Optimized Spatial Hashing for Collision Detection of Deformable Objects

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Vision Modeling and Visualization 2003
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Abstract

- We propose a new approach to collision and self–collision detection of dynamically deforming objects that consist of tetrahedrons
- The presented algorithm is integrated in a physically–based environment
  - be used in game engines and surgical simulators
- Using hash function
  - Not always provide a unique mapping of grid cells
    - Optimize the parameter
- The algorithm can detect collisions and self–collisions in environments of up to 20k tetrahedrons in real–time

Introduction

- The detection of collisions and self–collisions of deformable objects based on spatial hashing-1
  - Algorithm classifies all object primitives
    - Object primitives : vertices and tetrahedrons
    - Tetrahedrons → AABB
  - Using hash function
    - 3D boxes (cells) → 1D hash value
    - Each hash value contains a number of object primitives
    - Self-collision can be detected well

Introduction

- The detection of collisions and self–collisions of deformable objects based on spatial hashing-2
  - Using barycentric coordinates of a vertex with respect to a penetrated tetrahedron
    - To estimate the penetration depth for a pair of colliding tetrahedrons
    - Can be used for collision response

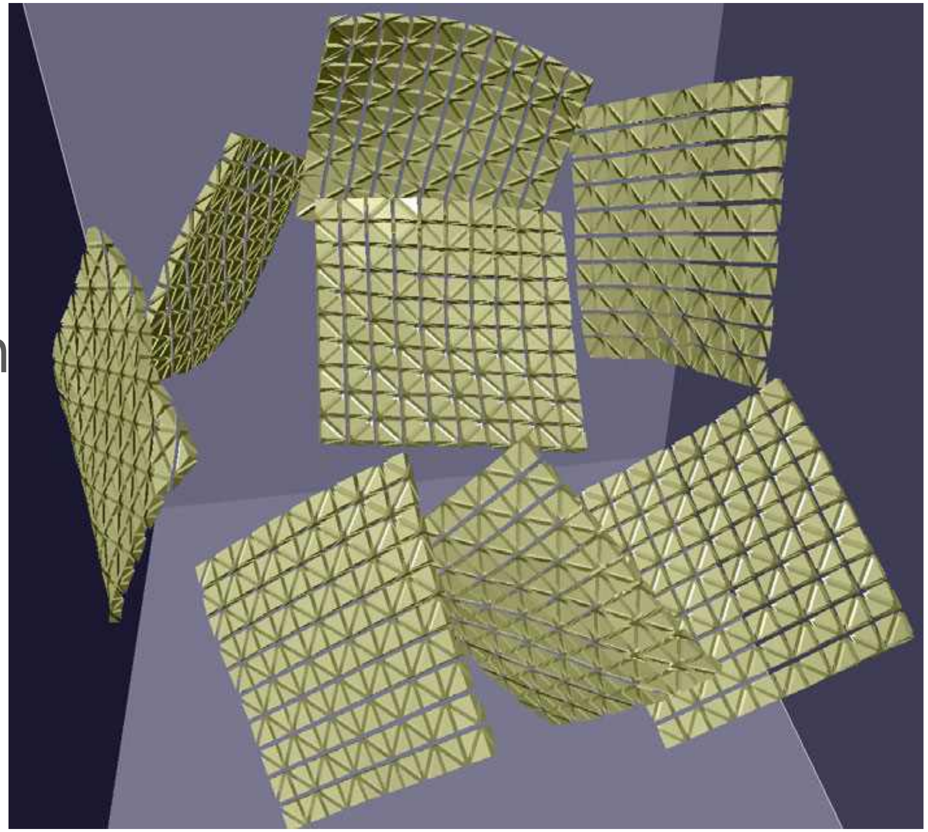
Introduction

- Using a hash function is very efficient
  - Do not need to Spatial Hashing
    - Pre–processing
      - To estimation that the global bounding box and the cell size

- The hash mechanism does not always provide a unique mapping of grid cells to hash table entries
  - the performance decreases
  - To reduce the number of index collisions
    - Optimized the parameters
      - Characteristics of the hash function, hash table size, and the cell size

Introduction

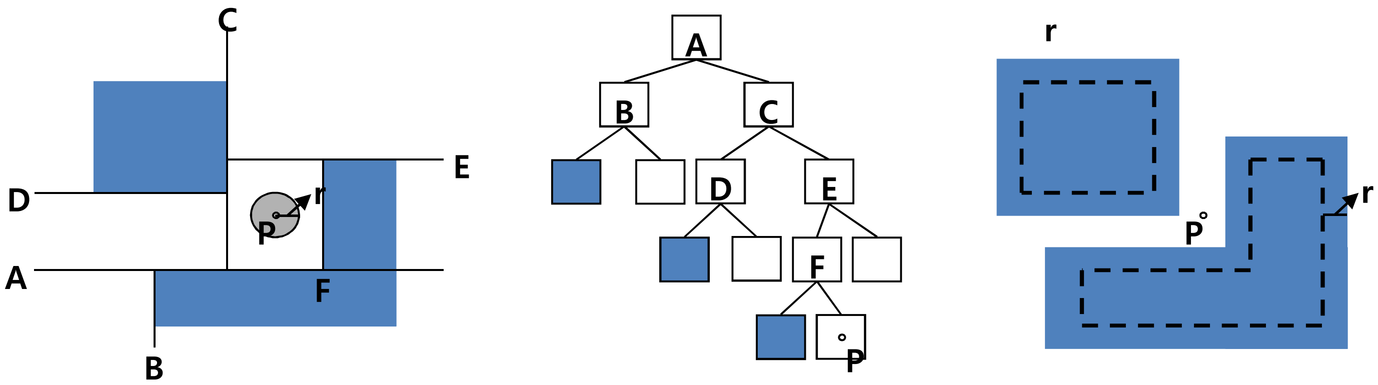
- The paper presents experimental results
  - using physically-based environments for deformable objects with varying geometrical complexity
- 20000 tetrahedrons can be tested for collisions and self–collisions in real–time on a PC



