# Ajay Kumar Garg Engineering College, Ghaziabad

## Department of Applied Sciences and Humanities

### MODEL SOLUTION: SESSIONAL TEST-2

Course: B.Tech
Session: 2017-18
Subject: LASER systems and Applications
Max Marks: 50

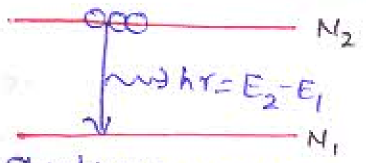
Semester: III
Section: CS-12,3, EN-1, 2, IT-1,2, EI
Sub. Code: ROE-033
Time: 2 hours

**Section-A**

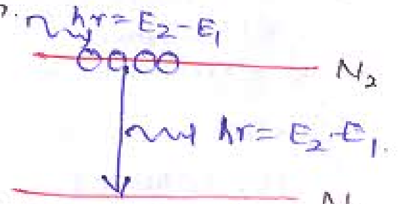
**A. Attempt all parts.** **(5X2= 10)**

1. What is the difference between spontaneous emission and stimulated emission of radiation?

Ans: If there are $N_0$ atoms in an assembly of atoms, which are present in two states (1 and 2). There are two possible types of transitions between 1 and 2, either atoms goes from $1 \to 2$ or from $2 \to 1$. In excited state, atom can stay only for short duration, after that it automatically come down to ground state. This process is called spontaneous emission. If before completion of life time a photon is interacting with an excited atom, it forcefully came to ground state emitting photon of excessive energy, process is known as stimulated emission.




Spontaneous emission




Stimulated emission

2. Calculate the number of modes of a laser beam of wavelength 5000A° in a cavity of length 50cm.

Ans :- In a resonant cavity, no. of axial modes are given as follows

$$L = \frac{n\lambda}{2}$$

$$n = \frac{2L}{\lambda}$$

$L = 50\,cm$, $\lambda = 5000\,A^\circ \Rightarrow n = \dfrac{2 \times 50\,cm}{5000 \times 10^{-8}\,cm} = \dfrac{2 \times \cancel{50}^{10}}{\cancel{5} \times 10^{-5}} = 2 \times 10^{6}$

$$\boxed{n = 2 \times 10^{6} \text{ axial modes}}$$

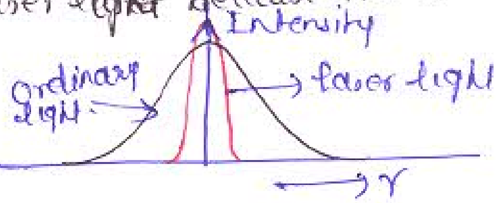
3 Compare laser light and ordinary light on the basis of intensity and monochromaticity.

Ans:- Laser light is highly intense and monochromaticity is also higher than ordinary light.

Intensity = $\frac{\text{Power}}{\text{Area}}$

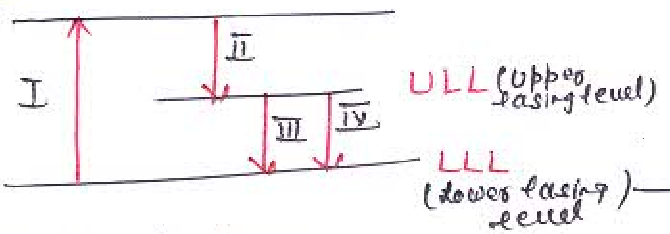
Since laser light is confined in a very small region. So, smaller the area greater will be intensity.

Monochromaticity is higher for laser light because the bandwidth is very small.




4 Why is a four level laser more efficient than a three level laser?

Ans:-



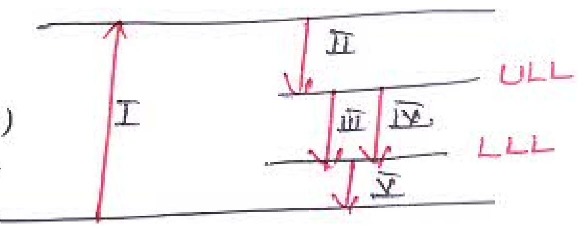

3-level laser

I → Pumping transition

II → Radiative / Non-radiative

III → spontaneous emission.

IV → stimulated emission (L.T)




4-level laser

I → Pumping transition

II → Radiative / Non radiative

III → spontaneous emission

IV → stimulated emission (Laser)

In a three level laser, since lower level is ground state which is most stable state, whereas in a four-level laser lower lasing level is a metastable state which is ideally empty. So it is easy to achieve population inversion in a 4-level laser and also easy to maintain the state.

