AKGEC/IAP/FM/02

Ajay Kumar Garg Engineering College, Ghaziabad

Department of MCA

SESSIONAL TEST-2 – SOLUTION

Course:	MCA	Semester:	I
Session:	2017-18	Section:	MCA-1
Subject:	Discrete Mathematics	Sub Code:	RCA-103
Max Marks:	25	Time:	1 hour

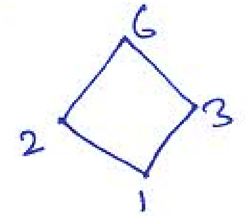
Note: Answer **all** the sections.

Section-A

Q1. Define lattice. Also draw a lattice which is bounded, complemented and distributed.

Sol. A partial order set in which every pair of elements has both a least upper bound and a greatest lower bound is called lattice.

A lattice which is bounded, complemented & distributed is shown below:



Q2. Define GLB and LUB.

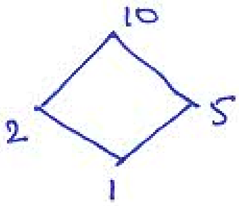
Sol. LUB: Let A be a subset of poset $(S, \leq)$. An element x is called least upper bound of A if x is an upper bound of A and $x \leq z$ whenever z is an upper bound of A.

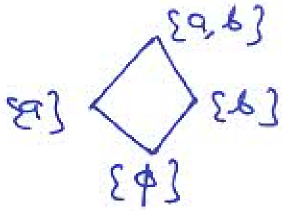
GLB: Let A be a subset of poset $(S, \leq)$. An element x is called greatest lower bound (GLB) of A if x is a lower bound of A and $y \leq x$ whenever y is a lower bound of A.

Q3. Explain bounded lattice with example.

Sol. A lattice $(L, \leq)$ is said to be bounded if it has a greatest element denoted by I and least element denoted by O.

Example:





Q4. Define complete DNF with example.

Sol. A DNF in n variables which contains 2^n terms is called the complete DNF in n variables.

Example: A complete DNF with 2 variable is

$$xy + x'y + xy' + x'y'$$

Q5. Write boolean expression of three variables which value is always 1.

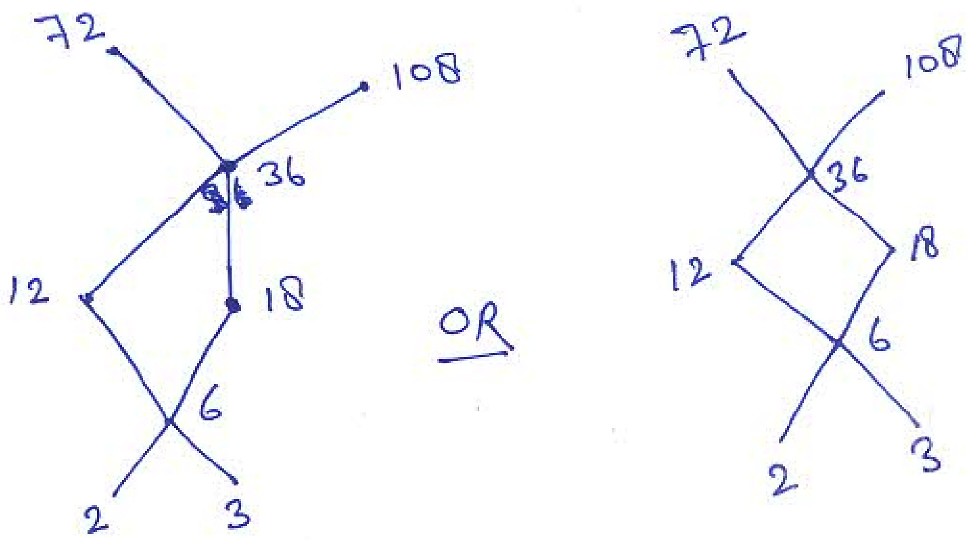
Sol. A boolean expression of three variables which value is always 1 is shown below:

$$x'y'z' + x'y'z + x'yz' + x'yz + xy'z' + xy'z + xyz' + xyz$$

Section-B

Q6. Draw a Hasse diagram for the poset S= { 2, 3, 6, 12, 18, 36, 72, 108} under the relation of divisibility. Also find Supremum and Infimum for the subset B= {6,12,18}.

