AJAY KUMAR GARG ENGINEERING COLLEGE, GHAZIABAD

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGG.

SOLUTION OF ST-2

SUBJECT- NETWORK ANALYSIS AND SYNTHESIS (REE-305)

BY

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SECTION-A

1. Laplace Transform is a powerful mathematical tool to convert the differential equation into algebraic equation.

The Laplace Transform of a time-function $x(t)$ is defined as

$$\mathcal{L}[x(t)] = X(s) \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt \quad \text{------- (i)}$$

where s is a complex variable and is equal to $s = \sigma + j\omega$

The Laplace transform defined as in equation (i) with $-\infty$ as lower limit for the integral is called two-sided or bilateral Laplace Transform.

If the lower limit is changed to '0' we get one-sided or unilateral Laplace transform

$$\mathcal{L}[x(t)] = \int_{0}^{\infty} x(t)\, e^{-st}\, dt$$

2. Initial value theorem :→ If $x(t)$ and its first derivative are Laplace transformable, then initial value of $x(t)$ is given by

$$x(0) = \lim_{t \to 0} x(t) = \lim_{s \to \infty} s X(s)$$

Final value theorem :→ If $x(t)$ and its first derivative are Laplace transformable, then the final value of $x(t)$ is given by

$$x(\infty) = \lim_{t \to \infty} x(t) = \lim_{s \to 0} s X(s)$$

3. Planar graph :→ A graph is said to be planar if it can be drawn on sheet of paper in such a way that no two branches of the graph cross each other.

Tree :→ It is defined as a connected subgraph of a connected graph which contains all the nodes of the original graph but does not contain any loop (closed path). In a tree of a graph there is one and only one path between every pair of nodes.

4. Duality in an electric network : Two networks N_1 and N_2 are said to be dual of each other, if mesh equations of N_1 and nodal equations of N_2. There is no dual relationship for the network having mutual inductances.

Some dual relationships are

Voltage $\longleftrightarrow$ Current

Resistors $\longleftrightarrow$ Conductors

Inductors $\longleftrightarrow$ Capacitors

KVL $\longleftrightarrow$ KCL

Mesh $\longleftrightarrow$ Node

Series $\longleftrightarrow$ Parallel

5. Bode Plot :→ A Bode Plot consists of two graphs: One is plot of the Logarithmic of magnitude of sinusoidal transfer function: the other is a plot of phase angle; both against frequency on a logarithmic scale.

Consider the network function

$$F(s) = \frac{N(s)}{D(s)} \quad ; \quad F(j\omega) = \frac{N(j\omega)}{D(j\omega)}$$

Magnitude $M(\omega) = |F(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|}$

$20 \log M(\omega) = 20 \log |N(j\omega)| - 20 \log |D(j\omega)|$ in db.

Phase angle $\phi(\omega)$ = Sum of phase angle of individual factors of $F(j\omega)$

SECTION-B

6. $x_1(t) = e^{-at} u(t)$

$$\mathcal{L}[x_1(t)] = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-st} \cdot dt = \int_0^{\infty} e^{-at} e^{-st} \cdot dt = \int_0^{\infty} e^{-(s+a)t} dt$$

This integral converges if $Re(s+a) > 0$

or, $\boxed{X_1(s) = \frac{1}{s+a}}$ ROC : i.e. $Re(s)\ \sigma > -a$

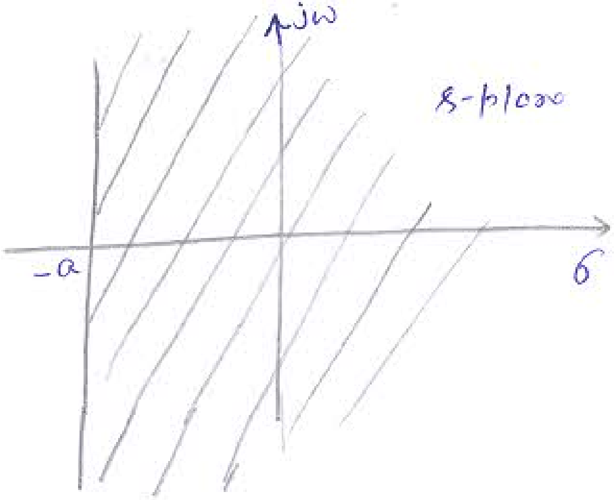
Now $x_2(t) = -e^{-at} u(-t)$

$$\mathcal{L}[x_2(t)] = \int_{-\infty}^{\infty} -e^{-at} u(-t) e^{-st} \cdot dt = -\int_{-\infty}^{0} e^{-(s+a)t} dt = -\int_{0}^{\infty} e^{(s+a)t} dt$$

