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Department of ECE

Model Solution of ST-2

Course:	B.Tech	Semester:	V
Session:	2017-18	Section:	EC1, EC2 & EC3
Subject:	Antenna & Wave Propagation	Sub. Code:	NEC-504
Max Marks: 50		Time:	2 hour

Note : Answer **all** the sections.

Section-A

(1) Question: Define array factor and classify various types of array?

(1) Answer: The array factor is defined as the ratio of magnitude of the resultant electric field to the maximum electric field.

$$A.F. = \frac{|E_T|}{|E_{max}|}$$ where $|E_T|$ – Magnitude of resultant field, $|E_{max}|$ – Magnitude of maxi field

Antenna array may be classified in two ways

(i) Linear Array → Ex. Broadside array, End fire Array etc.

(ii) Non-Linear Array → Ex. Binomial array, Chebyshev array.

(2) Question: A linear broadside array consists of four equal isotropic point sources with $\frac{\lambda}{3}$ spacing. Calculate Directivity & HPBW.

(2) Answer: Given $n = 4$, $d = \lambda/3$

length $L = (n-1)d = 3 \times \frac{\lambda}{3} = \lambda$

$$\text{Directivity } (D) = \frac{2L}{\lambda} = \frac{2 \times \lambda}{\lambda} = 2$$

$$\boxed{D = 2}$$

$$HPBW \Rightarrow BWFN = \frac{114.6}{L/\lambda} = \frac{114.6}{\lambda/\lambda} = 114.6$$

$$HPBW = \frac{BWFN}{2} = 57.3°$$

$$\boxed{HPBW = 57.3°}$$

(3) Question: Write the differences between resonant antenna and non resonant antenna.

(3) Answer: Differences b/w resonant & non resonant antenna are following.

(i) The resonant antenna have standing waves along its length, while non-resonant antenna, the incident waves are absorbed terminated impedance then no reflected wave.

(ii) V antenna is example of resonant antenna while rhombic antenna is example of non-resonant antenna.

(4) Question: Determine the directivity of a loop antenna whose radius is 0.5 m when it is operating in 0.9 MHz.

(4) Answer: Given $r = 0.5\,m$, $f = 0.9\,MHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{0.9\times10^6} \simeq 333\,m$$

Then Directivity $(D) = 0.682\left(\frac{C}{\lambda}\right)$

where $C = 2\pi r$

$$D = 0.682\left(\frac{2\pi\times0.5}{333}\right)$$

$$D = 2.1415/333$$

$$\boxed{D \simeq 0.06}$$

(5) Question: Write down the design eq^n. of log-periodic antenna.

(5) Answer: Scale factor is designated by τ. Dipole length & spacing are related as

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \cdots = \frac{R_n}{R_{n+1}} = \tau$$

$$\frac{L_1}{L_2} = \frac{L_2}{L_3} = \frac{L_3}{L_4} = \cdots = \frac{L_n}{L_{n+1}} = \tau$$

$$\& \quad \frac{S_1}{S_2} = \frac{S_2}{S_3} = \frac{S_3}{S_4} = \cdots = \frac{S_n}{S_{n+1}} = \tau$$

In general $\frac{S_{n+1}}{S_n} = \frac{L_{n+1}}{L_n} = \frac{R_{n+1}}{R_n} = \frac{1}{\tau} = K$

Spacing factor $(\sigma) = \frac{R_{n+1} - R_n}{2L_{n+1}}$

SECTION - B

(6) Question: A uniform linear array of 16 isotropic point sources with a spacing $\frac{d}{4}$. If the phase difference is $-90°$; Calculate (i) HPBW (ii) Beam Solid Angle (iii) Directivity (iv) Effective Aperture.

