Ajay Kumar Garg Engineering College, Ghaziabad
Department of Mechanical Engineering

SESSIONAL TEST-II

Course: B. Tech
Session: 2017-18
Semester: VII
Section: CE-1,2 ECE-1,2,3 EI-1, EN-1,2
Subject: Operations Research
Sub. Code: NOE-073
Max. Marks: 50
Time: 2 hour

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Section-A

Q1. what do you mean by critical path?

Ans (1) Critical path in a network diagram is a path on which all the activities have zero float. All such activities, having zero float, put in an increasing sequence of nodes make a critical path.

Q2. Explain various types of time used in PERT.

Ans (2) The different times are

t_m → It is the normal or moderate time period of complete execution of an activity.

t_o → It is an optimistic approach about completion time of an activity.

t_p → It is the time given for completion of an activity in which all the adverse calls are considered. It is a pessimistic approach for activity timings.

Q3. What do you mean by unbalanced & balanced transportation problem?

Ans (3) A transportation problem is said to be balance if total supply is equal to total demand and vice versa for unbalanced transportation problem.

By adding a dummy demand or supply of shortage amount an unbalanced problem is converted into a balanced problem.

Q4. What are the direct & indirect cost?

Ans (4) Direct costs are the cost of labours, cost of resources, general expenses.

Indirect cost involves office overheads, cost of visits by supervisors or concerned higher authorities. It is calculated for the entire project duration.

Q5. Excluding CPM & PERT what are the other network techniques you know? Explain any one.

Ans (5) The different networks techniques are minimal spanning tree, shortest path problem, maximal flow etc.

Maximal flow is used where some real life situations, which makes us plan to reschedule the given flow in a network, may be an electrical network, water lines etc. All such situations demand for timely changes, modifications and expansions.

Section-B

Q. (6) Find the optimal assignment so that the total working time to make five jobs is min.

T \ M	M_1	M_2	M_3	M_4	M_5
T_1	7	8	9	9	8
T_2	8	9	11	10	9
T_3	7	8	10	8	9
T_4	10	9	8	9	10
T_5	10	8	9	11	9

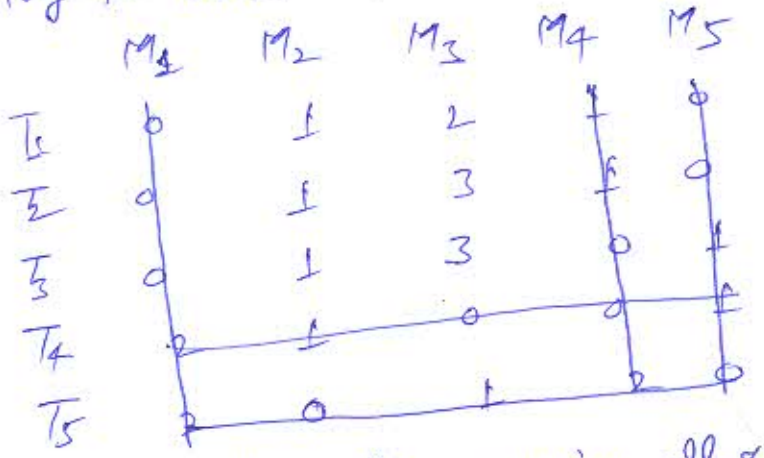
Ans (6) Step 1. Subtract the min of each row from all the entries of row.

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	2	1
T_2	0	1	3	2	1
T_3	0	1	3	1	2
T_4	2	1	0	1	2
T_5	2	0	1	3	1

Step 2. Subtract the min of each column from all the entries of above matrix.

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	1
T_4	2	1	0	0	1
T_5	2	0	1	2	0

Step 3. Try to cover all zeros with min. no of lines.



Step 4. There are five lines covering all zeros. The matrix at the end of step 3 is ready for assignment.

Step 5. There is no single zero in any row. We search for a single zero in columns. We get one in the second column. Mark that and delete the row.

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	1
T_4	2	1	0	0	1
T_5	2	0*	1	2	0

Again look for a single zero, we find in third column.

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	0
T_4	2	1	0	0	1
T_5	2	0	1	2	0

Again we have a single zero in fourth column.

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	0
T_4	2	1	0	0	1
T_5	2	0	1	2	0

Now, at this stage we do not get a single zero (the first of the second row, the first of the fifth column contain two zero). Any zero can be arbitrarily selected. We select cell (1,5).

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	1
T_4	2	1	0	0	1
T_5	2	0	1	2	0

Finally a single zero is found in the cell (2,1).

