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Department of Mechanical Engineering

Model Solution of Sessional Test-2

Course:	B.Tech	Semester:	VI
Session:	2017-18	Section:	ME-1,2, 3
Subject:	Computer Aided Design	Sub. Code:	NME-701
Max Marks: 50		Time:	2 hours

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Section-A

Ques-1(a)- Difference b/w DDA and Bresenham line drawing algo?

Ans

DDA Algo.	Bresenham line drawing
(1)- uses floating point real numbers	(1)- Integer Bresenham algo uses Integer value.
(2)- Rounding off is done	(2)- Rounding off is not needed
(3)-Computationally expensive due to use of real numbers and multiplication operation performed.	(3)- Less computation needed since Integers are added and subtracted.
(4)-DDA is slow as complex operations (multiplication and division) is performed	(4)- Faster as only addition and subtractions operations are performed.

Ques-2 Define Homogeneous Co-ordinate System?

Ans→ In homogeneous co-ordinate system a n directional vector can be represented as n+1 dimensional vector. eg. position vector of point P be [x, y, z] in cartesian co-ordinate, it can be represented as [x, y, z, w] in homogenous co-ordinate system. An extra co-ordinate is in the end generally w=1

→ Also it helps to unify different transformation operations as multiplicative transformation.

eg.- A line segment undergoes translation T, Rotation R and Scaling S simultaneously. The Transformation can be given as a 4x4 multiplicative matrix of-

$$T_r = T \times R \times S$$

Ques-3 Explain graphics standards?

Ans-→ previously software for producing graphics was mostly device dependent. Graphics software written for one type of hardware system was not portable to another type.

→ graphics standard were set to some portability issues to render the application software device independent.

→ It helped integrate and automate design and manufacturing process.

It provides

→ high interactivity

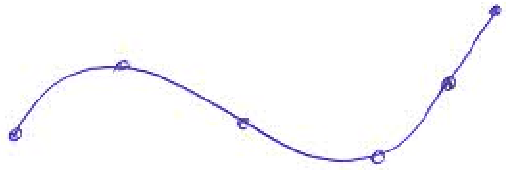
→ real time graphic data modification

→ support for geometric transformation.

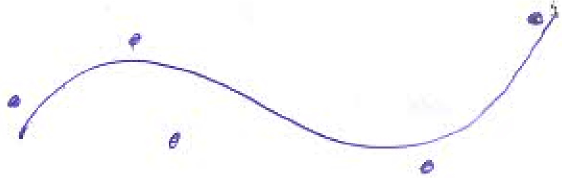
Ques 4- Define approximation and interpolation in curve design?

Ans- when polynomials are fitted to the control points without necessarily passing through any control point, resulting curve is said to approximate the control points.

Interpolation- Process of finding and evaluating a function whose graph (curve) goes through control points. The resulting curve is said to interpolate the data points.



Interpolate



Approximate

Ques-5- Write properties of Bezier Curve?

Ans- (1)- Does not use tangent vector for controlling curve shape like hermite cubic spline.

(2)- Reversing the sequence of control points does not change the shape of curve.

(3)- Invariant under geometric transformation.

(4)- Show convex hull properties.

(5)- Provides global control of curves.

SECTION (B)

8

P. T. O.

Ques 7- Derive mid point circle algo.

Ans- This method test the following halfway location between two pixels to determine if this midpoint is inside or outside the circle circumference.

