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DEPARTMENT OF CIVIL ENGINEERING

Sessional Test – II Solution

Course: B.Tech | Semester: V^{th}
Session: 2017-18 | Section: CE-01 & CE-02
Subject: Structural Analysis -II | Sub. Code: NCE 504
Max Marks: 50 marks | Time: 2 hr.

Sec A

1 (a) Define 'flexibility coefficient' & 'Stiffness coefficient'

Solⁿ. Flexibility coefficient:

Total displacement at i due to unit force at j is -

$$\Delta i = \delta_{i1} P_1 + \delta_{i2} P_2 + \ldots \delta_{in} P_n$$

Matrix formed by displacement element is called flexibility matrix & elements of matrix are called flexibility coefficient.

Stiffness coefficient:

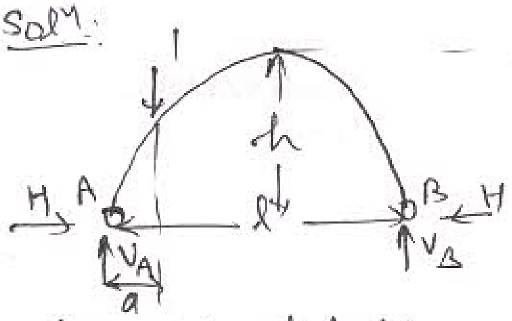
Total force P_i at i due to displacements $\Delta_1, \Delta_2, \ldots \Delta_n$ may be written as -

$$P_i = k_{i1}\Delta_1 + k_{i2}\Delta_2 + k_{i3}\Delta_3 + \ldots k_{in}\Delta_n$$

Matrix formed by force elements is called stiffness matrix & elements of matrix are called stiffness coefficient.

1(b) Draw I.L.D for horizontal thrust for two hinged arch.

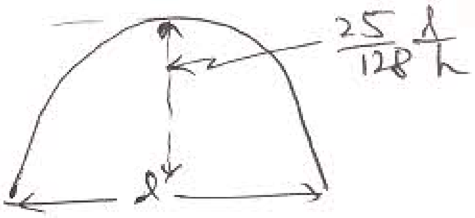
Soln:



(Two hinged Arch)

where - $h \rightarrow$ central rise

$l \rightarrow$ Span length



(I.L.D for H)

1(c) What is horizontal thrust developed in a semi-circular arch of radius R subjected to u.d.l of w/unit length over entire span. (EI = constt)

Soln: $\boxed{H = \frac{4}{3}\frac{WR}{\pi}}$

$R \rightarrow$ Radius of semicircular arch

$h \rightarrow$ central rise

1(d) State Muller Breslau principle.

Soln: If an internal stress component or reaction is considered to act through some small distance & thereby deflect or displace the structure, the curve of deflected structure will be to some scale the influence line for stress or reaction component.

1(e) What do you mean by stiffness matrix?

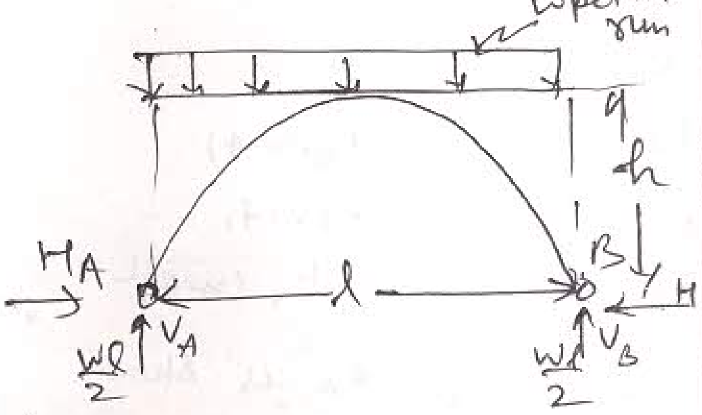
Soln: from, $[P] = \{P_L\} + [k][D]$

$[k] \rightarrow$ Stiffness matrix

$$[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2n} \\ | & | & | & & | \\ k_{n1} & k_{n2} & k_{n3} & \cdots & k_{nn} \end{bmatrix}_{n \times n}$$

Section B

2(a) A two hinged parabolic arch of span 'L' & rise 'h' carries a u.d.l of w per unit run over whole span. Find horizontal thrust at each support.



