AJAY KUMAR GARG ENGINEERING COLLEGE, GHAZIABAD
DEPARTMENT OF MECHANICAL ENGINEERING

SESSIONAL TEST – 2
MODEL SOLUTION

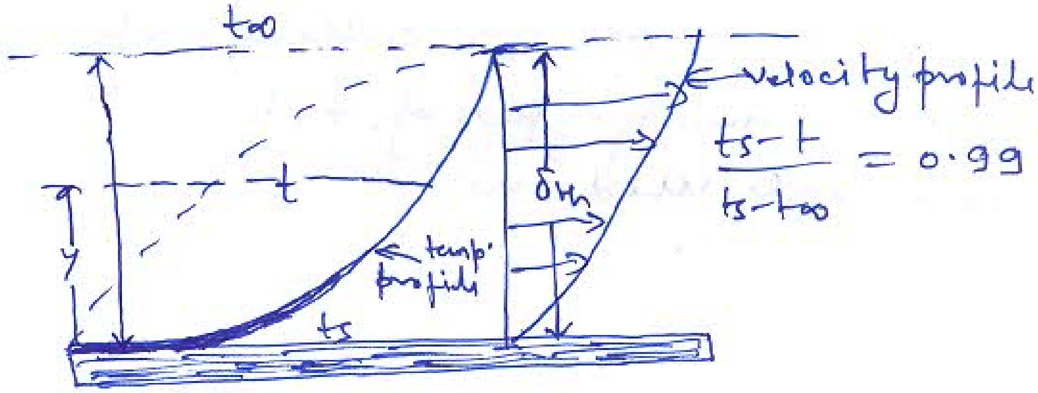
Course: B.Tech
Session: 2017-18
Subject: Heat and Mass Transfer
Max Marks: 50

Semester: V
Section: ME 1,2,3
Sub. Code: NME-504
Time: 2 hours

Section – A

Q-1 – Show with neat sketch, the temp. & velocity profile for the case when hot fluid is flowing over a cool flat plate.

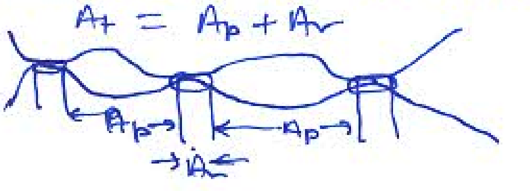
Solution –

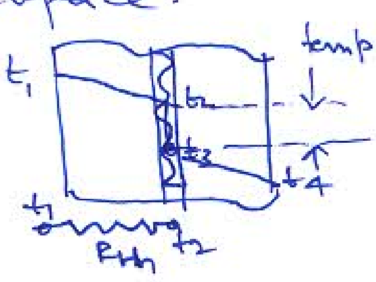


Flow of hot fluid over cool plate.

Q-2 – What do you understand by thermal contact resistance, how it differ from thermal resistance.

Solution – Thermal contact resistance – In real systems due to surface roughness & voids spaces the contact surfaces touch only at discrete locations. Thus the area available for heat flow at the interface will be small compared to geometric ~~face~~ area. Due to this reduced area and present of air voids, a large resistance to heat flow at the interface occurs. This resistance is known as thermal contact resistance & it cause temp. drop between two materials at the interface.





$$R_{th\,A-B} = \frac{t_2 - t_3}{Q/A}$$

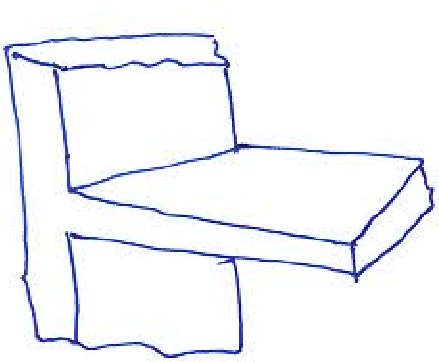
However, $R_{th} = \frac{dx}{KA}$

Thermal resistance that offered by the body itself.

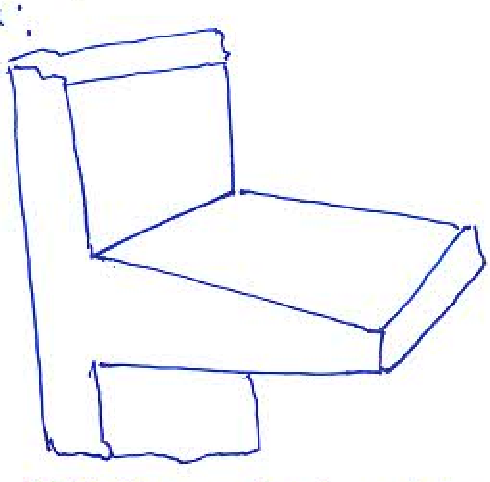
Q-3 → What do you mean by extended surfaces; explain with neat sketch the common type of surfaces which are used in practice?

Solution – Extended Surface – The surfaces which are having more surface area as compared to its volume are used as additional surface to increase the rate of heat transfer are called extended surface.

