# Implementing a Simple Anisotropic Rough Diffuse Material with Stochastic Evaluation

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**Abstract**

We describe a simple implementation of a rough diffuse microfacet BRDF built upon anisotropic Beckmann and anisotropic GGX distributions. We evaluate the BRDF by sampling a microfacet from the distribution of visible normals (VNDF) and evaluating diffuse reflectance on this normal. Our technique is stochastic and unbiased which makes it appropriate for Monte Carlo rendering.

# 1 Introduction

Rough diffuse materials have traditionally been cumbersome to implement due to the lack of closed-forms of existing models. Currently, rough diffuse materials implement the closed-form approximation of the Oren-Nayar model [ON94]. While this approach works fairly well in practice, it is limited to isotropic behaviors. Moreover, it models shadowing and masking effects using the $\vee$ groove cavity model of Torrance and Sparrow [TS67] that tends to be downplayed by the industry in favor of the Smith model [Smi67] because it reacts to roughness [HMD+14]. In the following sections, we introduce an implementation of rough diffuse materials that is suitable for Monte Carlo renderers. Our implementation computes the equations of the diffuse microfacet model that incorporates Smith shadowing effects and supports anisotropy [Hei14].
It carries the following properties

- It does not use precomputed or fitted data.
- It does not approximate the equations of the model, which are solved exactly.
- It works with anisotropic Beckmann and anisotropic GGX distributions.
- Its evaluation is stochastic but unbiased.

Note that, as any stochastic method, our implementation produces noise if the sampling rate is insufficient. As such, it is inadequate for real-time rendering purposes. For such use cases, a noise-free integration scheme should be employed, such as the deterministic (but biased) sampling scheme for Beckmann distributions that we introduced in LEADR mapping [DHI+13].

## 2 Rough Diffuse BRDF Model

| | |
|---|---|
| $\boldsymbol{\omega}_i$ | incident direction |
| $\boldsymbol{\omega}_o$ | outgoing direction |
| $\boldsymbol{\omega}_m$ | microfacet normal |
| $\boldsymbol{\omega}_g$ | geometric normal |
| $\rho(\boldsymbol{\omega}_o, \boldsymbol{\omega}_i)$ | BRDF |
| $D(\boldsymbol{\omega}_m)$ | distribution of normals (NDF) |
| $D_{\boldsymbol{\omega}_i}(\boldsymbol{\omega}_m)$ | distribution of visible normals (VNDF) |
| $G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)$ | masking function |
| $G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)$ | masking-shadowing function |

Table 1: Notation.

**Classic Definition** The equation of the rough diffuse BRDF is [Hei14] (Section 3.4)

$$\begin{aligned}
&\rho^{\text{diff}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \\
&= \frac{1}{\pi} \frac{1}{|\boldsymbol{\omega}_g \cdot \boldsymbol{\omega}_o| \, |\boldsymbol{\omega}_g \cdot \boldsymbol{\omega}_i|} \int_\Omega \langle \boldsymbol{\omega}_i, \boldsymbol{\omega}_m \rangle \, \langle \boldsymbol{\omega}_o, \boldsymbol{\omega}_m \rangle \, G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m) \, D(\boldsymbol{\omega}_m) \, d\boldsymbol{\omega}_m \\
&= \frac{1}{\pi} \frac{1}{|\boldsymbol{\omega}_g \cdot \boldsymbol{\omega}_o|} \int_\Omega \langle \boldsymbol{\omega}_o, \boldsymbol{\omega}_m \rangle \, \frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)} \, D_{\boldsymbol{\omega}_i}(\boldsymbol{\omega}_m) \, d\boldsymbol{\omega}_m,
\end{aligned} \tag{1}$$

which has no closed form and cannot be directly evaluated.

**Stochastic Definition** In order to evaluate the BRDF in a Monte Carlo rendering context, we use the reformulation we introduced to evaluate the diffuse SGGX microflake phase function [HDCD15]. Intuitively, our reformulation sums up the contributions due to visible microfacets $\boldsymbol{\omega}_m$ in $D_{\boldsymbol{\omega}_i}$. The contribution due to each visible microfacet depends on

1. its diffuse reflectance $\frac{1}{\pi} \langle \boldsymbol{\omega}_o, \boldsymbol{\omega}_m \rangle$,
2. the probability that it is not shadowed given that it is not masked, which is given by the fraction $\frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}$.