(6) Answer: Given $\delta = -90°$ (phase difference), $n = 16$

$d = \lambda/4$

$\because \delta = -90°$ so it is end fire array

$L = (n-1)d = \frac{15\lambda}{4}$

(i) $HPBW = \frac{57.3}{\sqrt{L/2\lambda}} = \frac{57.3}{\sqrt{\frac{15\lambda}{4 \times 2\lambda}}} = \frac{57.3}{\sqrt{15/8}}$

$HPBW = \frac{57.3}{1.8} = 41.82$

$HPBW \simeq 41.82°$

Directivity $(D) = \frac{4L}{\lambda} = \frac{4 \times 15 \times \lambda}{4 \times \lambda}$

$D = 15$

Effective Apature $(A_e) = \frac{D\lambda^2}{4\pi} = \frac{15 \cdot \lambda^2}{4\pi}$

$A_e = 1.194\lambda^2$

Beam solid angle $(\Omega_A) = \frac{4\pi}{D} = \frac{4\pi}{15}$

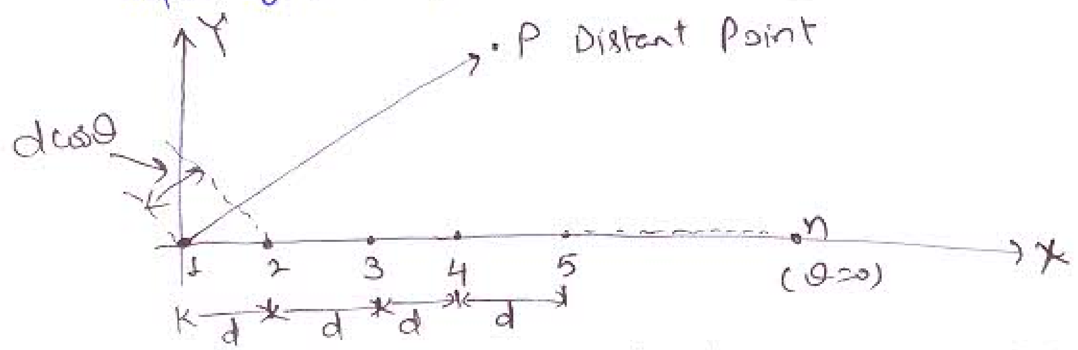
$\Omega_A = 0.845\ Sr$

$\boxed{HPBW = 41.82° \simeq 42°,\ D = 15,\ \Omega_A = 0.845\ Sr,\ A_e = 1.194\lambda^2}$ Ans

(7) Question: Derive the total field of linear array with n-isotropic point sources.

(7) Answer: If the individual elements of the array are spaced equally along a line & are fed with currents of equal amplitude & having a uniform progressive phase shift, then this array is called a linear array.

Fig. (a) shows a linear array of n elements with equal spacing (d) & fed with equal amplitude of current (E_0)



linear array with n-isotropic point sources with equal amplitude & spacing

The total far field (E_T) at a distant point P can be obtained by adding vectorially the fields of individual sources.

So $$E_T = E_0 e^{0j\psi} + E_0 e^{1j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + \cdots + E_0 e^{j(n-1)\psi} \quad -①$$

$$E_T = E_0 (1 + e^{j\psi} + e^{2j\psi} + \cdots + e^{(n-1)j\psi}) - ②$$

or $$E_T = E_0 \sum_{n=1}^{n} e^{(n-1)j\psi}$$

where E_T = Total far field

ψ = ~~total~~ phase difference of field at point P from adjacent sources = $(\beta d \cos\theta + \delta)$ rad.

d = spacing b/w sources.

