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Федеральное государственное бюджетное образовательное учреждение
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«Московский государственный технический университет
имени Н.Э. Баумана
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(МГТУ им. Н.Э. Баумана)

ФАКУЛЬТЕТ
КАФЕДРА

«Информатика и системы управления»
«Информационная безопасность»

ЛАБОРАТОРНАЯ РАБОТА 1:

«Теория игр и исследование операций»

Аналитический и численный методы решения антагонистической игры в смешанных стратегиях

ВАРИАНТ: 4

Студент:

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Преподаватель:

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Цель работы:

Изучить аналитический (обратной матрицы) и численный (Брауна-Робинсон) методы нахождения смешанных стратегий в антагонистической игре двух лиц в нормальной форме.

Постановка задачи:

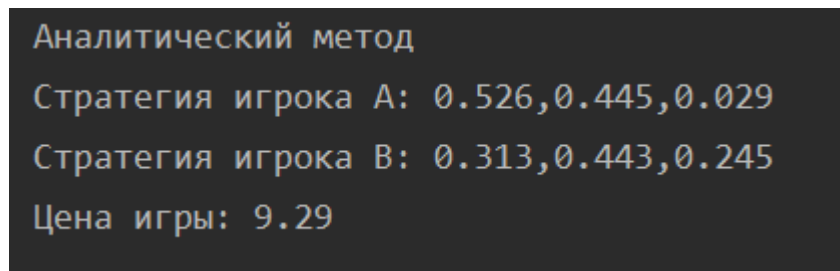
Задана матрица игры:

$$\begin{pmatrix} 17 & 4 & 9 \\ 0 & 16 & 9 \\ 12 & 2 & 19 \end{pmatrix}$$

Найдите цену игры и оптимальные стратегии обоих игроков методами обратной матрицы и Брауна-Робинсон. Сравните полученные результаты.

Ход работы:

Решение задачи аналитическим методом приведено на рисунке 1.



Аналитический метод
Стратегия игрока А: 0.526, 0.445, 0.029
Стратегия игрока В: 0.313, 0.443, 0.245
Цена игры: 9.29

Рисунок 1 – решение задачи аналитическим методом

Данное решение было посчитано по формулам

$$y = \frac{C^{-1} * u^T}{u * C^{-1} * u^T}$$

$$x = \frac{u * C^{-1}}{u * C^{-1} * u^T}$$

$$v = \frac{1}{u * C^{-1} * u^T}$$

где $u = (1, 1, \dots, 1) \in R^m$

Решение задачи методом Брауна – Робинсон представлено на рисунке 2.

| (index) | StratA | StratB | Win_A | Loose_B | Eps |
|---------|-------------------|-------------------|---------|---------|---------|
| 0 | [17, 0, 12] | [17, 4, 9] | '17.00' | '4.00' | '13.00' |
| 1 | [21, 16, 14] | [34, 8, 18] | '10.50' | '4.00' | '6.50' |
| 2 | [25, 32, 16] | [51, 12, 27] | '10.67' | '4.00' | '6.50' |
| 3 | [29, 48, 18] | [51, 28, 36] | '12.00' | '7.00' | '3.50' |
| 4 | [33, 64, 20] | [51, 44, 45] | '12.80' | '8.80' | '1.70' |
| 5 | [37, 80, 22] | [51, 60, 54] | '13.33' | '8.50' | '1.70' |
| 6 | [54, 80, 34] | [51, 76, 63] | '11.43' | '7.29' | '1.70' |
| 7 | [71, 80, 46] | [51, 92, 72] | '10.00' | '6.38' | '1.20' |
| 8 | [88, 80, 58] | [51, 108, 81] | '9.78' | '5.67' | '0.98' |
| 9 | [105, 80, 70] | [68, 112, 90] | '10.50' | '6.80' | '0.98' |
| 10 | [122, 80, 82] | [85, 116, 99] | '11.09' | '7.73' | '0.98' |
| 11 | [139, 80, 94] | [102, 120, 108] | '11.58' | '8.50' | '0.98' |
| 12 | [156, 80, 106] | [119, 124, 117] | '12.00' | '9.00' | '0.78' |
| 13 | [165, 89, 125] | [136, 128, 126] | '11.79' | '9.00' | '0.78' |
| 14 | [174, 98, 144] | [153, 132, 135] | '11.60' | '8.80' | '0.78' |
| 15 | [178, 114, 146] | [170, 136, 144] | '11.13' | '8.50' | '0.78' |
| 16 | [182, 130, 148] | [187, 140, 153] | '10.71' | '8.24' | '0.78' |
| 17 | [186, 146, 150] | [204, 144, 162] | '10.33' | '8.00' | '0.78' |
| 18 | [190, 162, 152] | [221, 148, 171] | '10.00' | '7.79' | '0.78' |
| 19 | [194, 178, 154] | [238, 152, 180] | '9.70' | '7.60' | '0.70' |
| 20 | [198, 194, 156] | [255, 156, 189] | '9.43' | '7.43' | '0.43' |
| 21 | [202, 210, 158] | [272, 160, 198] | '9.55' | '7.27' | '0.43' |