Sol^n

$$H = \frac{\int M_{ss}\, y\, dx}{\int y^2 dx}$$

$$y = \frac{4hx}{l^2}(l-x)$$

$$M_{ss} = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$\therefore H = \frac{\cancel{2}\int_0^{l/2}\left(\frac{wl}{2}x - \frac{wx^2}{2}\right)\frac{4hx}{l^2}(l-x)\,dx}{\cancel{2}\int_0^{l/2}\left[\frac{4hx}{l^2}(l-x)\right]^2 dx}$$

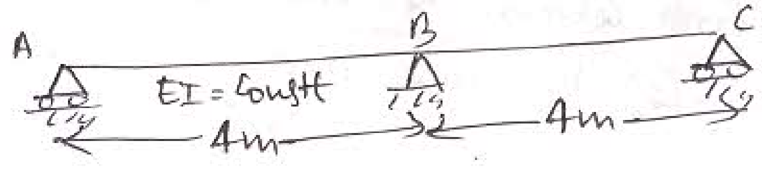
$$= \frac{\frac{w}{2}\frac{4h}{l^2}\int_0^{l/2}(lx-x^2)\cdot(l-x)\cdot x\,dx}{\frac{16h^2}{l^2}\int_0^{l/2}x^2(l-x)^2\,dx}$$

$$\boxed{H = \frac{wl^2}{8h}}$$

$l \to$ span of arch

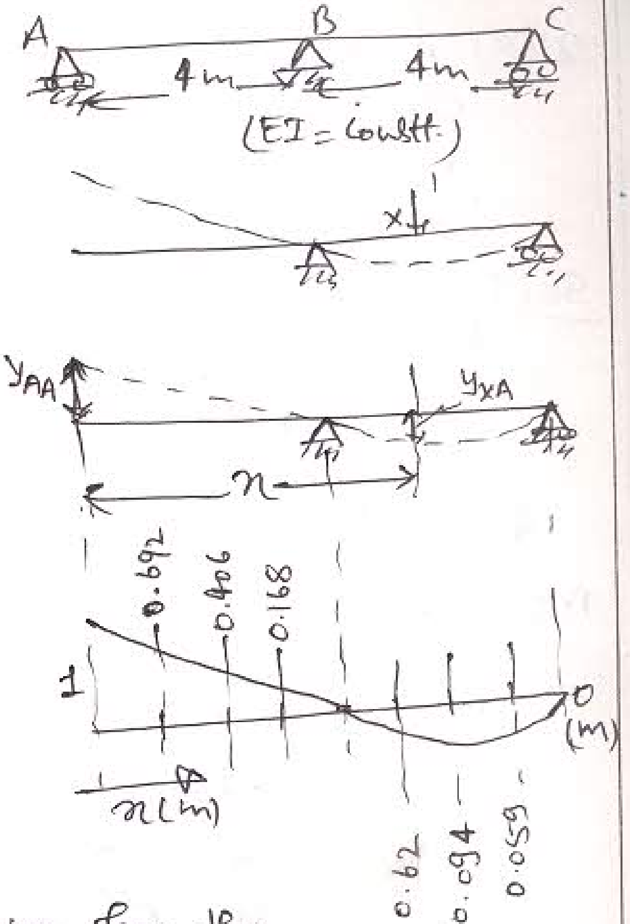
$h \to$ rise of arch.

2(b) Determine the influence line for R_A of the continuous beam given below. Compute ordinates at every 1m interval.



Sol^n.

Apply a unit vertical load at A then -



$$R_A = \frac{y_{XA}}{y_{AA}}$$

$R_B = 2\downarrow, \; R_C = 1\uparrow$

$$EI\,y'' = -1\times x \,|\, + R_B(x-4)$$

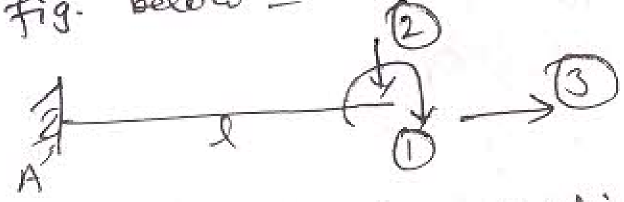
$$EI\,y'' = -x \,|\, + 2(x-4)$$

$$EI\,y' = -\frac{x^2}{2} + C_1 \,|\, + (x-4)^2$$

The I.L for R_A is shown in fig.