Related Work

- 1. Bounding Box

- 2. Collision Detection using BSP Trees



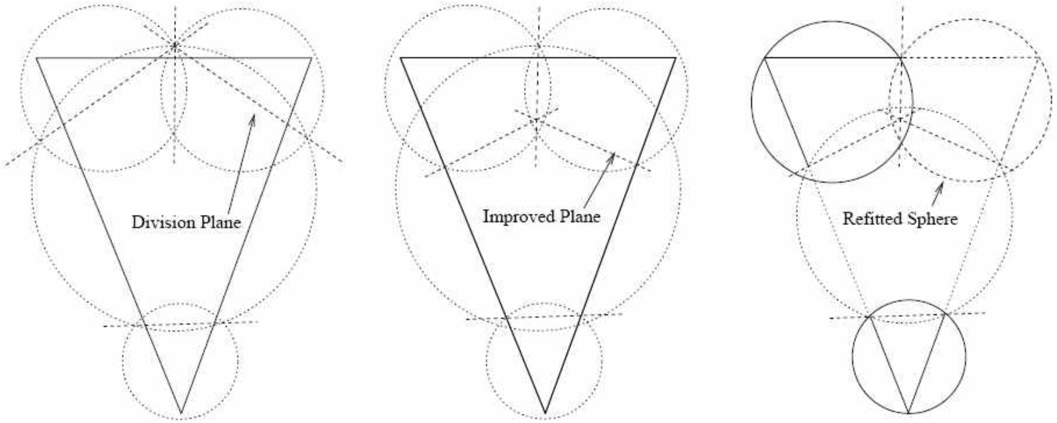
Related Work

- 3. Collision detection for Bounding Box
- 4. If collision detection is detected for bounding boxes
  - → collision detection for primitives

- Many types of BVs have been investigated

Related Work

- Sphere BV



tween Non-

- AABB BV
  - Efficient
    models

    - Journ
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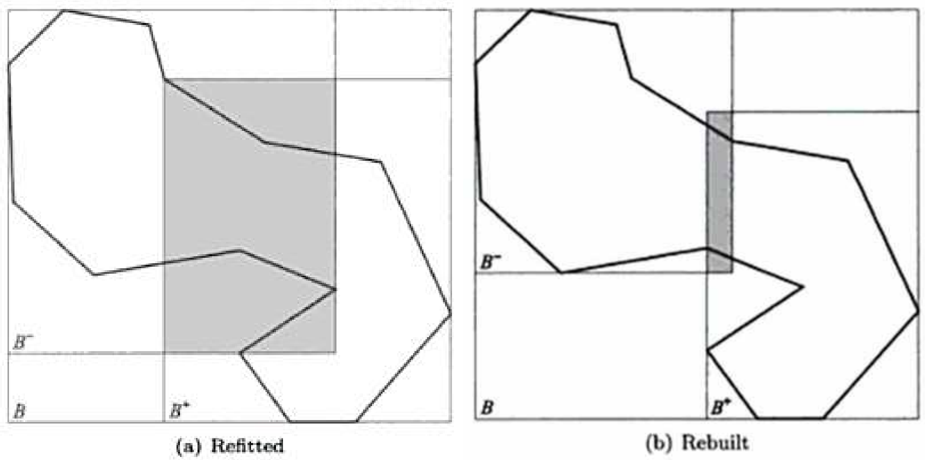
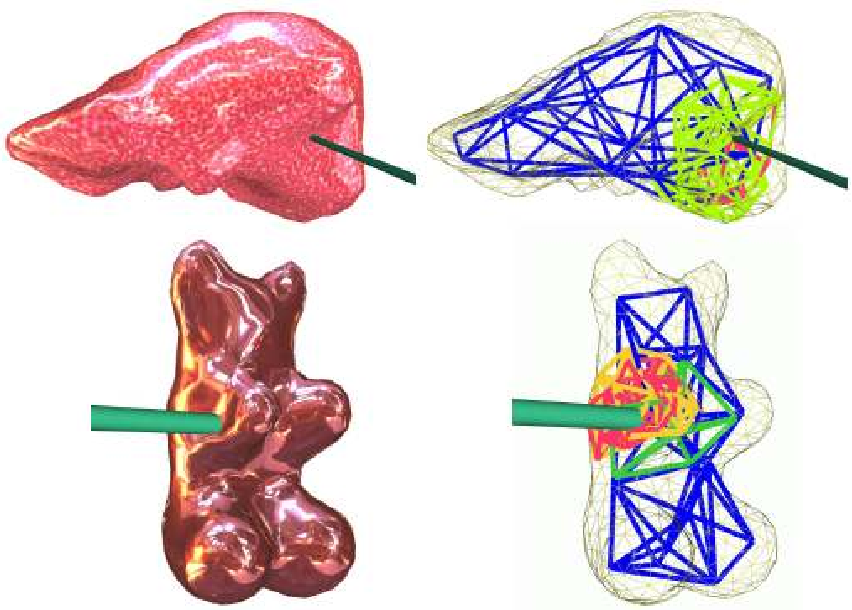


(a) Refitted (b) Rebuilt

eformable

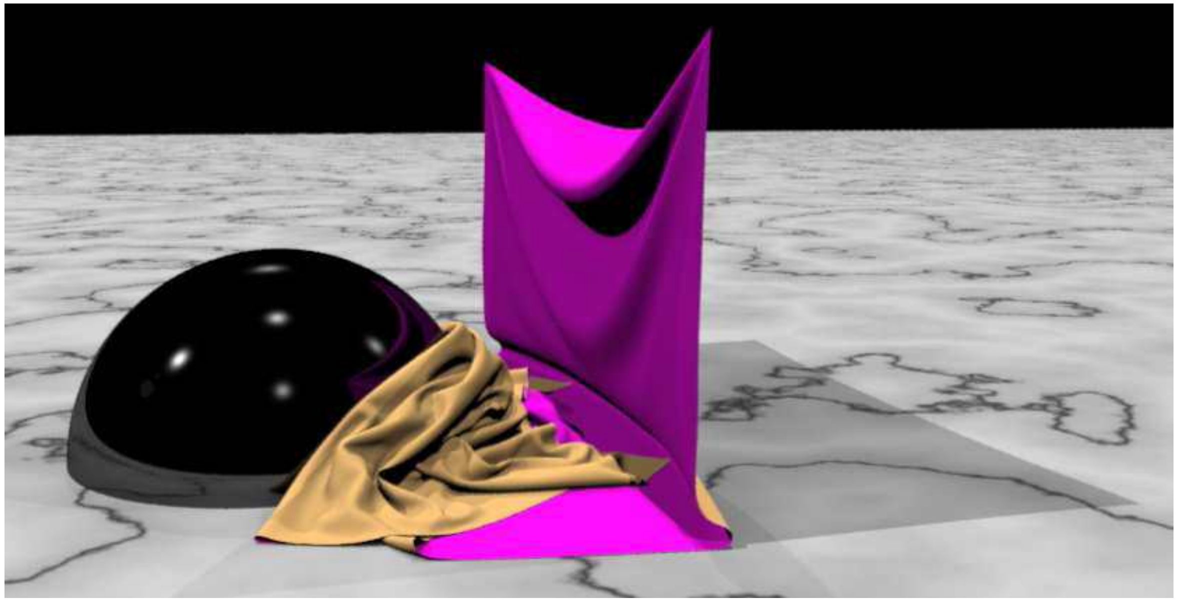
Related Work

- Physically–based simulation
  →computational surgery
  - Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
    - SIGGRAPH 2001
    - Gilles Debunne et al.