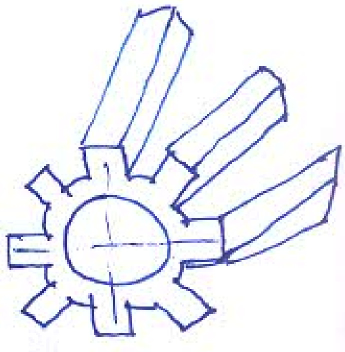
Some common types of extended surfaces that are commonly used are:



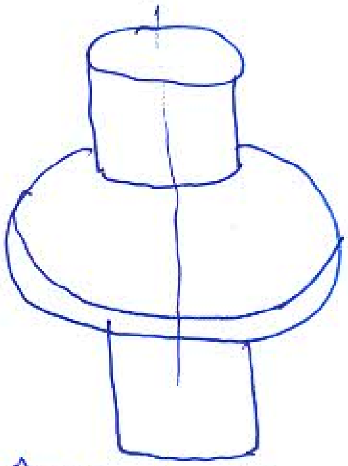
Uniform straight fin



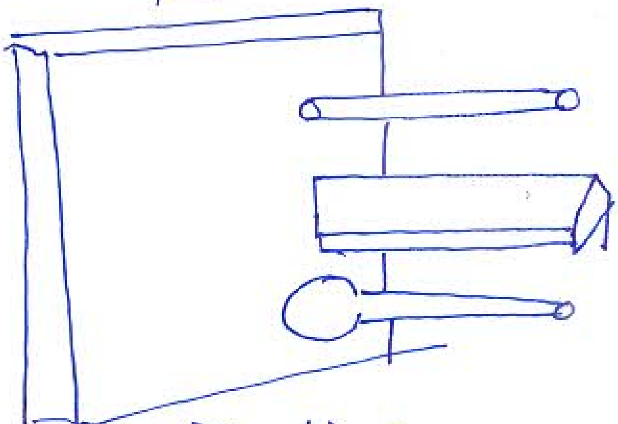
Tappered straight fin



Splines



Annular fin



Pin fins

Q-4 – What do you mean by effectiveness of a fin. Discuss the physical significance of effectiveness.

Solution – Effectiveness of fin –

$$\epsilon_{fin} = \frac{Q_{with\,fin}}{Q_{without\,fin}} = \frac{\sqrt{hPkA_{cs}}\,(t_0 - t_a)}{hA_{cs}\,(t_0 - t_a)}$$

$$\boxed{\epsilon_{fin} = \sqrt{\frac{Pk}{hA_{cs}}}}$$

So, effectiveness of fin is defined as the ratio of the fin heat transfer rate to the heat transfer rate that would exists without fin.

Effectiveness shows the use of fin is justified or not.

Q-5 — Explain the following in details:

Prandtl number, Nusselt number and its physical significance.

Solution —

Prandtl Number — It is the ratio of kinematic viscosity (ν) to the thermal diffusivity (α).

$$P_r = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k} = \frac{\nu}{\frac{k}{\rho C_p}} = \frac{\nu}{\alpha}$$

It connects link between the velocity field & temp. field & its value strongly influences relative growth of velocity & thermal boundary layer.

Nusselt Number — It is the ratio of heat flow rate by convection process under a unit temp. gradient to the heat flow rate by conduction process under a unit temp. gradient through a stationary thickness of L meters.

Thus, $$\text{Nusselt Number} = \frac{Q_{conv.}}{Q_{cond.}} = \frac{h}{k/L} = \frac{hL}{k}$$

It is used to measure convective heat transfer coefficient.

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— Ankita Maheshwari
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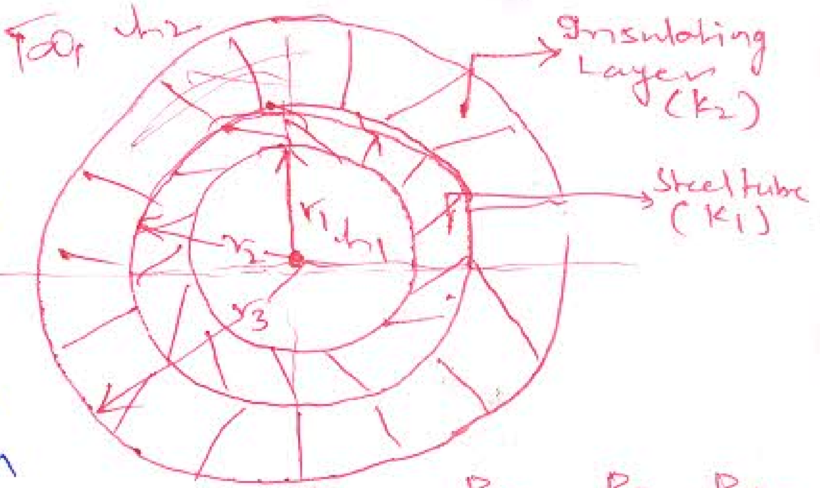
Section - B

Q-6 – A steel tube ($K = 45$ W/mK) of outside diameter 7.6 cm, and thickness 1.3 cm, is covered with an insulating material ($K = 0.2$ W/mK) of thickness 2 cm. A hot gas at 330°C, with convection coefficient of 200 W/m²K, is flowing inside the tube. The outer surface of the insulation is exposed to ambient air at 30°C, with convection coefficient of 50 W/m²K.

