# 2.5

# The Parallel Transport Frame


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Many tasks in computer games require generating a suitable orientation as an object moves through space. Let's say you need to orient a camera flying along a looping path. You'd probably want the camera to turn with the path and point along the direction of travel. When the path loops, the orientation of the camera should change appropriately, to follow the loop. You wouldn't want it to suddenly flip or twist, but turn only to match the changes of the path. The *parallel transport frame* method can help provide this "steady" orientation.

You can also use this technique in the generation of geometry. A common operation in 3D modeling is *lofting,* where a 2D shape is *extruded* along a path curve, and multiple sections made from the shape are connected together to produce 3D geometry. If the 2D shape was a circle, the resulting 3D model would be a tube, centered on the path curve. The same criteria apply in calculating the orientation of the shape as did with the camera—the orientation should "follow" the path and shouldn't be subject to unnecessary twist.

The parallel transport method gets its stability by incrementally rotating a coordinate system (the frame) as it is translated along a curve. This "memory" of the previous frame's orientation is what allows the elimination of unnecessary twist—only the minimal amount of rotation needed to stay parallel to the curve is applied at each step. Unfortunately, in order to calculate the frame at the end of a curve, you need to iterate a frame along the path, all the way from the start, rotating it at each step. Two other commonly used methods of curve framing are the *Frenet Frame* and the *Fixed Up* method [EberlyOl], which can be calculated analytically at any point on the path, in one calculation. They have other caveats, however, which will be described later.

## The Technique

A relatively simple numerical technique can be used to calculate the parallel transport frame [Glassner90]. You take an arbitrary initial frame, translate it along the curve, and at each iteration, rotate it to stay as "parallel" to the curve as possible.

Given:

a Curve C
an existing frame Fl at t-1
a tangent Tl at t-1 (the 1st derivative or velocity of C at t-1)
a tangent T2 at t

a new frame F2 at the next time t can be calculated as follows:

F2s position is the value of C at t.
F2s orientation can be found by rotating Fl about an axis A with angle Ot, where
A = Tl X T2 and
a = ArcCos((Tl •T2)/(|T1||T2|))

If the tangents are parallel, the rotation can be skipped (i.e., if Tl X T2 is zero) (Figure 2.5.1).

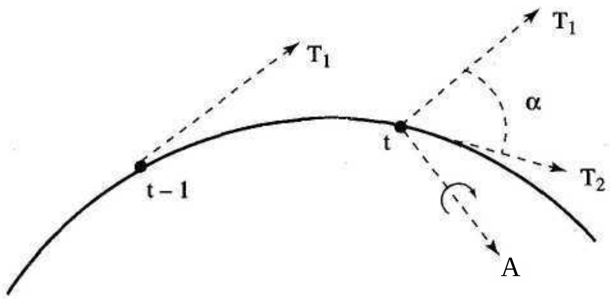


FIGURE 2.5.1 *The frame at t-1 is rotated about A by* Of *to calculate the frame at t.*

The initial frame is arbitrary. You can calculate an initial frame in which an axis lies along the tangent with the Fixed Up or the Frenet Frame method.

In some cases, you may find it desirable to use parallel transport to generate frames at a coarse sampling along the curve, and then achieve smooth rotation between the sample frames by using quaternion interpolation. Using quaternions is desirable anyway, since there is an efficient method of generating a quaternion from a rotation axis and angle [EberlyOl]. You can use the angle and axis shown previously to generate a rotation quaternion, and then multiply it with the previous frame's quaternion to perform the rotation.

## Moving Objects

You can orient a moving object with a single parallel transport rotation each time the object is moved, presumably once per frame. We need three pieces of information: the velocity of the object at the current and previous locations, and the orientation at the previous location. The velocities correspond to the tangents Tl and T2 shown previously.

For some tasks, the parallel transport frame may be too "stable." For example, an aircraft flying an S-shaped path on the horizontal plane would never bank. To achieve realistic-looking simulation of flight, you may need to use a different solution, such as simulating the physics of motion. Craig Reynolds describes a relatively simple, and thus fast, technique for orienting flocking "boids" that includes banking [Reynolds99]. Reynolds' technique is similar to parallel transport in that it also relies on "memory" of the previous frame.

## Comparison

The details here show how the parallel transport method we have looked at so far compares with the Frenet Frame and Fixed Up methods of curve framing.