$R_A \rightarrow \downarrow$ then -ve

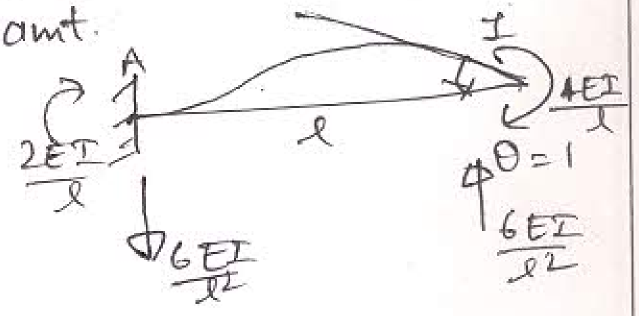
2(c) Generate stiffness matrix for the beam with reference to the co-ordinates shown in fig. below -



Sol^n. To develop stiffness matrix we will release the displacement component & calculate the required force.

For first column of matrix:

Release - 1 by unit amt.

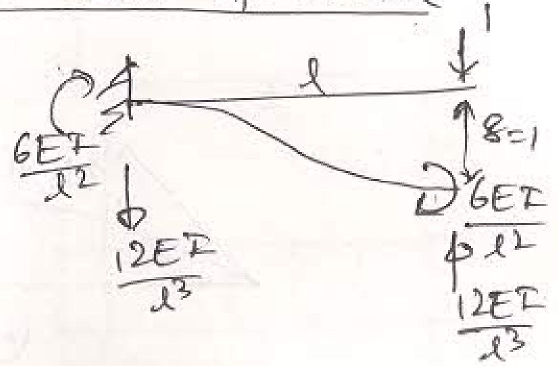


$$k_{11} = \frac{4EI}{l}$$

$$k_{21} = -\frac{6EI}{l^2}$$

$$k_{31} = 0$$

Release ② by unit for second column of matrix

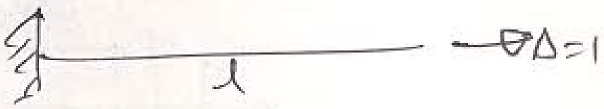


$k_{21} = \frac{6EI}{l^2}$

$k_{22} = \frac{-12EI}{l^3}$

$k_{23} = 0$

third column of matrix.



$k_{31} = 0$

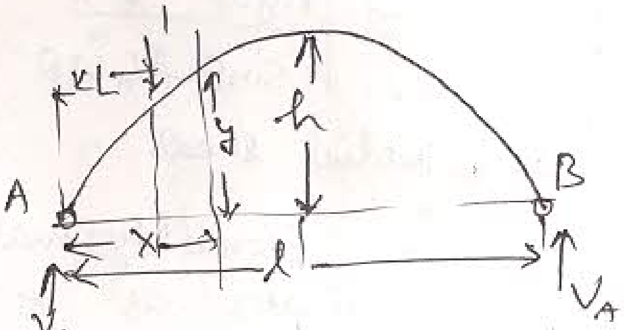
$k_{32} = 0$

$k_{33} = \frac{AE}{l}$

$$\therefore \; [k] = \begin{bmatrix} \frac{AEI}{l} & -\frac{6EI}{l^2} & 0 \\ -\frac{6EI}{l^2} & \frac{12EI}{l^3} & 0 \\ 0 & 0 & \frac{AE}{L} \end{bmatrix}_{3\times3}$$

2(d) Draw I.L.D for bending moment at any section, radial shear and normal thrust at any section for a typical two hinged symmetrical parabolic arch.

