. of components required to construct filter.





Design of IIR filters from analog filters

Indirect method

-> To design an analog prototype filter for the given specification.

-> To transform IIR filter from analog filter

-> Basic steps for analog to digital filter conversion.

i. Analog <-> digital

Ω_p ω_p -> passband frequency

Ω_s ω_s -> stopband ''

Ω_c ω_c -> cut-off '' (rad/sec)

ii. Derive the analog transfer function.

iii. Transform the analog transfer function into a equivalent digital filter transfer function

$$H(s) \rightarrow H(z)$$

$$\lvert H(j\Omega)\rvert - \lvert H(e^{j\omega})\rvert$$

-> Methods used for designing of IIR (digital) filter from analog filter.

- Impulse Invariant Transformation
- Bilinear Transformation.

Impulse Invariant Transformation :-

-> In this method, the impulse response of the digital filter can be obtained by sampling the impulse response of analog filter.

$$h(n) = h_a(nT) ; \left[h_a(t)\big|_{t=nT}\right]$$

where

$h(n)$ = Impulse response of digital filter.

$h_a(n)$ = Sampling impulse response of analog filter.

Design steps

i Find the transfer function of an analog filter $H_a(s)$ for given specifications using analog filter design techniques

- Butterworth filter design
- chebyshev filter design

ii select sampling rate of the digital filter

-> T sec's per sample.

iii Express the analog filter transfer function

$$\boxed{H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k}}$$

where C_k = Coefficients in partial fraction

P_k = poles of analog filter

N = order of the filter.

iv Find Z-transform of digital filter

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$$

For sampling rate

$$H(z) = \sum_{k=1}^{N} \frac{T C_k}{1-e^{P_k T} z^{-1}} \quad \text{for } T \gg 1$$

(P). Determine $H(z)$ for the analog transfer function

$$H(s) = \frac{2}{(s+1)(s+2)} \text{ using}$$

Impulse invariant method. Assume $T = 1$ sec.

Sol i) The transfer function of an analog filter

$$H(s) = \frac{2}{(s+1)(s+2)}$$

ii) The sampling period is $T = 1$ sec

iii Express $H(s)$ as the sum of single-pole filter

$$H(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k}$$

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{2}{(s+1)(s+2)} \times (s+1)\Big|_{s=-1}$$

$$A = \frac{2}{-1+2} = 2; \quad \boxed{A=2}$$

$B: \frac{2}{(s+1)\cancel{(s+2)}} \times \cancel{(s+2)} \Big|_{s=-2}$

$B = \frac{2}{-1} = -2, \quad \boxed{B=-2}$

$H(s) = \frac{2}{s-\underset{(P_1)}{(-1)}} - \frac{2}{s-\underset{(P_2)}{(-2)}}$

iv compute z-transform of the digital filter

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{P_k T} z^{-1}} = \frac{c_1}{1-e^{P_1 T} z^{-1}} + \frac{c_2}{1-e^{P_2 T} z^{-1}}$$

$P_1 = -1;\ P_2 = -2;\ T = 1$

$c_1 = 2;\ c_2 = -2$

$$H(z) = \frac{2}{1-e^{(-1)1} z^{-1}} - \frac{2}{1-e^{-2(1)} z^{-1}}$$

$$H(z) = \frac{2}{1-e^{-1} z^{-1}} - \frac{2}{1-e^{-2} z^{-1}}$$

$$H(z) = \frac{2}{1-0.3678z^{-1}} - \frac{2}{1-0.1353z^{-1}}$$

$$H(z) = \frac{2(1-0.1353z^{-1}) - 2[1-0.3678z^{-1}]}{(1-0.3678z^{-1})(1-0.1353z^{-1})}$$

$$= \frac{\cancel{2}-0.2706z^{-1}-\cancel{2}+0.7356z^{-1}}{1-0.1353z^{-1}-0.3678z^{-1}+0.0498z^{-2}}$$

$$\boxed{H(z) = \frac{0.465z^{-1}}{1-0.5031z^{-1}+0.0498z^{-2}}}$$

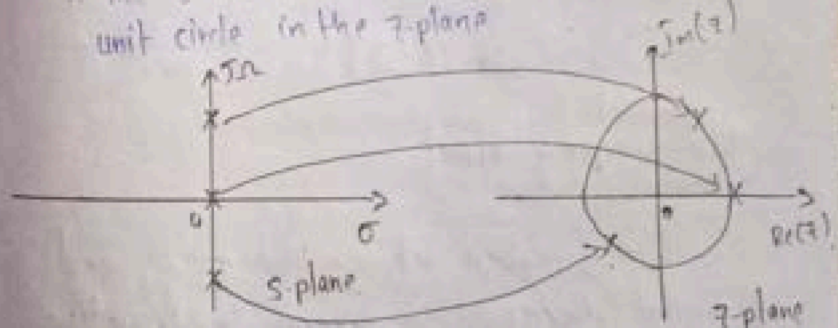
H(z) = Transfer function of the IIR digital filter using impulse invariant method.

ii Bilinear Transformation method

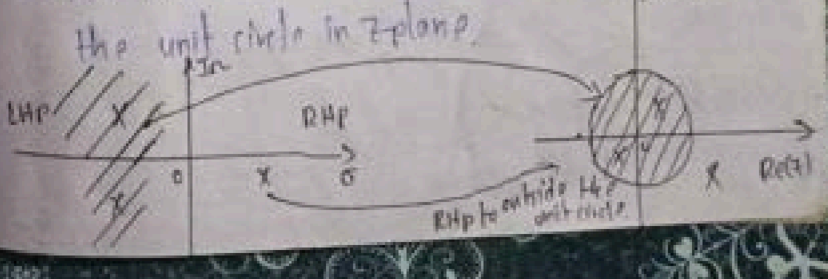
-> Bilinear transformation method is a most popular mapping method used to design IIR filter from an analog filter is called Bilinear Transformation.

properties

-> The $j\Omega$ axis on the s-plane maps onto the unit circle in the z-plane



-> The left half of s-plane maps ~~onto~~ inside the unit circle in z-plane.



