Ajay Kumar Garg Engg. College. Ghaziabad
Dept. of Mechanical Engineering
ST-2

Course - B. Tech | Semester - V

Session - 2017-18 | Section - EI

Subject - Fluid Mechanics | Sub Code - NCE-509

Max Marks - 50 | Time - 2 hours.

Solution

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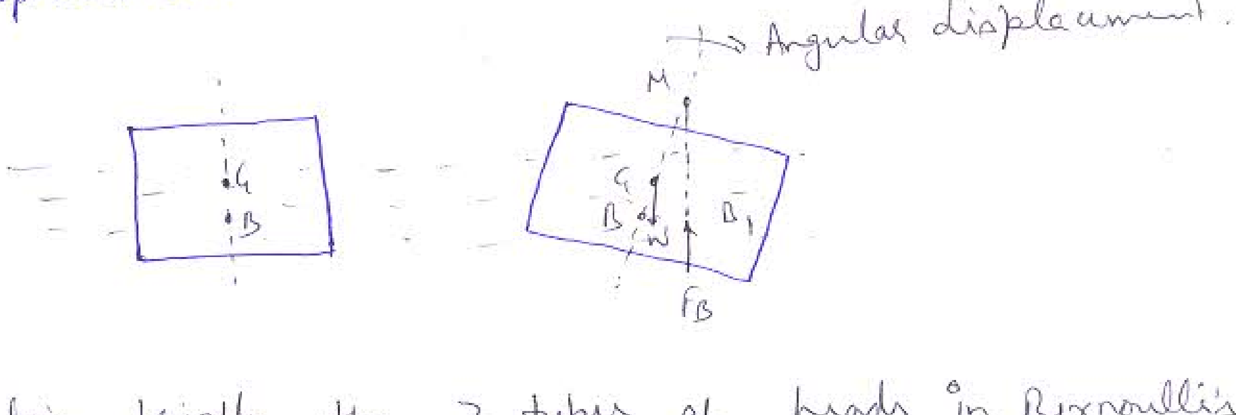
Section-A.

Q.1 - Define Metacentre with specific diagram.

Ans1 - It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.



Q.2. Explain briefly the 3 types of heads in Bernoulli's equation.

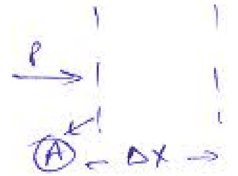
Ans1 - In. $\frac{P}{\rho g} + \frac{v^2}{2g} + z = c$

① Pressure head = $\frac{P}{\rho g}$ = $\frac{\text{Flow work}}{\text{weight of fluid.}}$ = $\frac{PA\,\Delta x}{\rho g (A \Delta x)}$

② Kinetic head. $\frac{v^2}{2g}$ = $\frac{\text{Kinetic Energy}}{\text{weight of fluid.}}$ = $\frac{\frac{1}{2}\rho \overset{\text{Volume}}{V} v^2}{\rho V g}$

(3) Potential head = z = $\frac{\text{Potential Energy}}{\text{weight of fluid.}}$ = $\frac{\rho g z V}{\rho V g}$

Flow work is defined as the work done by fluid element for a displacement Δx.

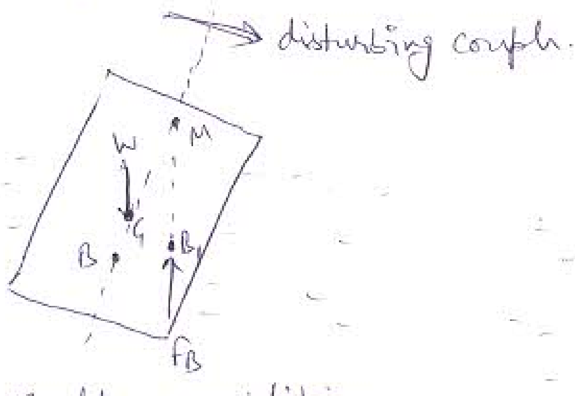


flow work = $(PA).\Delta x$

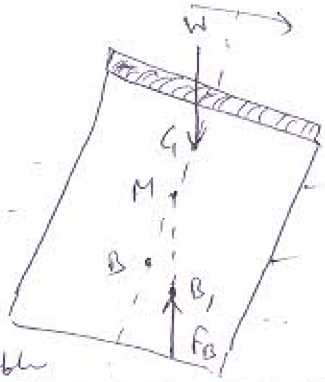
Q.3. Explain briefly the types of equilibrium for floating bodies.