Related Work

- Cloth modeling
  - Robust treatment of collisions, contact and friction for cloth animation
    - SIGGRAPH '02
    - R. Bridson et al.



Collision Detection Algorithm

- In a first pass
  - All vertices of all objects are classified with respect to these small 3D cells
- In a second
  - All tetrahedrons are classified with respect to the same 3D cells
- Intersection test
  - Using barycentric coordinates

Collision Detection Algorithm

- Collisions and self–collisions
  - Collisions
    - If
      - A vertex penetrates a tetrahedron
    - Then
      - Collision is detected
  - Self-collisions
    - If
      - A vertex penetrates a tetrahedron
      - The vertex and the tetrahedron belong to the same object
    - Then
      - Self-collisions is detected

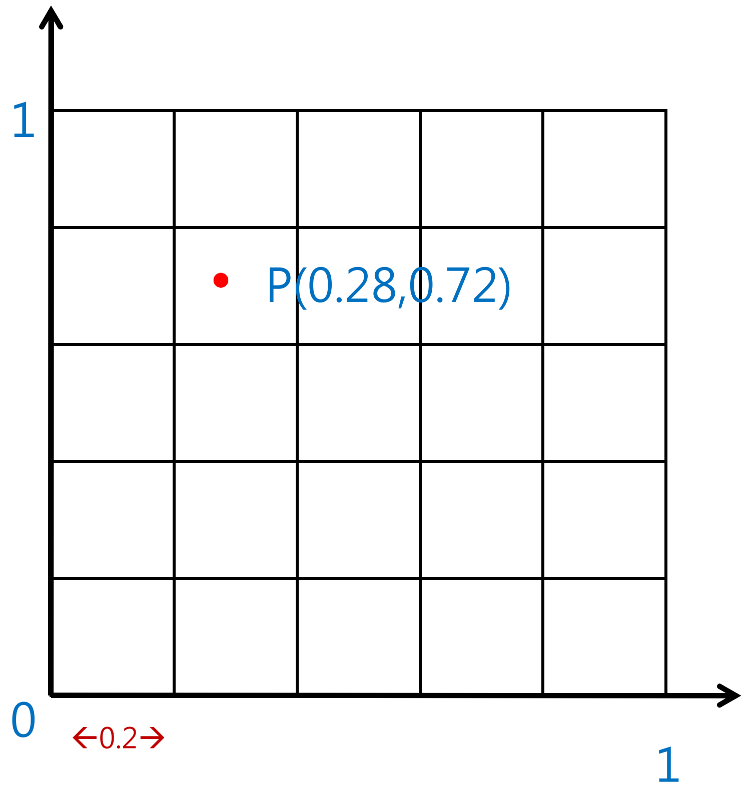
Collision Detection Algorithm

- Spatial Hashing of Vertices-1
  - position (x, y, z)

    → integer (i, j, k):

$$i=\lfloor x/l \rfloor, j=\lfloor y/l \rfloor, k=\lfloor z/l \rfloor$$

  - Example
    - P(0.28,0.72) → I(1,3)
    - i : 0.28/0.2 = 1.4 → 1
    - J : 0.72/0.2 = 3.6 → 3



Collision Detection Algorithm

- Spatial Hashing of Vertices-1
  - The hash function
    - Mapping the discretized 3D position(i, j, k) to a 1D index h
  - The vertex and object information is stored
    - In a hash table at this index h: h = hash(i, j, k)

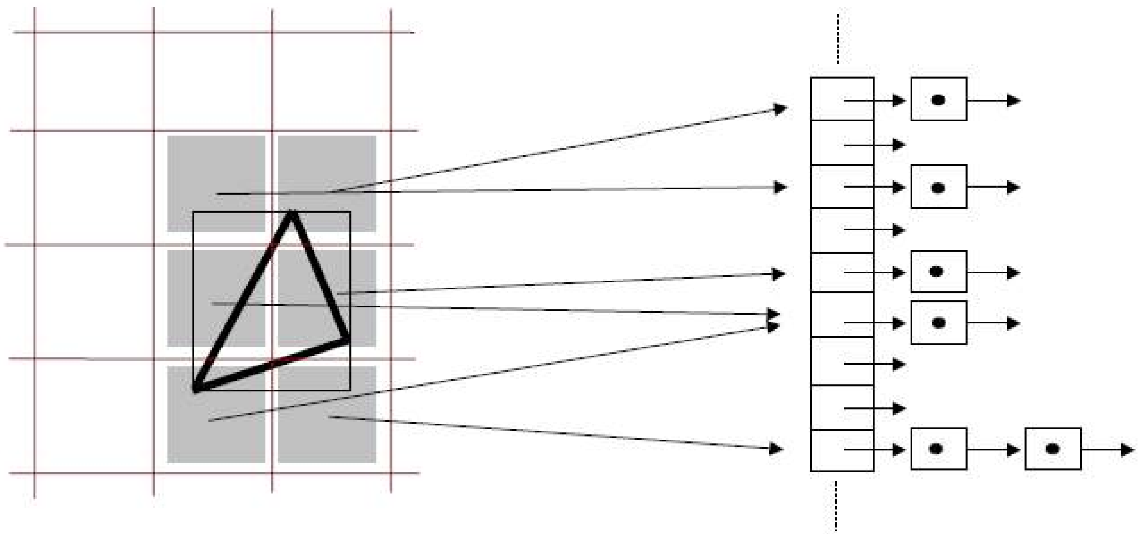
- In a first pass
  - Spatial Hashing of Vertices
  - Compute the AABBs of all tetrahedrons

Collision Detection Algorithm

- Spatial Hashing of Tetrahedrons-2
  - First,
    - The minimum and maximum values describing the AABB of a tetrahedron, are discretized
    - These values are divided by the user–defined cell size and rounded down to the next integer
  - Second,
    - Hash values are computed for all cells affected by the AABB of a tetrahedron

Collision Detection Algorithm

- Spatial Hashing of Tetrahedrons-2
  - All cells are traversed from the discretized minimum to the discretized maximum of the AABB
  - All vertices found at the according hash table index are tested for intersection



