I.1

THE AREA OF A SIMPLE POLYGON

Jon Rokne
University of Calgary
Calgary, Alberta, Canada

The formula for the area of a triangle is given in "Triangles" (Goldman, l990b). This was generalized by Stone (1986) to a formula for a simple polygon that is easy to remember.

Let $P_i = (x_i, y_i)$, $i = 1, \ldots, n$ be the counterclockwise enumeration of the vertices of the polygon as in Fig. 1.

The area of the polygon is then

$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{vmatrix},$$

where the interpretation of

$$\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{vmatrix}$$

is the summing of the products of the "downwards" diagonals and subtraction of the product of the "upwards" diagonals.

A specific example serves to clarify the formula. Consider the polygon in Fig. 2.

The area of this polygon is

$$\begin{aligned} A &= \frac{1}{2}\begin{vmatrix} 6 & 5 & 2 & 4 & 2 & 6 \\ 2 & 4 & 3 & 3 & 1 & 2 \end{vmatrix} \\ &= (6 \times 4 + 5 \times 3 + 2 \times 3 + 4 \times 1 + 2 \times 2 \\ &\quad - 5 \times 2 - 2 \times 4 - 4 \times 3 - 2 \times 3 - 6 \times 1)/2 \\ &= 7.5. \end{aligned}$$

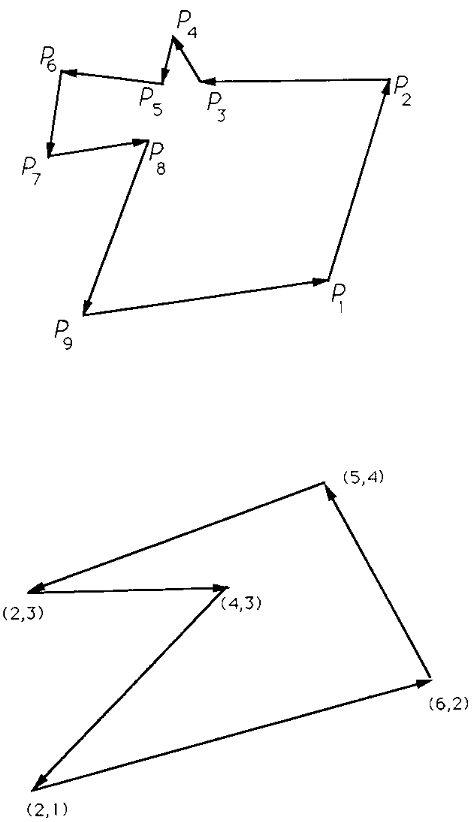

Figure 2.

See also IV.1 The Area of Planar Polygons and Volume of Polyhedra, Ronald N. Goldman