δ = phase difference in adjacent point sources

It is seen that field from sources 2, 3, 4 ---etc. are leading in phase by angles $\psi, 2\psi, 3\psi$ ---etc. respectively assuming source one as phase centre

Multiply eqn ② by $e^{j\psi}$

$$E_T \cdot e^{j\psi} = E_0 [e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots + e^{nj\psi}] - ③$$

Substract eqn ③ from eqn ②

$$E_T (1 - e^{j\psi}) = E_0 [1 - e^{jn\psi}]$$

$$E_T = E_0 \frac{[1 - e^{jn\psi}]}{[1 - e^{j\psi}]}$$

or $$E_T = E_0 \left[\frac{1 - e^{jn\frac{\psi}{2}} \cdot e^{jn\frac{\psi}{2}}}{1 - e^{j\frac{\psi}{2}} \cdot e^{j\frac{\psi}{2}}} \right]$$

$$E_T = E_0 \left[\frac{(-e^{jn\frac{\psi}{2}})(e^{jn\frac{\psi}{2}} - e^{-jn\frac{\psi}{2}})}{(-e^{j\frac{\psi}{2}})(e^{j\psi/2} - e^{-j\psi/2})} \right]$$

$$E_T = E_0 e^{j(n-1)\frac{\psi}{2}} \cdot \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}$$

or $$E_T = E_0 \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right] \cdot e^{j\phi}$$

where $\phi = \left[\frac{n-1}{2}\right]\psi$

$$\boxed{E_T = E_0 \left[\frac{\sin(\frac{n\psi}{2})}{\sin(\frac{\psi}{2})} \right] \angle \phi}$$

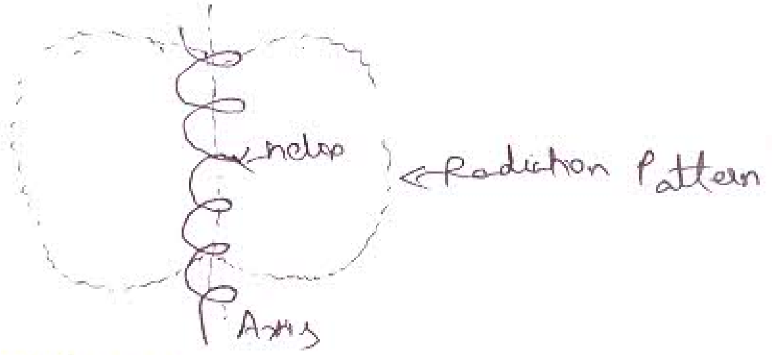
This eqn gives the total far field pattern of linear array of n- isotropic point source when source 1 is taken as reference point for phase.

(Q) Question: Classify the modes of operation of helical antenna and design a helical antenna operating in axial mode that gives a directivity of 14 dB at 2.4 GHz. For this helical antenna calculate the i/p impedance, HPBW, BWFN, and the axial ratio.

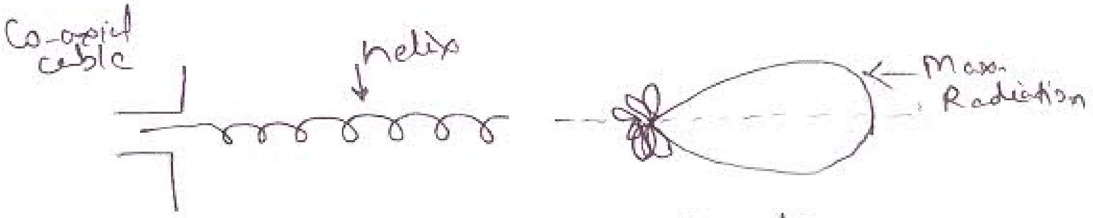
(Q) Answer: There are two modes of operation in helical antenna

(i) Normal Mode of operation → In this mode, the radiation is max in a direction normal to helix axis. This modes occurs when the dimensions of helix are small as compared with wavelength

$$\boxed{NL << \lambda}$$



(ii) Axial Mode →



Axial Mode of helical antenna

This mode provides max radiation along the axis of helix. i.e. there only a one major lobe.