Collision Detection Algorithm

- Intersection Test-1
  - If
    - **p** and *t* are mapped to the same hash index
    - **p** is not part of *t*
      - **p** : vertex , *t* : tetrahedron
  - Then
    - a penetration test has to be performed

Collision Detection Algorithm

- Intersection Test-2(The actual intersection test)
  - First,
    - **p** is checked against the AABB of *t*
  - second
    - Whether **p** is inside *t*
      - This test computes barycentric coordinates of **p** with respect to a vertex of ***t***

Parameters

- Optimize all these aspects of the algorithm
  - The characteristics of the hash function
  - The size of the hash table
  - The size of a 3D cell for spatial subdivision
  - The actual intersection test influence the performance of the algorithm

Hash Function

- The hash function has to work
  - Vertices of the same object, that are close to each other
  - Vertices of different objects, that are farther away

  - Hash function

$$hash(x,y,z) = (\, x\, p1 \textbf{ xor } y\, p2 \textbf{ xor } z\, p3) \textbf{ mod } n$$

  - where p1, p2, p3 are large prime numbers in our case 73856093, 19349663, 83492791
  - The value n is the hash table size

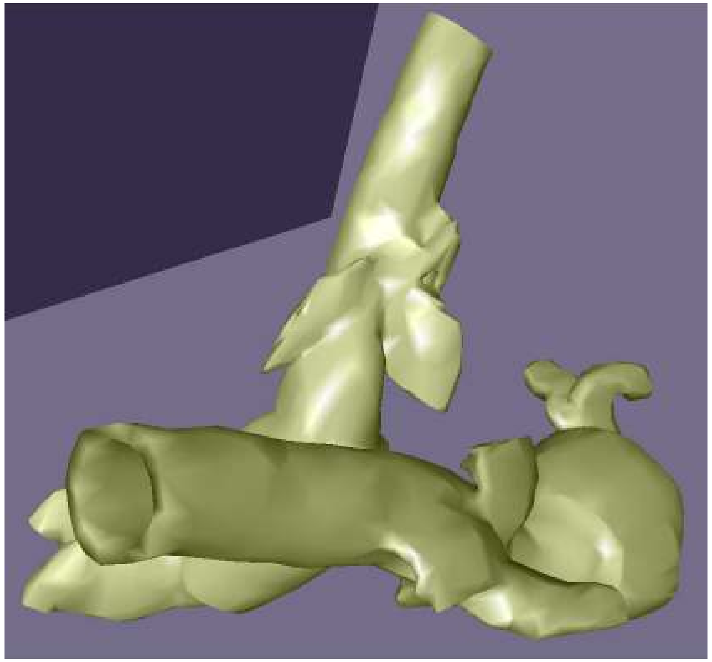
Hash Table Size

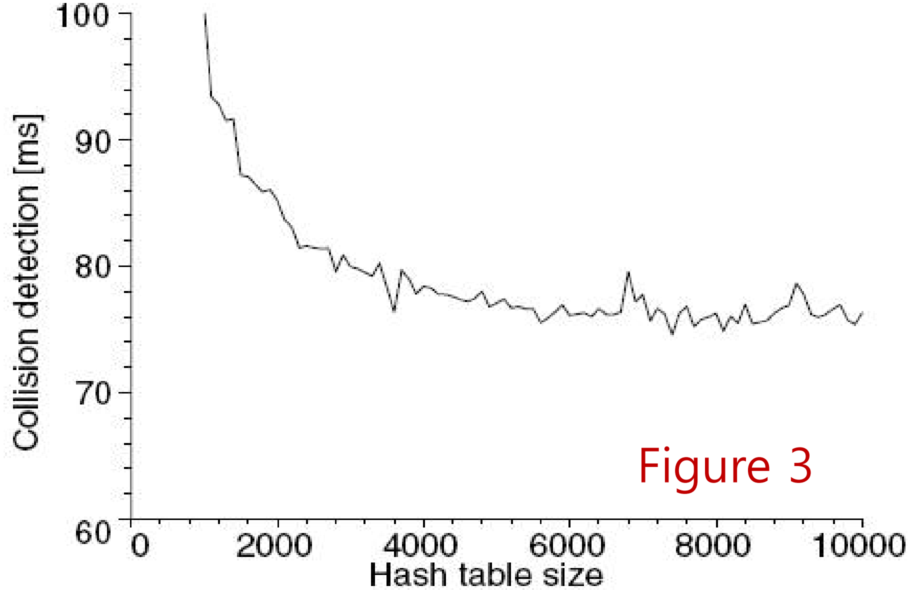
- Larger hash tables
  - reduce the risk of mapping different 3D positions to the same hash index
  - The algorithm generally works faster
  - The performance slightly decreases
    - due to memory management

- If (the hash table size > the number of object primitives)
  - the risk of hash collisions is minimal

Hash Table Size

- Performance of the collision detection algorithm for two deformable vessels
- An overall number of 5898 vertices and 20514 tetrahedrons

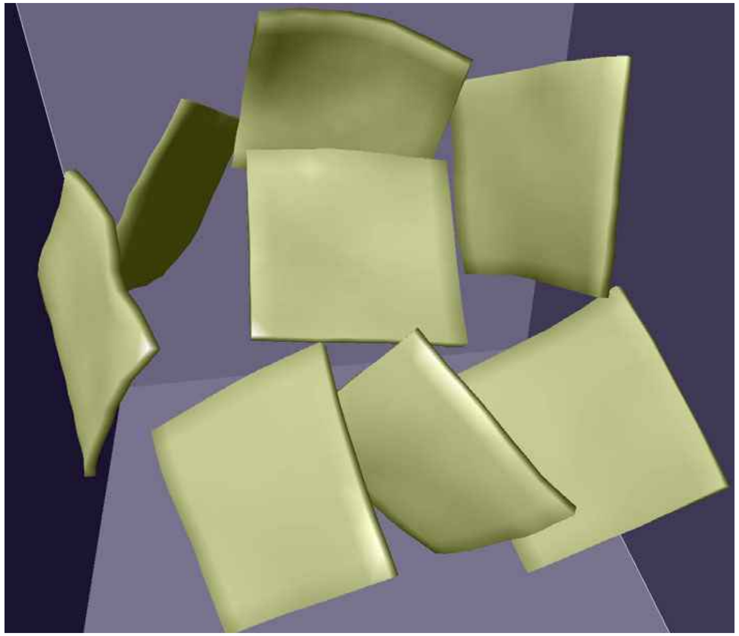




Parameters

Hash Table Size

- Performance of the collision detection algorithm for 100 deformable objects
- An overall number of 1200 vertices and 1000 tetrahedrons