Ans. The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(a) Stable equilibrium - If the point M is above G, the floating body will be in stable equilibrium as shown in.



Stable Equilibrium



Unstable

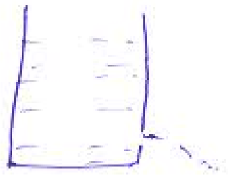
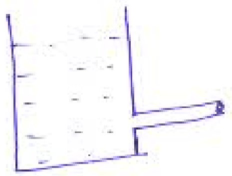
(b) Unstable Equilibrium- If the point M is below G', the floating body will be in unstable equilibrium. The disturbing couple is acting in the clockwise direction. The couple due to F_B & W is also acting in clockwise direction.

Thus, overturning the floating body.

(c) Neutral Equilibrium- If the point M coincide with the CG of the Body, the floating body will be in neutral equilibrium

Q4. Explain difference b/w Orifices & Mouthpieces?

An-

	Orifice	Mouthpiece
1.	Orifice is a small opening of any cross-section on the side or at the bottom of a tank, through which a fluid is flowing.	A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in tank or any other vessel.
2.	Coefficient of discharge is low C_d(0.60)	Coefficient of discharge is high C_d = 0.80
3.		

Ans5- Viscous flow:- Viscous flow is a non ideal flow in which viscosity of fluid is non-zero.

Turbulent flow:- When the velocity is increased or fluid is less, viscous, the fluid particles do not move in straight paths. The fluid particles move in random manner resulting in general mixture of the particles. This type of flow is called turbulent flow.

For pipe flow. $(Re) = \frac{\rho V D}{\mu} = 2000$

for flow over flat plate, $Re = 5 \times 10^5$

Reynold's Number = $\frac{\text{Inertia force}}{\text{Viscous force}}$

Section B

Q.6. Given-

depth of water = 0.5m

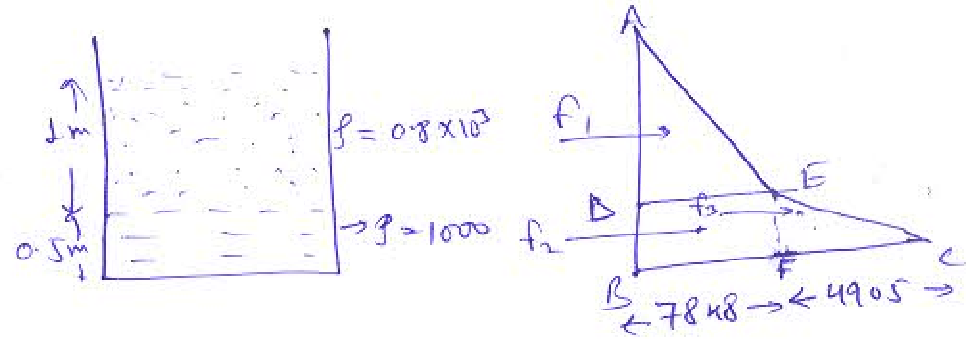
liquid = 1 m.

sp. gr. of liquid = 0.8

density of liquid = $(\rho_1) = 0.8 \times 1000 = 800\ kg/m^3$

Density of water $= (\rho_2) = 1000\ kg/m^3$

width of tank = 2 m.