5 Find the intensity of laser beam of 20mW power and having a diameter of $1.3 \times 10^{-3}$ m. Assume uniform intensity across the beam

Ans:-

$P = 20\,mW = 20 \times 10^{-3}\,W$

$d = 1.3 \times 10^{-3}\,m \Rightarrow r = \frac{1.3 \times 10^{-3}\,m}{2}$

$$I = \frac{P}{\pi r^2} = \frac{20 \times 10^{-3} \times 4}{3.14 \times 1.3 \times 1.3 \times 10^{-6}}$$

$$= \frac{8 \times 10^{-2}}{3.14 \times 1.3 \times 1.3 \times 10^{-6}} = \frac{8}{3.14 \times 1.3 \times 1.3} \times 10^{4}$$

$$I = 1.50 \times 10^{4}\,W/m^2$$

6 What do you mean by population inversion? Describe various methods to achieve it.

Ans:- In normal condition most of the atoms try to remain in ground state i.e stable state. Only few atoms exist in excited state. But to achieve ~~populati~~ laser action stimulated emission, more no. of atoms should be present in excited state. So that particular state where we have more atoms in excited state as compared to ground state is called Population inversion state.


a) Normal state

$N_1 >> N_2$

b) Population inversion state.

$N_2 >> N_1$

Process through which population inversion is achieved is called pumping process or pumping methods.

Types of pumping methods.

- Optical pumping
- Electric discharge pumping
- Atom-atom collision pumping
- Direct conversion
- Chemical pumping

# Optical Pumping : A light source is used to pump ground state atoms to upper lasing level, so that population inversion can be achieved. This pumping method is suitable for laser system with broad absorption and emission spectra. e.g. all solid state laser and liquid laser

* Electric discharge Pumping: This method is suitable for gaseous laser systems where electric discharge is passed through gaseous medium, accelerated electrons supply their energy to ground state atoms, then population inversion is achieved

$$\boxed{A + e^- \rightarrow A^*}$$

* Atom-atom collision pumping :- An inelastic collision between two different type of atoms in a gaseous laser help to achieve population inversion e.g He-Ne laser. First He-atoms get excited by electric discharge and then excited Helium collide with unexcited Neon and population inversion is achieved

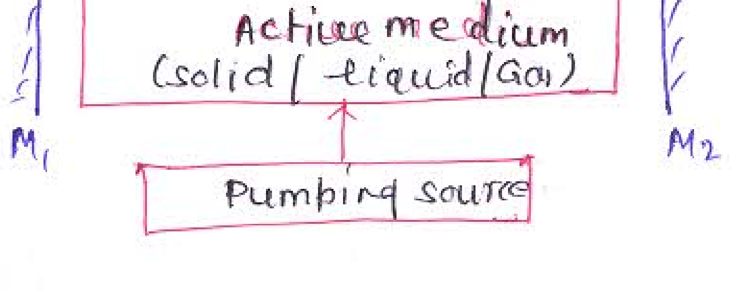
$$\boxed{\begin{array}{l} A + e^- \longrightarrow A^* \\ A^* + B \longrightarrow B^* \end{array}}$$

* Direct Conversion :- Used by semiconductor diode lasers where directly electrical energy is converted into light energy. A forward bias voltage is applied on the diode which will appear in the form of laser light

* Chemical Pumping : In this pumping method chemical reaction help to increase the population of excited state e.g HCl, HF laser etc.

7 What is the role of resonant cavity in a laser system? Describe various ~~methods to achieve it~~ types of resonant cavities and stability diagram.

Ans :-

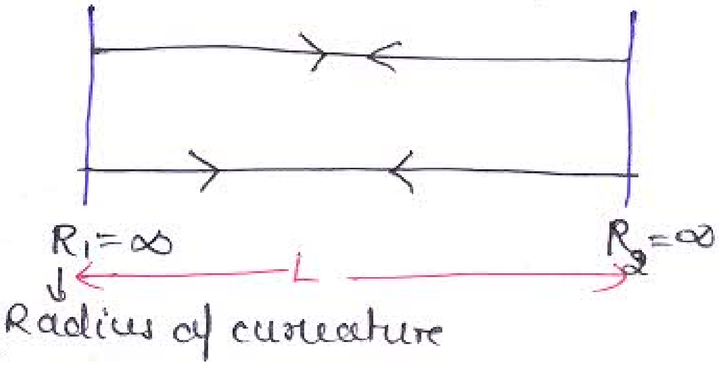


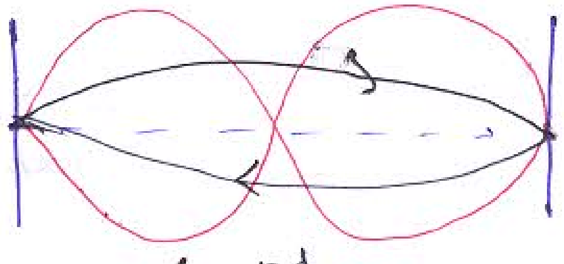
A laser system consist of three essential components: Active medium, pumping source and resonant cavity. A resonant cavity is a combination of two mirrors kept at two ends of the active medium in order to capture the photons within the active medium so that they can participate in light amplification process.