Ans: $y = \frac{4hx}{l^2}(l-x)$



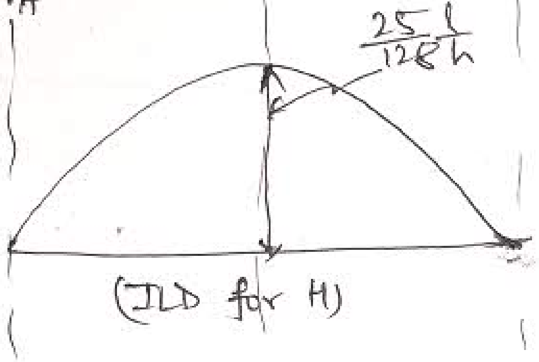
Unit load left of x

$N_x = H\cos\theta - V_B\sin\theta$

Unit load right of 'x',

$N_x = H\cos\theta + V_A\sin\theta$

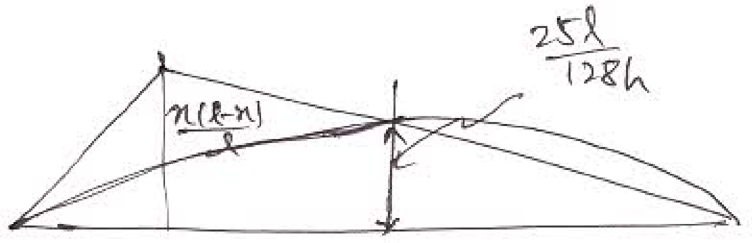
$N_x \rightarrow$ Normal thrust



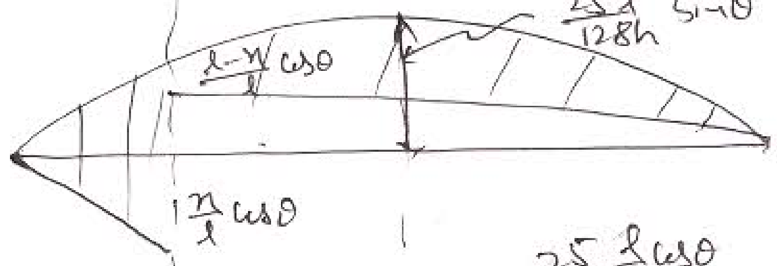
(ILD for H)

unit load left of 'x' –

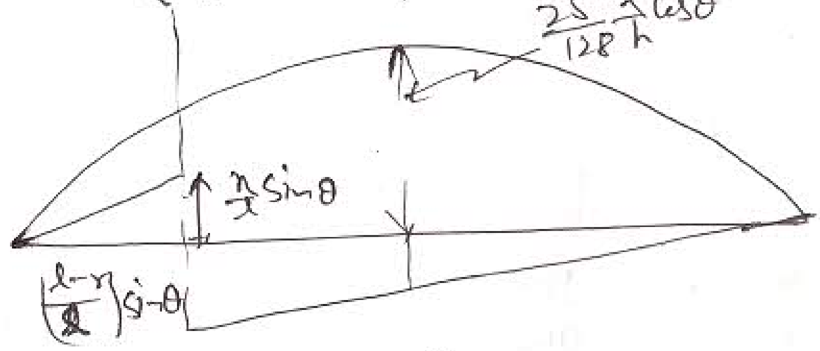
$Q_x = H\sin\theta + V_B\cos\theta$



(ILD for B.M)



(ILD for Q_x)



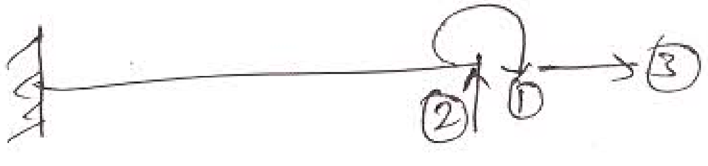
(I.L.D for N_x)

Unit load right of X

$$Q_x = H\sin\theta - V_A\cos\theta$$

$Q_x \rightarrow$ radial shear

2(e) Generate flexibility matrix for beam with reference to coordinates as in fig. below –