Bilinear Transformation (proof)

→ In Bilinear Transformation the relation b/w s and ϕ is given by

Assume $H(s) = \frac{b}{s+a}$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$sY(s) + aY(s) = bX(s)$$

→ Apply Inverse laplace transform

$$\frac{dy(t)}{dt} + ay(t) = bx(t) \quad (1)$$

→ solution of differential equation

$$y(t) = \int_{\tau=t_0}^{t} y'(\tau)\, d\tau + y(t_0)$$

→ By using trapezoidal method we can write

$$\int_{t_0}^{t} y'(\tau)\, d\tau = \frac{T}{2}\left[y'(t_0) + y'(t)\right]$$

let $t_0 = (n-1)T$, $t = nT$

$$y(nT) = \frac{T}{2}\left[y'(n-1)T + y'(nT)\right] + y(n-1)T$$

$$y(n) = \frac{T}{2}\left[bx(n-1) - ay(n-1) + bx(n) - ay(n)\right] + y(n-1)$$

→ Apply Z-transform

$$y(z) = \frac{T}{2}\left[bz^{-1}X(z) - az^{-1}y(z) + bX(z) - ay(z) + z^{-1}Y(z)\right]$$

$$= \frac{b\frac{T}{2}\left[\cancel{1+z^{-1}}\right]}{\left[\frac{T}{2}(\cancel{1+z^{-1}})\right]\left[\frac{2}{T}\frac{(1-z^{-1})}{1+z^{-1}}\right] + a}$$

$$= \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a}$$

$$\boxed{H(z) = H(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}}$$

Analysis:

-> let consider an analog filter with the transfer function $H(s) = \frac{b}{s+a}$

-> The transfer function of IIR digital filter can be obtained by replacing $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$.

-> $$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + a}$$

-> Relation b/w the frequencies in two domains

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

-> The warping effect

The relation between the analog and digital frequencies in bilinear transformation is given as,

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

-> For small value of ω

$$\boxed{\tan\frac{\omega}{2} = \frac{\omega}{2}}$$

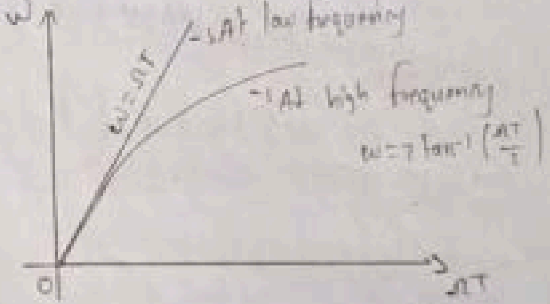
$$\Omega = \frac{2}{T} \times \frac{\omega}{2} = \frac{\omega}{T}$$

$$\boxed{\omega = T\Omega}$$

-> For lower frequencies, the relation b/w Ω and ω is linear.

-> For higher frequencies, the relation b/w Ω and ω is non linear.

-> This effect is known as warping effect



-> prewarping effect: The warping effect can be eliminated by prewarping the analog filter is known as pre..

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

-> Here we perform prescaling (or) prewarping

Advantages: -> provides one-to-one mapping
-> No aliasing effect
-> stable analog system -> stable digital system

Disadvantages:
-> produces distortion due to linearity
-> Both impulse and phase response can't preserved

Design steps:

i from given specifications, find prewarping analog frequencies

$$\boxed{\Omega = \frac{2}{T}\tan\frac{\omega}{2}}$$

ii Find Transfer function H(s) of the analog filter with Ω

iii Select the sampling period (time) of the digital filter. T:

iv Find transfer function of the digital filter.

$$\boxed{H(z) = H(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}}$$

(P) Design a chebyshev filter for the given specification using Bilinear Transformation.

$$0.8 \le |H(e^{j\omega})| \le 1; \quad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2; \quad 0.6\pi \le \omega \le \pi$$

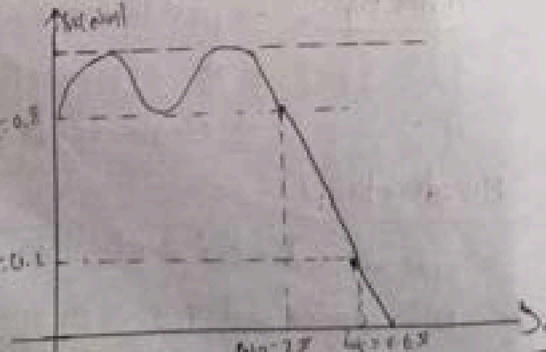
Sol: given data.

$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8$

$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$

$\omega p = 0.2\pi$

$\omega s = 0.6\pi$



$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8$

Squaring on Bs.

$\frac{1}{1+\varepsilon^2} = 0.64$

$0.64 + 0.64\varepsilon^2 = 1$

$0.64\varepsilon^2 = 1 - 0.64$

$= 0.36$

$\varepsilon^2 = \frac{0.36}{0.64}$

$\varepsilon^2 = 0.5625$

$\varepsilon = \sqrt{0.5625}$

$\boxed{\varepsilon = 0.75}$

$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$

Squaring on Bs

$\frac{1}{1+\lambda^2} = 0.04$

$0.04 + 0.04\lambda^2 = 1$

$0.04\lambda^2 = 1 - 0.04$

$= 0.96$

$\lambda^2 = \frac{0.96}{0.04}$

$\lambda^2 = 24$

$\lambda = \sqrt{24}$

$\boxed{\lambda = 4.89}$

Designing

i pre-warping analog frequencies (Ω_p, Ω_s)

$\omega p = 0.2\pi, T = 1$

$\Omega p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$

$= 2\tan\left(\frac{0.2\pi}{2}\right)$

$= 2 \times 0.0347$

$\boxed{\Omega p = 0.6498 \text{ rad/sec}}$

$\omega s = 0.6\pi, T = 1$

$\Omega s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$

$= \frac{2}{1}\tan\left(\frac{0.6\pi}{2}\right)$

$= 2 \times 1.3764$

$\boxed{\Omega s = 2.7528 \text{ rad/sec}}$

11) Transfer function of analog chebyshev filter

→ order of the filter

$$N \geq \frac{\cosh^{-1}(1/\varepsilon)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\cosh^{-1}\left(\frac{4.899}{0.75}\right)}{\cosh^{-1}\left(\frac{2.7528}{0.6495}\right)}$$

$$N \geq \frac{\cosh^{-1}(6.532)}{\cosh^{-1}(4.238)}$$

$$N \geq \frac{2.5639}{2.1230}$$

$N \geq 1.2076$

$\boxed{N = 2}$

→ Major and minor axis values of 'a' and b of an ellipse.