## Types of resonant cavities.

- Plane Parallel resonant cavity
- Confocal resonant cavity
- Concentric resonant cavity

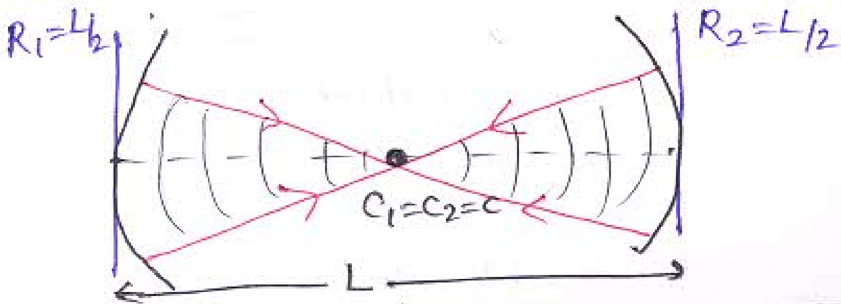
**Plane Parallel resonant cavity:** simplest typical resonators where two plane mirrors are kept at two ends of the active medium. One fully and other partially reflecting. This cavity covers maximum volume of the active medium but very sensitive to misalignment.



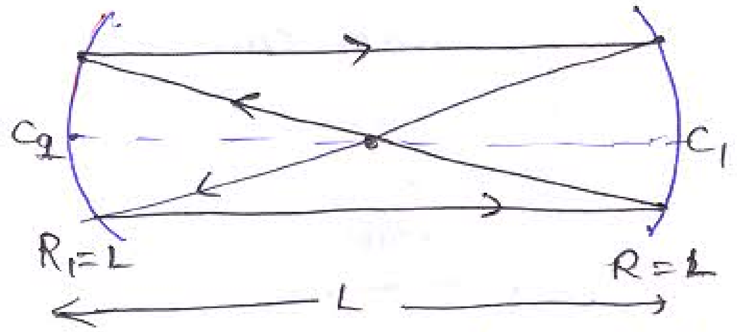
$$L=\frac{n\lambda}{2}$$

**Concentric resonant cavity:** — Instead of plane mirrors spherical mirrors are used with a common centre as shown in the figure.



Minimum volume is utilized by this cavity and it is not very sensitive to mis alignment.

Confocal resonant cavity: Again two spherical mirrors are kept at two ends with a common focus shown in the figure given below. Volume of the active medium utilized is inbetween plane-parallel and concentric resonant cavity




Criteria of stability and stability diagram:

A resonant cavity will be stable if it follows following condition $\boxed{0 \leq g_1 g_2 \leq 1}$

where $g_1 = 1 - \frac{L}{R_1}$

and $g_2 = 1 - \frac{L}{R_1}$

Plane Parallel :- $R_1 = \infty$, $R_2 = \infty$, $g_1 = 1$, $g_2 = 1$

$g_1 g_2 = 1 \rightarrow$ stable cavity

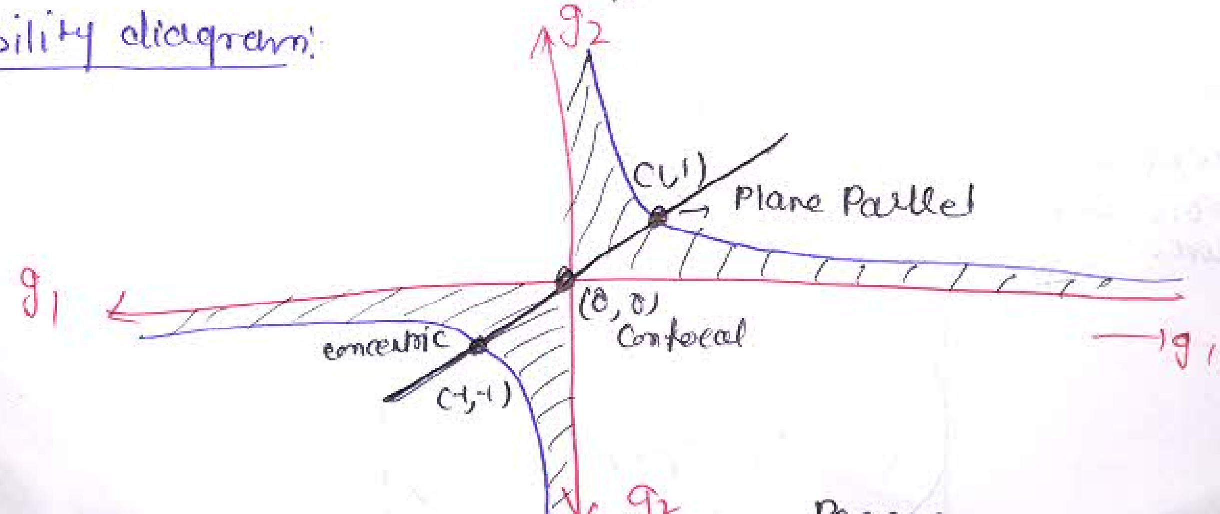
Concentric :- $R_1 = \frac{L}{2}$, $R_2 = \frac{L}{2}$, $g_1 = -1$, $g_2 = -1$