Calculate – (i) Heat loss to air from the 5 m long tube.

(ii) The temperature drop due to thermal resistance of hot gases, steel tube, the insulating layer and the outside air.

Solution —



Inner dia. of steel tube,

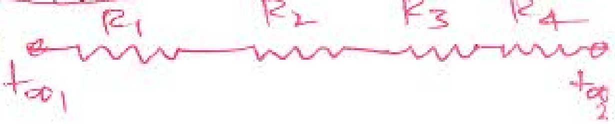
$d_1 = (7.6 - 2 \times 1.3)$ cm

$= 5$ cm

$r_1 = 2.5$ cm $= 0.025$ m

$r_2 = 3.8$ cm $= 0.038$ m

$r_3 = (3.8 + 2.0) = 0.058$ m.



Assumptions —

(i) Steady state conduction in radial directions.

(ii) No contact resistance.

(iii) Constant properties.

(i) Radial heat flow through the tube,

$$Q = \frac{T_{\infty_1} - T_{\infty_2}}{\sum R_{th}}$$

$A_1 = 2\pi r_1 L = 2\pi \times 0.025 \times 5 = 0.785$ m².

$A_2 = 2\pi r_3 L = 2\pi \times 0.058 \times 5 = 1.822$ m².

The various thermal resistances are:

$$R_1 = \frac{1}{h_1 A_1} = \frac{1}{200 \times 0.785} = 6.37 \times 10^{-3} \text{ K/W}.$$

$$R_2 = \frac{\ln r_2/r_1}{2\pi L K_1} = \frac{\ln \left(0.038/0.025\right)}{2\pi \times 5 \times 45} = 2.96 \times 10^{-4} \text{ K/W}$$

$$R_3 = \frac{\ln (r_2/r_3)}{2\pi L K_2} = \frac{\ln(0.058/0.038)}{2\pi \times 5 \times 0.2} = 0.0673 \text{ K/W}$$

$$R_4 = \frac{1}{h_4 A_2} = \frac{1}{50 \times 1.822} = 0.0109 \text{ K/W}.$$

Total resistance, $\Sigma R_{th} = R_1 + R_2 + R_3 + R_4$

$$= 84.94 \times 10^{-3} \text{ K/W}.$$

Total heat loss, $Q = \dfrac{330 - 30}{84.84 \times 10^{-3}} = 3531.8 \text{ W}.$

The temperature drop can be calculated by —

$$\Delta T_i = Q R_i$$

$$\Delta T_1 = Q \times R_1 = 3531.8 \times 6.37 \times 10^{-3}$$

$$= 22.5 \text{ K}$$

$$\Delta T_2 = Q \times R_2 = 1.045 \text{ K}$$

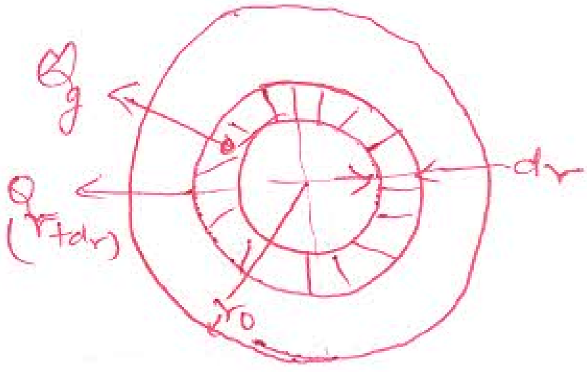
$$\Delta T_3 = Q \times R_3 = 237.68 \text{ K}$$

$$\Delta T_4 = Q \times R_4 = 38.77 \text{ K}$$

Ans —

Q-7 — Derive the expression of temperature distribution and heat flow rate in a solid sphere with uniform internal heat generation.

Solution—



Let us consider a solid sphere with uniform heat generation source $\dot{q}$ (w/m3).

The energy balance eqn.

$$Q_r + Q_g = Q_{(r+dr)}$$

$$Q_g = \dot{q} \times 4\pi r^2 dr$$

$$Q_r = -K\,4\pi r^2 \frac{dT}{dr}$$

$$Q_{(r+dr)} = -K 4\pi r^2 \frac{dT}{dr} - K\pi 4 \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) dr$$

$$Q_g = \frac{\partial}{dr}(Q_r)\,dr$$

$$q_g \times 4\pi r^2 dr = \frac{d}{dr}\left[-4\pi K r^2 \frac{dt}{dr}\right] dr$$

or $$\frac{d}{dr}\left(r\frac{\partial t}{\partial r}\right) + \frac{dt}{dr} + \frac{q_g . r}{K} = 0$$

Integrating both side, we get —

$$r\frac{dt}{dr} + t + \frac{q_g}{K}\frac{r^2}{2} = C_1$$

Again integrating —

$$rt + \frac{q_g}{K}\frac{r^3}{6} = C_1 r + C_2$$

B.C. are :— At, $r = 0$, $t = t_{max}$.