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$$

$$= (0.75)^{-1} + \sqrt{1+(0.75)^{-2}}$$

$$= 1.333 + 1.667$$

$\boxed{\mu = 3}$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$= 0.6498\left[\frac{3^{1/2} - 3^{-1/2}}{2}\right]$$

$$= 0.6498\left[\frac{1.732 - 0.577}{2}\right]$$

$\boxed{a = 0.3752}$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$= 0.6498\left(\frac{3^{1/2} + 3^{-1/2}}{2}\right)$$

$$= 0.6498\left(\frac{1.732 + 0.577}{2}\right)$$

$\boxed{b = 0.75}$

→ poles of chebyshev filter

$\boxed{s_k = a\cos\phi_k + j b\sin\phi_k}$

where $\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$; $k = 1, 2, \ldots N$

$k = 1, 2; N = 2$

$$\phi_1 = \frac{\pi}{2} + \left(\frac{2-1}{4}\right)\pi \Rightarrow \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} = \phi_1$$

$$\phi_2 = \frac{\pi}{2} + \frac{(4-1)\pi}{4} = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4} = \phi_2$$

$s_k = 0.3752\cos\phi_k + j\,0.75\sin\phi_k$; $k = 1, 2$

$s_1 = 0.3752\cos\phi_1 + j\,0.75\sin\phi_1$

$\boxed{s_1 = -0.2653 + j\,0.53}$

$s_2 = 0.3752\cos\phi_2 + j\,0.75\sin\phi_2$

$\boxed{s_2 = -0.2653 - j0.53}$

→ Denominator polynomial $D(s)$ of $H(s)$

$D(s) = (s+0.2653)^2 + (0.53)^2$

$\boxed{D(s) = s^2 + 0.5306s + 0.3513}$

→ Numerator polynomial $H(s)$:

$N(s) = \frac{0.3513}{\sqrt{1+(0.75)^2}} = \frac{0.3513}{1.25}$

$\boxed{N(s) = 0.2848}$ $\boxed{H(s) = \frac{N(s)}{D(s)} = \frac{0.2848}{s^2+0.5306s+0.3513}}$

iii Transfer function of digital chebyshev filter is

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{0.2848}{s^2+0.5306s+0.3513}$$

$$= \frac{0.2848}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\}^2 + 0.5306\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\} + 0.3513}$$

$$\boxed{H(z) = \frac{0.0522(1+z^{-1})^2}{1-1.3482z^{-1}+0.6079z^{-2}}}$$

Basic Structures of IIR

→ IIR systems are represented in four different ways.

1. Direct form - I and Direct form - II
2. cascade form structure
3. parallel form structure

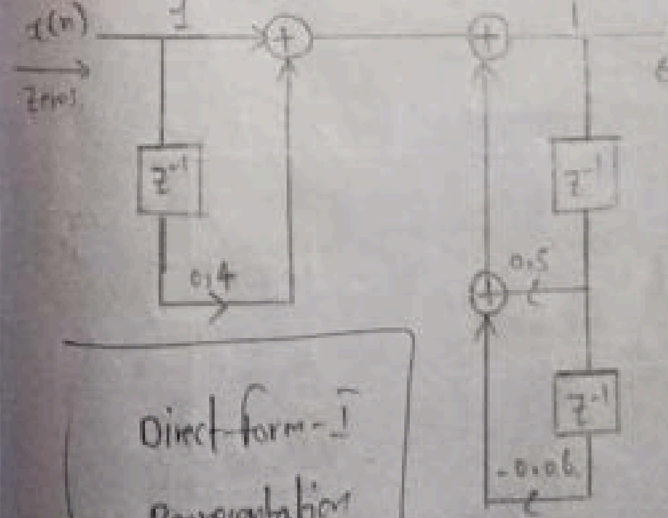
1. Direct forms Structures

ⓐ Direct form - I

ⓑ Direct form - II

① $H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-1}}$ zeros / poles

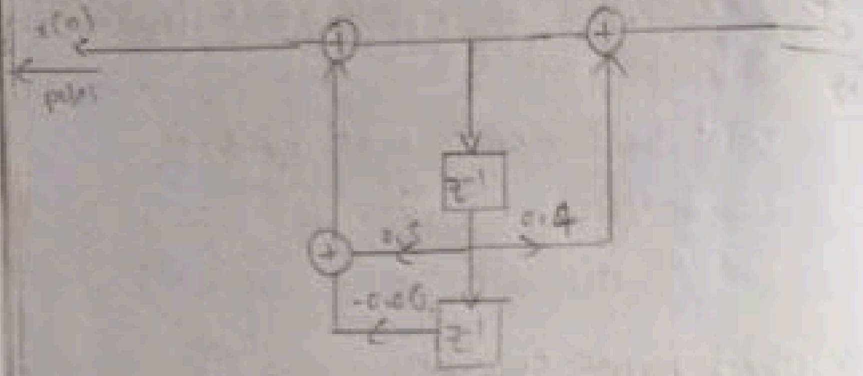
Sol. In a given transfer numerator is zeros and Denominator is poles.



Direct form - I Representation

In y(n) & x(n) denominator terms always (-) if comes it will be (+)

Direct form-II



Q. $H(z) = \dfrac{z^{-1}-3z^{-2}}{(10-z^{-1})(1+0.5z^{-1}+0.5z^{-2})}$

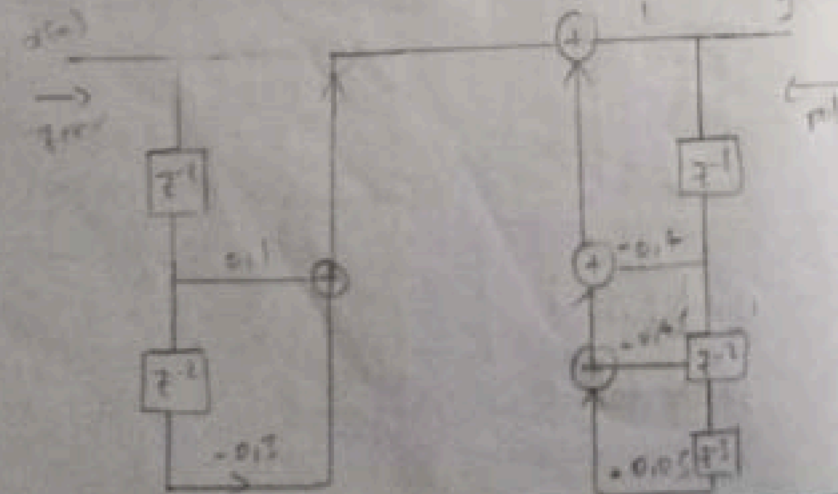
Sol:

$H(z) = \dfrac{z^{-1}-3z^{-2}}{10+4z^{-1}+4.5z^{-2}-0.5z^{-3}}$

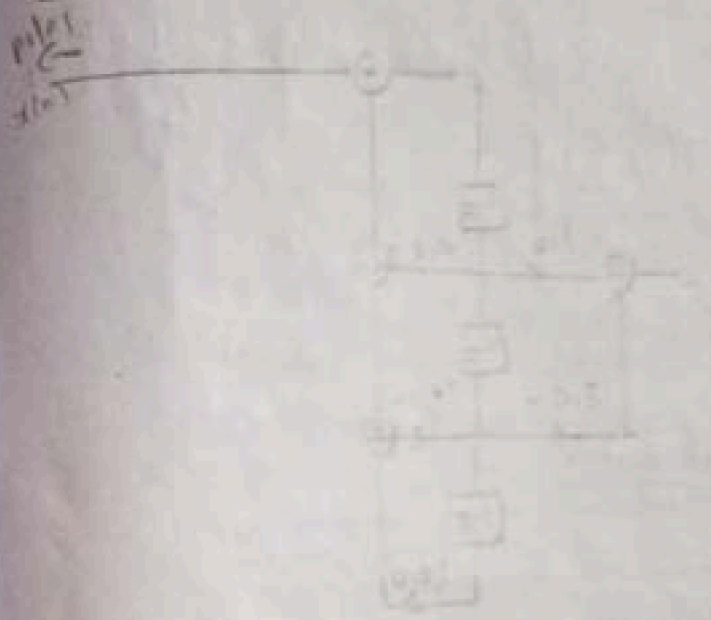
Divide with 10

$H(z) = \dfrac{0.1z^{-1}-0.3z^{-2}}{1+0.4z^{-1}+0.45z^{-2}-0.05z^{-3}}$ zeros / poles