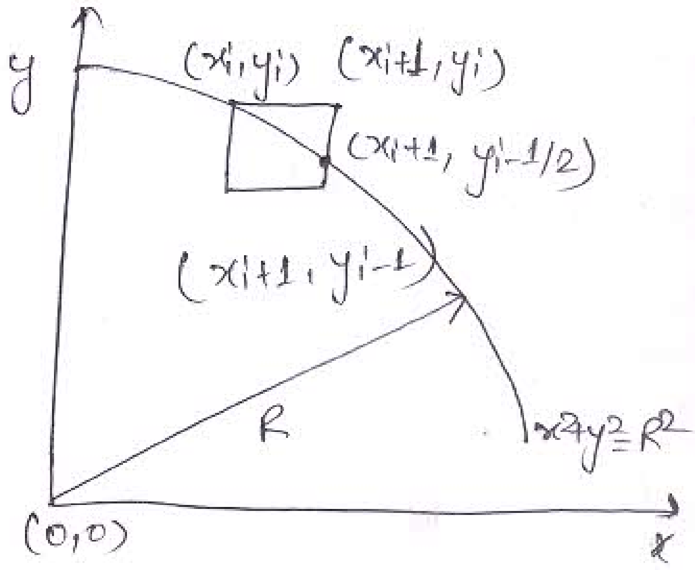
→ The algorithm steps are as follows-

Set the initial value of variables

Given circle center co-ordinates (h,k)

Shift the center to origin $(0,0)$

$error(e) = \frac{5}{4} - r$



→ Test to determine whether the entire circle has been scan converted i.e. $x \geq y$ stop.

→ compute location of next pixel

if $(e \geq 0)$ $x_{i+1} = x_i + 1$

$y_{i+1} = y_i - 1$

and $e_{i+1} = e_i + 2x_{i+1} + 1 - 2y_{i+1}$

else

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i$

$e_{i+1} = e_i + 2x_{i+1} + 1$

→ Plot remaining points by taking mirror image about $x = y$, $x = 0$ axis

Shift center back to its original position (h,k) and rest of the calculated points by translating points by h in x direction and k in y direction

⇒ Go to Step2

Q6 Find raster locations by Bresenham's line drawing algorithm for line segment with end points (3,2) and (8,6).

Sol Given end points (3,2) and (8,6)

$dx = x_2 - x_1 = 8 - 3 = 5$

$dy = y_2 - y_1 = 6 - 2 = 4$

$$\text{Slope} = \frac{dy}{dx} = \frac{4}{5} = 0.8 < 1$$

Hence the line lies in the 1st octant.

Integer Bresenham line drawing algorithm is

$e = 2dy - dx$

if $(e > 0)$

$e = e + 2(dy - dx)$

$y = y + 1$

$x = x + 1$

else $e = e + 2dy$

$y = y$

$x = x + 1$

x	y	e (error)	
3	2	$e = 2\times4 - 5 = 3$	$e > 0$
4	3	$e = 3 + 2(-1) = 1$	$e > 0$
5	4	$e = 1 + 2(-1) = -1$	$e < 0$
6	4	$e = -1 + 2(4) = 7$	$e > 0$
7	5	$e = 7 + 2(-1) = 5$	$e > 0$
8	6		

Q8 Find transformed cordinates of a plane triangular lamina having the vertices (5, 2), (3, 1) and (2, 2) rotated by 90° about the point (5, 2) in counter clockwise direction.

Sol Given points. (5, 2), (3, 1) and (2, 2)
Since points (5, 2) is fixed, we need to translate, rotate and translate back

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix}$$

$$T_R = T_1 \times R \times T_2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -3 & 1 \end{bmatrix}$$