& at, $r = R$, $t = t_w$

We get – $C_2 = 0$ & $C_1 = t_w + \frac{q_g}{6k} R^2$

So,

$$rt + \frac{q_g}{k} \cdot \frac{r^3}{6} = \left[t_w + \frac{q_g}{6k} R^2 \right] r$$

or $$\boxed{t = t_w + \frac{q_g}{6k} (R^2 - r^2)}$$

we get the temp. distribution is parabolic.

The max. temp. is at the centre where, $r = 0$, $t = t_{max.}$

$$t_{max.} = t_w + \frac{q_g}{6k} \cdot R^2$$

In diamensianless form –

$$\frac{t - t_w}{t_{max.} - t_w} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

From fourier's law –

$$Q = -kA \left(\frac{dt}{dr}\right)_{r=R}$$

$$= -k 4\pi R^2 \left\{ \frac{d}{dr} \left[t_w + \frac{q_g}{6k} (R^2 - r^2) \right] \right\}_{r=R}$$

or $$Q = -k 4\pi R^2 \left[\frac{q_g}{6k} (-2r) \right]_{r=R}$$

or $$Q = \frac{4}{3} \pi R^3 \times q_g$$

$$\frac{4}{3} q_g \pi R^3 = h \times 4\pi R^2 (t_w - t_a)$$

or $$t_w = t_a + \frac{q_g R}{3h}$$

$$t = t_a + \frac{q_g R}{3h} + \frac{q_g}{6k} (R^2 - r^2)$$

The max. temp. –

$$\boxed{t_{max.} = t_a + \frac{q_g}{3h} \cdot R + \frac{q_g}{6k} \cdot R^2} \text{ at } r = 0$$

Q-8 — An aluminium sphere weighing 6 kg and initially at temperature of 350 °C is suddenly immersed in a fluid at 30 °C with convection coefficient of 60 W/m²K. Estimate the time required to cool the sphere to 100 °C.

Take thermophysical properties of aluminium as:
Specific heat = 900 J/KgK, Density = 2700 Kg/m³.
Thermal conductivity = 205 W/mK.

Solution —

volume of sphere, $V = \frac{4}{3}\pi r_0^3 = \frac{m}{\rho}$

$= \frac{6}{2700}$

$r_0 = 0.0809$ m.

Characterstic length of the sphere,

$L_c = \frac{V}{A_s} = \frac{r_0}{3} = 0.0269$ m.

$Bi = \frac{hL_c}{k} = \frac{60 \times 0.0269}{205} = 7.89 \times 10^{-3}$

$Bi << 0.1$

Hence lumped parameter analysis is applicable

So,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{-\frac{h\tau}{\rho L_c \cdot c}\right\}$$

$$\frac{100-30}{350-30} = \exp\left\{\frac{-60\tau}{2700 \times 0.0269 \times 900}\right\}$$

$\boxed{\tau = 1655 \text{ sec.} = 27.6 \text{ min.}}$ Ans—

Q-9 – A fan provides air speed up to 50 m/s, is used in low speed wind tunnel with atmospheric air at 27°C. If this wind tunnel is used to study the boundary layer behavior over a flat plate up to $R_e = 10^8$.
What should be the minimum plate length?
At what distance from the leading edge would transition occur, if critical Reynold's number is $R_{cr} = 5 \times 10^5$?

Solution –

Properties of air at 27°C are approximately –
$\rho = 1.16$ kg/m³, $\mu = 184.6 \times 10^{-7}$ kg/m/s.

(i) $$R_{ex} = \frac{\rho V_\infty x}{\mu} = 10^8$$

$$x = \frac{10^8 \times 184.6 \times 10^{-7}}{1.16 \times 50} = 31.82 \text{ m.}$$

The minimum length of the plate for $R_{ex} = 10^8$ is 31.82 m.