| | | | | | |
|----|----------------------|----------------------|---------|--------|--------|
| 22 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '7.65' | '0.43' |
| 23 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.08' | '8.00' | '0.43' |
| 24 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.32' | '8.32' | '0.43' |
| 25 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.54' | '8.62' | '0.43' |
| 26 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.74' | '8.89' | '0.43' |
| 27 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.93' | '9.00' | '0.43' |
| 28 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.86' | '9.00' | '0.43' |
| 29 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.80' | '9.00' | '0.43' |
| 30 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.74' | '8.77' | '0.43' |
| 31 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.41' | '8.50' | '0.43' |
| 32 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.09' | '8.24' | '0.43' |
| 33 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.79' | '8.00' | '0.43' |
| 34 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.51' | '7.77' | '0.43' |
| 35 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.39' | '7.56' | '0.39' |
| 36 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.59' | '7.81' | '0.39' |
| 37 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.79' | '8.05' | '0.39' |
| 38 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.97' | '8.28' | '0.39' |
| 39 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.15' | '8.50' | '0.39' |
| 40 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.32' | '8.71' | '0.39' |
| 41 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.48' | '8.90' | '0.39' |
| 42 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.63' | '9.00' | '0.39' |
| 43 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.59' | '9.00' | '0.39' |
| 44 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.56' | '9.00' | '0.39' |
| 45 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.52' | '9.00' | '0.39' |
| 46 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.49' | '9.00' | '0.39' |

| | | | | | |
|----|----------------------|----------------------|---------|--------|--------|
| 47 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.46' | '9.00' | '0.39' |
| 48 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.43' | '8.90' | '0.39' |
| 49 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.30' | '8.80' | '0.39' |
| 50 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.18' | '8.71' | '0.39' |
| 51 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.06' | '8.62' | '0.39' |
| 52 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.94' | '8.53' | '0.39' |
| 53 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '8.44' | '0.39' |
| 54 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.73' | '8.36' | '0.39' |
| 55 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.63' | '8.29' | '0.39' |
| 56 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.53' | '8.21' | '0.39' |
| 57 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.43' | '8.14' | '0.39' |
| 58 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.34' | '8.07' | '0.34' |
| 59 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.38' | '8.00' | '0.34' |
| 60 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.49' | '8.13' | '0.34' |
| 61 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.60' | '8.26' | '0.34' |
| 62 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.70' | '8.38' | '0.34' |
| 63 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.80' | '8.50' | '0.34' |
| 64 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.89' | '8.62' | '0.34' |
| 65 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.98' | '8.73' | '0.34' |
| 66 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.07' | '8.84' | '0.34' |
| 67 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.16' | '8.94' | '0.34' |
| 68 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.25' | '9.00' | '0.34' |
| 69 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.23' | '9.00' | '0.34' |
| 70 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.21' | '9.00' | '0.34' |
| 71 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.19' | '9.00' | '0.34' |

