Curvature-Based Shading of Translucent Materials, such as Human Skin

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Abstract

The paper introduces a new approximate method for rendering translucent materials. We represent the surface around a point to be rendered in Monge's form using principal curvatures. The subsurface reflectance equation in the dipole diffusion approximation [Jensen et al. 2001] is then integrated over the surface. The outgoing radiance at the point is expressed as a function of principal curvatures and light vector components along the principal directions. This function can be precomputed as a 2D lookup table, which can be stored as a texture image. The paper presents preliminary results of our work on implementation of the model described.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

Keywords: subsurface scattering, rendering

1 Introduction

Light in translucent materials, such as human skin, marble, wax, fruits, undergoes significant subsurface scattering. Profound models for single and multiple scattering in such materials has been elaborated ([Hanrahan and Krueger 1993; Stam 2001] are examples). But early works on the topic had the shortcoming of assuming that light leaves surface at the same point where it entered it. This means that the same reflectance function, BRDF, is employed at different points of a translucent object, no matter what the geometry around a point is. However, subsurface scattering transfers light away from the entrance point, and the result depends on local geometry.

Adequate description of light reflection from translucent materials is given by the Bidirectional Surface Scattering Distribution Function (BSSRDF). Based on the light diffusion theory, Jensen et al. [2001] suggested the dipole-source model for BSSRDF. To integrate BSSRDF, they used a Monte-Carlo based approach, which is very expensive in terms of time and memory. Jensen and Buhler [Jensen and Buhler 2002] showed that the contribution of single scattering events to radiance is much smaller than that of multiple scattering. Using an hierarchical diffusion approximation, they achieved a better speed, but the approach does not allow real-time rendering.

Sloan et al. [2003] incorporated Jensen and Buhler's hierarchical diffusion approach in the method of pre-computed radiance transfer [Sloan et al. 2002]. Lighting can be changed interactively, and arbitrary object motion is possible, but pre-computing the radiance transfer vector, which includes calculating the subsurface scattering term with the hierarchical diffusion approach, is very time consuming. The pre-computation should be done for any particular shape, and therefore objects cannot be deformed interactively. Hao and Varshney [2004] separated the rendering pass from the surface integration of the main part of BSSRDF, subsurface light diffusion. They achieved good frame rates, but the object shape is fixed because the preprocessing stage involves integration over the mesh. Carr et al. [2003] and Lensch et al. [2002] employed resemblence of the surface integration in the subsurface reflection equation and a single radiosity gathering step. They demonstrated interactive rendering using contemporary desktop GPUs. Dachsbacher and Stamminger [2003] augment shadow maps with depth and irradiance information and take the dipole-source subsurface-scattering integral using a hierarchical filtering technique based on mip-maps. Mertens et al. [2003] render an irradiance map from the camera viewpoint and filter it using an importance sampling scheme. Banterle [2005] demonstrated impressive frame rates, but the approach is not physically based.

The diffusion approximation was extended to multilayered translucent materials by introducing the multipole model in Donner and Jensen [2005]. High quality of human-skin renderings obtained with this method was further improved by using the spectral scattering and absorption coefficients of skin components [Donner and Jensen 2006].

In this paper, we use the diffusion BSSRDF model [Jensen et al. 2001; Donner and Jensen 2005] and the idea of precomputing its surface integral at vertices of the polygon mesh of an object [Hao and Varshney 2004]. We assume illumination to be approximately constant over area of the order of the effective light scattering range and factor out the light vector components from *scattering neighborhood factors* (term introduced by Hao and Varshney [2004]). By representing the object surface using principal curvatures, we show that the remaining parts of scattering neighborhood factors become functions of curvatures. As a result, we express the outgoing radiance in terms of principal curvatures and light vector components using a two-variate function, which is precomputed and kept as three 2D lookup tables for red, green, blue colors in our implementation. Our shading model can be considered as a physically based variation of wrap lighting [Green 2004; Bredow 2002] for translucent materials and is simple enough to be used for real-time rendering with both desktop and embedded graphics hardware. We present here preliminary results of our investigation of the model and its implementation.