This mode occurs when circumference & spacing are order of one wavelength

$$\boxed{C = \lambda \quad \& \quad S = \lambda/4}$$

Given: $f = 2.4$ GHz, $D = 14$ dB

then $\lambda = \frac{c}{f} = \frac{3\times10^8}{2.4\times10^9} = 0.125$ m for axial mode

Let $C = \frac{3}{4}\lambda$ (assumption)

$$C = \frac{3}{4} \times 0.125 = 0.09375 \text{ m}$$

The helical parameters are given by

Input Impedance $Z_i = 140 \frac{C}{\lambda}$

$$HPBW = \frac{52\lambda^{3/2}}{C\sqrt{L}}$$

$\because L = NS$

$$BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}$$

$$AR(\text{Axial Ratio}) = 1 + \frac{1}{2N}$$

$$\text{Directivity}(D) = \frac{15NSC^2}{\lambda^3}$$

$$D = 25.11$$

I/P Impedance $Z_i = 140 \frac{C}{\lambda} = \frac{140 \times 0.09375}{0.125}$

$$Z_i = 105\,\Omega$$

$$HPBW = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}}$$

Let $NS = 10$

$$HPBW = \frac{52}{0.09375}\sqrt{\frac{(0.125)^3}{10}} = 7.75^\circ$$

$$BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}} = \frac{115}{0.09375}\sqrt{\frac{(0.125)^3}{10}}$$

$$= 17.14^\circ$$

$$A.R. = 1 + \frac{1}{2 \times 3} \quad \because \text{let } N = 3$$

$$AR = 1.167$$

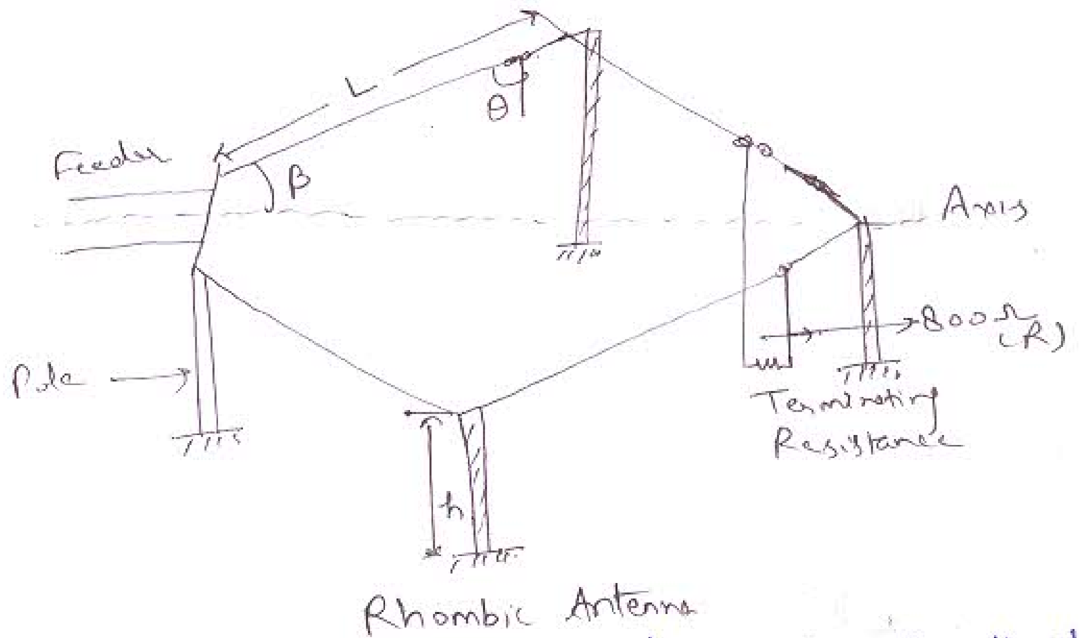
So

$$\boxed{Z_i = 105\,\Omega,\ HPBW = 7.75^\circ,\ BWFN = 17.14^\circ,\ AR = 1.167}$$

(9) Question: Explain Rhombic antenna & Horn antenna and also mention their applications.

