Ajay Kumar Garg Engineering College,Ghaziabad
Department of ECE
MODEL SOLUTION ST-2

Course: B.Tech	Semester: III
Session: 2017-18	Section: EC-1,EC-2,EC-3,EI
Subject: Signals and Systems	Sub.Code: REC-303
Max.Marks: 50	Time: 2 Hour

Prepared by: Krishna Rathore

Section - A

Q.1 Define
(i) Causal System
(ii) Stable System

Ans.1 (i) Causal System - Systems in which Output depends only on present and past Values of Input.

(ii) Stable System - Systems having bounded Output for bounded input.

Q.2 Find $U(t) \otimes \delta(t-2)$

Ans.2 $$U(t) \otimes \delta(t-2) = \int_{-\infty}^{\infty} U(t)\, \delta(t-2-\tau)\, d\tau$$

$$= U(t-2)$$

Q.3 Find the Fourier transform of $x(t) = \mathrm{sgn}(t)$

Ans.3
$$\mathrm{Sgn}(t) = U(t) - U(-t)$$

$$U(t) \xleftrightarrow{F.T} \frac{1}{j\omega} + \pi\,\delta(\omega)$$

$$u(-t) \xleftrightarrow{FT} \frac{-1}{j\omega} + \pi\delta(\omega)$$

$$\therefore \quad Sgn(t) \xleftrightarrow{FT} \frac{2}{j\omega}$$

Q.4 Find the DTFT of the following signal

$$x[n] = \left(\frac{1}{3}\right)^n u[n-3]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=3}^{\infty} \left(\frac{1}{3}\right)^n e^{-j\omega n}$$

$$= \left(\frac{1}{3}\right)^3 e^{-j3\omega} + \left(\frac{1}{3}\right)^4 e^{-j4\omega} + \dots \infty$$

$$= \frac{\left(\frac{1}{3}\right)^3 e^{-j3\omega}}{1 - \frac{1}{3}e^{-j\omega}}$$

Q.5 Prove that any signal $g(t)$ and its hilbert transform have same magnitude spectrum

$$g(t) \xleftrightarrow{FT} \hat{g}(t)$$

$$\hat{G}(\omega) = -j\, Sgn(\omega)\, G(\omega)$$

$$|\hat{G}(\omega)| = |-j\, Sgn(\omega)\, G(\omega)| \qquad [\because |Sgn(\omega)| = 1]$$

$$\therefore \quad |\hat{G}(\omega)| = |G(\omega)|$$

Section B

Q.6 State and prove the time differentiation property of CTFT.

Statement -:

$$x(t) \longrightarrow X(\omega)$$

$$\frac{dx(t)}{dt} \longrightarrow j\omega X(\omega)$$

Proof -:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Differentiating wrt t on both sides, we get

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, j\omega\, e^{j\omega t}\, d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{j\omega X(\omega)}\, e^{j\omega t}\, d\omega$$

$$\frac{dx(t)}{dt} \xleftrightarrow{F.T} j\omega X(\omega)$$

Hence proved

Q.7 A discrete time LTI System is given by the difference equation

$$y[n-2] - 5y[n-1] + 6y[n] = x[n-1]$$

Determine the impulse response $h[n]$ of the system.

$$y[n-2] - 5y[n-1] + 6y[n] = x[n-1]$$

Taking discrete time Fourier transform of both the sides,

$$e^{-jw2} Y(e^{jw}) - 5e^{-jw} Y(w) + 6Y(e^{jw}) = e^{-jw} X(e^{jw})$$

$$Y(e^{jw}) [e^{-jw2} - 5e^{-jw} + 6] = e^{-jw} X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{e^{-jw}}{e^{-jw2} - 5e^{-jw} + 6}$$

$$H(e^{jw}) = \frac{e^{-jw}}{e^{-jw2} - 3e^{-jw} - 2e^{-jw} + 6}$$

$$H(e^{jw}) = \frac{e^{-jw}}{e^{-jw}(e^{-jw} - 3) - 2(e^{-jw} - 3)}$$

$$\frac{H(e^{jw})}{e^{-jw}} = \frac{1}{(e^{-jw} - 3)(e^{-jw} - 2)}$$

Taking partial fractions, we get

$$\frac{1}{(e^{-jw} - 3)(e^{-jw} - 2)} = \frac{A}{e^{-jw} - 3} + \frac{B}{e^{-jw} - 2}$$