Sol. A Hasse diagram for a given poset S = {2, 3, 6, 12, 18, 36, 72, 108} under divisibility relation is shown below:



Supremum of Infimum of B = {6, 12, 18}

UB of {6,12,18} = {36, 72, 108}

LUB of {6,12,18} = {36}

Hence Supremum = {36}

LB of {6,12,18} = {2, 3, 6}

GLB of {6,12,18} = {6}

Hence Infimum = {6}

Q7. The complement of an element a in a bounded distributive lattice, if it exists, is unique.

Sol. Let L be a lattice and a_1 & a_2 are two complement of a & $a, a_1, a_2 \in L$

Then

$$\left.\begin{aligned} a \vee a_1 &= I \\ \& \ a \vee a_2 &= I \end{aligned}\right\} \text{ --- (1)}$$

and

$$\left.\begin{aligned} a \wedge a_1 &= 0 \\ a \wedge a_2 &= 0 \end{aligned}\right\} \text{ --- (2)}$$

Now we have

$$a_1 = a_1 \vee 0 = a_1 \vee (a \wedge a_2) \text{ from (2)}$$
$$= (a_1 \vee a) \wedge (a_1 \vee a_2) \text{ by distributive law}$$
$$= I \wedge (a_1 \vee a_2) \text{ by (1)}$$
$$= a_1 \vee a_2$$

Similarly,

$$a_2 = a_2 \vee 0 = a_2 \vee (a \wedge a_1) \text{ from (2)}$$
$$= (a_2 \vee a) \wedge (a_2 \vee a_1)$$
$$= I \wedge (a_2 \vee a_1) \text{ by (1)}$$
$$= a_2 \vee a_1$$
$$= a_1 \vee a_2 \text{ by commutative law}$$

Therefore $a_1 = a_2$

Hence proved.

Q8. **Obtain the disjunctive normal form of the following Boolean expression:**

$$(x + y')(y + z')(z + x')$$

Sol. Disjunctive normal form for the given exp. is

$(x+y')(y+z')(z+x')$

$\Rightarrow (xy + xz' + y'y + y'z')(z+x')$

$\Rightarrow (xy + xz' + 0 + y'z')(z+x')$ by $y'y = 0$

$\Rightarrow (xy + xz' + y'z')(z+x')$

$\Rightarrow (\cancel{xyz}\; xyz + xz'z + y'z'z + xyx' + xz'x' + y'z'x'$

$\Rightarrow xyz + 0 + 0 + 0 + xx'z' + y'z'x'$

$\Rightarrow \cancel{xyz}\; xyz + xx'z + y'z'x'$

$\Rightarrow xyz + 0 + x'y'z'$

$\Rightarrow xyz + x'y'z'$

Hence DNF is

$xyz + x'y'z'$

Q9. If $(B, +, *, ', 0, 1)$ is a boolean algebra and $a, b \in B$ then prove that

$$(a+b)' = a'b' \text{ or } (a \vee b)' = a' \wedge b'$$

Sol. To prove the complement of $a+b$ is $a'b'$, we shall have to prove that

$$(a+b) + a'b' = 1 \quad \text{——— (1)}$$

and $(a+b) * a'b' = 0$ ——— (2)

where 0 & 1 are identities for operation $*$ and $+$ respectively.

To prove (1), LHS

$(a+b) + a'b'$

$\Rightarrow (a+b+a') * (a+b+b')$

$\Rightarrow (1+b) * (a+1)$

$\Rightarrow 1 = R.H.S$

To prove (2), LHS

$(a+b) * a'b'$

$\Rightarrow a*a'*b' + b*a'*b'$

$\Rightarrow a*0 + 0*a'$

$\Rightarrow 0 + 0$

$\Rightarrow 0 = R.H.S$

Thus by (1) & (2), It is proved that

$(a+b)' = a'*b'$

Q10. Define algebraic definition of lattice and Boolean algebra. Also define relationship between Boolean algebra and lattice.

Sol. Lattice : Let L be a non empty set closed under two binary operation denoted by $\vee$ and $\wedge$. Then $(L, \wedge, \vee)$ is a lattice if the following axioms are satisfied for all $a, b, c \in L$.