(ii) For transition to occurs at –

$Re_{crit.} = 5 \times 10^5$

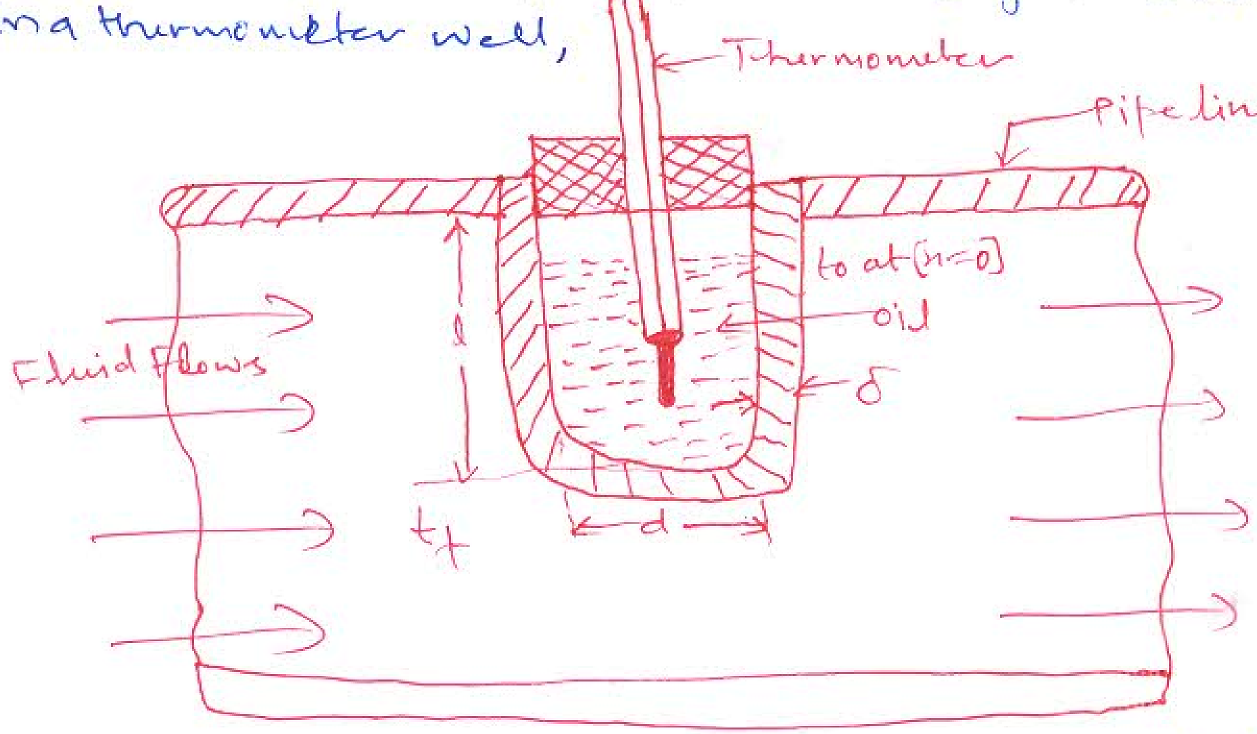
$$x_{crit.} = \frac{5 \times 10^5 \times 184.6 \times 10^{-7}}{1.16 \times 50} = 0.159 \text{ m.}$$

The transition from laminar to turbulent will occurs at $x_{crit.} = 0.159$ m.

Ans –

Q-10 — How the error is measured, in temperature measurement by thermometric well.

Solution— **Thermometer Well** — For estimating the error in the value of temp. measured by a thermometer dipped in a thermometer well,



the theory of extended surface is very helpful.

A thermometer well is defined as a small tube welded radially in to a pipeline through which a fluid whose temp. is to be measured is flowing.

The temp. distribution at any distance x measured from pipe wall along the thermometer well is given by—

$$\frac{\theta_x}{\theta_0} = \frac{t_x - t_f}{t_0 - t_f} = \frac{\cosh\left[m(l-x)\right]}{\cosh(ml)}$$

At $x = l$,

$$\frac{t_l - t_f}{t_0 - t_f} = \frac{\cosh\left[m(l-l)\right]}{\cosh(ml)} = \frac{1}{\cosh(ml)}$$

$\frac{1}{\cosh(ml)} \Rightarrow$ thermometric error

$$P = \pi(d+2\delta) \simeq \pi d$$

$$A_{cs} = \pi d \delta$$

$$\frac{P}{A_{cs}} = \frac{1}{\delta}$$

Then, $m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h}{k\delta}}$

So, $\frac{t_1 - t_f}{t_0 - t_f} = \frac{1}{\cosh\left(\sqrt{\frac{h}{k\delta}}\, l\right)}$