Based on this intuition, we rewrite Equation (1) as

$$\rho^{\text{diff}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \, \langle \boldsymbol{\omega}_o, \boldsymbol{\omega}_g \rangle = \lim_{N \to +\infty} \frac{1}{N} \sum_{n=1}^{N} \frac{1}{\pi} \langle \boldsymbol{\omega}_o, \boldsymbol{\omega}_m(n) \rangle \, \frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}, \tag{2}$$

where the samples $\boldsymbol{\omega}_m(1), \cdots, \boldsymbol{\omega}_m(N)$ are distributed according to $D_{\boldsymbol{\omega}_i}$.

# 3 Implementation

In this section, we explain how to implement the functions **eval**, **sample**, and **pdf**, which are required in a typical material plugin. We use the importance sampling technique of $D_{\boldsymbol{\omega}_i}$ with Beckmann and GGX distributions, which is explained in the supplemental material of Heitz and d'Eon [HD14] (note that Wenzel Jakob introduced a better technique for Beckmann distributions [Jak14]).

## 3.1 Stochastic Evaluation

From Equation (2), we observe that an unbiased estimator of the rough diffuse BRDF is obtained by sampling a normal $\boldsymbol{\omega}_m$ from $D_{\boldsymbol{\omega}_i}$ and evaluating the diffuse contribution of the light source to this normal $\frac{1}{\pi}\langle\boldsymbol{\omega}_o, \boldsymbol{\omega}_m\rangle$.

**Algorithm 1** Stochastic Evaluation

1: **function** EVAL($\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_o$)
2: $\quad\boldsymbol{\omega}_m \leftarrow \text{sample}(D_{\boldsymbol{\omega}_i})$
3: $\quad$**return** $\frac{1}{\pi}\langle\boldsymbol{\omega}_o, \boldsymbol{\omega}_m\rangle \frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}$
4: **end function**

## 3.2 Importance Sampling

Similarly, to sample $\rho^{\text{diff}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)\langle\boldsymbol{\omega}_o, \boldsymbol{\omega}_g\rangle$, we generate a sample $\boldsymbol{\omega}_m$ from $D_{\boldsymbol{\omega}_i}$, and sample a diffuse reflected direction $\boldsymbol{\omega}_o$ in the hemisphere centered on $\boldsymbol{\omega}_m$. The weight of the sample is $\frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}$.

**Algorithm 2** Importance Sampling

1: **function** SAMPLE($\boldsymbol{\omega}_i$)
2: $\quad\boldsymbol{\omega}_m \leftarrow \text{sample}(D_{\boldsymbol{\omega}_i})$
3: $\quad\boldsymbol{\omega}_o \leftarrow$ sample diffuse in the hemisphere centered on $\boldsymbol{\omega}_m$
4: $\quad\text{weight} \leftarrow \frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}$
5: **end function**

## 3.3 PDF for Multiple Importance Sampling

Since our evaluation is stochastic, we do not have access to the PDF required for Multiple Importance Sampling (MIS). We can compute an approximation numerically (if we do this, the integration has to be deterministic, not stochastic) but this would be costly. Given that the rough diffuse BRDF is low frequency, it is not necessary to have an accurate PDF. Indeed, MIS remains unbiased with inadequate PDFs, at the cost of slower convergence. In practice, we found that the PDF of a simple diffuse BRDF centered on the geometric normal results in acceptable variance reduction.

**Algorithm 3** PDF

1: **function** PDF(($\boldsymbol{\omega}_i, \boldsymbol{\omega}_o$))
2: $\quad$**return** $\frac{1}{\pi}\langle\boldsymbol{\omega}_o, \boldsymbol{\omega}_g\rangle$
3: **end function**

# 4 Results

Figures 1 and 2 show results computed in a retro-reflective configuration ($\boldsymbol{\omega}_i = \boldsymbol{\omega}_o$). We used the height-correlated form of the Smith masking-shadowing function (see Appendix A).