$$1 = A(e^{-jw} - 2) + B(e^{-jw} - 3)$$

$$A = 1, \quad B = -1$$

$$\frac{H(e^{jw})}{e^{-jw}} = \frac{1}{e^{-jw} - 3} - \frac{1}{e^{-jw} - 2}$$

$$H(e^{jw}) = \frac{1}{1-3e^{jw}} - \frac{1}{1-2e^{jw}}$$

$$h[n] = (3)^{-n} u(-n) - (2)^{-n} u(-n)$$

$$\left[\because a^{-n} u(-n) \xrightarrow{DTFT} \frac{1}{1-ae^{jw}} \right]$$

Q.8 Consider a discrete time system with input $x[n]$ and Output $y[n]$ where $y[n] = n[x(n)]^2$. Is this system

(i) linear or non linear

(ii) time variant or time invariant

(i) Let the Fourier transform of

$$x_1(n) \rightarrow y_1(n)$$

$$x_2(n) \rightarrow y_2(n)$$

$$x_1(n) + x_2(n) \rightarrow y_3(n) = y_1(n) + y_2(n)$$

$$= n[x_1(n)]^2 + n[x_2(n)]^2$$

$$= n[(x_1(n))^2 + (x_2(n))^2] \quad \text{— ①}$$

$$y_3'(n) = n[x_1(n) + x_2(n)]^2$$

$$= n[x_1^2(n) + x_2^2(n) + 2x_1(n)x_2(n)] \quad \text{— ②}$$

Since ① $\neq$ ②, the system is Non linear.

(ii)

$$y'(n-n_0) \xrightarrow[n \text{ by } n-n_0]{\text{Replace}} (n-n_0)[x(n-n_0)]^2 \quad — (1)$$

$$y''(n-n_0) \xrightarrow[n \text{ by } n_0]{\text{Delay}} n[x(n-n_0)]^2 \quad — (2)$$

Since (1) $\neq$ (2), the System is time Variant.

Q.9 Determine whether following systems are Causal & Stable

(i) $h[n] = (0.8)^n U[n+2]$

(ii) $h(t) = e^{-6|t|}$

(i) $h[n] = (0.8)^n U[n+2]$

For causality -: $h(n) = 0 \; ; \; n < 0$

$U(n+2) = 0 \; ; \; n < -2$

Therefore, the system is Non Causal

For Stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} (0.8)^n U(n+2) = \sum_{n=-2}^{\infty} (0.8)^n$$

$$= \frac{(0.8)^{-2}}{1-0.8} = \text{finite value}$$

Therefore, the system is stable.

(ii) $h(t) = e^{-6|t|}$

For causality, $h(t) = 0, \quad t < 0$

but $h(t) \neq 0, \quad t < 0$

$\therefore$ the system is Non Causal

For Stability

$$\int_{-\infty}^{\infty} e^{-6|t|} dt = \int_{-\infty}^{0} e^{6t} dt + \int_{0}^{\infty} e^{-6t} dt$$

$$= \left. \frac{e^{6t}}{6} \right|_{-\infty}^{0} + \left. \frac{e^{-6t}}{-6} \right|_{0}^{\infty}$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6} = \frac{1}{3} \quad \text{(finite)}$$

$\therefore$ the system is stable

Q. 10 The unit impulse response of a discrete time system is {1, 1/2, 1/4, 1/6, 1/8, 1} for an input sequence {1, 0, 1, 2, 5} then find the O/P sequence.