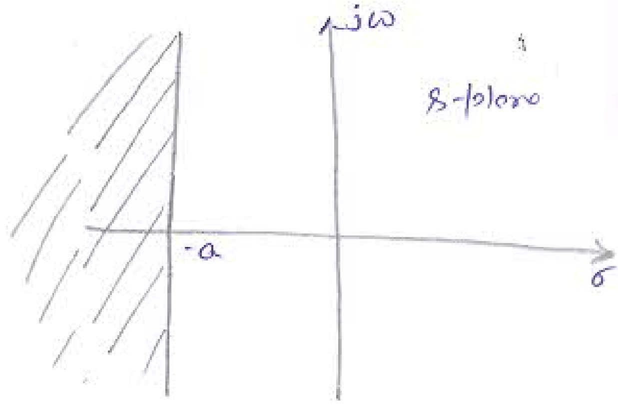
The above integral converges if $Re\ s+a < 0$ i.e. $Re(s) < -a$

$$\therefore X_2(s) = -\left[\frac{e^{(s+a)t}}{s+a}\right]_0^{\infty} = \frac{1}{s+a} \ ; \ ROC;\ Re(s)\ \sigma < -a$$

Here $\boxed{X_1(s) = X_2(s) = \frac{1}{s+a}}$

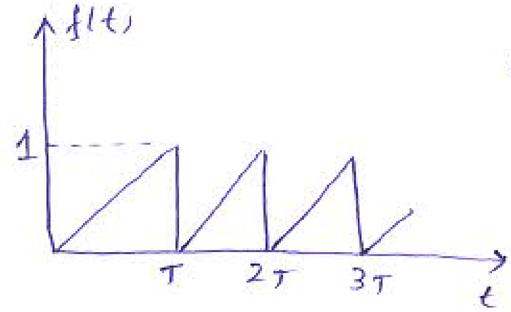


ROC of $X_1(s)$



ROC of $X_2(s)$

(7.)



This is a periodic function of Period T.

The Laplace Transform of a Periodic function $f(t)$ is given by

$$F(s) = \frac{1}{1-e^{-Ts}} F_1(s)$$

where $F_1(s)$ is Laplace Transform of first cycle $f_1(t)$

Now $f_1(t)$ is given by

$$f_1(t) = \frac{1}{T} r(t) - \frac{1}{T} r(t-T) - u(t-T)$$

$$= \frac{1}{T} t\,u(t) - \frac{1}{T}\left[(t-T)u(t-T)\right] - u(t-T)$$

$$F_1(s) = \frac{1}{T}\frac{1}{s^2} - \frac{1}{Ts^2} e^{-Ts} - \frac{1}{s} e^{-Ts}$$

$$= \frac{1}{Ts^2}\left[1 - e^{-Ts} - Ts\,e^{-Ts}\right]$$

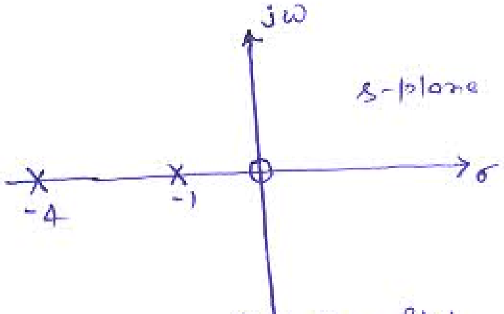
Hence $$F(s) = \frac{1}{1-e^{-Ts}}\left[\frac{1}{Ts^2}\left\{1 - e^{-Ts} - Ts\,e^{-Ts}\right\}\right]$$

(8) $$F(s) = \frac{3s}{(s+1)(s+4)}$$

Pole-Zero diagram

Zero at $s=0$

Poles at $s=-1$ and $s=-4$



Pole-Zero Plot

$$F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+4}$$

$$f(t) = K_1 e^{-t} + K_2 e^{-4t} \quad \text{------(i)}$$

$$K_1 = H_0 \frac{M_{01}}{M_{41}} e^{j(\phi_{01} - \phi_{41})}$$ Here $H_0 = 3$ $M_{01} = 1$, $M_{41} = 3$

$\phi_{01} = 180°$, $\phi_{41} = 0°$

$$K_1 = 3 \frac{1}{3} \times e^{j180°} = 1 \times (-1) = -1$$

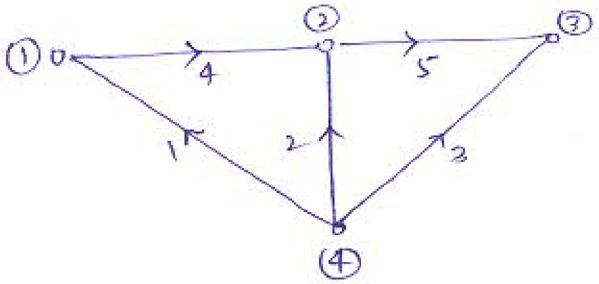
$$K_2 = H_0 \frac{M_{04}}{M_{14}} e^{j(\phi_{04} - \phi_{14})}$$ Here $H_0 = 3$ $M_{04} = 4$, $M_{14} = 3$