	M_1	M_2	M_3	M_4	M_5
T_1	0	1	2	1	0
T_2	0	1	3	1	0
T_3	0	1	3	0	1
T_4	2	1	0	0	1
T_5	2	0	1	2	0

We have the final answer.

Technician	Machine	Job Time (hrs)
T_1	M_5	8
T_2	M_1	8
T_3	M_4	8
T_4	M_3	8
T_5	M_2	8 = 40 hours.

Q. (7) Solve the transportation problem by VAM method.

From \ To	W_1	W_2	W_3	W_4	Supply
F_1	3	5	2	2	90
F_2	8	9	2	5	80
F_3	2	8	9	8	90
Demand	50	65	65	70	

Ans (1) Since demand = 250 & supply = 260
we have to add a dummy demand = 10 units

	W_1	W_2	W_3	W_4	W_5	Supply
F_1	3	5	2	2	0	90
F_2	8	9	2	5	0	80
F_3	2	8	9	8	0	90
Demand	50	65	65	70	10	

Step 1. We find penalty for each row & column. Select the highest penalty and make an allocation in the least cost cell.

	W_1	W_2	W_3	W_4	W_5	Supply	Penalty
F_1	3	5	~~90~~ 2	[70] 2	0	90/20	2
F_2	8	9	2	5	0	80	2
F_3	2	8	9	8	0	90	2
Demand	50	65	65	70/0	10		
Penalty	1	3	0	↑3	0		

Step 2. We repeat this process and make remaining allocations

	W_1	W_2	W_3	W_4	W_5	Supply	Penalty				
F_1	3	20 5	2	70 2	0	90/20/0	2	2	-	-	-
F_2	8	9	2	5	0	80/15/5/0	2	2	2	8 ←	1
F_3	2	8	9	8	0	90/40/0	2	2	2	2	6
Demand	50/0	65/45/40/0	60/0	70/0	10/0						
Penalty	1	3	0	3↑	0						
	1	3↑	0	—	0						
	6	1	7↑	—	0						
	6	1	—	—	0						
	6↑	1	—	—	—						

This completes the procedure. Now we calculate the cost of transportation.

Allocation	Unit Cost	Cost
$x_{12} = 20$	5	100
$x_{14} = 70$	2	140
$x_{22} = 05$	9	45
$x_{23} = 65$	2	130
$x_{25} = 10$	0	00
$x_{31} = 80$	2	160
$x_{32} = 40$	8	320
		835

Q.8(i) Write down the complete mathematical model of the following problem.

Ans 8(i). Min $Z = 10x_{11} + 2x_{12} + 20x_{13} + 11x_{14} + 12x_{21} + 7x_{22} + 9x_{23} + 20x_{24} + 4x_{31} + 14x_{32} + 16x_{33} + 18x_{34}$

Subjected to,

$$x_{11} + x_{12} + x_{13} + x_{14} = 15$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 25$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 10$$
$$x_{11} + x_{21} + x_{31} = 5$$
$$x_{12} + x_{22} + x_{32} = 15$$
$$x_{13} + x_{23} + x_{33} = 15$$
$$x_{14} + x_{24} + x_{34} = 15$$

$x_{ij} \geq 0$; $i = 1, 2, 3$ & $j = 1, 2, 3$ & 4.

Q.8(ii). Explain the different features of activities while making a network diagram.

Ans 8(ii). - An activity generally denoted using capital letters like A, B, C, ---etc.

- To an activity there is always a predecessor activity, except those activities which begin from initial node.
- To an activity, there is always a follower activity. The last event is no like such one.
- There can be one and only one activity between any two nodes.

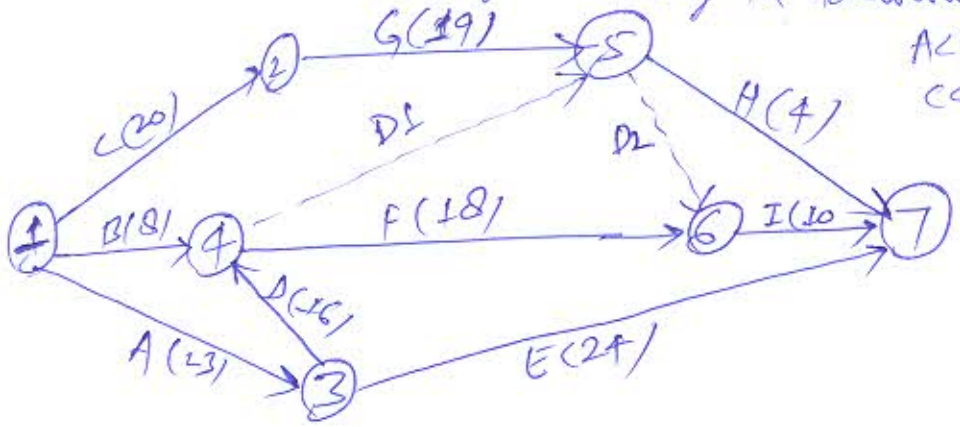
Q.9. Write down the differences between CPM and PERT.