$P_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753\ N/m^2$

F_1 = Area of $\Delta ADE \times$ width of tank

$= \frac{1}{2} \times AD \times DE \times 2 = \frac{1}{2} \times 1 \times 7848 \times 2.0$

$= 7848\ N$

f_2 = Area of rectangle DBFE × width of tank

$= 0.5 \times 7848 \times 2 = 7848\,N$

f_3 = Area of DEFC × width of tank

$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2 = 2452.5\,N$

$f = f_1 + f_2 + f_3$

$= 18148.5\,N$

(ii) Centre of Pressure (h^*) = Taking the moments of all forces About A. -

$$f \times h^* = f_1 \times \frac{2}{3} AD + f_2\left(AD + \frac{1}{2} BD\right) + f_3\left[AD + \frac{2}{3} BD\right]$$

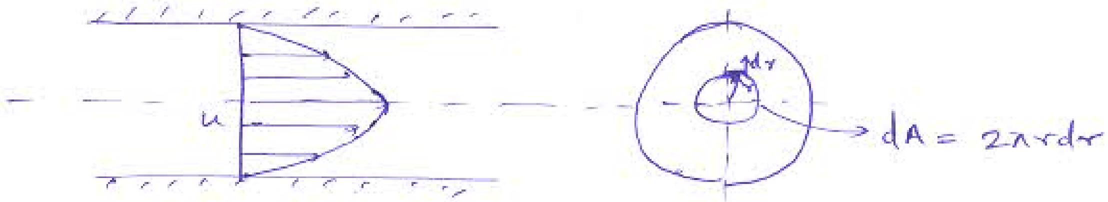
$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848\left(1 + \frac{0.5}{2}\right) + 2452.5\left[1 + \frac{2}{3} \times 0.5\right]$$

$$= 18312$$

$$h^* = \frac{18312}{18148.5} = 1.009\ m \text{ from top.}$$

Q.7. <u>Momentum</u> × <u>Correction factor</u> × It is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on avg. velocity across a section. It is denoted by β. Hence mathematically,

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on avg. velocity}}$$



$$u = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)(R^2 - r^2)$$

Elemental area $dA = 2\pi r dr$.

Rate of flow flowing through the ring $= dQ$
$= u \times 2\pi r dr$

momentum of fluid through ring per second $=$ mass $\times$ velocity
$= \rho \times dQ \times u$

$$= \rho \times 2\pi r dr \times u \times u$$

$$= 2\pi\rho u^2 r dr$$

$\therefore$ total actual momentum of the fluid per second across the section $= \int_0^R 2\pi\rho u^2 r\, dr$

$$= \frac{\pi\rho}{48\mu^2}\left(\frac{-\partial p}{\partial x}\right)^2 R^6$$

Momentum of the fluid per second based on avg. velocity

$$= \rho A\bar{u}.\bar{u} = \rho A\bar{u}^2$$

$$U_{avg.} = \frac{1}{2} \times \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

$$= \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

momentum / sec based on avg velocity –

$$= \rho \times \pi R^2 \times \left[\frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2\right]^2 =$$

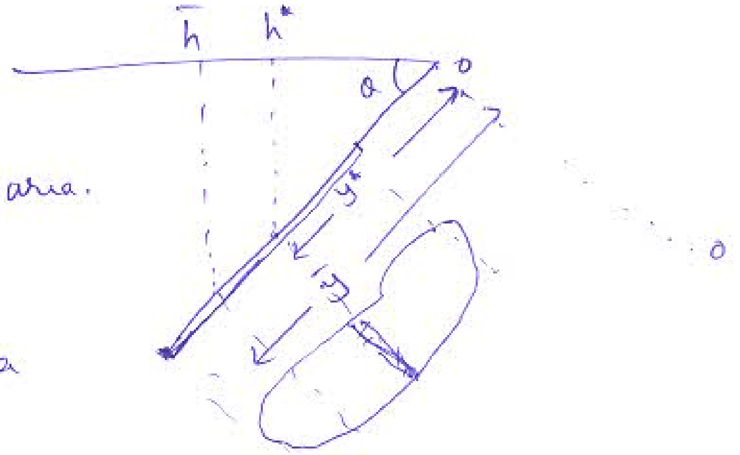
$$= \frac{1}{64\mu}(\rho\pi)\left(\frac{-\partial p}{\partial x}\right)^2 R^6$$

$$\beta = \frac{\frac{\pi \rho}{48\mu^2}\left(-\frac{\partial p}{\partial x}\right)^2 R^6}{\frac{\pi \rho}{64\mu^2}\left(\frac{-\partial p}{\partial x}\right)^2 R^6} = \frac{64}{48} = \frac{4}{3}$$

Q.8. Derive the formula for the position of centre of pressure when an inclined surface is submerged in liquid

Ans 8.

$$\frac{h}{y} = \sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$$



Total pressure on elemental area.