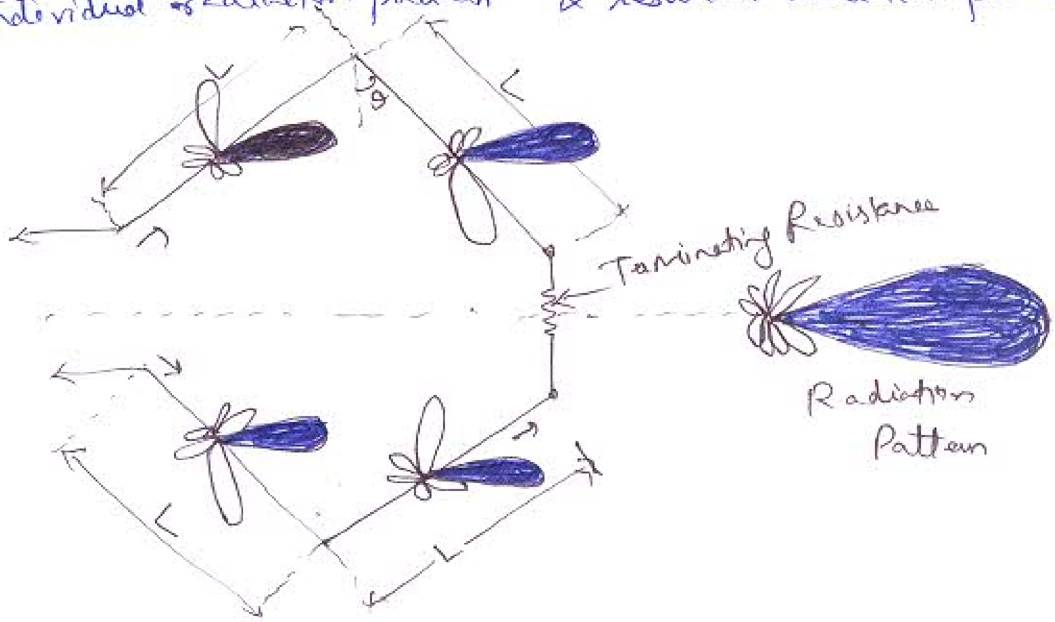
(9) Answer: Rhombic Antenna → It is non-resonant antenna & capable of operating over the entire range of frequencies of HF Band (3-30 MHz) for transmission or Reception.

It consist of four conductors joined to form a rhombus or diamond shape as shown below. The four arms of this antenna terminated in its characteristic impedance R. The Terminating resistance is often in the range of 800Ω & i/p impedance varies from 650 to 700Ω



Rhombic Antenna

The side of rhombus, the angle b/w the sides, the elevation and height etc. decide the radiation characteristic. Fig below shows the individual radiation pattern & resultant radiation pattern



height $h = \frac{\lambda}{4\sin\alpha}$ & $L = \frac{\lambda}{2\cos^2\theta}$

Applications: Used for HF application like Television, Telegraph, amateur radio, ship to coast & ship to aircraft communication

Horn Antenna → The propagation through WG have two difficulties

(i) The open end is a discontinuity which does not match the waveguide impedance & free space impedance

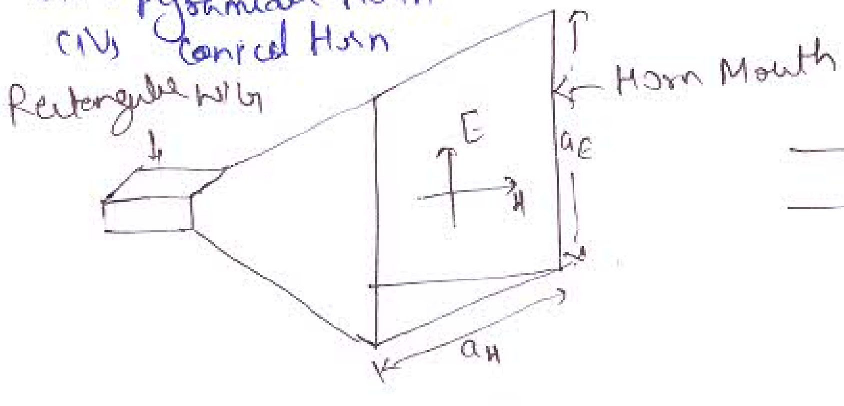
(ii) The diffraction around the edges will give poor radiation & a non-directive pattern

