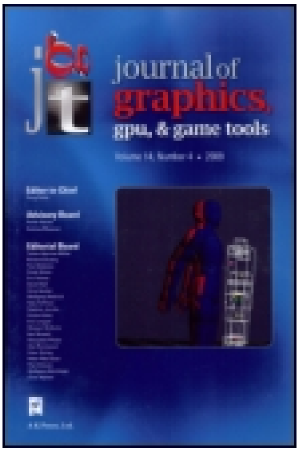

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Computing Vertex Normals from Polygonal Facets

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Abstract. The method most commonly used to estimate the normal vector at a vertex of a polygonal surface averages the normal vectors of the facets incident to the vertex considered. The vertex normal obtained in this way may vary depending on the tessellation of the polygonal surface since the normal of each incident facet contributes equally to the normal in the vertex. To overcome this drawback, we extend the method so that it also incorporates the geometric contribution of each facet, considering the angle under which a facet is incident to the vertex in question.

1. The Problem: Computing the Vertex Normal

Often in computer graphics, one needs to determine normal vectors at vertices of polygonal surfaces, e.g., for Gouraud or Phong shading [Gouraud 71], [Phong 75]. If the polygonal surface approximates a curved surface, whenever possible, vertex normals are provided from the analytical description of the underlying curved surface. Alternatively, a vertex normal N can be computed directly from the polygonal surface. The method commonly suggested is to average the normals N_i of the n facets incident into the vertex [Gouraud 71]:

$$N = \frac{\sum_{i=1}^{n} N_i}{|\sum_{i=1}^{n} N_i|}. \tag{1}$$

If the polygonal surface approximates a curved surface, the normals are only accurate if the facets have regular shape and the orientation does not change much between adjacent polygons. Applying this method, the resulting

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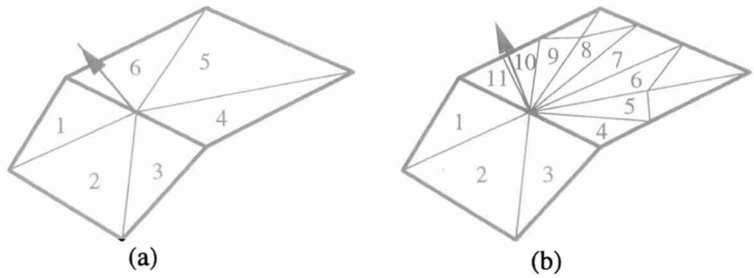

Figure 1. Computed vertex normal for two different meshes of the same geometric situation.

normal vectors depend on the meshing of the surface since the normal of each incident facet contributes equally to the vertex normal. Consequently, if the meshing changes due, e.g., to adaptive tessellation of a deforming surface, the resulting normals will change.

Consider the example shown in Figure 1. Using Equation (1), the computed normal vector varies depending on the tessellation of the polygonal surface. In Figure 1(a), the facets 4, 5, and 6 contribute three of six normals. In Figure 1(b), the facets 4 to 11 contribute eight of eleven normals, despite the fact that the surfaces are the same.

2. The Solution: Average Weighted by Angle

To obtain a result that depends only on the local geometric situation at the vertex, and not on the meshing, we need to consider the spatial contribution of each facet. To do this, we weight the contribution of the normal of each facet by the incident angle of the corresponding facet into the vertex. In symbols,

$$N = \frac{\sum_{i=1}^{n} \alpha_i N_i}{\mid \sum_{i=1}^{n} \alpha_i N_i \mid}, \tag{2}$$

where α_i is the angle under which the i^{th} facet is incident to the vertex in question and is computed as the angle between the two edges of the i^{th} facet incident in the vertex.

One might consider using other measures, e.g., the area of the polygons, for the weighting factors α_i. However, only the use of the angle as weighting factor results in a normal vector that depends exclusively on the geometric situation around the vertex. Consider the example of Figure 1 again. Since $\alpha_4+\alpha_5+\alpha_6$ in (a) is equal to $\alpha_4+\alpha_5+...+\alpha_{11}$ in (b), the normals computed for (a) and (b) by Equation (2) are identical. Thus, applying Equation (2) leads to results that are independent of the tessellation of the polygonal surface. Moreover, the normal at a vertex stays the same regardless of any additional tessellation of the surface.

The proposed extension can be easily incorporated into implementations based on Equation (1), e.g., as suggested by Glassner [Glassner 94]. Of course, this leads to an increase in processing time, but the additional computations are fast and need to be done only once for a polygonal surface.

For the numerical computation, one must be concerned about division by zero in Equation (2). For some applications, e.g., collision detection, one only needs the direction of the normal, so the normalization step can be skipped. Otherwise, if all normals of incident facets lie in the same hemisphere, their average can never be zero. In general, this is the case if the polygonal surface is two-manifold in the vertex, i.e., the facets incident in the vertex form a circular list with each element intersecting its predecessor and successor in exactly one edge, and no other facets in the list.

Another concern is degeneracies in the mesh, i.e., polygons with no area. If the vertex angle of a polygon is 0°, the weight is zero and the polygon will not contribute to the average. Consequently, the result is not affected. If, however, the vertex angle of a degenerate polygon is 180°, e.g., the apex of a zero-height triangle, there is insufficient information at the vertex to produce the geometrically correct result. Such degeneracies should be avoided. Note that the traditional method, Equation (1), would also produce an incorrect result, although a different one.

Other applications that compute vertex properties from polygonal facets could also potentially benefit from weighting by angle. For example, this technique could be used to compute the radiosity value in vertices from incident elements [Cohen, Greenberg 85] in order to reconstruct a continuous radiosity function from a radiosity solution that was performed with constant elements.

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