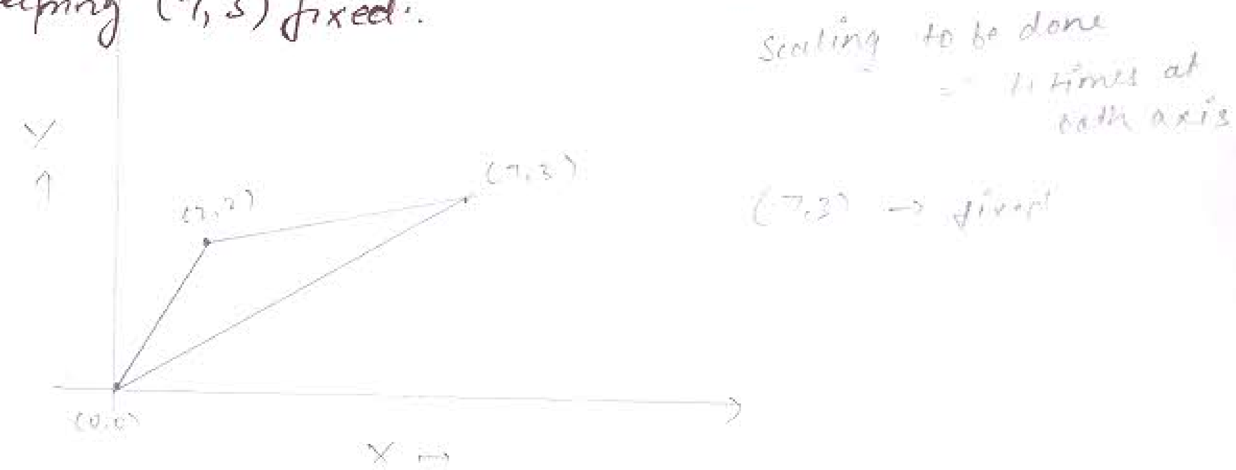
New cordinates can be given as

$$(x', y', 1) = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -3 & 1 \end{bmatrix}$$

in homogenous cordinate system.

$$= \begin{bmatrix} 5 & 2 & 1 \\ 6 & 0 & 1 \\ 5 & -1 & 1 \end{bmatrix}$$

Q9 Using scaling matrix magnify the triangle with vertices (0,0) (2,2) and (7,3) to 4 times its size in both directions
Ques-4. Keeping (7,3) fixed.



Given :- Triangle coordinates (0,0), (2,2) & (7,3)

Fixed coordinate → (7,3)

Let, S_x & S_y are scaling factor.

$\therefore S_x = S_y = 4$

As, scaling to be done keeping (7,3) fixed, then firstly, we have to translate (7,3) to origin.

Translation matrix $T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & -3 & 1 \end{bmatrix}$

Then, after trasaltion, scaling to be done:-

Scalining matrix $S_1 = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then, after scaling again translation should done from origin to (7,3)

Translation matrix $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 3 & 1 \end{bmatrix}$

So, overall transformation matrix $= [R] = [T_1] \times [S_1] \times [T_2]$

$$[R] = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ -21 & -9 & 1 \end{bmatrix}$$

So, Other coordinates values after scaling

(1). $[x_1' \ y_1' \ 1] = [x \ y \ 1][R]$

$$= [0 \ 0 \ 1]\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ -21 & -9 & 1 \end{bmatrix}$$

$$= [-21 \ -9 \ 1]$$

$\therefore$ $x_1' = -21$ $\quad (-21, -9)$

$y_1' = -9$

(2) $[x_2' \ y_2' \ 1] = [x \ y \ 1][R]$

$$\Rightarrow [2 \ 2 \ 1]\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ -21 & -9 & 1 \end{bmatrix}$$