$f = \rho g dA \bar{h}$

Pressure force on whole area

$f = \int df = \int \rho g h dA$

$f = \rho g \sin\theta \bar{y} \times A$

$= \rho g A \bar{h}$ $\qquad \bar{h} = \bar{y}\sin\theta$

Centre of Pressure - (h^*)

Pressure force on the strip $(df) = \rho g h dA$

$= \rho g y \sin\theta dA.$

Moment of the force, df, about 0-0

$= df \times y = \rho g y \sin\theta dA \times y$

$= \rho g \sin\theta \, y^2 dA$

Sum of moments of all such forces about 0-0.

$= \int \rho g \sin\theta \, y^2 dA = \rho g \sin\theta \int y^2 dA$

$\int y^2 dA$ = MoI of the surface about $0\text{-}0 = I_0$

$\therefore$ Sum of moments of all forces about $0\text{-}0 = \rho g \sin\theta I_0$

Moment of the total force, F, about 0-0 is also given by $= F \times y^*$

{where y^* distance of centre of pressure from 0-0}

Equating the two values given by- eqⁿ.

$$F \times y^* = \rho g \sin\theta \, I_0$$

$$y^* = \frac{\rho g \sin\theta \, I_0}{F}$$

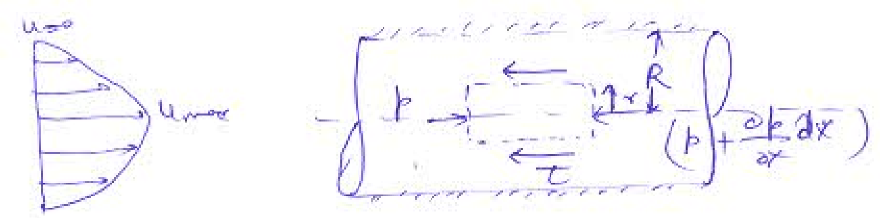
$$y^* = \frac{h^*}{\sin\theta} \quad , \quad F = \rho g A\bar{h}$$

I_0 by theorem of parallel axis $= I_G + A\bar{y}^2$

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

(9). Derive eqⁿ of motion for laminar flow through pipes & find average velocity with suitable diagram?

Ans-



Elemental surface Area. $(A) = 2\pi r \Delta x$

$$p(\pi r^2) - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 = \tau \, 2\pi r \Delta x$$

$$-\frac{\partial p}{\partial x} \pi r^2 \Delta x = \tau \, 2\pi r \, \Delta x$$

$$\tau = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2}$$

Putting $\tau = \mu \frac{du}{dy}$

$y = R - r$

$dy = -dr$

$$\tau = -\mu \frac{du}{dr} = \frac{-\partial p}{\partial x} \frac{r}{2}$$

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} \frac{r^2}{2} + c$$

B.C. at $r = R$ $U = 0$

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} \frac{R^2}{2} + c \Rightarrow c = \frac{-1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\frac{R^2}{2}$$

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

$$\boxed{u = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)(R^2 - r^2)}$$

Average velocity = discharge of the fluid across the section by the area of the pipe πR^2

$$dQ = u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)\, 2\pi r\, dr$$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)\, 2\pi r\, dr$$

$$= \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$$

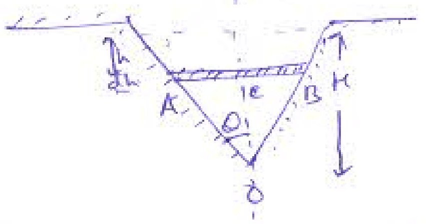
Q6. Define notches & weir? Which have better advantage? Derive equation of a triangular notch?