Direct form - I



Direct form-II



Q. $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-1) = x(n) + \frac{1}{2}x(n-2)$

Sol: Take z transform on B.S.

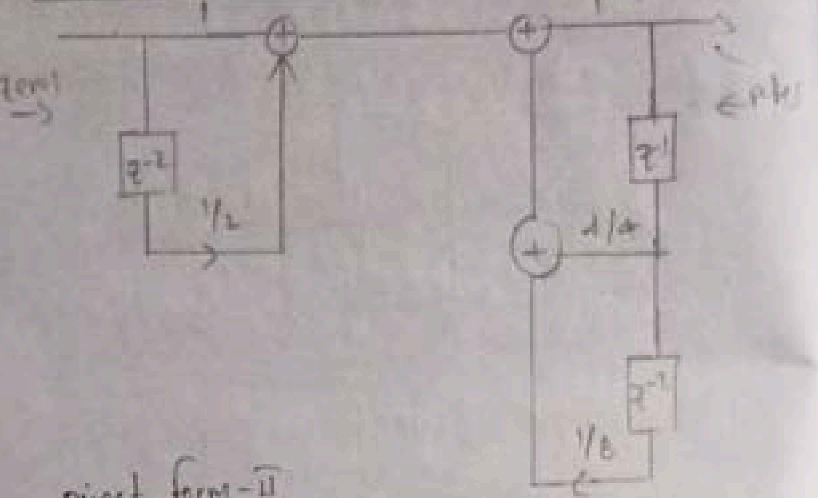
$y(z) - \frac{1}{4}z^{-1}y(z) + \frac{1}{8}z^{-2}y(z) =$

$x(z) + \frac{1}{2}z^{-2}x(z)$

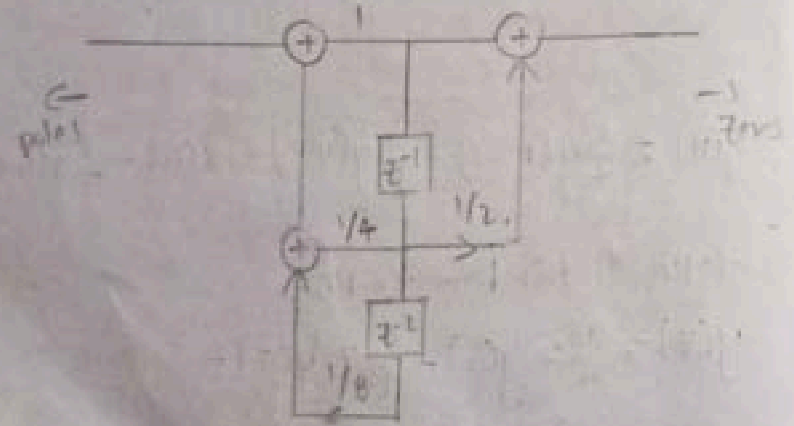
Take y(z) and x(z) out.

$$\frac{y(z)}{x(z)} = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

Direct form-I



Direct form-II



ii Cascade form

→ cascade form structure: $H(z) = \underbrace{H_1(z)}_{D.F-II} \cdot \underbrace{H_2(z)}_{D.F-II}$

D.F: direct form

(P). $H(z) = \dfrac{1 + \frac{7}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}}$

Sol: write above eq in $H(z) = H_1(z) \cdot H_2(z)$, for this we need to find roots for both numerator and denominator.

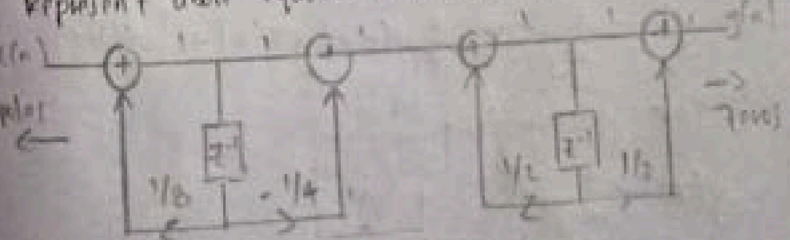
$$H(z) = \frac{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 - \frac{5}{8}z^{-1} + \frac{1}{16}z^{-2}} \begin{array}{l} \to -\frac{1}{4}, -\frac{1}{2} \\ \to \frac{1}{8}, \frac{1}{2} \end{array}$$

$$H(z) = \frac{\left(1 + \frac{1}{4}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{8}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$H(z) = \frac{\left(1 + \frac{1}{4}z^{-1}\right)}{\left(1 - \frac{1}{8}z^{-1}\right)} \cdot \frac{\left(1 + \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$\quad (H_1(z)) \quad (H_2(z))$$

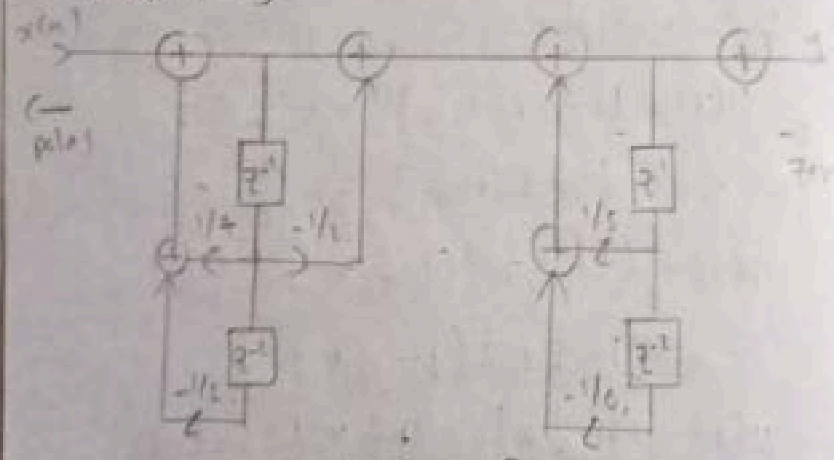
Represent above equation in direct form II