$g_1 g_2 = 1 \rightarrow$ Stable

Confocal :- $R_1 = L$, $R_2 = L$, $g_1 = 0$, $g_2 = 0$

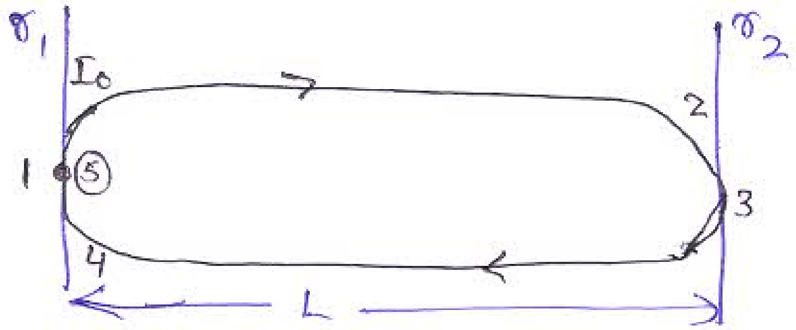
$g_1 g_2 = 0 \longrightarrow$ stable.

stability diagram:



8. What do you mean by loop gain? Derive threshold condition of laser oscillation.

Ans:- Loop Gain :- Loop gain means the gain in light energy during the round trip of photons from one mirror to the second and then from second to one.




Let the energy (intensity) of beam at the starting point

$$I_1 = I_0 \quad \text{——} ①$$

At ②
$$I_2 = I_0 e^{(\gamma-\alpha)L}$$

$\gamma$ – Gain coefficient
$\alpha$ – Loss coefficient

$$I_3 = I_0 e^{(\gamma-\alpha)L} \cdot r_2$$

$$I_4 = I_0 (e^{(\gamma-\alpha)\cdot 2L} \cdot r_2$$

$$I_5 = I_0 e^{(\gamma-\alpha)\cdot 2L} r_2 \cdot r_1$$

So gain during the round trip,

$$G = \frac{I_5}{I_1} = \frac{I_0 e^{(\gamma-\alpha)\cdot 2L} \cdot r_1 r_2}{I_0}$$

$$G = e^{(\gamma-\alpha)\cdot 2L} r_1 r_2$$

If $I_5 \geq I_1$ only then we will get laser beam.

$$e^{(\gamma-\alpha)\cdot 2L} \cdot r_1 r_2 \geq 1$$

$$e^{(\gamma-\alpha)\cdot 2L} \geq \frac{1}{r_1 r_2}$$

$$(\gamma-\alpha) 2L \geq \ln\left(\frac{1}{r_1 r_2}\right)$$

$$(\gamma-\alpha) \geq \frac{1}{2L} \ln\left(\frac{1}{r_1 r_2}\right)$$

$$\gamma \geq \alpha + \frac{1}{2L} \ln\left(\frac{1}{r_1 r_2}\right)$$

Threshold condition for laser oscillator

$$\boxed{\gamma = \alpha + \frac{1}{2L} \ln\left(\frac{1}{r_1 r_2}\right)}$$

9 The coherence length for a light source is $2.945 \times 10^{-2}$ m and its wavelength is 5890 $A^0$. Calculate
i) No. of oscillation corresponding to coherence length.
ii) Coherence time
iii.) Spectral width.

Ans: i) $L_c = n\lambda$

$$n = \frac{L_c}{\lambda}$$

Given $L_c = 2.945 \times 10^{-2}$, $\lambda = 5890 A^0$

$$n = \frac{2.945 \times 10^{-2}}{5890 \times 10^{-10}} = 5 \times 10^{4} \text{ oscillations}$$

ii) $L_c = c T_c$

$$T_c = \frac{L_c}{c} = \frac{2.945 \times 10^{-2}}{3 \times 10^{8}} = 9.82 \times 10^{-11} s$$

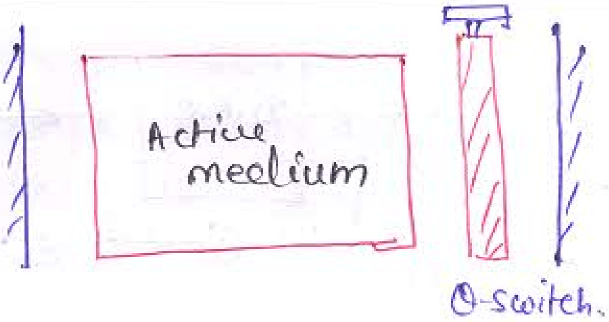
iii) $$L_c = \frac{\lambda^2}{\Delta\lambda}$$