### *The Frenet Frame*

The Frenet Frame is built from three orthogonal axes:

- The tangent of the curve
- The cross-product of the tangent, and the second derivative
- Another vector generated from the cross-product of the prior two vectors

The Frenet Frame is problematic for the uses already discussed because it cannot be calculated when the second derivative is zero. This occurs at points of inflection and on straight sections of the curve [Hanson95]. Clearly, not being able to calculate a frame on a straight section is a big problem for our purposes. In addition, the frame may spin, due to changes in the second derivative. In the case of an S-shaped curve, for example, the second derivative points into the curves, flipping sides on the upper and lower halves. The resulting Frenet Frames on the S-shaped curve will flip in consequence. Figure 2.5.2 shows what this means graphically; instead of continuous

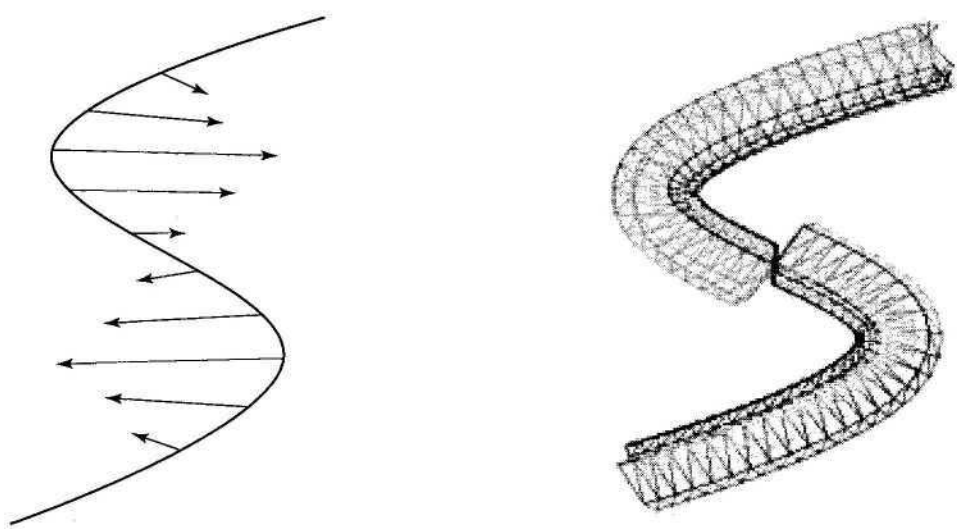

**FIGURE 2.5.2** *Second derivative on an S-shaped curve, and Frenet Frame generated tube from the same curve.*

geometry, we have a discontinuity where the second derivative switches sides. If this was a flock of birds, they would suddenly flip upside down at that point.

### *The Fixed Up Method*

In the case of the Fixed Up method, the tangent T and an arbitrary vector V (the Fixed Up vector) are used to generate three axes of the resulting frame, the direction D, up U, and right R vectors [EberlyOl].

$$D = T / |T|$$
$$R = D \times V / |D \times V|$$
$$U = R \times D$$

A problem with the Fixed Up method occurs when the tangent and the arbitrary vector chosen are parallel or close to parallel. When T and V are parallel, the cross-product of D and V is zero and the frame cannot be built. Even if they are very close, the twist of the resulting vector relative to the tangent will vary greatly with small changes in T, twisting the resulting frame. This isn't a problem if you can constrain the path—which may be possible for some tasks, like building the geometry of freeways, but may not be for others, like building the geometry of a roller coaster.

Figure 2.5.3 shows a comparison of a tube generated using parallel transport with one using the Fixed Up method. In the upper and lower sections of the curve, the cross-product of tangent and the Fixed Up vector is coming out of the page. In the middle section, it is going into the page. The abrupt flip causes the visible twist in the generated geometry.

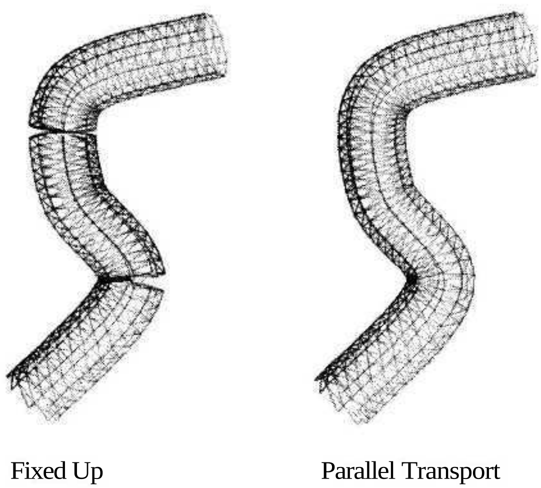


**FIGURE 2.5.3** *Comparison of Fixed Up and parallel transport.*

## Conclusion

For unconstrained paths—for example, flying missiles or looping tracks—parallel transport is one method that you can use to keep the tracks from twisting and the missiles from flipping.

## References

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