For less Error — ml should be high —

means — (i) Large value of h

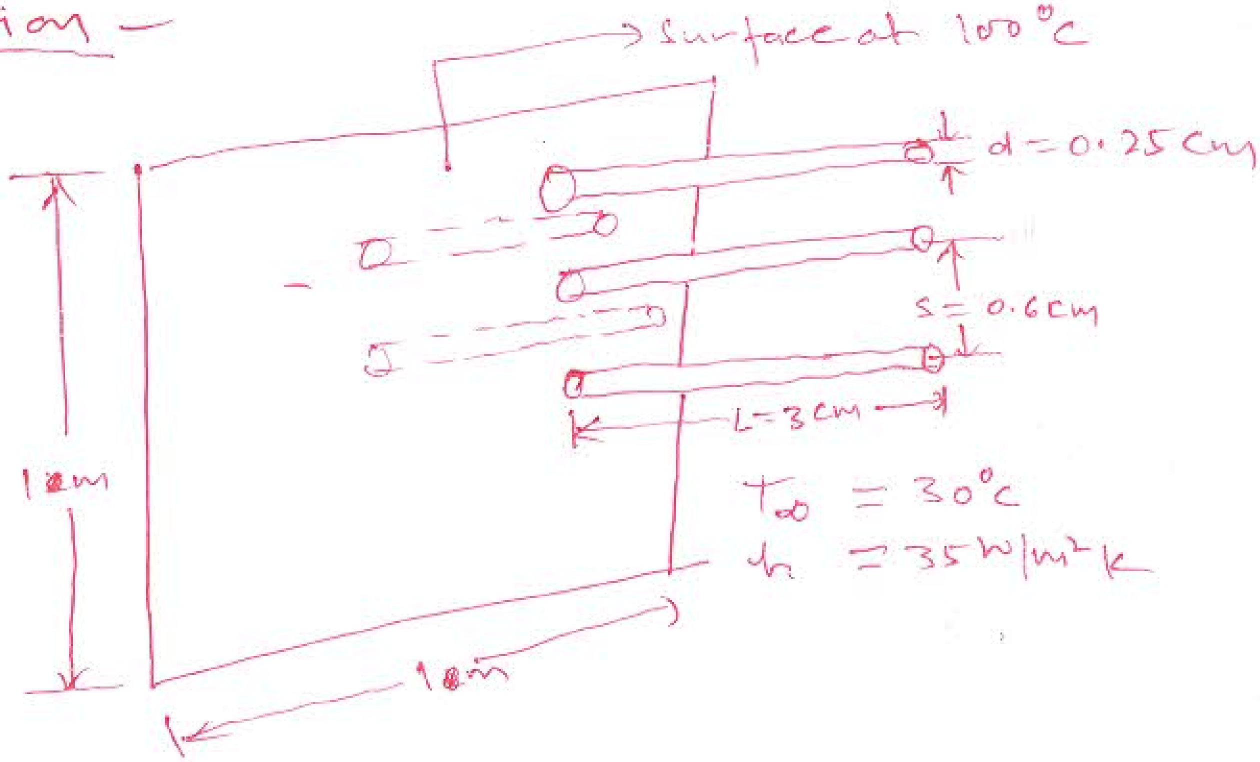
(ii) Small value of K

(iii) Long & thin well is used or may be placed Obliquely.

Section-C

Q. 11 – A hot surface at 100°C is to be cooled by attaching 3 cm long, 0.25 cm diameter aluminium fins ($K = 237\ W/mK$) to it, with a centre to centre distance of 0.6 cm. The temperature of surrounding air is 30°C and the heat transfer coefficient on surface is 35 W/m^2K. Calculate the rate of heat transfer from the surface for a 1m x 1m section of the plate. Also determine the overall effectiveness of the fins.

Solution –



Assumptions –

(i) Steady state conditions.

(ii) Finite long fin but, L >>> d, hence assuming insulated tip.

(iii) Constant properties.

Number of fins in a row $= \frac{W}{S} = \frac{100}{0.6} = 166.67 \cong 167$

Similarly No of fins in a column $= \frac{100}{0.6} = 167$

These fins are in matrix of $n \times n$, thus —

Total Number of fins, $M_{fin} = 167 \times 167 = 27,889$ fins/m²

For a fin, $A_c = \frac{\pi}{4} d^2 = 4.9 \times 10^{-6}\ m^2$

$P = \pi d = 7.8 \times 10^{-3}\ m.$

$m = \sqrt{\frac{hP}{kA_c}} = 15.37$

Heat transfer rate from all fins —

$Q_{n\text{-}fins} = M_{fin} \sqrt{hPkA_{cs}}\ (T_0 - T_\infty) \tanh(mL)$

$= 27\,889 \times \sqrt{35 \times 7.8539 \times 10^{-3} \times 237 \times 4.9 \times 10^{-6}}$
$\times (100 - 30) \times \tanh(15.37 \times 0.03)$