2 The Diffuse Model for Subsurface Scattering

Light reflection by translucent materials is described by BSSRDF, which relates the radiance L_o outgoing from a point x_o in a direction ω_o to the incident radiance L_i as [Nicodemus et al. 1977]

$$L_o(x_o,\omega_o) = \int_S \int_{\Omega^+(x_i)} L_i(x_i,\omega_i) \times \times S(x_i,\omega_i,x_o,\omega_o)(N_i\cdot\omega_i)\, d\omega_i dA(x_i) \quad (1)$$

Here, N_i is the normal to the surface at an entrance point x_i; ω_i is the incidence direction, S means the object surface around the point x_o, $\Omega^+(x_i)$ is the set of such directions ω_i that $N_i\cdot\omega_i > 0$.

The BSSRDF in the diffusion approximation is given by [Jensen

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et al. 2001]

$$S(x_i,\omega_i,x_o,\omega_o)=\frac{1}{\pi}F_t(N_o\cdot\omega_o)\,R_d(x_i,x_o)\,F_t(N_i\cdot\omega_i)\,. \tag{2}$$

where N_o is the normal to the surface at the exit point x_o, F_t is the Fresnel transmittance factor, $R_d(x_i,x_o)$ is the diffuse reflectance function, for which the dipole [Jensen et al. 2001] and multipole [Donner and Jensen 2005] approximation are successfully used. In this paper we use R_d in the dipole approximation - see [Jensen et al. 2001; Jensen and Buhler 2002] for detailed expressions. We do not indicate the dependence of the Fresnel transmittance on the refractive index explicitly for brevity.

3 Curvature-Based Precomputation of Scattering Neighborhood Factors

Inspired by Hao and Varshney [2004], we change the order of integration in the subsurface scattering equation and get

$$\begin{aligned}L_o(x_o,\omega_o) &= \int_\Omega\int_{S^+(\omega_i)} L_i(x_i,\omega_i)\,S(x_i,\omega_i,x_o,\omega_o)\times\\ &\times(N_i\cdot\omega_i)\,V(x_i,\omega_i)\;dA(x_i)\,d\omega_i \end{aligned}\tag{3}$$

where $S^+(\omega_i)$ is the surface part for which $N_i\cdot\omega_i>0$. $V(x_i,\omega_i)$ is the visibility function, which takes a value of 0 if the point x_i is shadowed in the direction ω_i by other parts of the surface (we consider only self-shadowing here) and 1 otherwise.

The diffuse subsurface scattering reflectance $R_d(x_i,x_o)$ falls off steeply at a distance of the effective light scattering range l_d, which is usually small. The domain of surface integration in 3 can be reduced to a few l_d in size. This reduced integration domain can be called *scattering neighborhood* [Hao and Varshney 2004]. Therefore, we assume that distances to light sources are much greater than l_d. We neglect variation of L_i over the scattering neighborhood and replace $L_i(x_i,\omega_i)$ with $L_i(x_o,\omega_i)$ in eq. 3.

We assume that the object surface can be represented (at least, locally) in Monge's form [Weisstein] as

$$x_3=f(x_1,x_2), \tag{4}$$

where $f(x_1,x_2)$ is a smooth function (if it is not, we can approximate it with such representation to some acceptable accuracy). Here x_j, $j=1,2,3$, are the components of a point x in three-dimensional space.

Noting that [Schey 1992]

$$dA(x)=\sqrt{1+\left(\frac{\partial f}{\partial x_1}\right)^2+\left(\frac{\partial f}{\partial x_2}\right)^2}dx_1dx_2$$

and

$$N=\frac{1}{\sqrt{1+\left(\frac{\partial f}{\partial x_1}\right)^2+\left(\frac{\partial f}{\partial x_2}\right)^2}}\cdot\left(-\frac{\partial f}{\partial x_1},-\frac{\partial f}{\partial x_2},1\right), \tag{5}$$

and substituting eq. 2 into 3, we arrive at the following formula for the radiance:

$$L_o(x_o,\omega_o) = \frac{1}{\pi}F_t(N_o\cdot\omega_o)\int_\Omega L_i(x_o,\omega)p(\omega)\,d\omega, \tag{6}$$

$$\begin{aligned}p(\omega) &= \int_{S^+(\omega)} R_d(x,x_o)\,F_t(N\cdot\omega)\,(-\omega_1\frac{\partial f}{\partial x_1}-\\ &-\omega_2\frac{\partial f}{\partial x_2}+\omega_3)V(x,\omega)\;dx_1dx_2,\end{aligned}\tag{7}$$

Here we omitted the subscript "i" for the point and direction of incidence and designate their components as x_j and ω_j, $j=1,2,3$, respectively. The integration domain $S^+(\omega)$ can now be described as the area where

$$-\omega_1\frac{\partial f}{\partial x_1}-\omega_2\frac{\partial f}{\partial x_2}+\omega_3>0.$$