Ⓟ. $H(z) = \dfrac{\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)\left(1-\frac{1}{5}z^{-1}+\frac{1}{6}z^{-2}\right)}$

Sol $\underbrace{\dfrac{\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}}_{H_1(z)} \cdot \underbrace{\dfrac{1}{\left(1-\frac{1}{5}z^{-1}+\frac{1}{6}z^{-2}\right)}}_{H_2(z)}$

→ Representing in direct form - II



Ⓟ obtain cascade form from

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

Sol: $y(n) - \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

Taking z-transform

$y(z) = \cancel{\frac{3}{4}y(z)z^{-1}}$

$\frac{3}{4}z^{-1}y(z) - \frac{1}{8}z^{-2}y(z) + x(z) + \frac{1}{3}z^{-1}x(z)$

$y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = x(z)\left[1+\frac{1}{3}z^{-1}\right]$

$\dfrac{y(z)}{x(z)} = H(z) = \dfrac{1+\frac{1}{3}z^{-1}}{\underbrace{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}}$

find roots
calc with
calculator

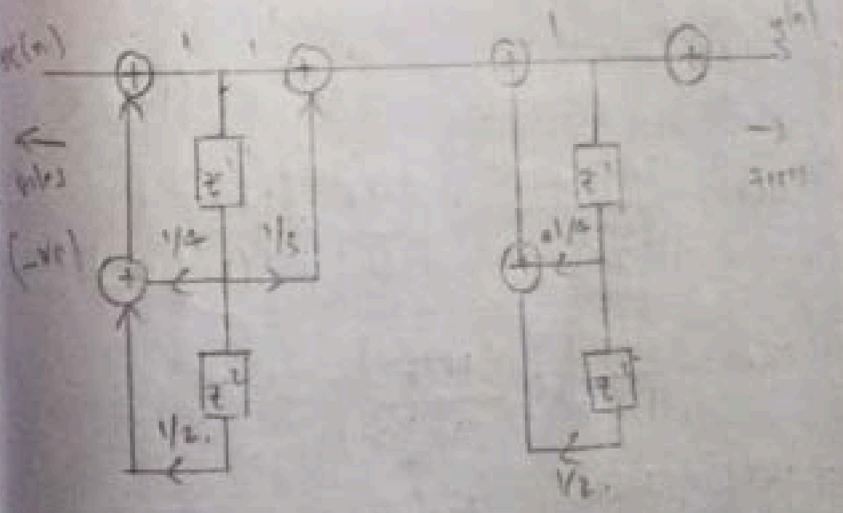
$\frac{1}{4}, \frac{1}{2}$

→ we get roots: $\frac{1}{4}, \frac{1}{2}$

$H(z) = \dfrac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1} \pm \frac{1}{2}z^{-2}}$

$H(z) = H_1(z)\, H_2(z)$

$H(z) = \dfrac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}-\frac{1}{2}z^{-2}} \cdot \dfrac{1}{1-\frac{1}{4}z^{-1}-\frac{1}{2}z^{-2}}$



(P) obtain cascade form

$$y(n) = \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

sol. Taking z-transform.

$$Y(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) = X(z) + 3z^{-1}X(z) + 2z^{-2}X(z)$$

$$Y(z)\left\{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right\} = X(z)\left\{1 + 3z^{-1} + 2z^{-2}\right\}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+2z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

$-2, -1$

$-\frac{1}{4}, \frac{1}{2}$

Find roots for both Numerator and denominator

$1+3z^{-1}+2z^{-2}$ $\qquad$ $1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}$

roots: 1, 2. $\qquad$ roots: $\frac{1}{2}, -\frac{1}{4}$

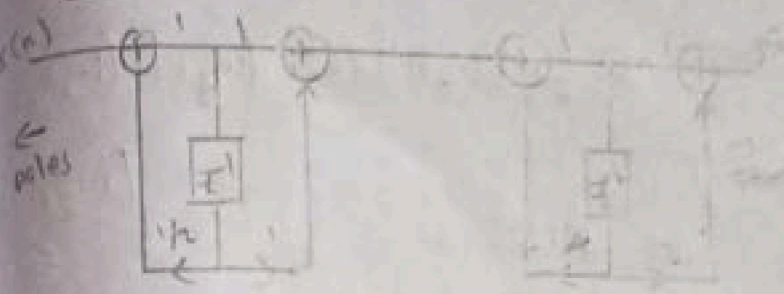
$(1+z^{-1})(1+2z^{-1})$ $\qquad$ $\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)$

$$H(z) = \frac{1+3z^{-1}+2z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} \cdot \frac{(1+z^{-1})(1+2z^{-1})}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z) = \frac{(1+z^{-1})}{\left(1-\frac{1}{2}z^{-1}\right)} \cdot \frac{(1+2z^{-1})}{\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z) = H_1(z) \quad H_2(z)$$

Direct form-II Representation



iii) parallel form Realization of IIR.

(P). obtain parallel form realization for

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)} \quad -(1)$$

$$H(z) = H_1(z) + H_2(z) + H_3(z) + \dots + H_m(z)$$

equation (1) can be simplified by using partial fraction

$$H(z) = \frac{A}{\left(1+\frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{4}z^{-1}\right)} + \frac{C}{\left(1+\frac{1}{8}z^{-1}\right)} \quad -(2)$$

$$H(z) = \frac{A\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right) + B\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right) + C\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)}$$

$$(1+z^{-1})(1+2z^{-1}) = A\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)+B\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)+C\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right) \quad (2)$$

let $z^{-1}=4$ sub in above eq.

$(1+4)(1+2(4)) =$

$A\cdot 0+B\left(1+\frac{1}{2}4\right)\left(1+\frac{1}{8}\cdot 4\right)+0$

$B(1+2)\left(1+\frac{1}{2}\right)$

$B(3)(1.5)$

$B=\frac{45}{4.5}=10$

$\boxed{B=10}$

let $z^{-1}=-8$

$(1+(-8))(1+2(-8)) = 0+0+C\left[1+\frac{1}{2}(-8)\right]\left[1-\frac{1}{4}(-8)\right]$

~~$A\left(1-\frac{1}{4}(-8)\right)\left(1+\frac{1}{8}(-8)\right)+B($~~

$= C[1-4][1+2]$

$(-7)(-15) = C[-9]$

$105 = C[-9]$

$C = \frac{105}{-9/3} = -\frac{35}{3}$

$\boxed{C=\frac{-35}{3}}$

let $z^{-1}=-2$ for A

$(1-2)(1+2(-2)) = A\left(1-\frac{1}{4}(-2)\right)\left(1+\frac{1}{8}(-2)\right)+0+0$