Ans (9) - CPM is used for repetitive jobs like planning the construction of house. On the other hand PERT is used for non-repetitive jobs like planning the assembly of the space platform.

- CPM is a deterministic model while PERT is a probabilistic model with uncertainty in activity duration.
- PERT is said to be event-oriented while CPM is activity oriented.
- PERT is used for planning & scheduling research programmes while CPM is employed in construction & business problems.

Q10. Construct the network diagram having the following constraints.

Ans 10.

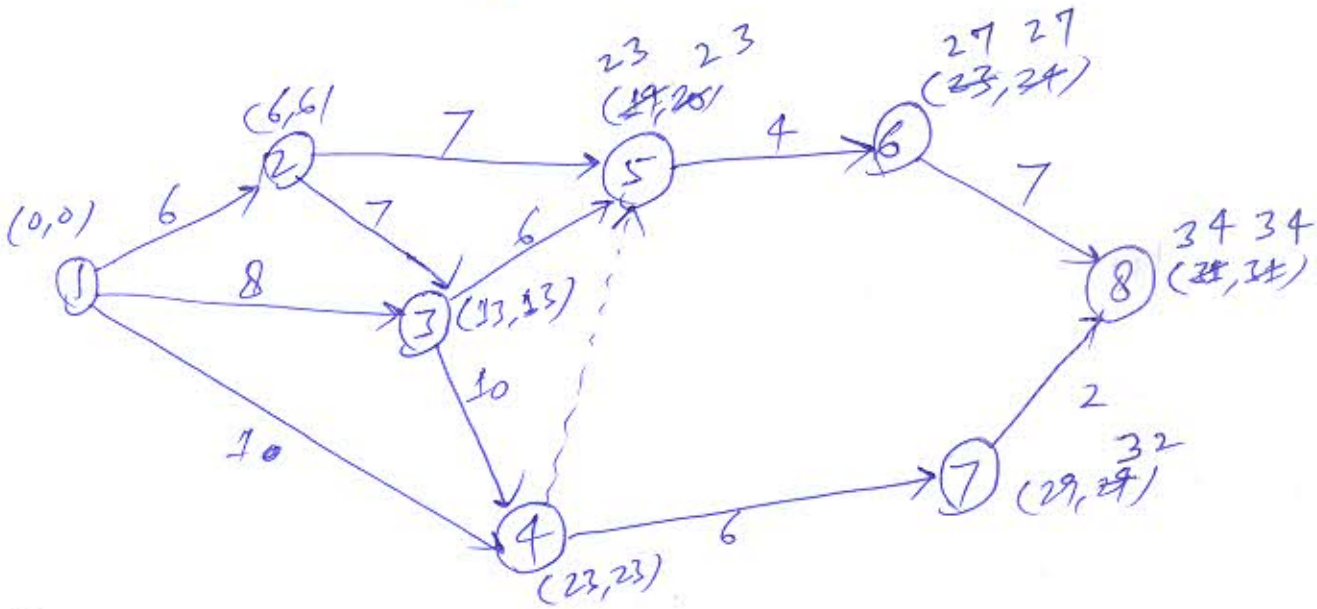


A<D, E. B, D<F; C<G; B, G<H; F, G<I

Section- C

Q11. Draw a network diagram and find the critical path. Also calculate the start timings, finishing timings and total float at each activity.

Ans 11. The network diagram is as follows.



First, at the node, $E_1 = 0$

$E_2 = E_1 + t_{12} = 6$

$E_3 = \max(6+7, 8) = 13$

$E_4 = \max(13+10, 10) = 23$

$E_5 = \max(13+6, 23) = 23$

$E_6 = E_5 + t_{56} = 27$

$E_7 = E_4 + t_{47} = 29$

$E_8 = \max(29+2, 27+7) = 34$

At the last node. $E_8 = L_8 = 34$

Now taking backward pass.