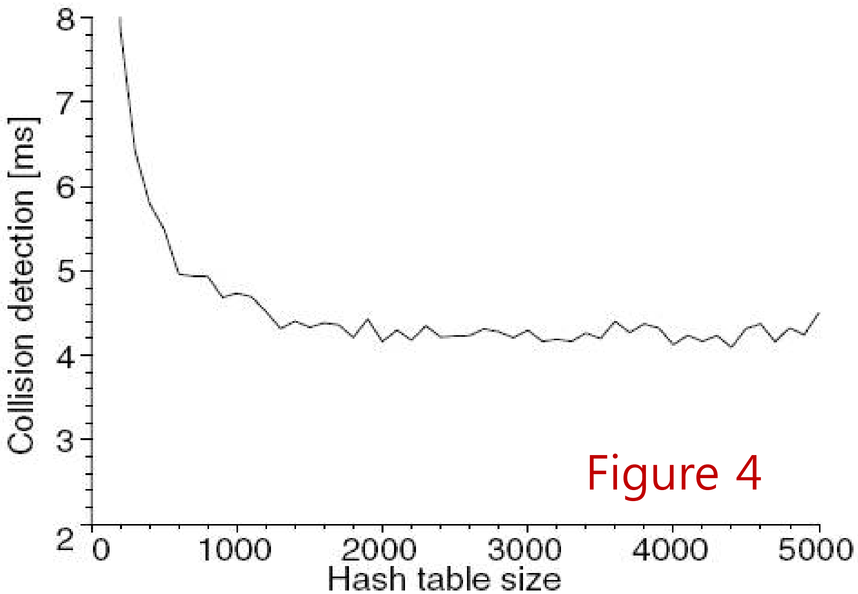


Figure 4

Hash Table Size

- NO re–initialization of hash table in each simulation step
  - These would reduce the efficiency
  - To avoid this problem
    - each simulation step is labeled with a unique time stamp
  - be performed during the simulation
    - would be comparatively costly for larger hash tables

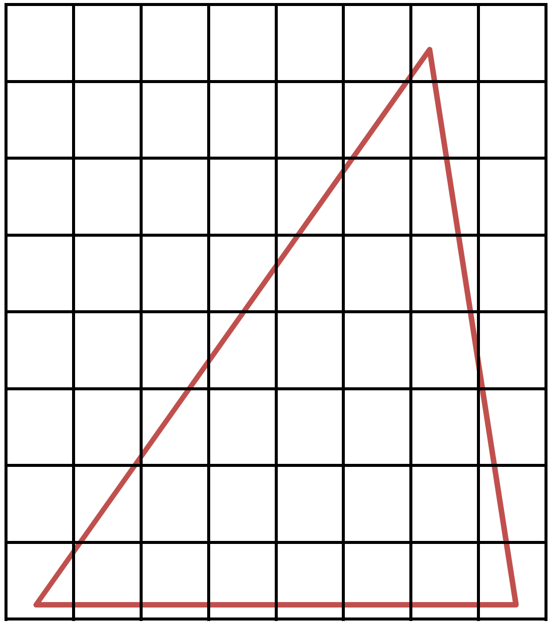
Grid Cell Size

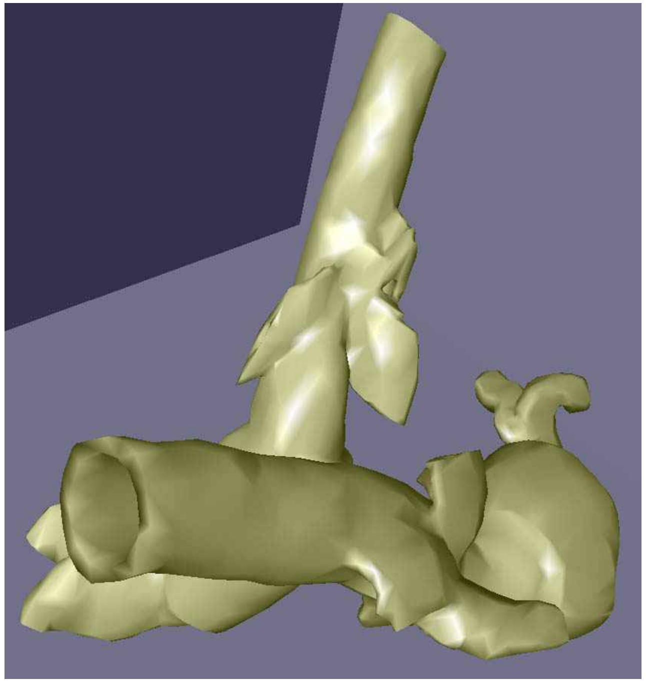
- The grid cell size
  : used for spatial hashing
  - Influences the number of object primitives
    - Mapping to the same hash index

- In case of larger cells,
  → (cell width size << tetrahedron's edge length)
  - The number of primitives per hash index increases
    - The intersection test slows down

Grid Cell Size

- If (cell size << tetrahedron size)
  → (cell width size << tetrahedron's edge length)
  - The tetrahedron
    - Covers a larger number of cells
    - has to be checked against vertices in a larger number of hash entries





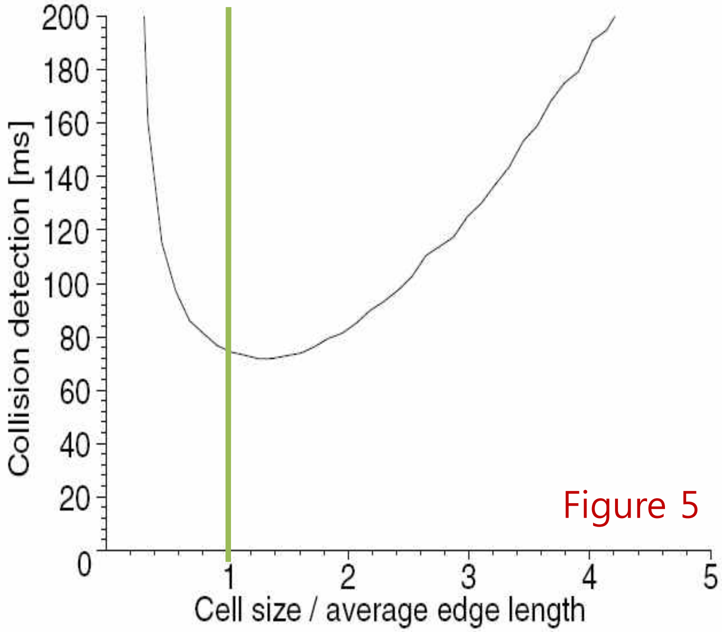
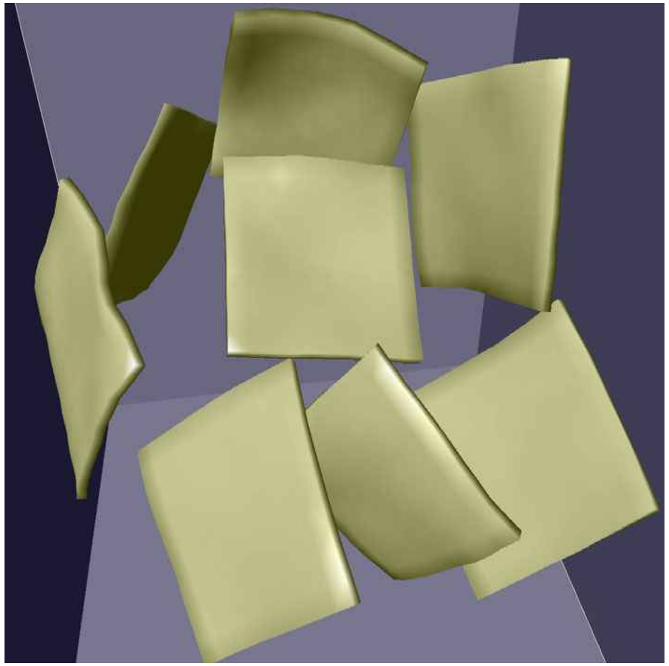