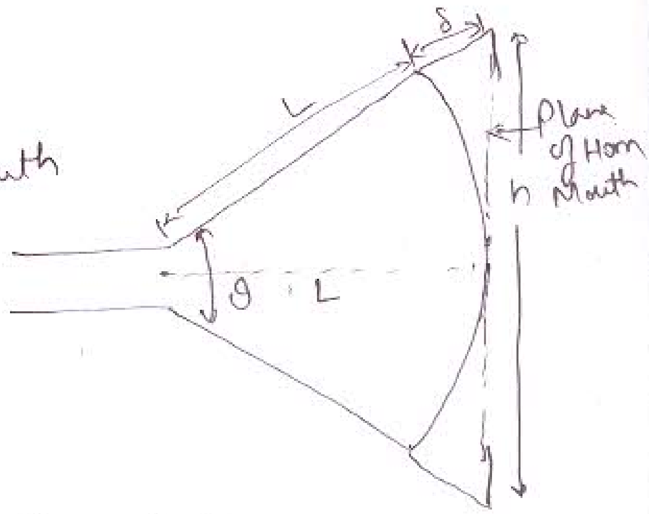
In order to improve the above difficulties the mouth of the waveguide is flared out (tapered) in the form of a horn. This flared is called a horn.

Horn antenna is simply a radiating element having the shape of a horn. The horn is a hollow pipe of different cross sections which are tapered (flared) to a large opening.

There are various types of horn antenna

(i) Sectorial E plane Horn
(ii) Sectorial H plane Horn
(iii) Pyramidal Horn
(iv) Conical Horn





Geometry of Horn Antenna

Length $L = \frac{h^2}{8\delta}$

HPBW; $\theta_E = \frac{56\lambda}{h}$ degree, $\theta_H = \frac{67\lambda}{W}$ degree

Directivity. $D = \frac{7.5Wh}{\lambda^2} = \frac{7.5A}{\lambda^2}$

Power Gain $G_p = \frac{4.5hW}{\lambda^2} = \frac{4.5A}{\lambda^2}$

Application → (i) They are used as primary antenna for paraboloids
(ii) Extensively used in microwave region

(10) Question → Write down the applications of loop antenna & explain how loop antenna acts as a direction finder.

(10) Answer → Loop antenna application are following
(i) Radio Receivers
(ii) U.HF Transmitters
(iii) Direction finder
(iv) Determination of sense.

Loop Antenna as a direction finder → Loop antenna has a directional properties where as a simple vertical antenna has not.

A loop antenna of area $A = lb$ used as a receiver shown in fig. (a)

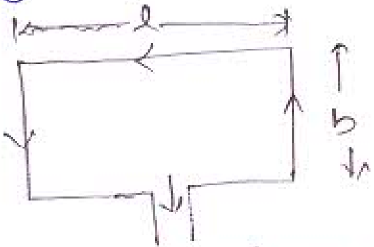


fig. (a) Loop antenna as direction finder

In order to determine the direction of unknown transmitter, a vertical loop antenna capable of rotating about vertical axis and attached with receiver is used.