$$\Rightarrow [-13 \ -1 \ 1]$$

$\therefore$ $x_2' = -13$ $\quad [-13, -1]$

$y_2' = -1$

[3] $[x_3' \ y_3' \ 1'] = [x_3 \ y_3 \ 1]$

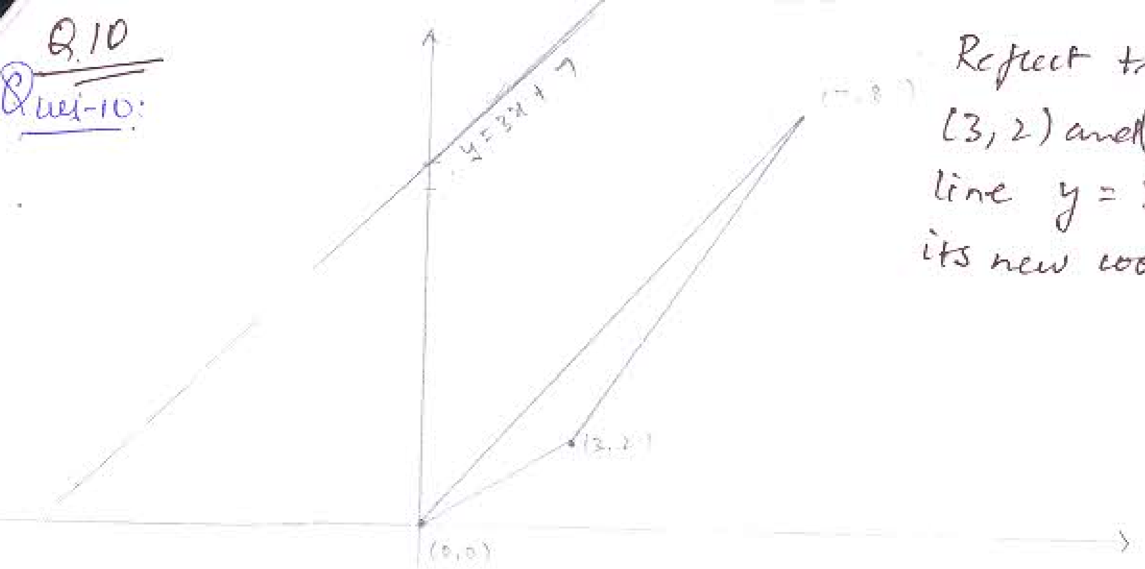
As (x_3, y_3) is fixed.

$\Rightarrow$ $(7, 3)$

Ans

Q.10
Ques-10:

Reflect triangle (0,0) (3,2) and (7,8) about line $y = 3x+7$ and write its new coordinates.



Following are the steps your solution.

Step 1: Translates the line so its passes through origin.
so, −7 units in y-direction ∴ $T_y = -7$ $T_x = 0$

$$\text{Translation matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 1 \end{bmatrix} = [T_1]$$

Step 2:- Rotate the line so, line alligns to Y-axis.
∴ $\theta' = \tan^{-1}[3] \Rightarrow 72^\circ$ ∴ $\theta = 90 - \theta' \Rightarrow 18^\circ$

$$\text{Rotation matrix } [R_1] = \begin{bmatrix} \cos(\theta) & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.951 & 0.309 & 0 \\ -0.309 & 0.951 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3:- Reflection about y-axis will take place.

$$\text{Reflection matrix } [A_1] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4:- The line is rotated back to its original inclination.
Therefore, rotate the line θ° clockwise.

$$\text{Rotation matrix} = [R_2] = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.951 & -0.309 & 0 \\ 0.309 & 0.951 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5:- The line is translated back to it's original posⁿ

$T_y = 7$, $T_x = 0$

$$\text{Translation matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix} = [T_2]$$

Overall rotation matrix [R] will be:-

$$[R] = [T_1] \times [R_1] \times [A_1] \times [R_2] \times [T_2]$$

$$\Rightarrow \begin{bmatrix} -0.8089 & 0.5877 & 0 \\ 0.5877 & 0.8089 & 0 \\ -4.114 & 1.3376 & 1 \end{bmatrix}$$

New coordinates:-

(1) $$[x_1' \; y_1' \; 1] = [0 \; 0 \; 1] \begin{bmatrix} -.8089 & 0.587 & 0 \\ 0.5877 & 0.809 & 0 \\ -4.114 & 1.338 & 1 \end{bmatrix} \Rightarrow [-4.114 \;\; 1.338 \;\; 1]$$

$$\Rightarrow (-4.114, 1.338)$$

(2) $$[x_2' \; y_2' \; 1] = [3 \; 2 \; 1] \begin{bmatrix} -.8089 & 0.588 & 0 \\ 0.5877 & 0.809 & 0 \\ -4.114 & 1.338 & 1 \end{bmatrix}$$

$$\Rightarrow [-5.365 \;\; 4.718 \;\; 1] \Rightarrow (-5.365, 4.718)$$

(3) $$[x_3' \; y_3' \; 1] = [7 \; 8 \; 1] \begin{bmatrix} -0.8089 & 0.588 & 0 \\ 0.5877 & 0.809 & 0 \\ -4.114 & 1.338 & 1 \end{bmatrix}$$

$$\Rightarrow [-5.074 \;\; 11.922 \;\; 1] = (-5.074, 11.922)$$

Ans

Q11 Draw Biezer curve with following control points (2,3), (4,5), (7,−7) and (11,7) and find 4 points on curve besides the one mentioned.