| | | | | | |
|----|----------------------|----------------------|---------|--------|--------|
| 72 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.18' | '9.00' | '0.34' |
| 73 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.16' | '9.00' | '0.34' |
| 74 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.15' | '9.00' | '0.34' |
| 75 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.13' | '8.95' | '0.34' |
| 76 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.00' | '8.83' | '0.34' |
| 77 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.87' | '8.72' | '0.34' |
| 78 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.75' | '8.61' | '0.34' |
| 79 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.63' | '8.50' | '0.34' |
| 80 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.51' | '8.40' | '0.34' |
| 81 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.39' | '8.29' | '0.34' |
| 82 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.31' | '8.19' | '0.31' |
| 83 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.40' | '8.30' | '0.31' |
| 84 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.49' | '8.40' | '0.31' |
| 85 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.58' | '8.50' | '0.31' |
| 86 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.67' | '8.60' | '0.31' |
| 87 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.75' | '8.69' | '0.31' |
| 88 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '8.79' | '0.31' |
| 89 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.91' | '8.88' | '0.31' |
| 90 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.99' | '8.97' | '0.31' |
| 91 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.07' | '9.00' | '0.31' |
| 92 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.05' | '9.00' | '0.31' |
| 93 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.04' | '9.00' | '0.31' |
| 94 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.03' | '9.00' | '0.31' |
| 95 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.02' | '9.00' | '0.31' |
| 96 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.01' | '9.00' | '0.31' |
| 97 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.00' | '9.00' | '0.31' |

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|-----|----------------------|----------------------|---------|--------|--------|
| 98 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.99' | '9.00' | '0.31' |
| 99 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.02' | '9.00' | '0.31' |
| 100 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.11' | '9.09' | '0.22' |
| 101 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.03' | '9.02' | '0.22' |
| 102 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.95' | '8.95' | '0.22' |
| 103 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.88' | '8.88' | '0.22' |
| 104 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.80' | '8.82' | '0.22' |
| 105 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.73' | '8.75' | '0.22' |
| 106 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.65' | '8.69' | '0.22' |
| 107 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.58' | '8.63' | '0.22' |
| 108 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.53' | '8.57' | '0.22' |
| 109 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.48' | '8.53' | '0.22' |
| 110 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.43' | '8.49' | '0.22' |
| 111 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.38' | '8.45' | '0.22' |
| 112 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.34' | '8.41' | '0.22' |
| 113 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.29' | '8.37' | '0.20' |
| 114 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.35' | '8.43' | '0.20' |
| 115 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.41' | '8.50' | '0.20' |
| 116 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.46' | '8.56' | '0.20' |
| 117 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.52' | '8.63' | '0.20' |
| 118 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.57' | '8.69' | '0.20' |
| 119 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.63' | '8.75' | '0.20' |
| 120 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.68' | '8.81' | '0.20' |
| 121 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.73' | '8.87' | '0.20' |
| 122 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.78' | '8.93' | '0.20' |
| 123 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '8.98' | '0.20' |
| 124 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.88' | '9.04' | '0.20' |
| 125 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.93' | '9.10' | '0.19' |
| 126 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.98' | '9.15' | '0.14' |
| 127 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.02' | '9.08' | '0.14' |
| 128 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.95' | '9.01' | '0.14' |
| 129 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.87' | '8.94' | '0.14' |
| 130 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.79' | '8.87' | '0.14' |
| 131 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.72' | '8.80' | '0.14' |
| 132 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.65' | '8.74' | '0.14' |
| 133 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.57' | '8.67' | '0.14' |
| 134 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.50' | '8.61' | '0.14' |
| 135 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.43' | '8.54' | '0.14' |
| 136 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.36' | '8.48' | '0.14' |
| 137 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.31' | '8.42' | '0.14' |
| 138 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.37' | '8.48' | '0.14' |
| 139 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.42' | '8.54' | '0.14' |
| 140 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.48' | '8.60' | '0.14' |
| 141 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.53' | '8.66' | '0.14' |
| 142 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.58' | '8.72' | '0.14' |
| 143 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.63' | '8.78' | '0.14' |
| 144 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.68' | '8.83' | '0.14' |
| 145 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.73' | '8.89' | '0.14' |
| 146 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.78' | '8.95' | '0.14' |
| 147 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '9.00' | '0.14' |