(i) Commutative law:

$$a \vee b = b \vee a \quad \& \quad a \wedge b = b \wedge a$$

(ii) Associative law:

$$a \vee (b \vee c) = (a \vee b) \vee c \quad \& \quad a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

(iii) Absorption law:

$$a \wedge (a \vee b) = a \quad \text{or} \quad a \vee (a \wedge b) = a$$

Boolean algebra : Let B be a non empty set equipped with two binary operation denoted by $+$ & $*$, one unary operation denoted $'$ (complement) and two special symbol denoted by 0 & 1. Then $(B, +, *, ', 0, 1)$ is a boolean algebra if following axioms are satisfied for all $a, b, c \in B$

(i) closure : $\forall\, a, b \in B$

$$a + b \in B \quad \& \quad a * b \in B$$

(ii) Commutative :

$$a + b = b + a \quad \& \quad a * b = b * a$$

(iii) Associative :

$$a + (b + c) = (a + b) + c \quad \& \quad a * (b * c) = (a * b) * c$$

(iv) Distributive:

$$a*(b+c) = (a*b) + (a*c)$$

and $a+(b*c) = (a+b)*(a+c)$

(v) Identity:

$$a+0 = a \quad \text{and} \quad a*1 = a$$

(vi) complement:

$$a+a' = 1 \quad \text{and} \quad a*a' = 0$$

Relationship between Boolean algebra & Lattice.

From the above definiti of Lattice & Boolean algebra, it is clear that any lattice becomes boolean algebra if it satisfied the following two conditions.

(i) Complemented: let 0 & 1 be least & last elements and $a', a \in L$ where a' is a complement of a.

$$a \vee a' = 1 \quad \text{and} \quad a \wedge a' = 0$$

(ii) Distributed: $\forall\, a, b, c \in L$

$$a*(b+c) = a*b + a*c$$

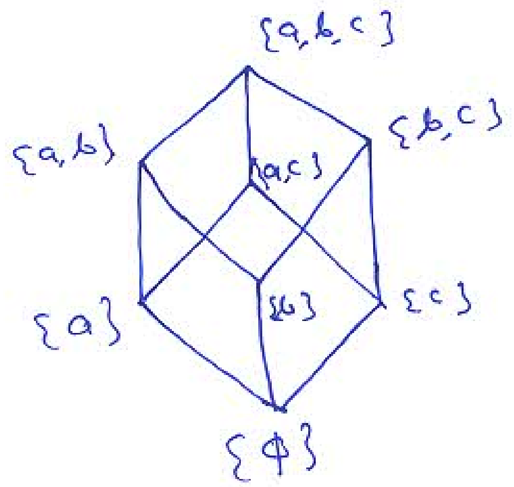
and $a+(bc) = ab + ac$

Section-C

11. Let S = { a, b, c} and A = P(S) the power set of S. Draw Hasse diagram of the poset A with the partial order $\subseteq$ (set inclusion). Find Least & Last elements and upper bound, lower bound, least upper bound, greatest lower bound for the subset S of set A. Also verify that the Hasse diagram is lattice or not.

Sol. Hasse diagram for a given poset is

$P(S) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$



Least Element $= \{\phi\}$

Last Element $= \{a, b, c\}$

upper bound of S $= \{a, b, c\}$

Lower bound of S $= \{\phi\}$

Least upper bound of S $= \{a, b, c\}$

Greatest lower bound of S $= \{\phi\}$

The given hasse diagram is lattice because for all pair of elements, the least upper bound and greatest lower bound exists.

Example.

Let us consider the non comparable element of the given poset is ~~[illegible]~~ $\{\{a\}, \{b\}, \{c\}\}$

Then

$UB = \{a, b, c\}$

$LB = \{\phi\}$

$LUB = \{a, b, c\}$

$GLB = \{\phi\}$

Similar for $\{\{a, b\}, \{a, c\}, \{b, c\}$

$UB = \{a, b, c\}$

$LB = \{\phi\}$

$LUB = \{a, b, c\}$

$GLB = \{\phi\}$

Hence the given diagram is lattice.

Q12. Simplify the boolean expression $f(w,x,y,z) = \sum m(0,1,3,5,8,10,13,15)$ by using k-map. Also draw the logic diagram of the simplified expression.