$(-1)(1-4) = A\left(1+\frac{1}{2}\right)\left(1-\frac{1}{4}\right)$

$3 = A(1.5)(0.75)$

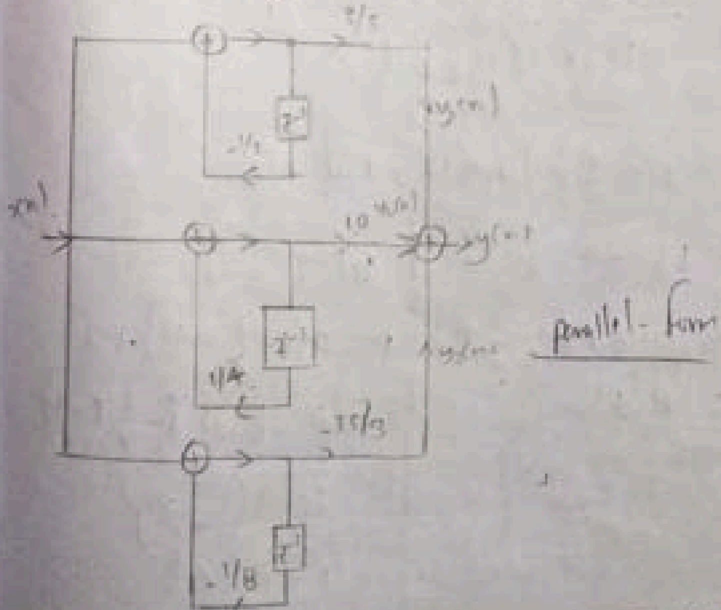
$3 = A(9/8)$

$A=\frac{3\times 8}{9}=\frac{8}{3}$

$\boxed{A=\frac{8}{3}}$

sub A, B, C in eq (2)

$$H(z)=\frac{\frac{8}{3}}{1+\frac{1}{2}z^{-1}}+\frac{10}{1-\frac{1}{4}z^{-1}}+\frac{-35/3}{1+\frac{1}{8}z^{-1}}$$



parallel form

(P). Realize the parallel form of

$$H(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

Sol

$$H(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

→ partial fraction method:

To find A and B

$$H(z) = \frac{A}{(1+\frac{1}{2}z^{-1})} + \frac{B}{(1-\frac{1}{4}z^{-1})} \quad -①$$

$$H(z) = \frac{A(1-\frac{1}{4}z^{-1}) + B(1+\frac{1}{2}z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$1 = A(1-\tfrac{1}{4}z^{-1}) + B(1+\tfrac{1}{2}z^{-1}) \quad -②$$

let $z^{-1} = 4$

$1 = 0 + B(1+\frac{1}{2}4)$

$1 = B(3)$

$\boxed{B = \frac{1}{3}}$

let $z^{-1} = -2$

$1 = A(1-\frac{1}{4}(-2))$

$1 = A(\frac{3}{2})$

$1 = A(3/2)$

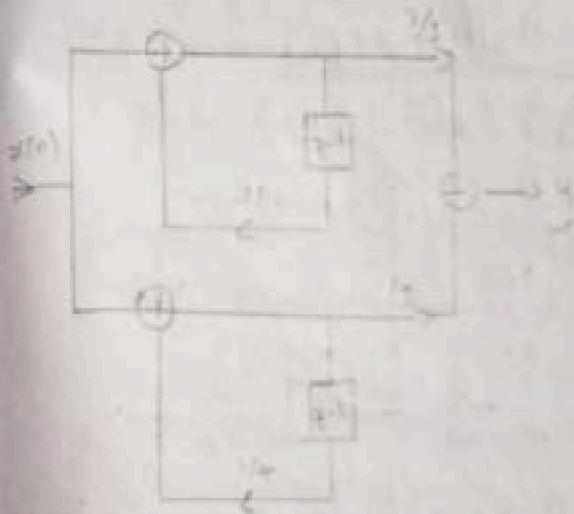
$\boxed{A = 2/3}$

Sub A and B values in eq ①

$$H(z) = \frac{\frac{2}{3}}{1+\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1-\frac{1}{4}z^{-1}}$$

$H(z) = H_1(z) + H_2(z)$

→ Parallel form representation



Frequency Transformation

→ The frequency transformation method is used to design a high pass filter (HPF), Bandpass filter (BPF), Band reject filter (BRF) from a low pass filter is called frequency transformation method.

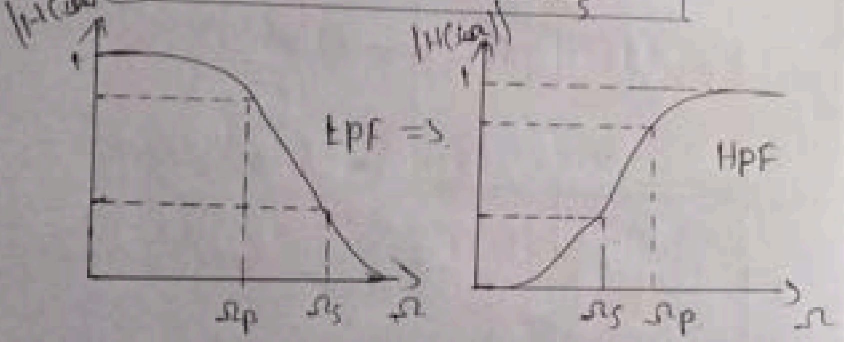
$H_{LPF}(s)$ —
- $H_{HPF}(s)$
- $H_{BPF}(s)$
- $H_{BRF}(s)$

* LPF to HPF

The high pass filter can be ~~replaced~~ by designed by replacing $s \to \frac{\Omega_c}{s}$ in the transfer function of LPF.

Ω_c = cutt-off frequency

$$\boxed{H_{HP}(s) = H_{LP}(s)\Big|_{s \to \frac{\Omega_c}{s}}}$$



* LPF to BPF

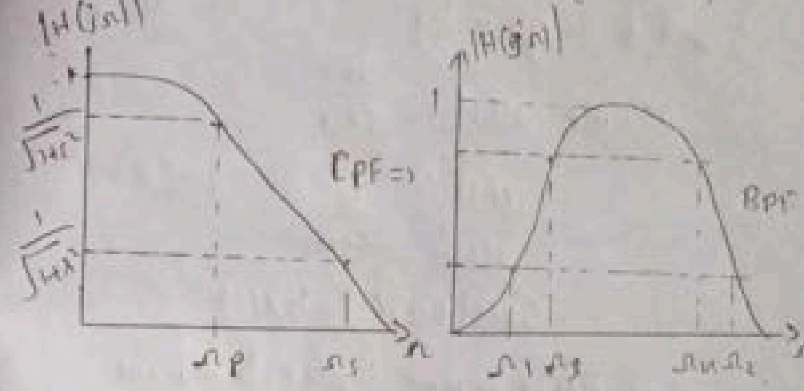
: The Band pass filter can be designed (or) obtained by replacing $s \to \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}$ in the transfer function of LPF.