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|-----|----------------------|----------------------|---------|--------|--------|
| 148 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.88' | '9.05' | '0.14' |
| 149 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.93' | '9.11' | '0.14' |
| 150 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.97' | '9.16' | '0.13' |
| 151 | [1667, 1859, 1435] | [1740, 1730, 1782] | '10.02' | '9.17' | '0.12' |
| 152 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.98' | '9.14' | '0.12' |
| 153 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.94' | '9.10' | '0.12' |
| 154 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.90' | '9.07' | '0.12' |
| 155 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.87' | '9.04' | '0.12' |
| 156 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.83' | '9.01' | '0.12' |
| 157 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.79' | '8.97' | '0.12' |
| 158 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.75' | '8.94' | '0.12' |
| 159 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.72' | '8.91' | '0.12' |
| 160 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.68' | '8.88' | '0.12' |
| 161 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.65' | '8.85' | '0.12' |
| 162 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.61' | '8.82' | '0.12' |
| 163 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.58' | '8.79' | '0.12' |
| 164 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.55' | '8.76' | '0.12' |
| 165 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.51' | '8.73' | '0.12' |
| 166 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.48' | '8.71' | '0.12' |
| 167 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.45' | '8.68' | '0.12' |
| 168 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.41' | '8.65' | '0.12' |
| 169 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.38' | '8.62' | '0.12' |
| 170 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.35' | '8.60' | '0.12' |
| 171 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.32' | '8.57' | '0.12' |
| 172 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.36' | '8.61' | '0.12' |
| 173 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.40' | '8.66' | '0.12' |
| 174 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.43' | '8.70' | '0.12' |
| 175 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.47' | '8.74' | '0.12' |
| 176 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.51' | '8.78' | '0.12' |
| 177 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.54' | '8.82' | '0.12' |
| 178 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.58' | '8.86' | '0.12' |
| 179 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.62' | '8.90' | '0.12' |
| 180 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.65' | '8.94' | '0.12' |
| 181 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.69' | '8.98' | '0.12' |
| 182 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.72' | '9.02' | '0.12' |
| 183 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.76' | '9.05' | '0.12' |
| 184 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.79' | '9.09' | '0.12' |
| 185 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.82' | '9.13' | '0.12' |
| 186 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.86' | '9.17' | '0.12' |
| 187 | [1667, 1859, 1435] | [1740, 1730, 1782] | '9.89' | '9.20' | '0.09' |

Рисунок 2 - Решение задачи методом Брауна – Робинсон

Получившиеся результаты представлены на рисунке 3.

```
Итеративный метод
Цена игры: 9.25
Количество итераций: 188
Стратегия игрока А: 0.511,0.441,0.048
Стратегия игрока В: 0.319,0.537,0.144
```

Рисунок 3 – результат численного метода

Предельная оценка цены игры была посчитана, как среднее арифметическое:

$$\frac{\min(\overline{v}[k]) + \max(\underline{v}[k])}{2}$$

На рисунке 4 представлен график зависимости величины погрешности от номера итерации. Можно видеть, что с увеличением номера итерации погрешность уменьшается и график зависимости имеет гиперболическую форму. Итерационным методом можно приблизить величину ошибки к сколь угодно малому значению. В нашей задаче величина ошибки не превышает 0.01.