$\phi_{04} = 180°$ $\phi_{14} = 180°$

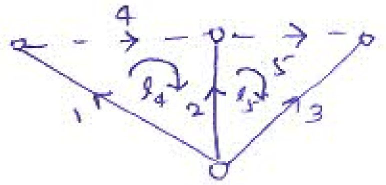
$$= \frac{3 \times 4}{3} = 4$$

$$\boxed{f(t) = (-1)\, e^{-t} + 4\, e^{-4t}}$$

(9.)

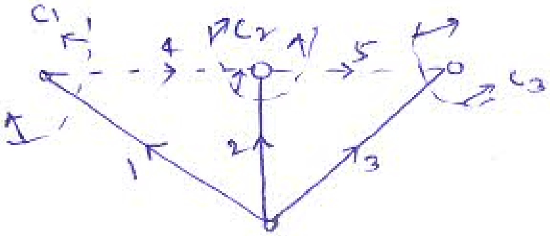


From question
1, 2 and 3 are tree branches



The basic loop matrix

$$B_f = \begin{array}{c} \\ l_4 \\ l_5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$



The basic cut-set matrix

$$Q_f = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(10)

(10) Routh-Hurwitz stability Criteria

The necessary and sufficient Condition, for a network to be stable, is that each term of first Column of Routh array of its characteristics equation of the network/system be positive if $a_0 > 0$. If this Condition is not met, the network/system is Unstable and number of sign changes of the terms of first Column of the Routh array Corresponds to the number of roots of the characteristics equation lie in the right half of the s-plane.

The given characteristic equation

$$Q(s) = 2s^5 + s^4 + 6s^3 + 3s^2 + s + 1$$

Routh array

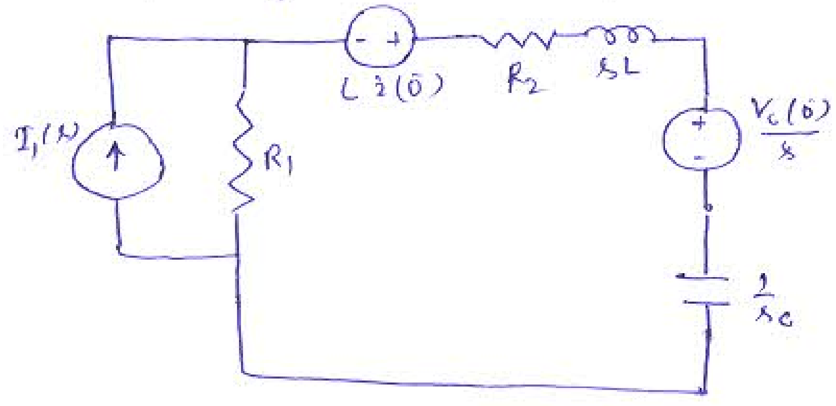
$$\begin{array}{c|ccc}
s^5 & 2 & 6 & 1 \\
s^4 & 1 & 3 & 1 \\
s^3 & \epsilon & -1 & \\
s^2 & \frac{3\epsilon+1}{\epsilon} & 1 & \\
s & \frac{-3\epsilon-1}{\epsilon} - \epsilon\Big/\frac{3\epsilon+1}{\epsilon} & & \\
s^0 & 1 & &
\end{array}$$

as $\epsilon \to 0$ the term $\frac{3\epsilon+1}{\epsilon}$ is positive

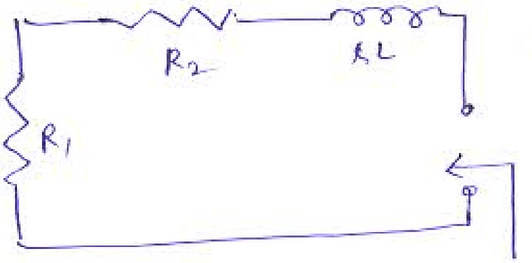
but $\frac{-3\epsilon-1-\epsilon^2}{\epsilon} \times \frac{\epsilon}{3\epsilon+1}$ becomes negative

Therefore, there are two changes in the sign terms of first Column of Routh array. Hence the system is unstable and two roots lie in right half of s-plane.