$= 15.046 \times 10^{3}\ W.$

Heat transfer rate from unfined portion —

$A_{without\,fin} = 1\,m^2 - M_{fin} \times A_c$

$= 1\,m^2 - 27,889 \times 4.9 \times 10^{-6}$

$= 0.863\ m^2$

$Q_{without\,fin} = h\,A_{without\,fin} \times (T_0 - T_\infty)$

$= 35 \times 0.863 \times (100 - 30)$

$= 2114.6\ W.$

Total heat transfer from the surface,

$Q_{total} = Q_{with\,fin} + Q_{without\,fin} = 15.046 \times 10^{3} + 2114.6$

$= 17.16 \times 10^{3}\ W.$

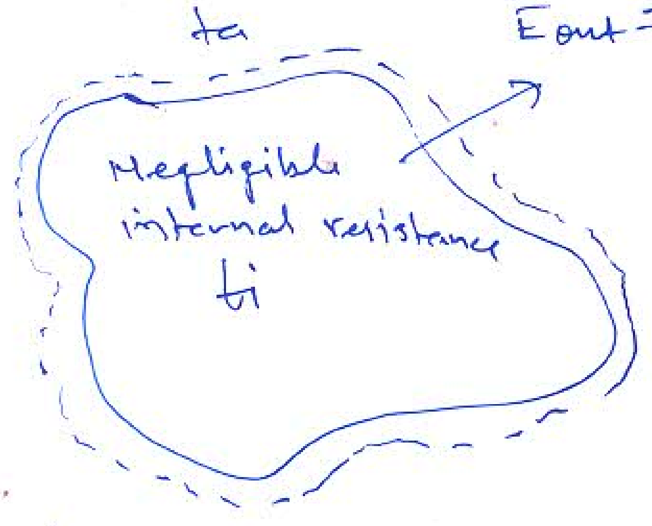
Overall effectiveness —

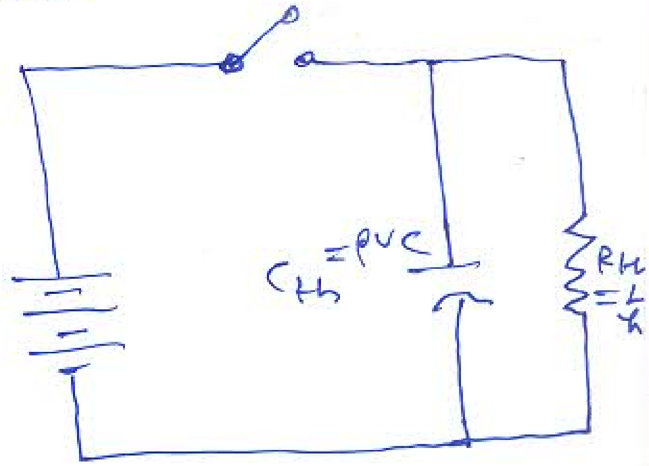
$\epsilon_{overall} = \frac{Q_{total}}{Q_{without\,fin}} = \frac{17.16 \times 10^{3}}{35(1) \times (100 - 35)} = 7.0$

$\boxed{\epsilon_{overall} = 7.0}$ Ans —

Q-12 – Discuss the mechanisms of lumped parameter analysis and derive the expression for temperature distribution and total heat transfer.

Solution – All solids have a finite thermal conductivity and there will be always a temp. gradient inside the solid whenever heat is added or removed.
However, for solids of large thermal conductivity, with the surface areas that are large in proportion to their volume like plates & thin mettalic wires, the internal resistance $\frac{L}{KA}$ can be assumed to be small or negligible in comparision with the ~~conduction~~ convection resistance $\left(\frac{1}{hA}\right)$ at the surface. The process in which internal resistance is assumed negligible with its surface ~~area~~ resistance, the temp. in this process, is considered to be uniform at a given time. Such analysis is called lumped parameter analysis.





Lumped system
$\tau = 0, \ t = t_i$
$\tau > 0, \ t = f(\tau)$

Circuit for lumped capacitance

The transient response of the body can be determined by —

$$Q = -\rho V c \frac{dt}{d\tau} = h A_s (t - t_a)$$

or $$\int \frac{dt}{(t-t_a)} = -\frac{h A_s}{\rho V c} \int d\tau$$

or $$\ln (t - t_a) = -\frac{h A_s}{\rho V c} \tau + C_1$$

The boundary conditions are:

