V. 4

INTERSECTION OF THREE PLANES

Ronald Goldman
University of Waterloo
Waterloo, Ontario, Canada

Let each plane be defined by a point P_k and a unit normal, vector $\mathbf{V}_k$, k = 1, 2, 3. Then the unique point of intersection can be written in closed form as

$$P_{\text{Int}} = \{ (P_1 \cdot \mathbf{V}_1)(\mathbf{V}_2 \times \mathbf{V}_3) + (P_2 \cdot \mathbf{V}_2)(\mathbf{V}_3 \times \mathbf{V}_1) + (P_3 \cdot \mathbf{V}_3)(\mathbf{V}_1 \times \mathrm{V}_2)\} / \mathrm{Det}(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3).$$

If two of the given planes are parallel there is no intersection. In this case the denominator $\mathrm{Det}(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)$ is zero.