Ans 10. **Notch** - is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top surface of the opening.

Weir - is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

⇒ Notch is of small size while the weir is of a bigger size. notch is made of metallic plate while weir is made of concrete or masonary structure.

Discharge over a triangular notch or weir -



$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{H-h}$$

$$AC = (H-h)\tan\frac{\theta}{2}$$

Area of the strip $= 2(H-h)\tan\frac{\theta}{2}\,dh$

the theoretical velocity of water through strip $= \sqrt{2gh}$

$$dQ = Cd \times \text{Area of strip} \times \text{velocity (theoretical)}$$

$$= 2Cd \times (H-h)\tan\frac{\theta}{2}\sqrt{2gh} \times dh$$

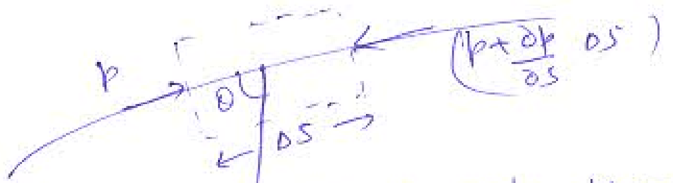
$$Q = \int dQ = \int_0^H 2Cd\,(H-h)\tan\frac{\theta}{2}\sqrt{2gh}\,dh$$

$$= 2Cd \times \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= \frac{8}{15}\,Cd\tan\frac{\theta}{2}\sqrt{2g}\,H^{5/2}$$

Section C

Q.11 Derive Euler's eqⁿ of motion along streamline?



(along streamline) Newton's second law: (No viscous force) only pressure force & body force.

$$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right) dA\, ds$$

$$-\frac{\partial p}{\partial s} ds - \rho g \cos\theta\, ds = \rho\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right) ds$$

is $V = V(s,t)$

$$\cos\theta = \frac{dz}{ds} = \frac{\Delta z}{\Delta s}$$

$$-\frac{\partial p}{\partial s} ds - \rho g \cdot \frac{\Delta z}{\Delta s} ds = \rho\left(\frac{dV}{dt} + \frac{V dV}{ds}\right) ds$$

Steady state, $\frac{dV}{dt} = 0$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + V\frac{\partial V}{\partial s} = 0$$

or $$\frac{\partial p}{\rho} + g\,dz + V\,dV = 0$$

This is known as Euler's eqⁿ of motion

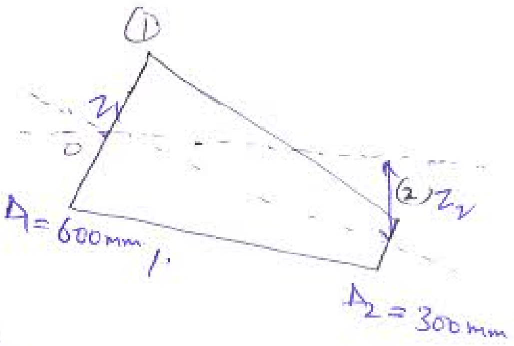
Q 12.

$L = 100\text{m}$

$D_1 = 600\text{mm}$

$A_1 = \frac{\pi}{4} D_1^2 = 0.2827\ \text{m}^2$

$D_2 = 300\text{mm}$ $A_2 = 0.07068\ \text{m}^2$



$p_1 = 19.62\ \text{N/cm}^2 = 19.62 \times 10^4\ \frac{\text{N}}{\text{m}^2}$

$Q = 50\ \frac{\text{liter}}{\text{s}} = \frac{50 \times 10^{-3}}{\text{s}}\ \text{m}^3$

$z_1 = 0$, $z_2 = \frac{10}{3}$

$Q = A_1 V_1 = A_2 V_2$

$V_1 = 0.177$ m/s $\quad V_2 = 0.707$ m/s.

Applying Bernoulli's eqn -

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + 0 = \frac{p_2}{\rho g} + \frac{(.707)^2}{2 \times 9.81} - \frac{10}{3}$$

$p_2 = 22.857$ N/cm^2 Ans.