$$\boxed{H_{BP}(s) = H_{LP}(s)\Big|_{s \to \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}}}$$

$$\Omega_r = \frac{\Omega_s}{\Omega_p} = \min\{|A|, |B|\}$$

where,

$$A = \frac{-\Omega_1^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)}, \quad B = \frac{\Omega_2^2-\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)}$$



LPF → BRF

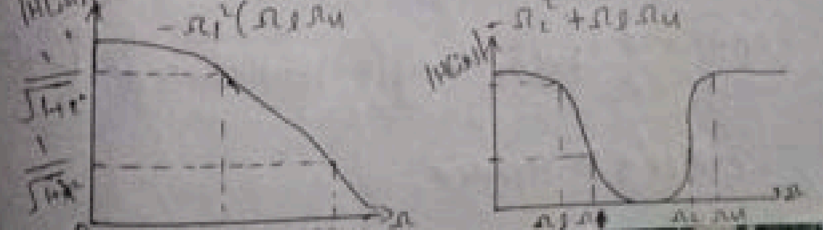
The Band Reject filter can be designed by replacing $s \to \frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_l\Omega_u}$ in the transfer function of LPF.

$$\boxed{H_{BR}(s) = H_{LP}(s)\Big|_{s \to \frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_l\Omega_u}}}$$

$$\Omega_r = \frac{\Omega_p}{\Omega_s} = \min\{|A|, |B|\}$$

where,

$$A = \frac{\Omega_1(\Omega_u-\Omega_l)}{-\Omega_1^2+\Omega_l\Omega_u}, \quad B = \frac{\Omega_2(\Omega_u-\Omega_l)}{-\Omega_2^2+\Omega_l\Omega_u}$$



unit-5

Quantization errors in Digital signal processing :

Number Representation :

-> In DSP, a number 'N' can be represented to any desired format using number system.

-> Types of Number Representation

i Fixed-point Representation

ii Floating-point Representation

① Fixed point Representation :

-> The position of the binary point is fixed is called fixed-point representation.

Ex In Binary , In Decimal

$$\underbrace{11}_{\text{Integer part}}\overset{\uparrow}{\underset{\text{point}}{.}}\underbrace{01011}_{\text{Fractional part}} \qquad \underbrace{3}_{\text{Integer}}\underset{\text{point}}{.}\underbrace{34375}_{\text{fractional}}$$

→ Representation of Negative number in fixed-point Representation :-

* sign - magnitude form
* one's compliment form
* Two's compliment form.

* Sign- magnitude form :-

→ The most - significant bit (MSB) is set to.

'1' → To represent the -ve sign

'0' → To represent the +ve sign.

Ex: $(-1.25)_{10} \Rightarrow (11.01)_2$

$(+1.25) \Rightarrow 01.01$

* one's compliment form :-

→ In this negative number is obtained by complementing all the bits.

Ex: $(0.875)_{10} \Rightarrow (0.111000)_2$

$(-0.875)_{10} \Rightarrow (1.000111)_2$

* Two's compliment form

→ The negative number is obtained by complementing all the bits and adding one to the LSB.

Ex $(0.875)_{10} \Rightarrow (0.111000)_2$

$$\begin{array}{r} 1.000111 \\ 1+ \\ \hline (-0.875)_{10} \Rightarrow 1.001000 \\ \hline \end{array}$$

(2) Floating - point Representation :-

→ A positive number is represented as

$F = 2^e . M$.

where, M → matissa $\frac{1}{2} \leq m < 1$

e → Exponent (+ or -)

Ex:

$2.25 = 2^2 \times 0.25$

$F = 2^{010} \times 0.01$

$0.125 = 2^0 \times 0.001$

$= 2^{000} \times 0.001$

→ -ve floating point is represented by mantissa as fixed - point number.

Fixed point Representation	Floating-point Representation
-> Fast and inexpensive implementation	-> slow and expensive implementation
-> Limited dynamic range.	-> Large dynamic range.
-> low power consumption	-> High power consumption.
-> less flexible.	-> More flexible.
-> overflow occurs in addition process.	-> overflow does not occurs.
-> Round-off errors occur for only for addition.	-> Round-off errors occur for both addition and multiplication.

Interpolation (or) Upsampling

Def Increasing the sampling rate by of discrete time systems is called interpolation.

-> Interpolation is also called as upsampling.

-> The sampling rate of DTS can be increased by a factor I by placing I-1.

-> Mathematically upsampling can be represented as

$$y(n) = \begin{cases} x\left(\frac{n}{I}\right) & ; n = 0, \pm I, \pm 2I \ldots \\ 0 & , \text{otherwise.} \end{cases}$$

-> Symbol for up sampler is

$$x(n) \longrightarrow \boxed{\uparrow I} \longrightarrow y(n)$$

$x(n) = \{1, 2, 3, 1, 2, 3\}$

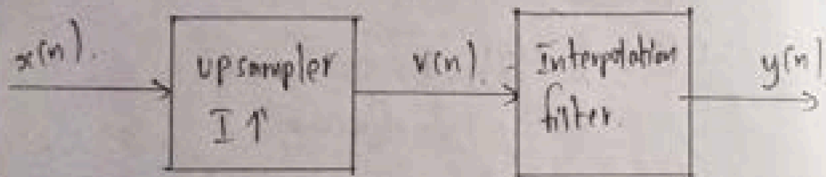
let $I = 2$

$y(n) = x\left(\frac{n}{2}\right) = \{1, 0, 2, 0, 3, 0, 1, 0, 2, 0, 3 \ldots\}$

let $I = 3$

$y(n) = x\left(\frac{n}{3}\right) = \{1, 0, 0, 2, 0, 0, 3, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0\}$

- Usually Anti-imaging filter can be kept after the upsamper to remove unwanted image developed due to upsampling
- The anti-imaging filter and upsampler together is called interpolator.
- Upsampler is used to increase the sampling rate.
- Anti-imaging filter is used to remove unwanted images.