The antenna is rotated through 360°. There are two maximas at $\theta = 0°$ & $180°$ and two minima at $\theta = 90°$ & $270°$. Thus when the coil is rotated maximum signal is heard in receiver when the loop antenna coil is set along the direction of transmitter. On the other hand, no signal is heard when the loop is right angle to the direction of incoming waves

The loop antenna ~~to~~ when used as a direction finder is unable to distinguish b/w bearing of a distant transmitter and its reciprocal bearing. The reason is that when the loop is rotated to either ~~a~~ null or maximum signal position, there is uncertainty whether the signal is arriving from left ~~to the~~ or right, front or back directions of the loop.

There are two types of direction finder

(i) Adcock direction finder

(ii) Bellini - Tosi direction finder. → It use radio-goniometer for obtaining bearing.

SECTION- C

(11) Question → Write the far field for $\lambda/4$ & $\lambda/2$ and also derive the radiation resistance for $\lambda/2$ antenna.

(11) Answer → The far field $H\phi$ & $E\theta$ of a symmetrical, centre fed, thin linear antenna of length are given as:

$$H\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos\left(\frac{\beta L\cos\theta}{2}\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right] \quad \text{—(A)}$$

$$\& \quad E\theta = \frac{j60[I_0]}{r}\left[\frac{\cos\left(\frac{\beta L\cos\theta}{2}\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right] \quad \text{—(B)}$$

where r → distant to distant point P

I_0 → peak value of current

Field for $\frac{\lambda}{4}$ antenna → By putting $L = \lambda/4$ in eqn (A) & (B)

$$H\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right) - \frac{1}{\sqrt{2}}}{\sin\theta}\right]$$

$$E\theta = \frac{j60[I_0]}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right) - 1/\sqrt{2}}{\sin\theta}\right]$$

For Half wave ($\lambda/2$) fields are

$$H\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]$$

& $$E_\theta = \frac{j60[I_0]}{r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]$$

where E_θ & H_ϕ are in time phase.

The max value of Poynting vector S_{max}

$$S_{max} = |E_\theta|\,|H_\phi|$$

The avg value of Poynting vector will be

$$S_{av} = \frac{E_\theta}{\sqrt{2}} \times \frac{H_\phi}{\sqrt{2}} = \frac{1}{2}|E_\theta|\,|H_\phi|$$

Now $$S_{av} = \frac{1}{2}\,\frac{60[I_0]}{r}\cdot\frac{[I_0]}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$S_{av} = \frac{15 I_0^2}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

The current distribution is not uniform

so $\frac{I_0}{\sqrt{2}} = I_{rms}$, S_{av} becomes

& $$S_{av} = \frac{15(\sqrt{2}\,I_{rms})^2}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$S_{av} = \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos^2(\pi/2\cos\theta)}{\sin\theta}\right]^2 \left(\frac{W}{m^2}\right)$$

Now radiated power for $\lambda/2$ antenna is

$$P = \int_0^{2\pi}\int_0^{\pi} S_{av}\,ds \quad \text{where } ds = r^2\sin\theta\, d\theta\, d\phi$$

$$P = \int_0^{2\pi}\int_0^{\pi}\frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\right]\cdot r^2\sin\theta\, d\theta\, d\phi$$

$$P = 60 I_{rms}^2\int_0^{\pi}\frac{\cos^2(\pi/2\cos\theta)}{\sin\theta}\,d\theta \quad \left[\because \int_0^{2\pi} d\phi = 2\pi\right]$$

$$P = 60 I_{rms}^2\int_0^{\pi}\left[\frac{1+\cos(\pi\cos\theta)}{2\sin\theta}\right]d\theta$$

$$\boxed{P = 60 I_{rms}^2 \times 1.219} \quad \text{—} \ (A)$$

Now we know that $P = I_{rms}^2 \times R_r$ — (B) for a antenna

where $R_r \rightarrow$ Radiation Resistance

compare eq^n (A) & (B)

$R_r = 60 \times 1.219$

$$\boxed{R_r = 73\Omega}$$

This is the radiation resistance for $\frac{\lambda}{2}$ antenna.