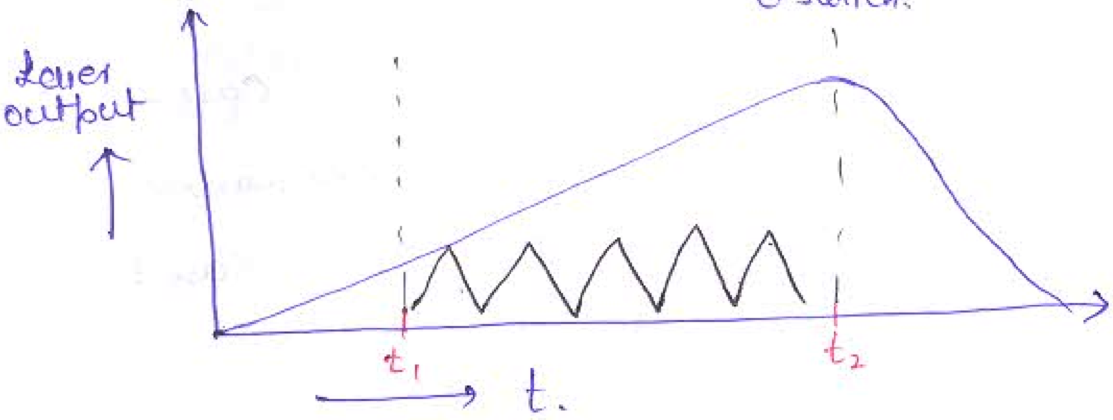
$$\Delta\lambda = \frac{\lambda^2}{L_c} = \frac{(5890 \times 10^{-10})^2}{2.945 \times 10^{-2}}$$

$$\boxed{\Delta\lambda = 0.1178 A^0}$$

10 What is the meaning of Q-switching in laser system? Describe various methods to achieve it.

Ans:- Q-switching meaning → switching of the quality factor between maximum and minimum. In Q-switching method short pulses are generated by controlling the quality factor. Switch is having only two state either the switch is on means quality factor minimum, or switch is off meaning quality factor maximum