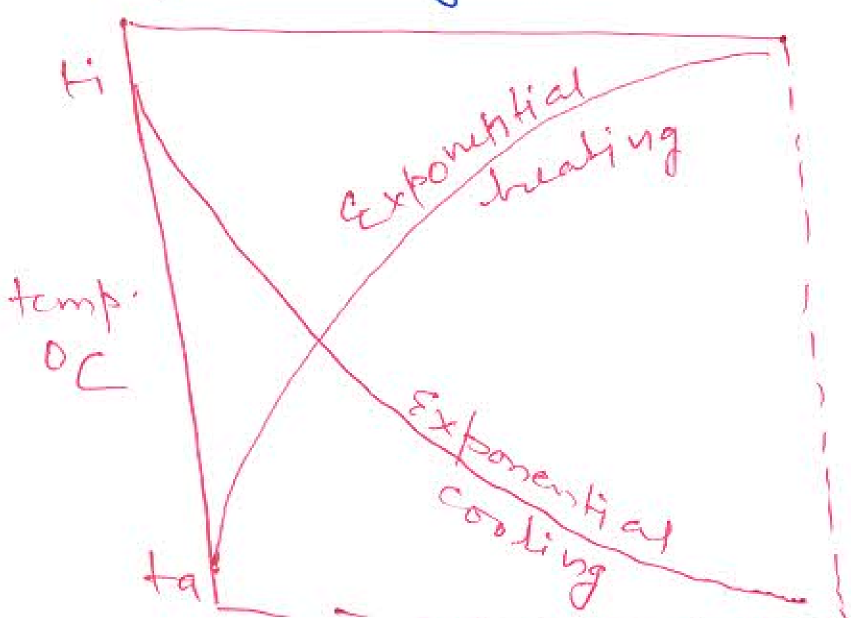
At, $\tau = 0$, $t = t_i$

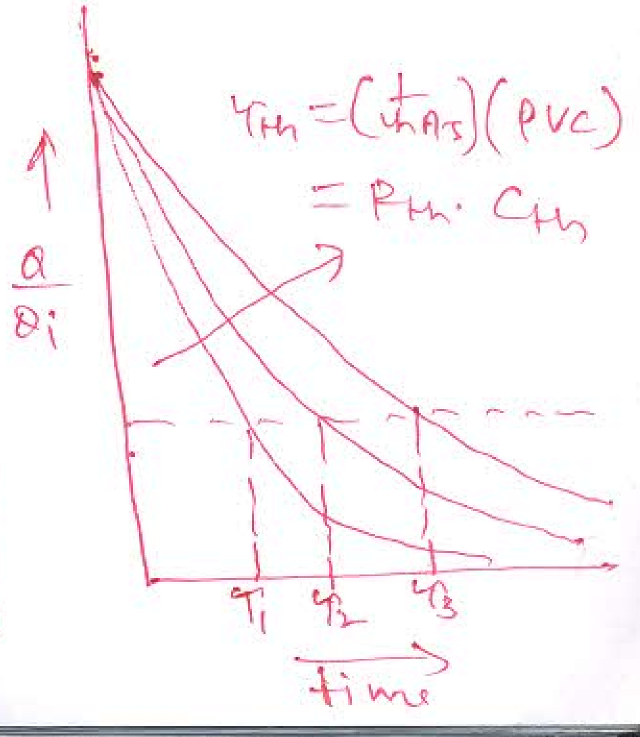
we get, $C_1 = \ln (t_i - t_a)$

Hence, $$\ln (t - t_a) = -\frac{h A_s}{\rho V c} \tau + \ln (t_i - t_a)$$

$$\boxed{\frac{t - t_a}{t_i - t_a} = \frac{Q}{Q_i} = \exp\left[-\frac{h A_s}{\rho V c} \tau\right]}$$

This is the temp. distribution in the body for Newtonian heating or cooling and it indicates that temp. rises exponentially with time.





$$\frac{\rho V c}{h A_s} = \tau_{th} \Rightarrow \text{thermal time constant.}$$

Its value is indicative of, how fast a body will response to a change in the environmental temperature.

$$\tau_{th} = \left(\frac{1}{h A_s}\right)(\rho V c) = R_{th} \cdot C_{th}$$

So, $$\frac{\theta}{\theta_i} = \exp\left(-\frac{\tau}{\tau_{th}}\right)$$

Figure indicates that any increase in R_{th} & C_{th} will cause a solid to respond more slowly to change in its thermal environment & will increase the time required to attain the thermal equilibrium $(\theta = 0)$

The power of exponential i.e. $\frac{h A_s}{\rho V c}\tau$ can be arrange in diamensionless form as follows:

$$\frac{h A_s}{\rho V c}\tau = \left(\frac{h V}{k A_s}\right)\left(\frac{A_s^2}{\rho V^2 c} k \tau\right) = \left(\frac{h L_c}{k}\right)\left(\frac{\alpha \tau}{L_c^2}\right)$$

$$= Bi \cdot Fo$$

So, $$\boxed{\frac{\theta}{\theta_i} = \frac{t - t_a}{t_i - t_a} = e^{-Bi\,Fo}}$$

Instantaneous heat flow rate and total heat transfer —

The instantaneous ~~heat~~ heat flow —

$$Q_i = \rho V c \frac{dt}{d\tau} = \rho V c \frac{d}{d\tau}\left[t_a + (t_i - t_a)\exp\left\{-\frac{h A_s}{\rho V c}\tau\right\}\right]$$

$$Q_i = \rho V c\left[(t_i - t_a)\left\{-\frac{h A_s}{\rho V c}\right\}\exp\left\{-\frac{h A_s}{\rho V c}\tau\right\}\right]$$

$$Q_i = -h A_s (t_i - t_a)\exp\left[-\frac{h A_s}{\rho V c}\tau\right]$$

$$\boxed{Q_i = -h A_s (t_i - t_a)\, e^{-Bi\,Fo}}$$

Total heat transfer —

$$Q' = \int_0^{\tau} Q_i \, d\tau$$

$$= \int_0^{\tau} -hA_s(t_i - t_a) \exp\left[-\frac{hA_s}{\rho V c}\tau\right] d\tau$$

$$= \left[-hA_s(t_i - t_a) \frac{\exp\left(\frac{-hA_s}{\rho V c}\right)\tau}{-\frac{hA_s}{\rho V c}}\right]_0^{\tau}$$

$$= \rho V c (t_i - t_a)\left[\exp\left\{-\frac{hA_s}{\rho V c}\tau\right\} - 1\right]$$

$$\boxed{Q' = \rho V c (t_i - t_a)\left[e^{-B_i F_o} - 1\right]}$$

By -

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Ankita Maheshwari
Mayank Tiwari
M.E.