Sol Given control points (2,3), (4,5), (7,−7) and (11,7)

Since 4 control points are given, the bezier curve can be represented with an equation of degree 3.

General parametric eq. of Biezer curve is given as.

$$P(u) = P_0(1-u)^n + C(n,1)P_1 u(1-u)^{n-1} \dots C(n,n-1)P_{n-1}u^{n-1}(1-u) + P_n u^n.$$

For a cubic biezier curve.

$$P(u) = (1-u)^3 P_0 + 3(1-u)^2 u P_1 + 3(1-u)u^2 P_2 + u^3 P_3$$

Equation for X coordinate.

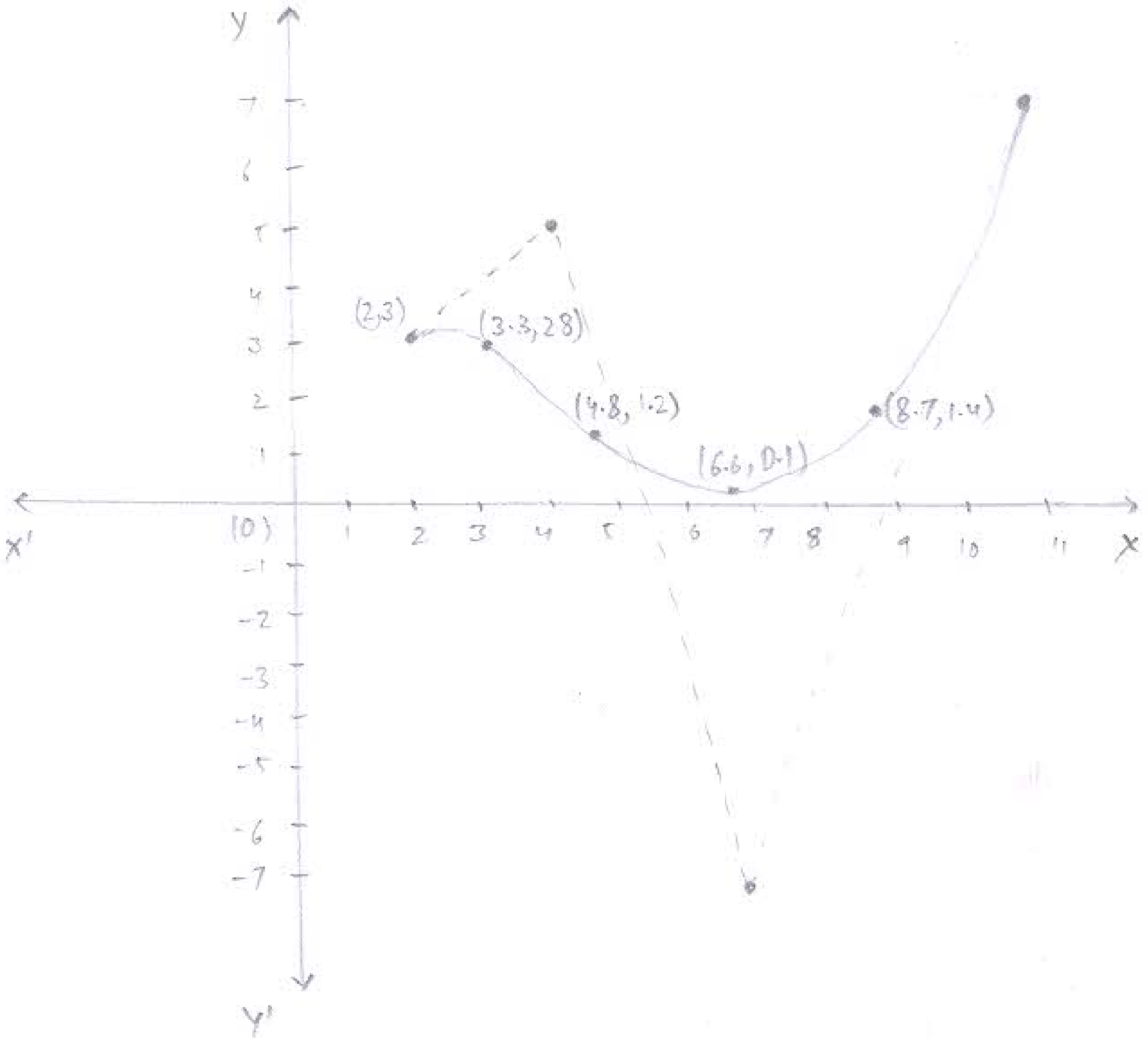
$$P_x(u) = 2(1-u)^3 + 3.4(1-u)^2 u + 3.7(1-u)u^2 + 11u^3$$