Figure 5



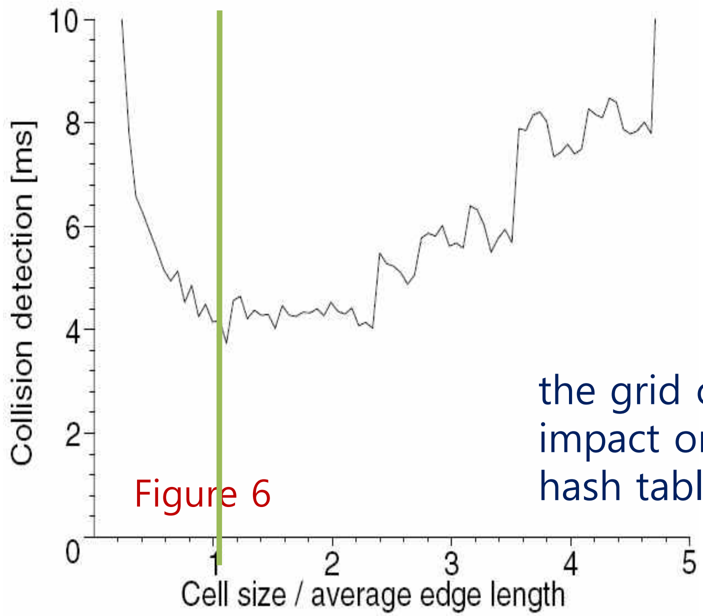
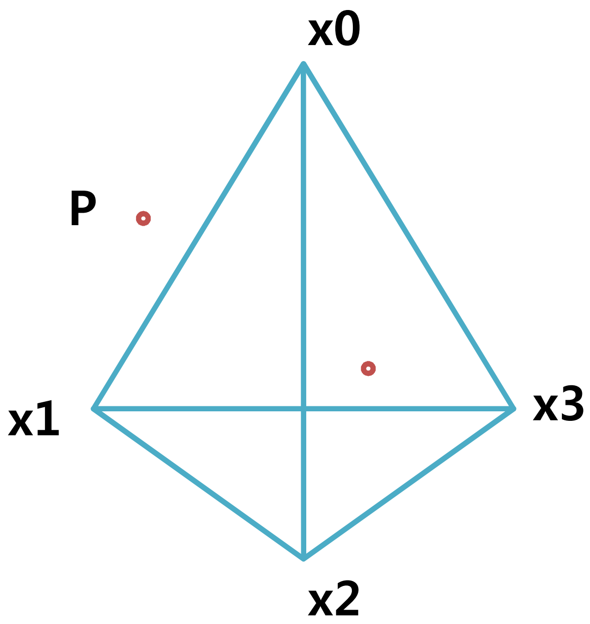


Figure 6

the grid cell size has a more impact on the performance than hash table size or hash function

Intersection Test

- Compare two tests for detecting whether a vertex **p** penetrates a tetrahedron t
  - Barycentric coordinates test
  - Half–space test
    - Checks whether a vertex is in the positive or negative half–space of oriented faces of a tetrahedron
  - barycentric–coordinate test is faster than the half–space test
    - Using Barycentric coordinates test



<tetrahedron t>

P is a vertex of another tetrahedron.

Intersection Test

- Barycentric coordinates test
  - Barycentric coordinates with respect to **x0**

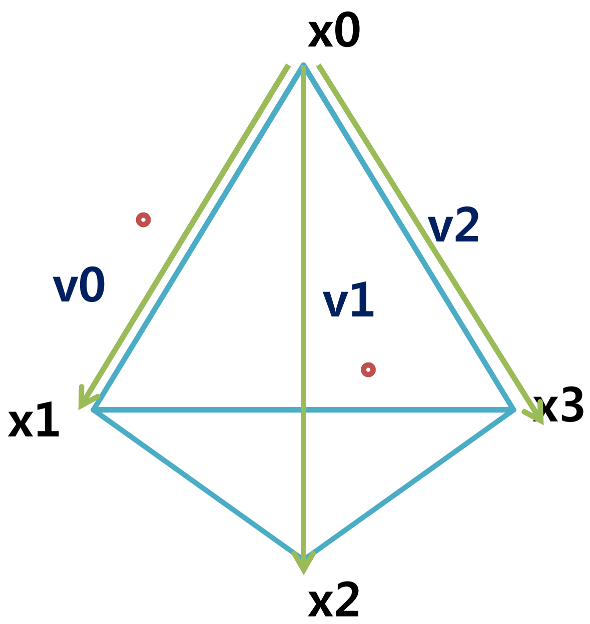
$$\beta = (\beta_1, \beta_2, \beta_3)^T$$

$$\mathbf{p} = \mathbf{x}_0 + \mathbf{A}\beta$$

$$\mathbf{A} = [\mathbf{x}_1 - \mathbf{x}_0, \mathbf{x}_2 - \mathbf{x}_0, \mathbf{x}_3 - \mathbf{x}_0]$$

$$\mathbf{P} = \mathbf{X0} + \beta_1 \cdot \mathbf{V}_1 + \beta_2 \cdot \mathbf{V}_2 + \beta_3 \cdot \mathbf{V}_3$$