To calculate the visibility function $V(x_1,x_2,\omega)$, we should find the point of intersection of the surface and the ray going from the surface point (x_1,x_2,x_3) in the direction ω. We substitute the components of a point on the ray, $x_1+\omega_1 t$, $x_2+\omega_2 t$, $x_3+\omega_3 t$, into eq. 4 to get

$$x_3+\omega_3 t=f(x_1+\omega_1 t,x_2+\omega_2 t). \tag{8}$$

The visibility function $V(x_1,x_2,\omega)$ is zero if eq. 8 has a positive solution t; it is one otherwise.

The function $p(\omega)$ is very similar to the scattering neighborhood factor of Hao and Varshney [2004]. The difference is that we assumed $L_i(x,\omega)$ to be constant in the scattering neighborhood and factor it out of the surface integral. This will help us to further simplify the subsurface scattering integration in the sequel.

Let us do a linear change of space coordinates such that x_o becomes the origin $(0,0,0)$, and the (x_1,x_2) plane is tangent to the object surface at $(0,0,0)$. The Taylor series expansion of $f(x_1,x_2)$ then begins from quadratic terms. Choosing the x_1 and x_2 axes along the principal curvature directions so that t_1 and t_2 are unit vectors in those directions, we get the following local surface representation

$$f(x_1,x_2)=\frac{1}{2}\left(k_1x_1^2+k_2x_2^2\right) \tag{9}$$

where k_1,k_2 are the principal curvatures, and we assume $k_1\geq k_2$. Equation 4 is accurate to the second order in x_1, x_2, but more terms of the Taylor expansion can be also taken into account, as discussed in section 5.

The length l_d is usually small in comparison with the dimensions of the object to be rendered. For example, l_d for human skin (tissue) is not more than a few millimeters. Therefore, we assume that l_d is much smaller than the object characteristic dimensions, such as curvature radiuses. In other words, we assume that the curvature radiuses k_1^{-1} and k_2^{-1} at each point are much greater than the effective light scattering range l_d - that is,

$$k_1l_d,k_2l_d\gg 1$$

The expression for the scattering neighborhood factor p with an accuracy of the first order in the small parameters k_1l_d, k_2l_d becomes

$$\begin{aligned}p(\omega,k_1,k_2) &= \int_{S^+(\omega)} R_d(x)F_t(N\cdot\omega)\,(-\omega_1k_1x_1-\\ &-\omega_2k_2x_2+\omega_3)V(x,\omega)\;dx_1dx_2,\end{aligned}\tag{10}$$

and $S^+(\omega)$ is defined now by the condition

$$-\omega_1k_1x_1-\omega_2k_2x_2+\omega_3>0.$$

Solving eq. 8 with $f(x_1,x_2)$ given by 9, we find the following. When k_1 or k_2 are negative (the surface is convex), the visibility function $V(x_1,x_2,\omega)$ is equal to 1 everywhere for any ω (which is trivial). When the surface is not convex (one of k_1, k_2 or both are positive), the surface is not illuminated at all from all (if both k_1, k_2 are positive) or some (if $k_1>0>k_2$) directions ω, defined by inequality $k_1\omega_1^2+k_2\omega_2^2>0$. This is not realistic because in reality only small local part of the surface is represented by equation

(5). Therefore, higher-order surface derivatives should be taken into account to get an accurate integral.

However, our experiments showed that a good practical solution is to replace a positive curvature (k_1 or k_2 or both, whichever happens to be positive) in the above equation for p with 0. This roughly approximates the effect of self-shadowing and makes it possible to remove the visibility function from the integral. Therefore, from now on we omit the visibility function $V(x_1, x_2, \omega)$ and assume that k_1 and k_2 are replaced with $\min(k_1, 0)$ and $\min(k_2, 0)$, respectively.