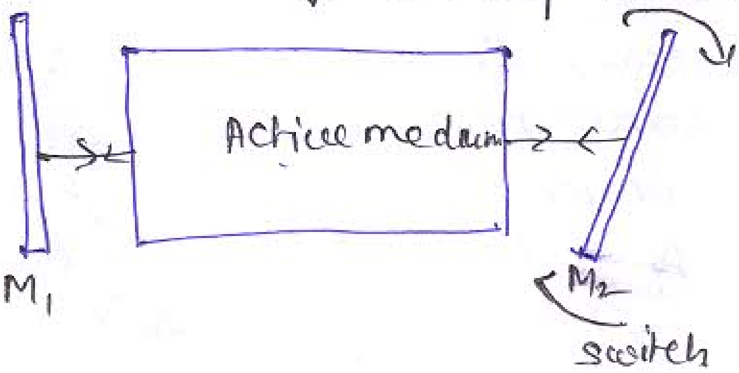


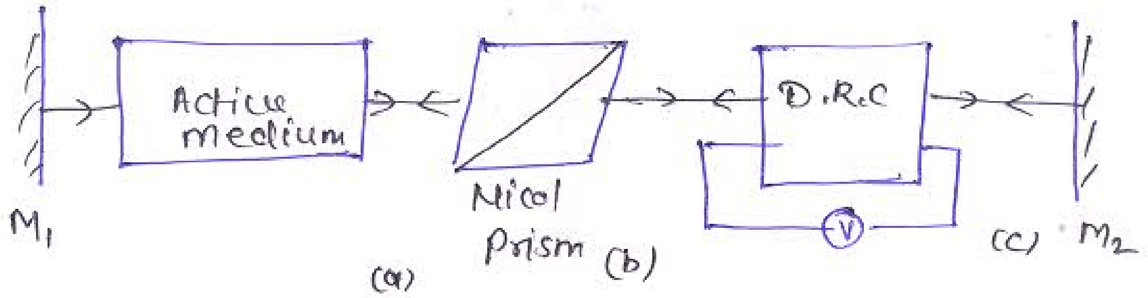

## Types of Q-switching

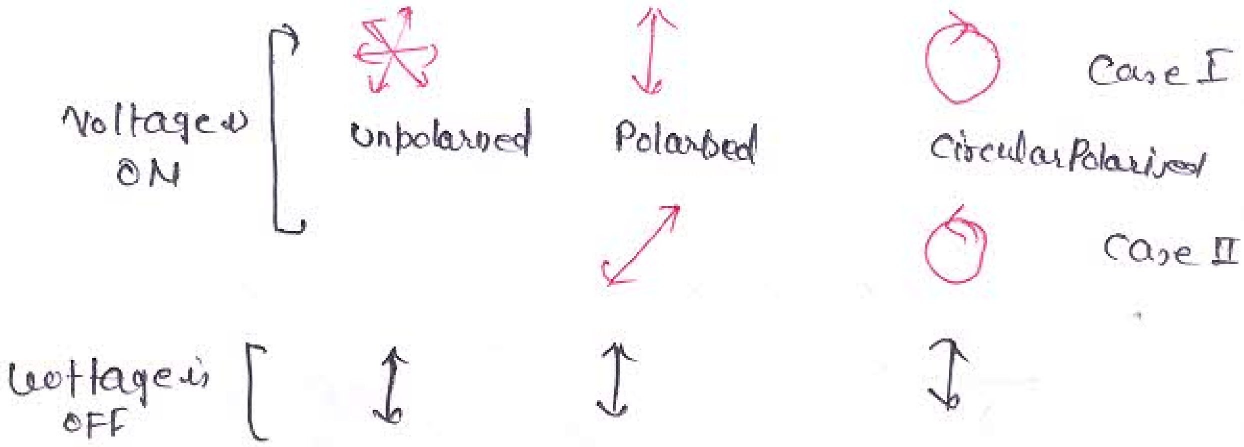
- Mechanical switching
- Electro-optic switching
- Acoustic optic switching
- Saturable absorber

**Mechanical switching:** In this method one of the mirror is a rotating mirror. One mirror is fixed and the second one is rotated in such a manner that population of atoms will go on building but two mirror are not parallel to each other, they will become parallel only when we get saturation level of atoms, and only then output is obtained in this way, rotating mirror is the switch.

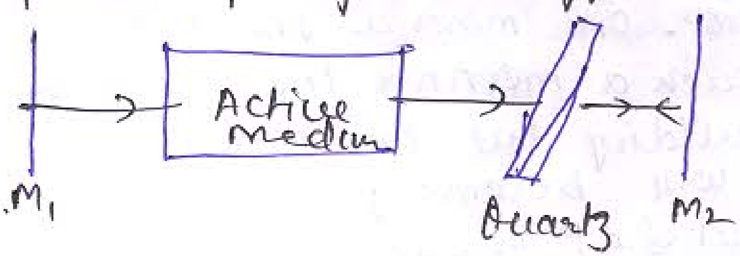



**Electro-optic switch:-** In this a double refracting crystal will act as a switch. The crystal will show double refraction only if voltage is applied on the crystal. As soon as voltage is off, no double refraction. So this crystal will control the quality factor.




<u>Acoustic optic switching:</u> A quartz crystal will act as a switch here because it will act as a grating if a radio frequency signal is applied. So switch will be on if radio frequency signal is ON and OFF if signal is off. In this which laser output is obtained in the form of short pulse of high energy



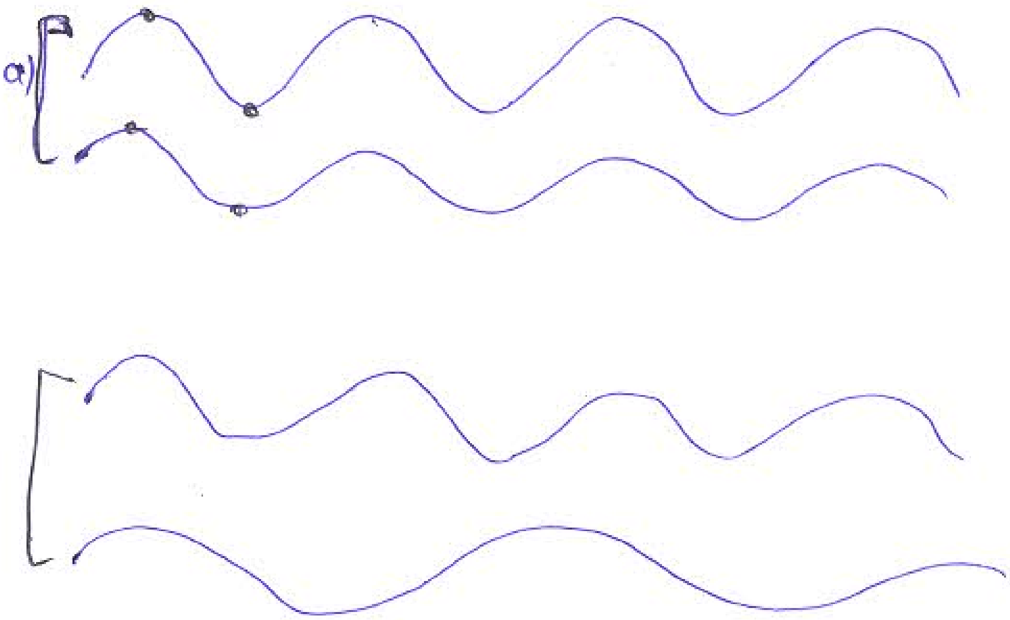

<u>Saturable Absorber switching:-</u> A dye solution will act as a switch here because it will keep on absorbing the photons upto the saturation level, then it will become transparent and photons will pass through it and the entire light energy is obtained in the form of a single pulse of very high energy but short duration.