Sol. The given boolean expression is

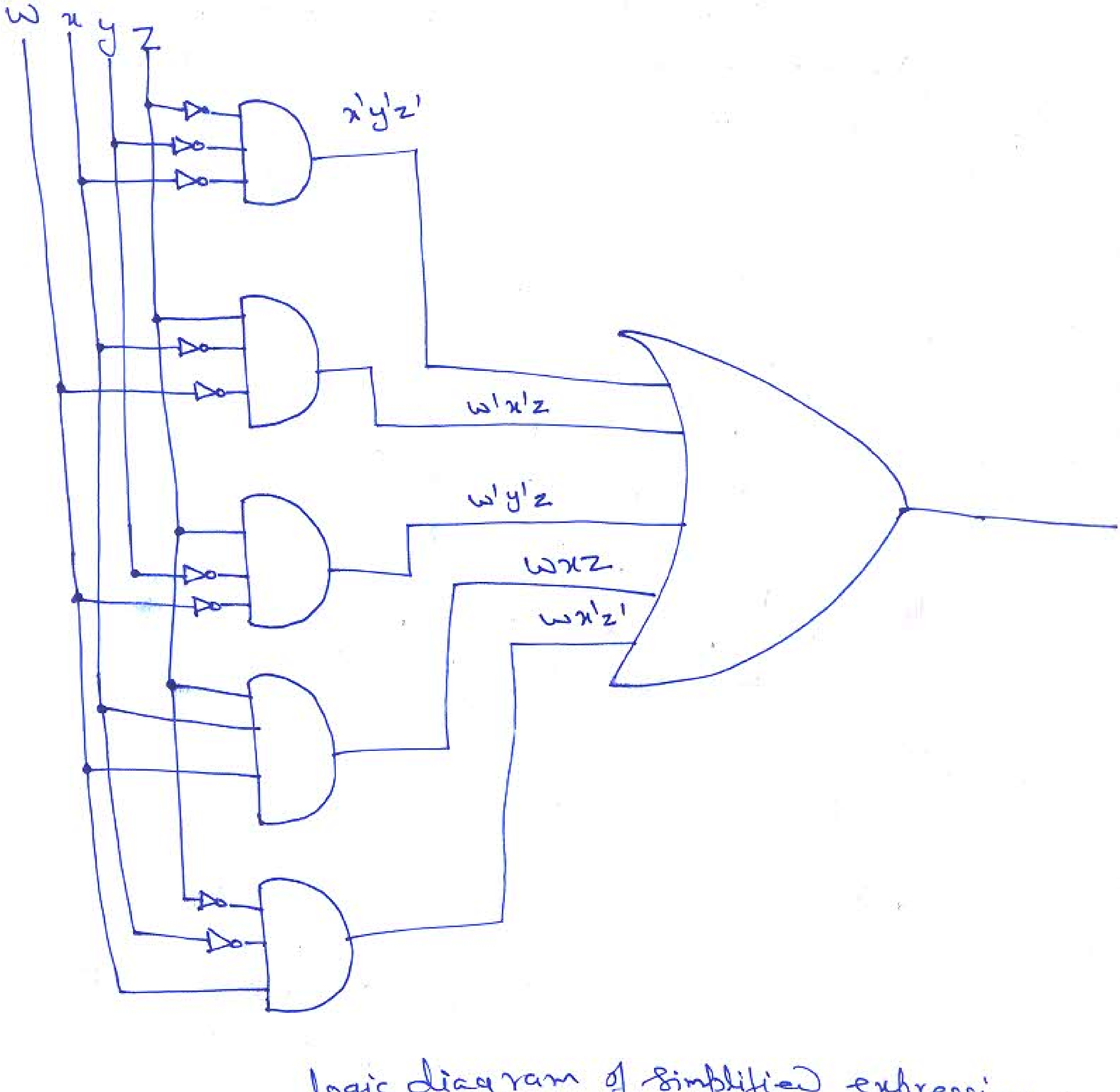
$$f(w,x,y,z) = \sum m(0,1,3,5,8,10,13,15)$$

wx \ yz	00	01	11	10
00	1	1	1	0
01	0	1	0	0
11	0	1	1	0
10	1	0	0	1

Hence the simplified expression for given boolean expression is

$$f(w,x,y,z) = x'y'z' + w'x'z + w'y'z + wxz + wx'z'$$

The logic diagram for the simplified expression is mentioned on next page.



logic diagram of simplified expression