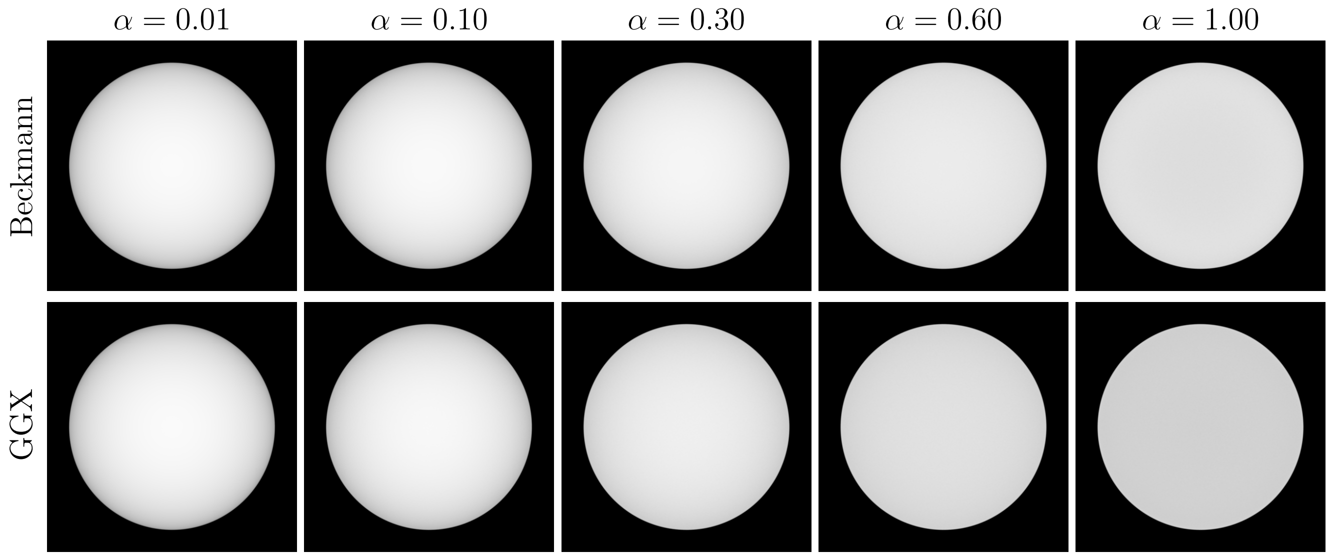


Figure 1: Isotropic rough diffuse BRDFs.

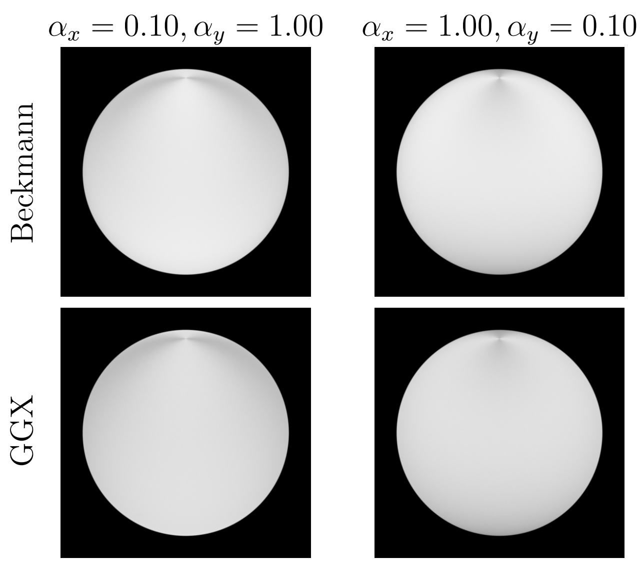


Figure 2: Anisotropic rough diffuse BRDFs.

# References


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# A Masking and Shadowing

Note that Equation (1) is independent of the masking-shadowing model. Usually, the Smith model is used. In this case the masking function is

$$G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m) = \frac{\chi^+(\boldsymbol{\omega}_i \cdot \boldsymbol{\omega}_m)}{1 + \Lambda(\boldsymbol{\omega}_i)}. \tag{3}$$

**Uncorrelated Masking and Shadowing** If the uncorrelated form of the Smith masking-shadowing function is used, then we have

$$G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m) = G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)\, G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m), \tag{4}$$

$$\frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)} = G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m). \tag{5}$$

**Height-Correlated Masking and Shadowing** If the height-correlated form of the Smith masking-shadowing function is used, then we have

$$G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m) = \frac{\chi^+(\boldsymbol{\omega}_i \cdot \boldsymbol{\omega}_m)\ \chi^+(\boldsymbol{\omega}_o \cdot \boldsymbol{\omega}_m)}{1 + \Lambda(\boldsymbol{\omega}_i) + \Lambda(\boldsymbol{\omega}_o)} \tag{6}$$

$$= \frac{G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m) G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m) + G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m) - G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m) G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}, \tag{7}$$

$$\frac{G_2(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)} = \chi^+(\boldsymbol{\omega}_o \cdot \boldsymbol{\omega}_m)\, \frac{1 + \Lambda(\boldsymbol{\omega}_i)}{1 + \Lambda(\boldsymbol{\omega}_i) + \Lambda(\boldsymbol{\omega}_o)} \tag{8}$$

$$= \frac{G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m)}{G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m) + G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m) - G_1(\boldsymbol{\omega}_o, \boldsymbol{\omega}_m) G_1(\boldsymbol{\omega}_i, \boldsymbol{\omega}_m)}. \tag{9}$$