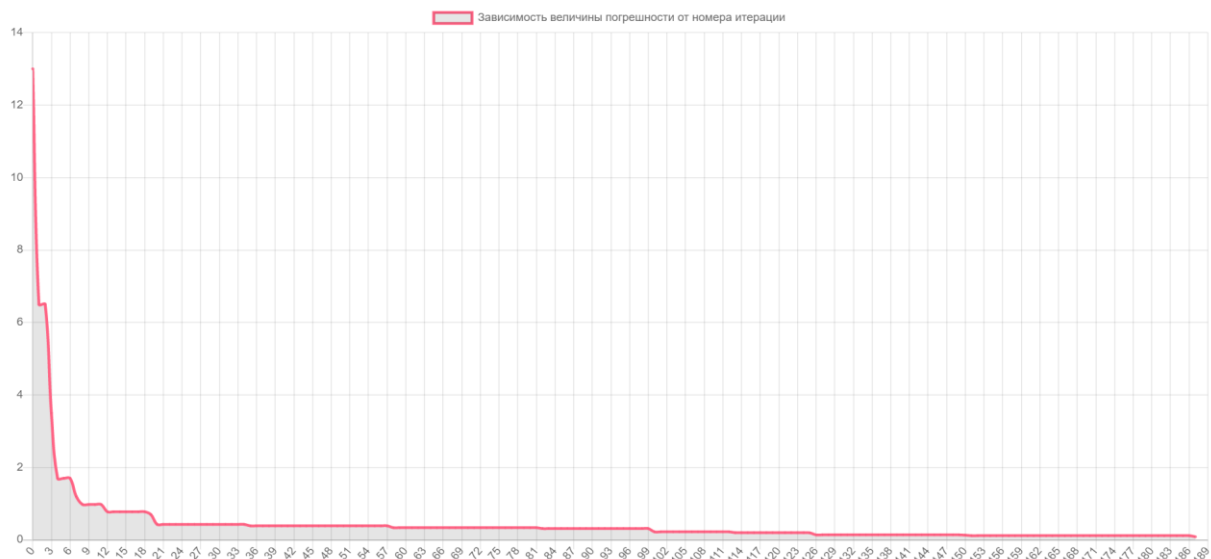


Рисунок 4 – график зависимости величины погрешности от номера итерации

Для того, чтобы убедиться, что верхняя и нижняя цены игры с каждой

итерацией приближаются к одному значению, построим графики (рисунок 5) зависимости данных величин от номера итерации.

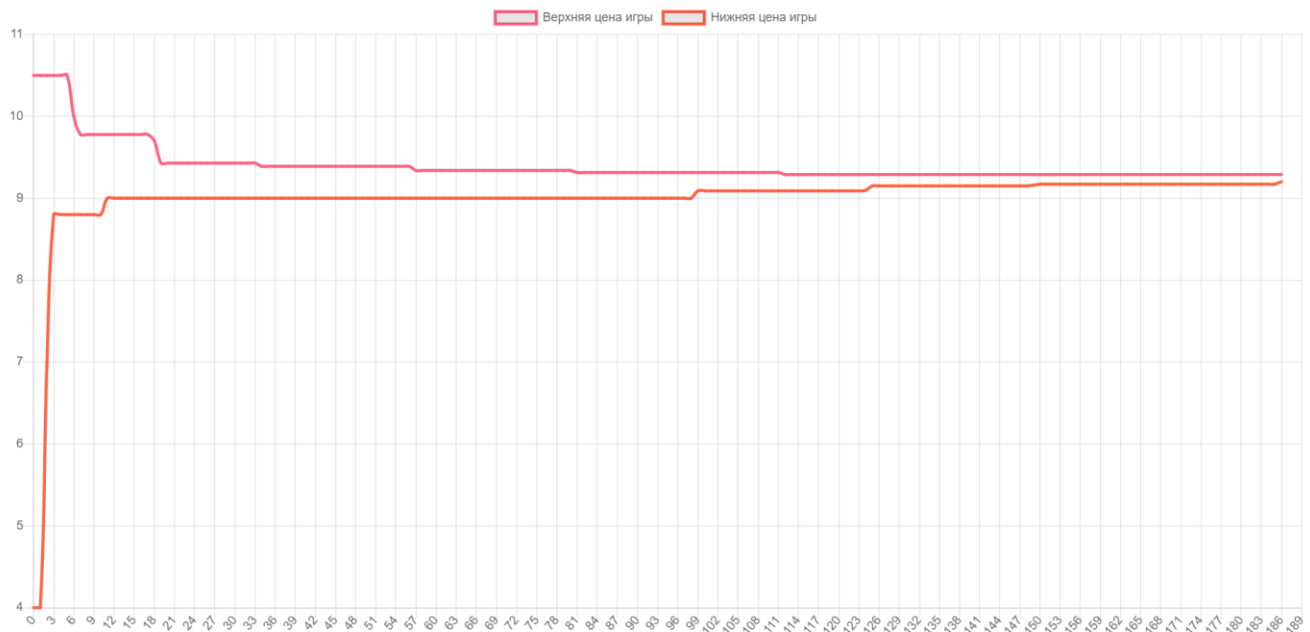


Рисунок 5 – график зависимости верхней и нижней цены игры от номера итерации

Можно заметить, что, при увеличении количества итераций в численном методе Брауна-Робинсон, разница между верхней и нижней ценой игры уменьшается, что подтверждает теорему:

$$\lim_{k \rightarrow \infty} \min(\bar{v}[k]) = \lim_{k \rightarrow \infty} \max(\underline{v}[k])$$