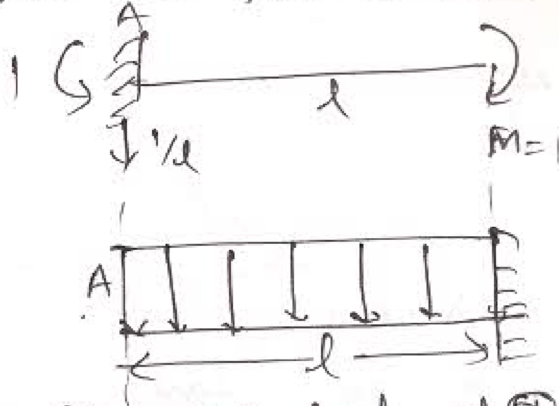


To generate first column give unit force at 1

$\delta_{11} = \left[\frac{l}{EI}\right]$

$\delta_{21} = \frac{l^2}{2EI}$

$\delta_{31} = 0$

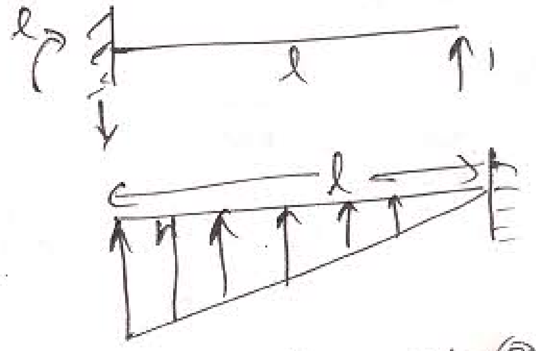


To generate second column give unit load at 2

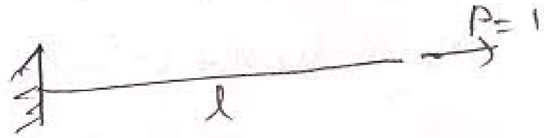
$\delta_{12} = \frac{l^2}{2EI}$

$\delta_{22} = \frac{l^3}{3EI}$

$\delta_{32} = 0$



To generate third column give unit force at 3



$\delta_{13} = 0$

$\delta_{23} = 0$

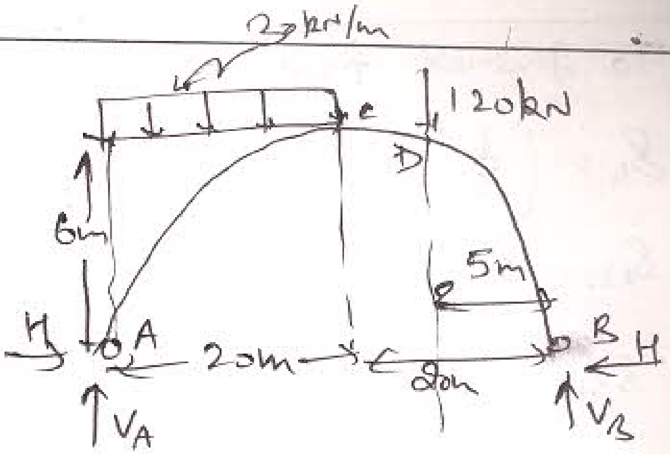
$\delta_{33} = \frac{AE}{l}$

$$\delta = \begin{bmatrix} \frac{l}{EI} & \frac{l^2}{2EI} & 0 \\ \frac{l^2}{2EI} & \frac{l^3}{3EI} & 0 \\ 0 & 0 & \frac{AE}{l} \end{bmatrix}_{3\times 3}$$

Sec-C

Q1 For two hinged Arch as shown in fig. Find out horizontal thrust, Max +ve & -ve B.M, S.F & thrust (normal) at 10m from right support.

Sol^n: $H = H_1 + H_2$



H_1 is due to udl on left half.

$\therefore H_1 = \frac{wl^2}{16h} = \frac{30 \times (40)^2}{16 \times 6}$

$H_1 = 500kN$

H_2 is due to pt. load.

$H_2 = \frac{5}{8}\frac{wl}{h}k(1-k)(1+k-k^2)$, $k = \frac{5}{40} = 0.125$

$H_2 = 60.669\,kN$

$\boxed{H = 560.669\,kN}$

$\therefore V_A = \frac{30 \times 20 \times 30 + 120 \times 5}{40} = 465\,kN$

$V_B = 255\,kN$

$M_x = V_A x + \left(-\frac{30x^2}{2}\right) - Hy$

$= 465x - 15x^2 - 560.669 \times \frac{4 \times 6x(40-x)}{(40)^2}$

$M_x = 465x - 15x^2 - 8.41(40 - x^2)$

For x to be max:

$\frac{dm_x}{dx} = 0 \Rightarrow \boxed{x = 9.757m}$

$\therefore \boxed{M_{9.757} = 627.388\,kN\text{-}m}$

For Part DB:

$M_x = V_B x - Hy \Rightarrow M_x = -81.4 - 8.41x^2$

$\frac{dM_x}{dx} = 0, \Rightarrow \boxed{x = 4.839m}$

$M_{4.839} = -196.96m$

Max +ve B.M = 627.388 kN-m

Max -ve B.M = - 605.766 kN-m

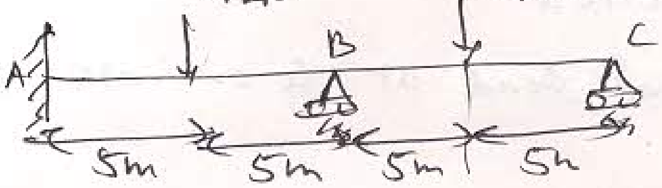
$$\tan\theta = \frac{dy}{dx} = 0.3$$

$$\boxed{\theta = 16.69}$$

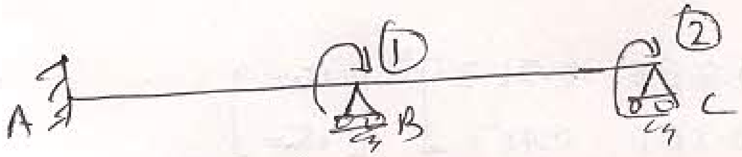
$$N = V\sin\theta + H\cos\theta = (255-120)\sin\theta + H\cos\theta$$
$$= 575.815 \text{ kN}$$