$$= 2(1-u)^3 + 12(1-u)^2 u + 21(1-u)u^2 + 11u^3$$

$$P_y(u) = 3(1-u)^3 + 15(1-u)^2 u + (-21)(1-u)u^2 + 7u^3$$

4 points lying on curve will be.

for	X coordinate	Y coordinate.
$u = 0.2$	3.32	2.84
$u = 0.4$	4.88	1.24
$u = 0.6$	6.68	0.12
$u = 0.8$	8.72	1.40


y
x'
x
y'
(0)
(2,3)
(3.3,2.8)
(4.8,1.2)
(6.6,0.1)
(8.7,1.4)

Q 12 Derive Hermite matrix.

Parametric eq. of a cubic spline curve is

$$P(u) = \sum_{i=0}^{3} c_i u^i \qquad 0 \le u \le 1$$

Or

$$P(u) = c_3 u^3 + c_2 u^2 + c_1 u + c_0 \quad \text{—— (1)}$$

where u is the parametric and c_i are the polynomial coefficients.

In matrix form

$$P(u) = [u^3 \; u^2 \; u \; 1] \, [c_3 \; c_2 \; c_1 \; c_0]^T \quad \text{—— (2)}$$

Tangent vector at any point can be given as (differentia-ting with respect u)

$$P'(u) = 3c_3 u^2 + 2c_2 u + c_1 \quad \text{—— (3)}$$

Applying boundary condition u = 0 and u = 1 in eq. (1) and (3), we get

$$\left.\begin{aligned} u = 0 \quad & P(0) = c_0 \\ u = 1 \quad & P(1) = c_0 + c_1 + c_2 + c_3 \\ u = 0 \quad & P'(0) = c_1 \\ u = 1 \quad & P'(1) = 3c_3 + 2c_2 + c_1 \end{aligned}\right\} \text{—— (4)}$$

Solving above equations we get

$$\left.\begin{aligned} c_0 &= P(0) \\ c_1 &= P'(0) \\ c_2 &= 3(P(1) - P(0)) - 2(P'(0) - P'(1)) \\ c_3 &= 2(P(0) - P(1)) + P'(0) + P'(1) \end{aligned}\right\} \text{—— (5)}$$

let $P(0) = P_0$, $P'(0) = P'_0$
$P(1) = P_1$, $P'(1) = P'_1$ } — (6)

Substituting (5) and (6) in (1) and rearranging

$$P(u) = (2u^3 - 3u^2 + 1)P_0 + (-2u^3 + 3u^2)P_1 + (u^3 - 2u^2 + u)P'_0 + (u^3 - u^2)P'_1 \quad \text{— (7)} \quad 0 \le u \le 1$$

In matrix form -

$$P(u) = [F_1(u)\ F_2(u)\ F_3(u)\ F_4(u)]\ [P_0\ P_1,\ P'_0\ P'_1]^T \quad \text{— (8)}$$

$$P = [F][B]$$

where f is blending function matrix, can be function explained as

$$F = \begin{bmatrix} 2u^3 & -3u^2 & +1 \\ -2u^3 & +3u^2 & \\ u^3 & -2u^2 & +u \\ u^3 & -u^2 & \end{bmatrix}^T$$

$$= [u^3\ u^2\ u\ 1] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Here $\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is the Hermite matrix