$L_8 = 34$

$L_7 = L_8 - t_{78} = 34 - 2 = 32$

$L_6 = 34 - 7 = 27$

$L_4 = 23 - 0 = 23$

$L_5 = 27 - 4 = 23$

$L_3 = \min(23-10, 23-6) = 13$

$L_2 = \min(13-7, 23-7) = 6$

$L_1 = \min(6-6, 13-8, 23-10) = 0$

Activity	Duration	Start E_S	L_S	Finish E_F	L_F	Total Float
1-2	6	0	0	6	6	0*
1-3	8	0	5	8	13	5
1-4	10	0	13	10	23	13
2-3	7	6	6	13	13	0*
2-5	7	6	16	13	23	10
3-4	10	13	13	23	23	0*
3-5	6	13	17	19	23	4
4-5	0	23	23	23	23	0*
4-7	6	23	26	29	32	3
5-6	4	19	23	23	27	0*
6-8	7	23	27	30	34	0*
7-8	2	29	32	31	34	3

$E_F(1-2) = E_S(1-2) + t_{1-2} = 0 + 6 = 6$

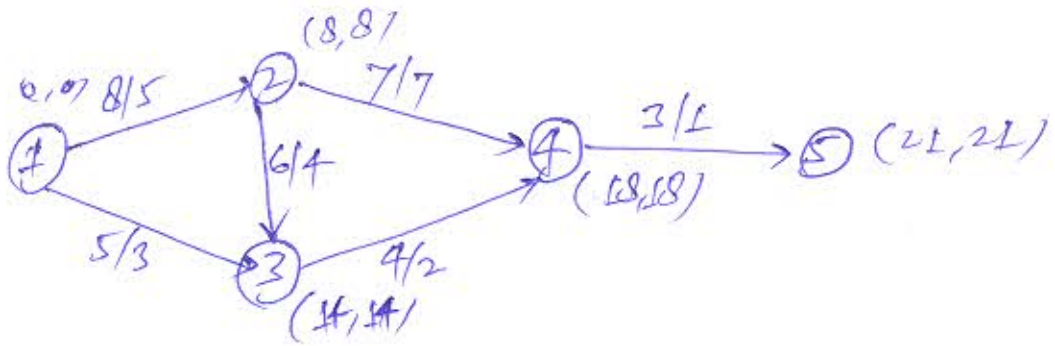
$L_S(1-2) = L_F(1-2) - t_{1-2} = 6 - 6 = 0$

Critical Activities are 1-2, 2-3, 3-4, 4-5, 5-6 & 6-8.
(Having zero float).

& Corresponding critical path is 1-2-3-4-5-6-8.

Q12- Find the optimal cost and time of the project completion. The indirect cost is Rs 500/day.

Ans (12)



Activity	t_n	t_c	C_n	C_c	Cost/slope
1-2	8	5	1000	2800	600/day
1-3	5	3	1200	1800	300/day
2-3	6	4	1800	2200	200/day
2-4	7	7	1000	1000	—
3-4	4	2	1200	3600	1200/day
4-5	3	1	500	2700	1100/day

First make the regular CPM table.

Activity	Duration	Start E_S	Start L_S	Finish E_F	Finish L_F	Float
1-2	8	0	0	8	8	0*
1-3	5	0	9	5	14	9
2-3	6	8	8	14	14	0*
2-4	7	8	11	15	18	3
3-4	4	14	14	18	18	0*
4-5	3	18	18	21	21	0*

Critical path is 1-2-3-4-5

Step 1. The different paths are

(a) 1-2-4-5 of duration 18 days
(b) 1-2-3-4-5 " 21 "
(c) 1-3-4-5 " 12 days

Step 2. The least cost is 200/- per day on critical path. ~~Their~~ for activity 2-3. This activity can be compressed for 2 days.

Total reduction in indirect cost = 2 × 500 = 1000/-

Expenditure = 2 × 200 = 400/-

Saving = 1000 − 400 = 600/- for 2 days.

Step 3. Next path for contraction having min cost is 1-2 at the cost. 600/day.

Indirect cost = 500 × 3 = 1500/-

Expenditure = 600 × 3 = 1800/-

The duration is 16 days.

Step 4. Next activity is 4-5 reduce to 2 days.

Indirect cost = 500 × 2 = 1000/-

Expenditure = 1100 × 2 = 2200/-

Step 5. Activity 3-4 reduce by 1 day.

Indirect cost = 500 × 1 = 500/-

Expenditure = 1200 × 1 = 1200/-

duration = 13 days.

No further crashing can be done.

No.	Activity Crashed	No of days	Cost	C_c	Direct cost $C_c + C_n$	duration	Indirect cost	Total cost
1	—	—	—	—	6700	21	10500	17200
2	2-3	2	200×2=400	400	400+6700	19	9500	16600
3	1-2	3	600×3=1800	2200	2200+6700	16	8000	16900
4	4-5	2	1100×2=2200	4400	4400+6700	14	7000	18100
5	3-4	1	1200×1=1200	5600	5600+6700	13	6500	18800

Min cost of project is 16 600 (duration = 19 days)

& time " is 13 days (cost = 18800/-)