By rotating the system of coordinates around the x_3 axis at an angle α such that

$$\sin\alpha = \frac{-\omega_2 k_2}{\sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}}, \quad \cos\alpha = \frac{-\omega_1 k_1}{\sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}},$$

we get

$$\begin{aligned} p(\omega, k_1, k_2) &= \int_{-\infty}^{\infty}\int_{x_c}^{\infty} F_t\left(\xi^{-1}\left(\omega_3 + \sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}x_1\right)\right) \times \\ &\times \left(\omega_3 + \sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}x_1\right) R_d(x)\, dx_1 dx_2, \end{aligned}$$

where $x_c = -\omega_3/\sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}$ and ξ is a combination of k_j, ω_j, x_j, $j = 1, 2$ that comes from the normalizing factor of eq. 5. By numerical analysis, we found that in most cases F_t in the above expression can be replaced with its average value $\langle F_t \rangle$ without noticeable loss of accuracy. Thus, we reduce eq. 10 to

$$\begin{aligned} p(\omega, k_1, k_2) &= \frac{\alpha'}{4\pi^2} \times \\ &\times J(\omega_3, l_u\sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}, \sqrt{3(1-\alpha')}, B), \end{aligned}$$

where

$$J(a, b, \beta, B) = I(a, b, \beta) + I(a, Bb, B\beta),$$

and

$$\begin{aligned} I(a, b, \beta) &= \int_{-\infty}^{\infty}\int_{-a/b}^{\infty} (a + bx_1)\langle F_t \rangle \times \\ &\times (\beta + \frac{1}{\sqrt{1 + x_1^2 + x_2^2}})\frac{e^{-\beta\sqrt{1+x_1^2+x_2^2}}}{1 + x_1^2 + x_2^2}\, dx_1 dx_2 \end{aligned}$$

where $\beta = \sqrt{3(1-\alpha')}$, $B = 1 + \frac{4}{3}A$, and A, α' and l_u are defined in [Jensen et al. 2001; Jensen and Buhler 2002]. Then the expression for $L_o(\omega_o)$ becomes

$$\begin{aligned} L_o(\omega_o) &= \frac{\alpha'}{4\pi^2} F_t(N_o \cdot \omega_o) \int_\Omega L_i(\omega) \times \\ &\times J\left(\omega_3, l_u\sqrt{\omega_1^2 k_1^2 + \omega_2^2 k_2^2}\right) d\omega \qquad (11) \end{aligned}$$

where we omitted the explicit dependence on the material parameters B and α'.

When illumination comes from a number of light sources, such as point, directional and cone ones, the integral over ω becomes a sum over light sources.

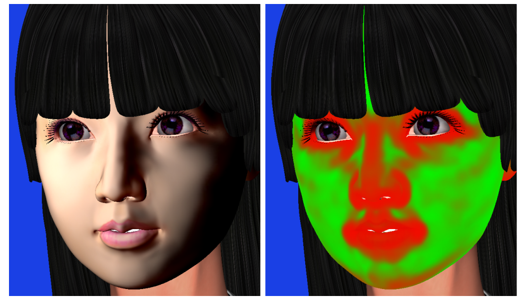

Figure 1: *A screenshot from an interactive application using the method proposed (left) and color visualization of curvature (right). Green and red colors correspond to small and big curvature, respectively. The neck is rendered using only texture. Model courtesy of Interactive Brains, Inc.*

4 Results

We implemented the proposed technique with an additional simplification: we ignore the azimuthal ω dependence of $L_o(\omega_o)$ in 11. That is, we replace the expression under the square root in eq. 11 with $(1-\omega_3^2)(k_1^2+k_2^2)/2$. We calculate per-vertex radiance in software (function I is precalculated and kept as 2D lookup table for each of the red, green, blue colors) and then do the Gouraud shading using an nVidia N7600GS hardware accelerator. The rendering time per frame was 0.45 sec for three point light sources and the face model with 44K polygons (Fig. 1) on an Intel IV 3 GHz PC. Notice that appearance is smooth despite the fact that curvatures were calculated from the mesh and therefore have some noise.

5 Future work

As future work, the suggested method should be implemented in full scope, without simplifications described in section 4 and with calculation of per-vertex values in a vertex shader. The function I will be precalculated and encoded in a texture image, which should be sampled by a pixel shader.

Our method can be generalized to higher-order surface derivatives. Then the above reasoning can be repeated, leading to expressions similar to eq. 11, but the corresponding functions will have more than two arguments.

In the case of low-frequency illumination, the scattering neighborhood factors can be represented in a spherical-harmonic basis in a way similar to that used in Hao and Varshney [2004].

6 Conclusions

In this paper we proposed an approximate method for precomputation of the surface integral in the subsurface scattering equation in terms of principal curvatures. We demonstrate that the approach can give plausible rendering of translucency by using a simplified version of our approach, which can be called "physically based wrap lighting." Implementing the method in full scope is one of the directions of further work.