Ans:

$$h(n) = \{1, 1/2, 1/4, \underset{\uparrow}{1/6}, 1/8, 1\}$$

$$x(n) = \{1, 0, \underset{\uparrow}{1}, 2, 5\}$$

$$y(n) = x(n) * h(n)$$

By Tabular method

	1	1/2	1/4	1/6 ↓	1/8	1
1	1	1/2	1/4	1/6	1/8	1
0	0	0	0	0	0	0
→ 1	1	1/2	1/4	1/6	1/8	1
2	2	1	1/2	1/3	1/4	2
5	5	5/2	5/4	5/6	5/8	5

$$y(n) = \{1, 1/2, 5/4, 8/3, 51/8, \underset{\uparrow}{25/6}, 41/24, \tfrac{25}{12}, \tfrac{21}{8}, 5\}$$

Ans

Section-C

Q.11 Find the fourier transform of a gaussian signal.

Ans:

$$g(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-t^2/2\sigma^2} \quad \text{Gaussian signal}$$

Differentiating w.r.t 't'

$$d/dt[g(t)] = \frac{1}{2\pi\sqrt{\sigma}} \left\{ \frac{-1}{2\sigma^2} \cdot 2t \; e^{-t^2/2\sigma^2} \right]$$

$$= \frac{-t}{\sigma^2} \underbrace{\left[\frac{1}{2\pi\sqrt{\sigma}} e^{-t^2/2\sigma^2} \right]}_{g(t)}$$

$$\frac{d}{dt} g(t) = -\frac{t}{\sigma^2}[g(t)]$$

Taking fourier transform from both sides

$$j\omega\, G(\omega) = \frac{-1}{\sigma^2}\, j \frac{d}{d\omega} G(\omega)$$

Integrating both sides

$$\int_{-\infty}^{\omega} \frac{\frac{d}{d\omega}[G(\omega)]}{G(\omega)}\, d\omega = \int_{-\infty}^{\omega} -\sigma^2\omega\, d\omega$$

$$\int_{0}^{\omega} \frac{\frac{d}{d\omega}[G(\omega)]}{G(\omega)}\, d\omega = \int_{0}^{\omega} -\sigma^2\omega\, d\omega$$

$$\ln G(\omega)\Big|_0^{\omega} = \frac{-\sigma^2\omega^2}{2}\Big|_0^{\omega}$$

$$\ln G(\omega) - \ln G(0) = -\sigma^2\omega^2/2$$

since $G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$

$$G(0) = \underbrace{\int_{-\infty}^{\infty} g(t)\, dt}_{\text{Area} = 1} = 1$$

$$\therefore \ln G(\omega) - 0 = (-\sigma^2\omega^2/2)$$

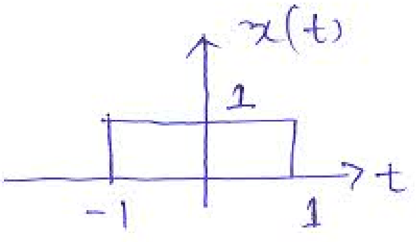
$$\boxed{G(\omega) = e^{-\sigma^2\omega^2/2}}$$

Q.12. If $x(t) = \begin{cases} 1 & -1 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$

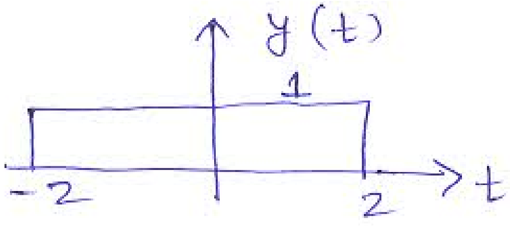
$y(t) = \begin{cases} 1 & -2 \le t \le 2 \\ 0 & \text{elsewhere} \end{cases}$

find $x(t) * y(t)$ and plot the result.

Ans:



$$x(t) = u(t+1) - u(t-1)$$



$$y(t) = u(t+2) - u(t-2)$$

$$x(t) * y(t) = [u(t+1) - u(t-1)] * [u(t+2) - u(t-2)]$$

$$= \{u(t+1) * u(t+2)\} - \{u(t+1) * u(t-2)\}$$
$$- \{u(t-1) * u(t+2)\} + \{u(t-1) * u(t-2)\}$$

$$= r(t+3) - r(t-1) - r(t+1) + r(t-3)$$

since $u(t) * u(t) = \int_{-\infty}^{\infty} u(t)\,dt = r(t)$

Plotting of result