$$\beta = \mathbf{A}^{-1}(\mathbf{p} - \mathbf{x}_0)$$



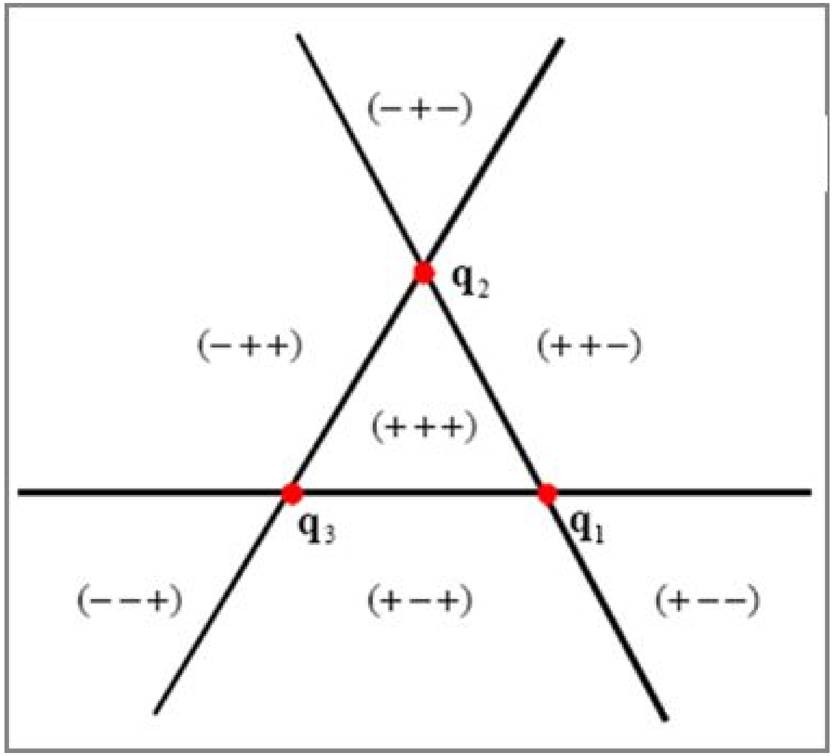
<tetrahedron t>

Intersection Test

- Barycentric coordinates
  → Triangle

  $$\beta = (\beta_1, \beta_2, \beta_3)^T$$

- Barycentric coordinates
  → Tetrahedron
  - if $\beta_1 + \beta_2 + \beta_3 \leq 1$
    $\beta_1 \geq 0,\ \beta_2 \geq 0,\ \beta_3 \geq 0$
  - then
    - The vertex is inside the tetrahedron



Time Complexity

- Let n be the number of primitives
  - Primitives : vertices and tetrahedrons
- Time complexity : $O(n^2)$
- The goal of our approach : $O(n)$
- During the first pass takes $O(n)$ time
  - All vertices are inserted into the hash table

Time Complexity

- In the second pass takes : $O(n \cdot p \cdot q)$
  - *p* is the average number of cells intersected by a tetrahedron
  - *q* is the average number of vertices per cell

  - If the cell size is chosen to be proportional to the average tetrahedron size *p* is a constant
  - If there are no hash collisions *q* is a constant
    - hash collisions : different primitives mapping same hash index

- Therefore
  - The time complexity of the algorithm turns out to be linearly dependent on the number of primitives

Results

- The performance is independent from the number of objects
  - It only depends on the number of object primitives

setup	objects	tetras	vertices
A	100	1000	1200
B	8	4000	1936
C	20	10000	4840
D	2	20514	5898
E	100	50000	24200

setup	ave [ms]	min [ms]	max [ms]	dev [ms]
A	4.3	4.1	6.5	0.24
B	12.6	11.3	15.0	0.59
C	30.4	28.9	34.4	1.25
D	70.0	68.5	72.1	0.86
E	172.5	170.5	174.6	1.08