SECTION C

11 What do you mean by coherence? Explain temporal and spatial coherence. Prove that temporal coherence is related to monochromaticity and spatial coherence related to size of the source.

Ans: coherence means either zero or constant phase difference or crest to crest or trough to trough correspondance as shown in the figure below.

a)



Coherent

b

Incoherent

Coherence

↓ Temporal coherence

↓ Spatial Coherence.

Temporal coherence:- If the phase difference between two points which are present in a plane parllel to direct of propagation, ~~then~~ is independent of time, then they are said to have temporal coherence. Suppose two points A and B are present in a plane parllel to direction of propagation at time $t$.

If phase diff. $\Delta\phi_1 = \phi_{B_1} - \phi_B$

and at time $t'$ $\Delta\phi' = \phi_B' - \phi_A'$

and $\Delta\phi = \Delta\phi'$ ⟶ temporal Coherence

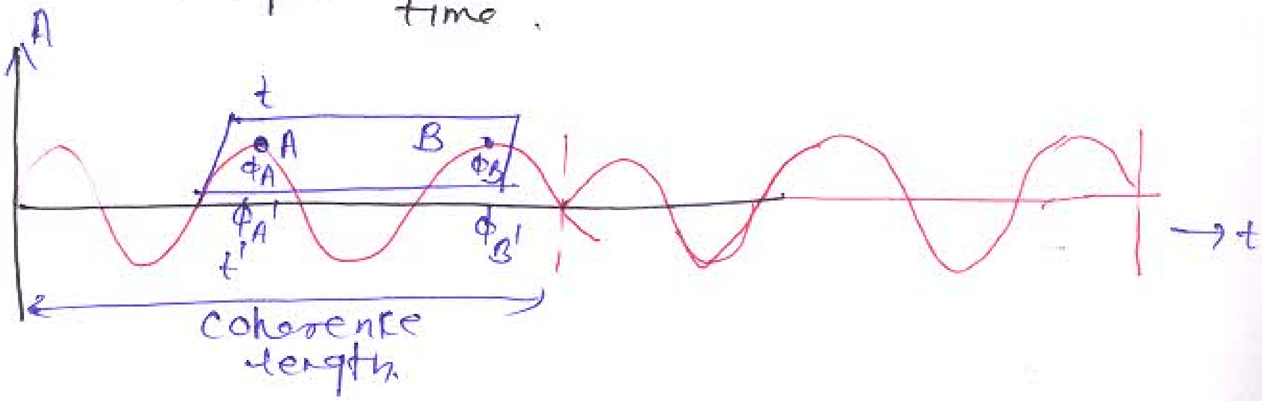
Time upto which light remains coherent is called coherence time.

For a light source, $c = \nu\lambda$

Distance travelled in coherence time is called Coherence length.

$$L_c = c\,T_c$$

$L_c \downarrow$ coherence length, $T_c \downarrow$ Coherence time.




$$\nu = \frac{c}{\lambda}$$

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

Magnitude $\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$

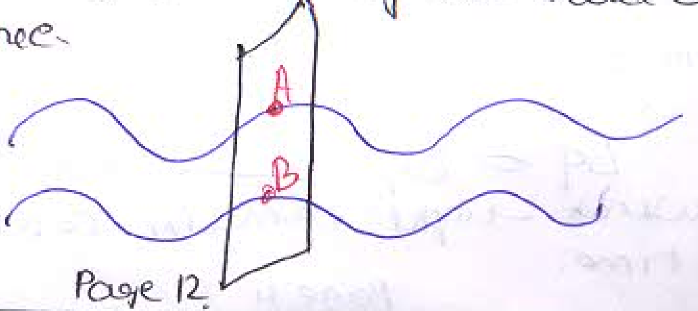
$$T_c = \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\,\Delta\lambda}$$

$$L_c = c\,T_c = \frac{\lambda^2}{\Delta\lambda}$$

$$\boxed{L_c = \frac{\lambda^2}{\Delta\lambda}}$$

Smaller the value of $\Delta\lambda$ greater will be the value of coherence length so temporal coherence is related to monochromaticity.