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References

BANTERLE, F. 2005. A fast translucency appearance model. In *SIGGRAPH '05: ACM SIGGRAPH 2005 Posters*, ACM Press, New York, NY, USA, 41.

BREDOW, R. 2002. Renderman on film. In *SIGGRAPH 2002 Course Notes. Course 16.*, 6,7.

CARR, N. A., HALL, J. D., AND HART, J. C. 2003. Gpu algorithms for radiosity and subsurface scattering. In *HWWS '03: Proceedings of the ACM SIGGRAPH/EUROGRAPHICS conference on Graphics hardware*, Eurographics Association, Aire-la-Ville, Switzerland, Switzerland, 51–59.

DACHSBACHER, C., AND STAMMINGER, M. 2003. Translucent shadow maps. In *EGRW '03: Proceedings of the 14th Eurographics workshop on Rendering*, Eurographics Association, Aire-la-Ville, Switzerland, Switzerland, 197–201.

DONNER, C., AND JENSEN, H. W. 2005. Light diffusion in multi-layered translucent materials. In *SIGGRAPH '05: ACM SIGGRAPH 2005 Papers*, ACM Press, New York, NY, USA, 1032–1039.

DONNER, C., AND JENSEN, H. W. 2006. A spectral bssrdf for shading human skin. In *Proceedings of the Eurographics Symposium on Rendering*, 409–417.

GREEN, S. 2004. *GPU Gems: Programming Techniques, Tips, and Tricks for Real-Time Graphics*. Addison-Wesley Professional, March, ch. Real-Time Approximations to Subsurface Scattering, 264–266.

HANRAHAN, P., AND KRUEGER, W. 1993. Reflection from layered surfaces due to subsurface scattering. In *SIGGRAPH '93: Proceedings of the 20th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 165–174.

HAO, X., AND VARSHNEY, A. 2004. Real-time rendering of translucent meshes. *ACM Trans. Graph. 23*, 2, 120–142.

JENSEN, H. W., AND BUHLER, J. 2002. A rapid hierarchical rendering technique for translucent materials. *ACM Trans. Graph. 21*, 3, 576–581.

JENSEN, H. W., MARSCHNER, S. R., LEVOY, M., AND HANRAHAN, P. 2001. A practical model for subsurface light transport. In *SIGGRAPH '01: Proceedings of the 28th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 511–518.

LENSCH, H. P. A., GOESELE, M., BEKAERT, P., KAUTZ, J., MAGNOR, M. A., LANG, J., AND SEIDEL, H.-P. 2002. Interactive rendering of translucent objects. In *PG '02: Proceedings of the 10th Pacific Conference on Computer Graphics and Applications*, IEEE Computer Society, Washington, DC, USA, 214.

MERTENS, T., KAUTZ, J., BEKAERT, P., REETH, F. V., AND SEIDEL, H.-P. 2003. Efficient rendering of local subsurface scattering. In *PG '03: Proceedings of the 11th Pacific Conference on Computer Graphics and Applications*, IEEE Computer Society, Washington, DC, USA, 51.

NICODEMUS, F. E., RICHMOND, J. C., HISA, J. J., GINSBERG, I. W., AND LIMPERIS, T. 1977. *Geometrical Considerations and Nomenclature for Reflectance*. Monograph number 160. National Bureau of Standards.

SCHEY, H. M. 1992. *Div, Grad, Curl, and All That: An Informal Text on Vector Calculus*. W. W. Norton.

SCHLICK, C. 1994. An inexpensive brdf model for physically-based rendering. *Computer Graphics Forum 13*, 3, 233–246.

SLOAN, P.-P., KAUTZ, J., AND SNYDER, J. 2002. Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments. *ACM Trans. Graph. 21*, 3, 527–536.

SLOAN, P.-P., HALL, J., HART, J., AND SNYDER, J. 2003. Clustered principal components for precomputed radiance transfer. In *SIGGRAPH '03: ACM SIGGRAPH 2003 Papers*, ACM Press, New York, NY, USA, 382–391.

STAM, J. 2001. An illumination model for a skin layer bounded by rough surfaces. In *Proceedings of the 12th Eurographics Workshop on Rendering Techniques*, Springer-Verlag, London, UK, 39–52.

WEISSTEIN, E. W. Monge's form. MathWorld–A Wolfram Web Resource, http://mathworld.wolfram.com/MongesForm.html.