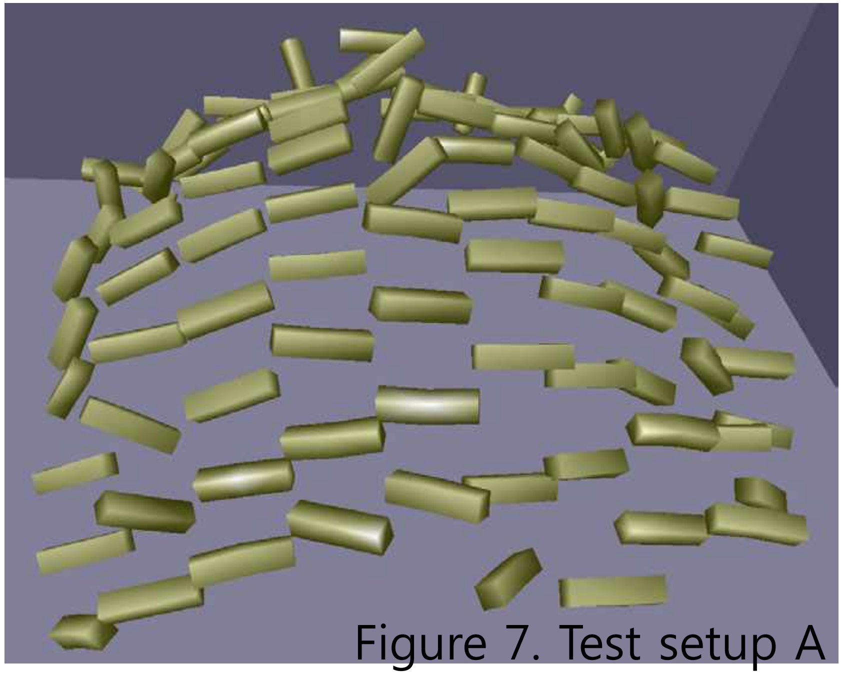


Figure 7. Test setup A

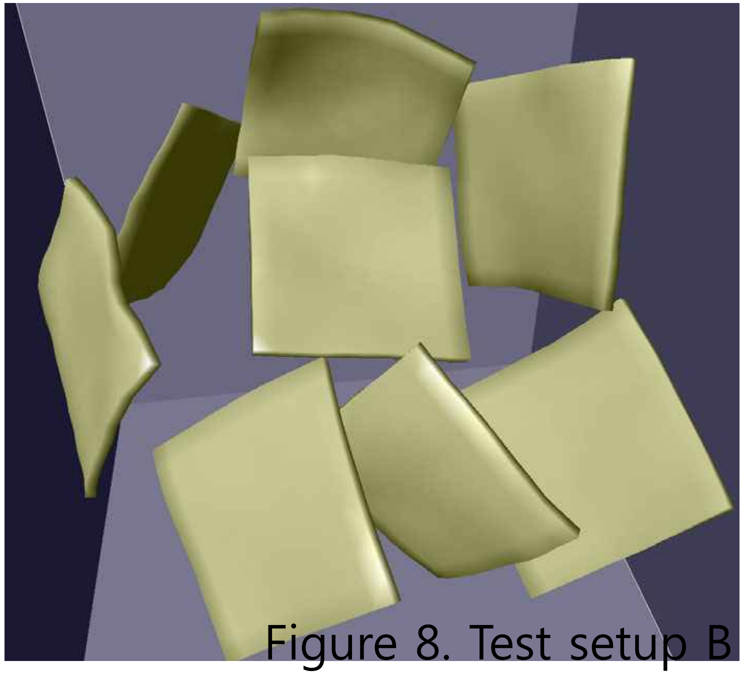


Figure 8. Test setup B

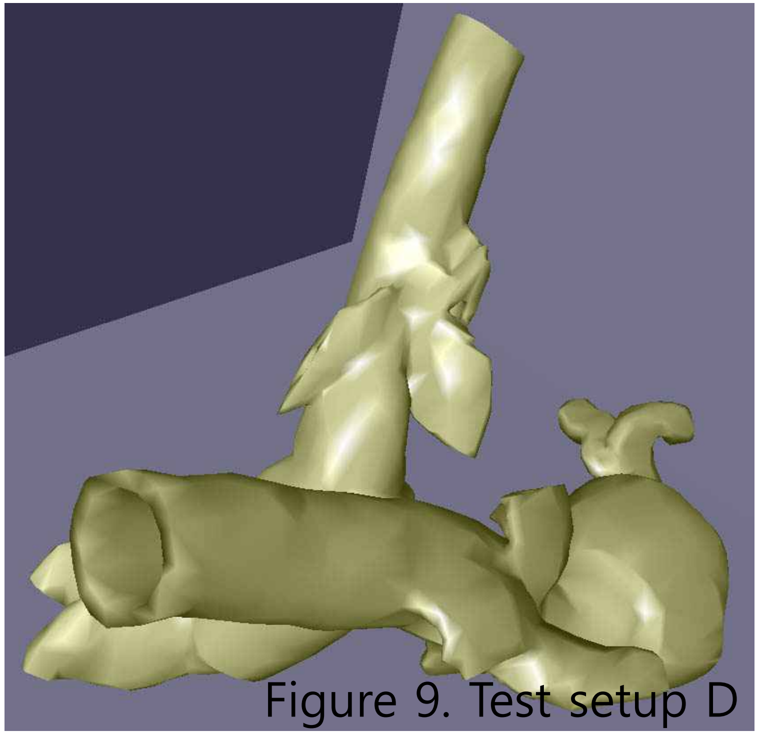


Figure 9. Test setup D

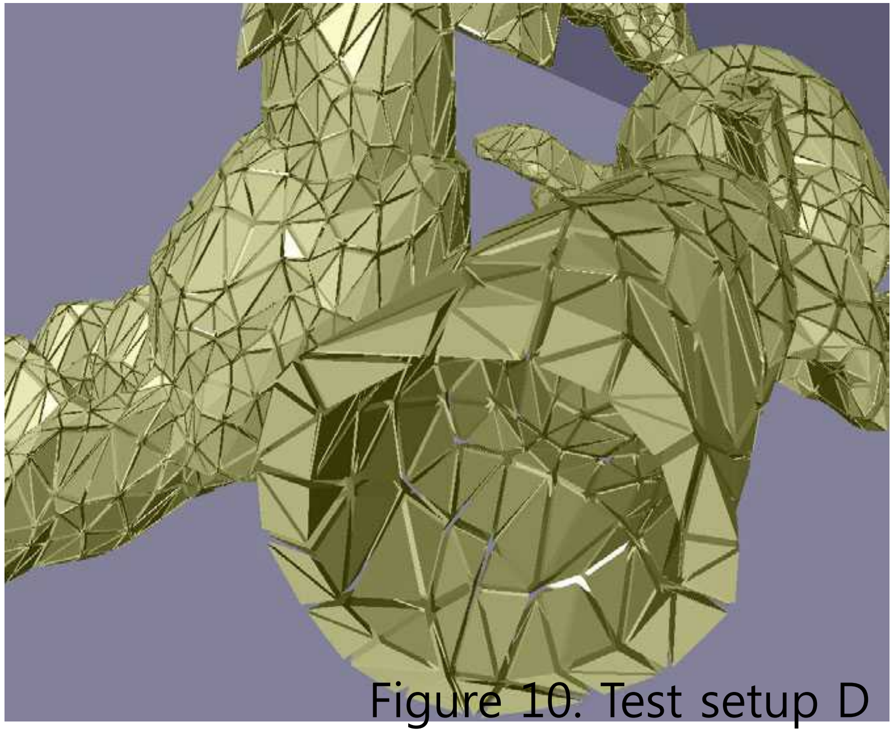


Figure 10. Test setup D

Discussion

- The proposed algorithm
  - Detects whether a vertex penetrates a tetrahedron

- Does NOT detect whether an edge intersects with a tetrahedron
  - The performance of the algorithm would decrease significantly
    - The relevance of an edge test is unclear in case of densely sampled objects
  - It is hard to do collision response in case of penetrating edges

Discussion

- Tetrahedrons are usually mapped to several hash indices
  - Leads to a larger number of elements in the hash table
  - decreasing the performance of the algorithm

- The comparison of the performance with other CD
  - = It is difficult
  - RAPID [9], PQP [18], and SWIFT [7]
    - These are NOT optimized for deformable objects
    - They work with data structures
      - That can be pre–computed for rigid bodies
    - But they have to be updated in case of deformable objects

OngoingWork

- Correct collision response based on our algorithm
  - Our algorithm provides the exact position of a vertex inside a penetrated tetrahedron
  - we can easily derive the penetration depth

- For real–time simulation of deformable objects
  → can be used in game engines or surgical simulators
  - Completed with the collision response(above mentioned)
    - the framework will handle interacting deformable models of up to several thousand tetrahedrons in real–time

Conclusion

- We have introduced
  - Detecting collisions and self–collisions of dynamically deforming objects
  - Origin : computing the global bounding box of all objects and explicitly performing a spatial subdivision
  - Ours : using a hash function that maps 3D cells to a hash table
  - Actual vertex-in-tetrahedron test
    - Using barycentric coordinates
      - Using this information
        : can be used for physically–based collision response
  - optimized the parameters
  - 20k tetrahedrons can be processed in real–time