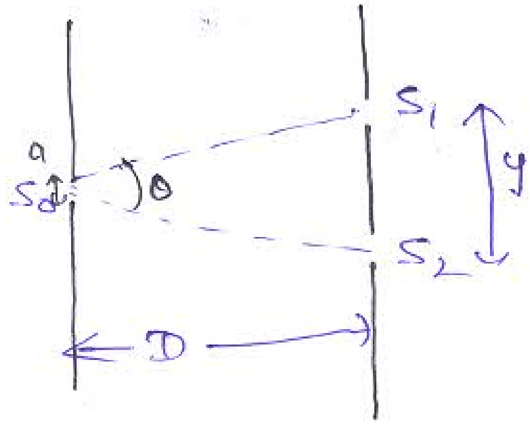
**Spatial Coherence**: If two points which are present in a plane perpendicular to direction of propagation can maintain a constant phase difference irrespective of time, then they are said to have spatial coherence.




$t$, $\Delta\phi = \phi_B - \phi_A$

$t'$, $\Delta\phi' = \phi_B' - \phi_A'$

$\Delta\phi = \Delta\phi'$ → spatial




$\angle S_1S_0S_2 = \frac{y}{D}$

Angular spread $= \frac{y}{D}$

According to the theory of diffraction

$a\sin\theta = \lambda$

$\sin\theta = \lambda/a$

$\sin\theta \approx \theta = \lambda/a$ because θ is very small

If $\frac{y}{D} \leq \frac{\lambda}{a}$

then there will be coherence,

$$\boxed{y_{max} = \frac{\lambda D}{a}}$$

Maximum separation upto which coherence can be maintained is called Coherence length.

12 Derive Einstein relation between Einstein's coefficients. Why stimulated emission is more probable at higher wavelengths? An atom has two atomic levels separated by 2.26 eV in energy. Calculate the temp. at which $N_2/N_1$ of two levels will be half.

Ans: If an assemble of atoms have two states 1 and 2, then the population of atoms can be written as

$$N_1 = N_0 e^{-E_1/kT} \quad \text{—①}$$

$$N_2 = N_0 e^{-E_2/kT} \quad \text{—②}$$

Transition between ① and ②, will achieve thermal equilibrium if $P_{12} = P_{21}$

$$P_{12} = B_{12} N_1 u(\nu) \quad \text{—③}$$

↳ Einstein coeff. of stimulated absorpn.

$$(P_{21})_{sp.en} = A_{21} N_2$$

↳ Einstein coeff. of spontaneous emissn

$P_{21})_{st.em} = B_{21} N_2 \, u(\nu)$

↳ Einstein coeff. of st. emission

At thermal equilibrium

$$P_{12} = P_{21}$$

$$B_{12} N_1 \, u(\nu) = A_{21} N_2 + B_{21} N_2 \, u(\nu)$$

$$u(\nu)\left(B_{12} N_1 - B_{21} N_2\right) = A_{21} N_2$$

$$u(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$= \frac{A_{21} N_2}{B_{21} N_2 \left[\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1\right]}$$

$$u(\nu) = \frac{A_{21}/B_{21}}{\left[\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1\right]}$$

$\boxed{B_{12} = B_{21}}$ → At thermal equilibrium.

$$u(\nu) = \frac{A_{21}/B_{21}}{[N_1/N_2 - 1]}$$

$$\frac{N_1}{N_2} = e^{+h\nu/kT}$$

So $$\boxed{u(\nu) = \frac{A_{21}/B_{21}}{e^{h\nu/kT} - 1}} \quad \text{—(4)}$$

According to Planck's radiation law

$$\boxed{u(\nu) = \frac{8\pi h\nu^3/c^3}{e^{h\nu/kT} - 1}} \quad \text{—(5)}$$

comparing (4) and (5) $$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \quad \text{—(6)}$$

From above equation it is clear that $B_{21}$ is directly proportional to $\lambda^3$. So higher wavelength means greater probability to get laser beam.

Numerical

According to Maxwell Boltzmann's distibution law.

$$\frac{N_2}{N_1} = e^{-hc/\lambda kT} = e^{-(E_2-E_1)/kT}$$

Given $E_2 - E_1 = 2.26\,eV = 2.26 \times 1.6 \times 10^{-19} = 3.616 \times 10^{-19}\,J$

$\frac{N_2}{N_1} = \frac{1}{2}$, $k = 1.38 \times 10^{-23}\,J/k$

$$\frac{1}{2} = e^{-\Delta E/kT}$$

$$\frac{1}{2} = e^{-3.616\times10^{-19}/1.38\times10^{-23}\,T}$$

$$= e^{-2.62\times10^{4}/T}$$

$$\ln\left(\frac{1}{2}\right) = -\frac{2.62\times10^{4}}{T}$$

$$T = -\frac{2.62\times10^{4}}{\ln(1/2)} = -\frac{2.62\times10^{4}}{-0.693}$$

$$= 3.78\times10^{4}$$

$$\boxed{T = 3.78\times10^{4}\,k}$$