Вывод:

В ходе проделанной лабораторной работы было приведено два решения (аналитическое и численное) антагонистической игры в смешанных стратегиях. Цена игры, посчитанная аналитическим методом, отличается от цены игры, которая посчитана численным методом, на 0.43%. Таким образом, несмотря на то, что метод Брауна-Робинсон является приближенным методом решения, цена игры отличается на незначительную величину. Также, преимуществом данного метода является его применимость к произвольной игре, а аналитический метод применим лишь к вполне смешанным играм.

ПРИЛОЖЕНИЕ А

Файл lab1.js:

```
let matrix = [[17, 4, 9], [0, 16, 9], [12, 2, 19]];

let x0 = [17, 4, 9];
let x1 = [0, 16, 9];
let x2 = [12, 2, 19];

let x = [x0, x1, x2];

let y0 = [17, 0, 12];
let y1 = [4, 16, 2];
let y2 = [9, 9, 19];

let y = [y0, y1, y2];

let strategyA = [...y0]; // выигрыши А, при условии выбора y1
let strategyB = [...x0]; // проигрыши В, при условии выбора x1
let wins = [];
let loses = [];
let eps = 13;
let counter = 1;
let epsilons = [];
let priceA = [];
let priceB = [];

let strategyAVector = {
  0: 1,
  1: 0,
  2: 0
};
let strategyBVector = {
  0: 1,
  1: 0,
  2: 0
};

// При равенстве элементов в строке, выбираем следующую стратегию случайно
function getRandomInt(max) {
  return Math.floor(Math.random() * Math.floor(max));
}

// Выбираем максимальный выигрыш
function findMaxWin(strategy) {
  let maxWin = strategy[0];
  let maxIdx = 0;
  if (strategy[0] === strategy[1] && strategy[1] === strategy[2]) {
    maxIdx = getRandomInt(3);
  } else {
    strategy.forEach(function (item, idx) {
      if (item >= maxWin) {
        maxWin = item;
        maxIdx = idx;
      }
    })
  }
}
```

```

        wins.push(strategy[maxIdx] / counter);
        strategyAVector[maxIdx]++;
        return maxIdx;
    }

    // Выбираем минимальный проигрыш
    function findMinLoose(strategy) {
        let minLoose = strategy[0];
        let minIdx = 0;
        if (strategy[0] === strategy[1] && strategy[1] === strategy[2]) {
            minIdx = getRandomInt(3);
        } else {
            strategy.forEach(function (item, idx) {
                if (item <= minLoose) {
                    minLoose = item;
                    minIdx = idx;
                }
            })
        }

        loses.push(strategy[minIdx] / counter);
        strategyBVector[minIdx]++;
        return minIdx;
    }

    //Вычисляем эпсилон
    function findE() {
        return eps = Math.min(...wins) - Math.max(...loses);
    }

    //Суммируем стратегии
    function sum(strategy, neutrality) {
        for (let i = 0; i < 3; i++) {
            strategy[i] += neutrality[i];
        }
        return strategy;
    }

    let table = [];

    function algorithm() {
        let k = findMaxWin(strategyA);
        let l = findMinLoose(strategyB);
        eps = findE();
        epsilons.push(eps);
        table.push({
            StratA: strategyA,
            StratB: strategyB,
            Win_A: wins[wins.length - 1].toFixed(2),
            Win_B: loses[loses.length - 1].toFixed(2),
            Eps: eps.toFixed(2)
        });
    }