(12) Question → Derive the directivity of circular loop antenna & prove directivity is 3/2 for small loop antenna

(12) Answer → The directivity (D) of an antenna is defined as the ratio of maximum radiation intensity to average radiation intensity

$$D = \frac{\text{Max. Radiation Intensity}}{\text{Avg. Radiation Intensity}}$$

For a loop antenna, the max radiation intensity is given by

$= S_r \times R^2$ — (A)

where S_r is poynting vector for a far field of loop antenna is given by

$$S_r = \frac{15\pi (\beta r I_0)^2 J_1^2 (\beta r \sin\theta)}{R^2}$$

where $R \rightarrow$ distance to field point

$J_1 \rightarrow$ Bessels fu^n of first order

$r \rightarrow$ radius of loop antenna

from eq^n (A)

$$= S_R \times R^2 = \frac{15\pi (\beta r I_0)^2 J_1^2 (\beta r \sin\theta)}{R^2} \cdot R^2$$

$$= 15\pi (\beta r I_0)^2 J_1^2 (\beta r \sin\theta) \quad \text{— (B)}$$

Avg. Radiation Intensity is $\frac{P}{4\pi}$

Now $\frac{P}{4\pi} = \dfrac{30\pi^2 (\beta r I_0)^2 \int_0^{\pi} J_1^2(\beta r \sin\theta) \sin\theta \, d\theta}{4\pi}$ — (C)

where P is radiated power of loop antenna which is given by $P = 30\pi^2 (\beta r I_0)^2 \int_0^{\pi} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta$

Now from eq^n (1)

$$D = \frac{15\pi(\beta r I_0)^2 J_1^2(\beta r \sin\theta)}{\left[\dfrac{30\pi^2 (\beta r I_0)^2 \int_0^{\pi} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta}{4\pi}\right]}$$

$$D = \frac{2 J_1^2(\beta r \sin\theta)}{\int_0^{\pi} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta}$$

$$D = \frac{2 J_1^2(\beta r \sin\theta)}{\frac{1}{\beta r}\int_0^{2\beta r} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta}$$

$$D = \frac{2\beta r J_1^2(\beta r \sin\theta)}{\int_0^{2\beta r} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta}$$

$$D = \frac{2(C/\lambda)\left[J_1^2\{(C/\lambda)\sin\theta\}\right]}{\int_0^{2C/\lambda} J_1^2\{(C/\lambda)\sin\theta\}\sin\theta \, d\theta} \quad \because (\beta r = \tfrac{C}{\lambda})$$

$$\boxed{D = \frac{2(C/\lambda)\left[J_1^2((C/\lambda)\sin\theta)\right]}{\int_0^{2C/\lambda} J_2(y)\, dy}}$$

where $J_2(y) = J_1^2\{\frac{C}{\lambda}\sin\theta\}\sin\theta$

For small loop $\left(\frac{C}{\lambda} < \frac{1}{3}\right)$

$$D = \frac{2 J_1^2(\beta r \sin\theta)}{\int_0^{\pi} J_1^2(\beta r \sin\theta)\sin\theta \, d\theta} \quad \text{— (D)}$$

for small loop $J_1(x) \simeq x/2$

$$J_1^2(\beta r \sin\theta) \simeq \left(\frac{\beta r \sin\theta}{2}\right)^2$$

put this value in eq^n (D)

$$D = \frac{2\left(\frac{\beta r \sin\theta}{2}\right)^2}{\int_0^{\pi}\left(\frac{\beta r \sin\theta}{2}\right)^2 \sin\theta \, d\theta}$$

$$= \frac{2 \sin^2\theta}{2\int_0^{\pi} \sin^3\theta \, d\theta}$$

$$= \frac{2 \times 1}{2 \cdot 2/3} = 3/2$$

$\therefore$ [$\sin^2\theta = 1$ for $\theta = 90$ in loopantenna]

for small loop

$$\boxed{D = \frac{3}{2}}$$ hence proved