$$Q = V\cos\theta - H\sin\theta = (255-120)\cos\theta - H\sin\theta$$
$$= -31.800 \text{ kN.}$$

3 (b) Analyse the continuous beam shown below by Stiffness matrix method & draw B.M.D take EI = constt.



Soln.



Here is only two independent components are rotations at B & C.

So d.o.f = 2

locking joints B & C, the fixed end moments due to applied loads are -

$$M'_{AB} = -\frac{240 \times 5 \times (5)^2}{(10)^2} = -300 \text{ kN-m}$$

$$M'_{BA} = \frac{240 \times 5 \times (5)^2}{(10)^2} = 300 \text{ kN-m}$$

$$M'_{BC} = \frac{-120 \times 5 \times (5)^2}{(10)^2} = -150 \text{ kN-m}$$

$$M'_{CB} = \frac{120 \times 5 \times (5)^2}{(10)^2} = 150 kN\text{-}m$$

$$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = 0$$

$$P_1' = 300 - 150 = 150 \text{ kN-m}$$

$$P_2' = 150 \text{ kN-m}$$

To generate first column of the stiffness matrix -

$$k_{11} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$$

$$k_{21} = \frac{2EI}{10} = 0.2EI$$

To generate second column of the stiffness matrix -

$$k_{12} = \frac{2EI}{10} = 0.2EI$$

$$k_{22} = \frac{4EI}{10} = 0.4EI$$

for no external load at co-ordinates ① & ②

$$P_1 = P_2 = 0$$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = - \begin{bmatrix} 0.8EI & 0.2EI \\ 0.2EI & 0.4EI \end{bmatrix}^{-1} \begin{bmatrix} 150 \\ 150 \end{bmatrix}$$

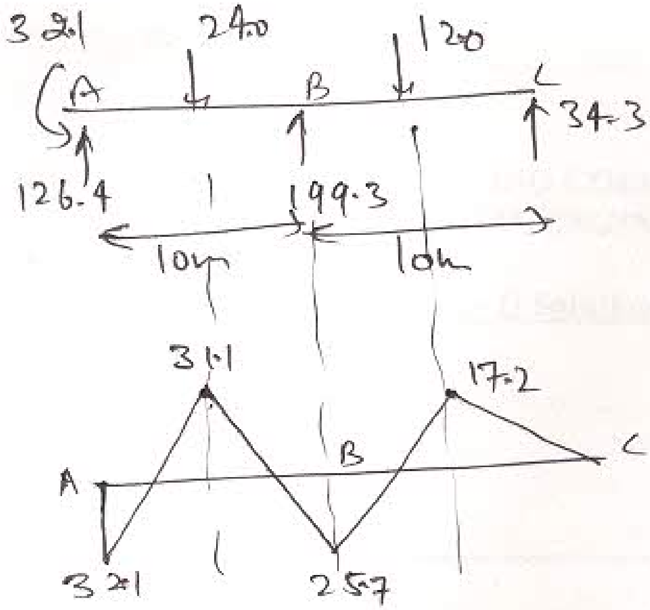
$$= \frac{1}{EI} \begin{bmatrix} -107.14 \\ -321.43 \end{bmatrix} \times 10^{-1}$$

$$M_{AB} = 300 + \frac{2EI}{10} \left(-\frac{107.14}{EI} \right) \times 10^{-1} = 321 \text{ kN-m}$$

$$M_{BA} = 300 + \frac{0.2EI}{10} \left(2 \times -\frac{107.14}{EI} \right) = 257 \text{ kN-m}$$

$$M_{BC} = -150 + \frac{0.2EI}{10} \left[2 \left(-\frac{107.14}{EI} \right) + \left(-\frac{321.43}{EI} \right) \right] = -257 \text{kN-m}$$

$$M_{CB} = 0$$



(B·M·D)