```

```

while (eps > 0.1) {
    counter++;
    strategyA = sum(strategyA, y[1]);
    strategyB = sum(strategyB, x[k]);

    k = findMaxWin(strategyA);
    l = findMinLoose(strategyB);
    eps = findE();
    epsilons.push(eps);

    let minWin = Math.min(...wins);
    let maxLoose = Math.max(...Looses);
    priceA.push(minWin);
    priceB.push(maxLoose);

    table.push({
        StrataA: strategyA,
        StrataB: strategyB,
        Win_A: wins[wins.length - 1].toFixed(2),
        Win_B: Looses[Looses.length - 1].toFixed(2),
        Eps: eps.toFixed(2)
    })
}
strategyAVector[k]--;
strategyBVector[l]--;

let minWin = Math.min(...wins);
let maxLoose = Math.max(...Looses);

console.table(table);
let priceAverage = (minWin + maxLoose) / 2;

let vecA = [(strategyAVector[0] / counter).toFixed(3), (strategyAVector[1] / counter).toFixed(3), (strategyAVector[2] / counter).toFixed(3)];
let vecB = [(strategyBVector[0] / counter).toFixed(3), (strategyBVector[1] / counter).toFixed(3), (strategyBVector[2] / counter).toFixed(3)];

console.log('Итеративный метод ' + '\n'
    + 'Цена игры: ' + priceAverage.toFixed(2) + '\n' +
    'Количество итераций: ' + counter + '\n' +
    'Стратегия игрока A: ' + vecA + '\n' +
    'Стратегия игрока B: ' + vecB);

let cU = [120 / 3566, 170 / 3566, 94 / 3566]; //Результат умножения вектора u на матрицу, обратную к C
let uC = [202 / 3566, 171 / 3566, 11 / 3566]; //Результат умножения матрицы, обратной к C на вектор u
let uCu = [uC[0] + uC[1] + uC[2]]; //Вычисление знаменателя
let xAn = [(uC[0] / uCu).toFixed(3), (uC[1] / uCu).toFixed(3), (uC[2] / uCu).toFixed(3)];
let yAn = [(cU[0] / uCu).toFixed(3), (cU[1] / uCu).toFixed(3), (cU[2] / uCu).toFixed(3)];
let cAn = [(1 / uCu).toFixed(2)];

console.log('Аналитический метод\n' +
    'Стратегия игрока A: ' + xAn + '\n' +
    'Стратегия игрока B: ' + yAn + '\n' +
    'Цена игры: ' + cAn);

```

```
    return eps;  
}  
algorithm();
```

ПРИЛОЖЕНИЕ В

Файл graphics.html

```
<!DOCTYPE html>
<html lang="en">
<head>
  <meta charset="UTF-8">
  <title>Graph</title>
  <script src="https://cdn.jsdelivr.net/npm/chart.js@2.8.0"></script>
  <script src="lab1.js"></script>
</head>
<body>
<canvas id="myChart"></canvas>

<script>
  let numbers = [];
  let i = 0;

  while (i < 190) {
    numbers.push(i);
    i++;
  }

  let ctx = document.getElementById('myChart').getContext('2d');

  //Строим график зависимости величины погрешности от номера итерации
  let chart = new Chart(ctx, {
    type: 'line',
    data: {
      labels: numbers,
      datasets: [{
        label: 'Зависимость величины погрешности от номера итерации',

        //backgroundColor: 'rgb(255, 99, 132)',
        borderColor: 'rgb(255, 99, 132)',
        data: epsilons,
        pointRadius: 1,

      }]
    },
  });

  //Строим графики зависимости цены игры от номера итерации
  let mixedChart = new Chart(ctx, {
    type: 'line',
    data: {
      datasets: [{
        label: 'Верхняя цена игры',
        data: priceA,
        pointRadius: 1,
        borderColor: 'rgb(255, 99, 132)',
        fill: false
      }, {
        label: 'Нижняя цена игры',
        data: priceB,
        type: 'line',
      }]
    }
  });
```

```
        pointRadius: 1,  
        borderColor: 'rgb(255, 99, 66)',  
        fill: false  
      },  
      labels: numbers  
    },  
  });  
</script>  
</body>  
</html>
```