(11) **Superposition theorem** :→ If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal impedances

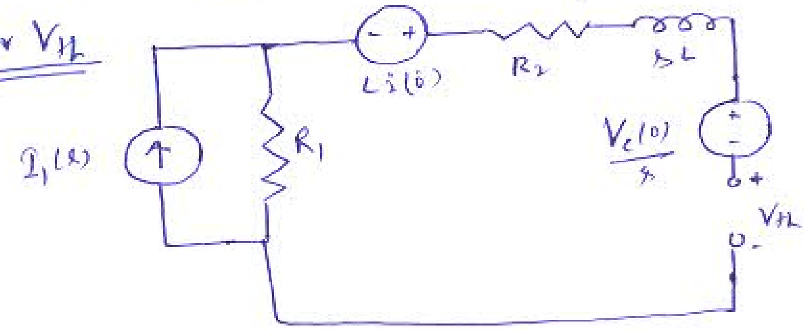


For Z_{th} : first we will remove the capacitor from the circuit and replace all independent sources by their internal impedances



$$Z_{th} = R_1 + R_2 + sL$$

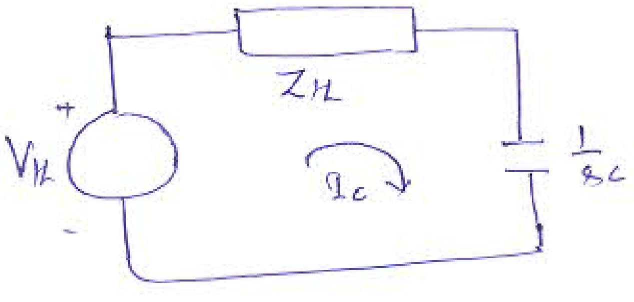
For V_{th}



Apply KVL for

$$I_1(s) R_1 + L\,i(0) - \frac{V_c(0)}{s} - V_{th} = 0$$

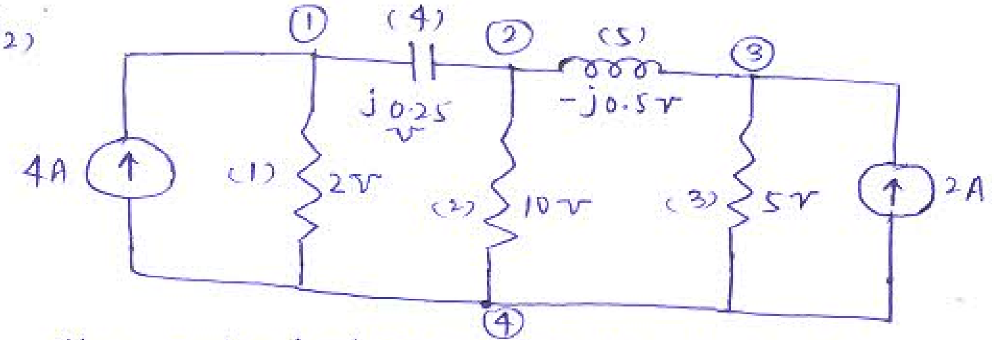
or, $\boxed{V_{th} = I_1(s) R_1 + L\,i(0) \mp \frac{V_c(0)}{s}}$



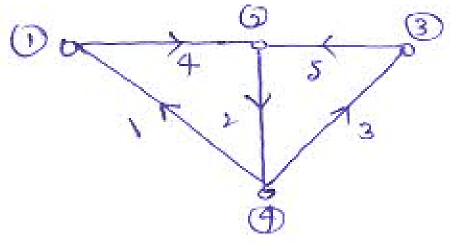
$$I_c = \frac{V_{th}}{Z_{th} + \frac{1}{sC}}$$

$$= \frac{I_1(s) R_1 + L\,i(0) - \frac{V_c(0)}{s}}{R_1 + R_2 + sL + \frac{1}{sC}}$$

(12)



The oriented graph of the network Taking node 4 as reference node



The reduced incidence matrix A

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array} ; \quad A^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Branch Admittance matrix Y_b

$$Y_b = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & j0.25 & 0 \\ 0 & 0 & 0 & 0 & -j0.5 \end{bmatrix} \end{array}$$

Now multiplication of A and Y_b matrices

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & j0.25 & 0 \\ 0 & 0 & 0 & 0 & -j0.5 \end{bmatrix}$$

$$A Y_b = \begin{bmatrix} -2 & 0 & 0 & j0.25 & 0 \\ 0 & 10 & 0 & -j0.25 & j0.5 \\ 0 & 0 & -5 & 0 & -j0.5 \end{bmatrix}$$

Now $A Y_b A^T = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10-j0.25 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix}$

We know that

$I_n = -\left(A Y_b V_s + A I_s \right)$

where symbols have their usual meaning

Since $A Y_b V_s = 0$

hence $I_n = -A I_s = -\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ (Taking current source entering the node positive)

Node equations in matrix form

$$\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10-j0.25 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where V_1, V_2, and V_3 are node voltages