→ Spectrum of upsampled signal

- The z-transform of the signal y(n) is given by

$$y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x\left(\frac{n}{I}\right) z^{-n}$$

$\boxed{\text{let } \frac{n}{I} = v,\ n = \frac{v}{I}}$

$$= \sum_{v=-\infty}^{\infty} x(v) z^{-vI}$$

replace v by n

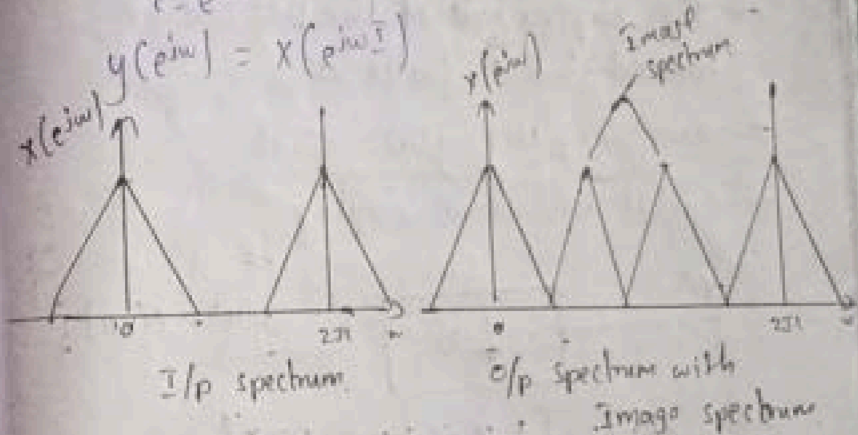
$$y(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-nI}$$

$$= \sum_{n=-\infty}^{\infty} x(n) (z^{I})^{-n}$$

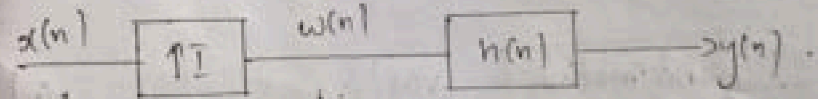
$$= X(z^{I})$$

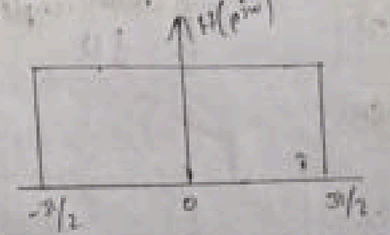
$z = e^{jw}$

$$y(e^{jw}) = X(e^{jwI})$$



Anti-Imaging filter





- The filter which is used for removing image spectrum is known as anti-imaging filter.

Down sampling (or) Decimation :

Def : Reducing the sampling rate of a discrete time systems is called down sampling.

- Down sampling is also called as Decimation.
- The sampling rate of the DTS can be reduced by a factor D
- Down sampling $y(n) = x(nD)$
- symbol for down sampler.

$$x(n) \longrightarrow \boxed{\downarrow D} \longrightarrow y(n) = x(nD)$$

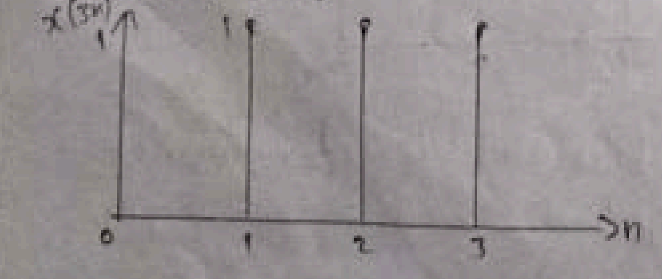
Ex : $x(n) = \{1, 2, 3, 1, 2, 3, 1, 2, 3\}$

$x(2n) = \{1, 3, 2, 1, 3, 2\}$

$x(3n) = \{1, 1, 1, 1 \ldots\}$

$$x(n) \longrightarrow \boxed{\begin{matrix}\text{Decimation} \\ \text{filter } h(n)\end{matrix}} \xrightarrow{v(n)} \boxed{\begin{matrix}\text{down sampler} \\ \downarrow D\end{matrix}} \longrightarrow y(n)$$

If $D = 3$ then,



- The decimator consist of two blocks i.e. Decimation filter, down sampler etc.

Spectrum of down sampled signal

$$\frac{T'}{T} = D \Rightarrow T' = TD$$

where T = sampling period

D = down sampling.

T' = New sampling period

- derivation of down sampling.

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{—} (1)$$

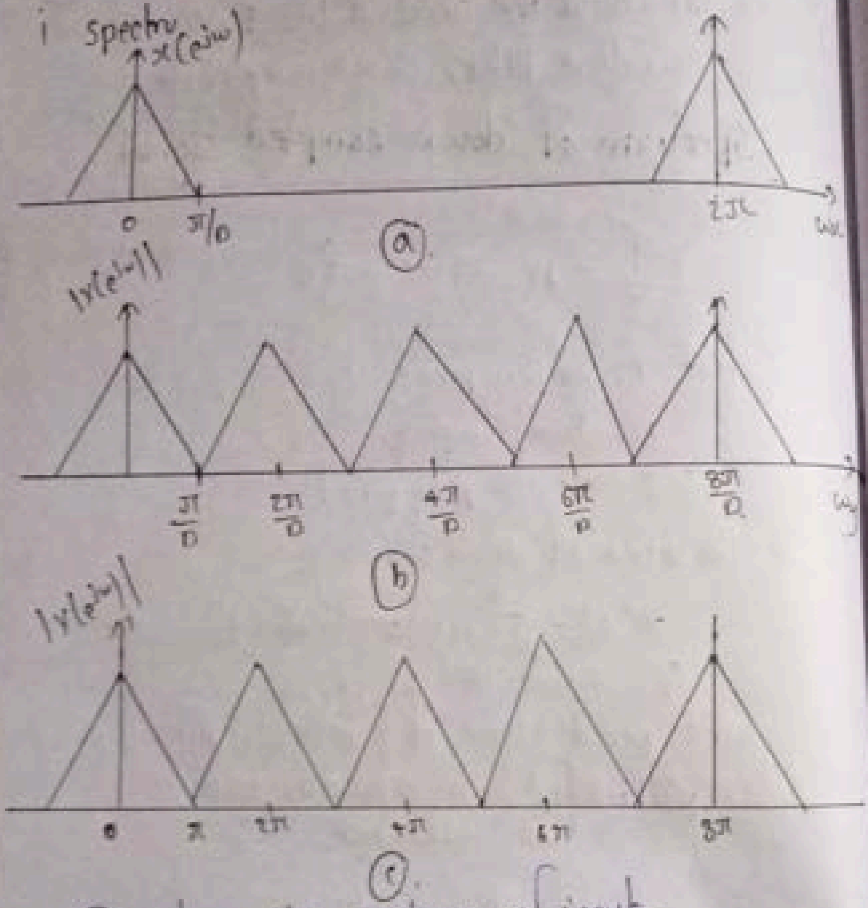
- The periodic train of impulses is given by

$$p(n) = \begin{cases} 1, & n = 0, \pm D, \pm 2D \\ 0, & \text{otherwise} \end{cases}$$

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D} \quad n = -\infty \text{ to } \infty$$

$$x'(n) = x(n)\, p(n)$$

$$Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(e^{j(\omega - 2\pi k/D)}\right)$$



(a) - shows the spectrum of input.

(b) - shows the " " output.

(c) - shows the Normalized "

Antialiasing filter

→ In antialiasing filter, input and output spectrums are overlapped.