Ajay Kumar Garg Engineering College, Ghaziabad

Department of ECE

Model Solution Sessional Test-2

Course:	B.Tech	Semester:	V
Session:	2017-18	Section:	EC-1,2,3 EI-1
Subject:	CONTROL SYSTEM 1	Sub. Code:	NIC 501
Max Marks:	50	Time:	2 hour

Section A

Question 1. Explain Eigen value and Eigen vector.

Ans) The characteristic eq^n $|\lambda I - A| = 0$

gives solution in terms of Eigen value, $\lambda_1, \lambda_2, \lambda_3 \cdots \lambda_n$

These eigen values, are further used to calculate eigen vectors $x_1, x_2, x_3 \cdots x_n$ which then helps to calculate model matrix.

Question 2. What is Controllability and Observability?

Ans) A system is said to be controllable, if the controlability test matrix "$|U|$" $\neq 0$, where

$$U = [B : AB : A^2B : \ldots\ldots A^{n-1}B]$$

A is system matrix and B is input matrix.

A system is said to be observable, if the obsenability test matrix "$|V|$" $\neq 0$, where

$$[V] = [C^T : A^TC^T : (A^T)^2C^T \ldots (A^T)^{n-1}C^T]$$

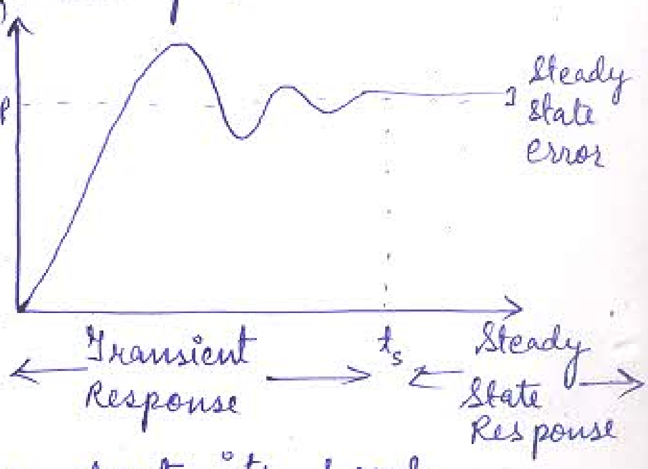
Question 3) What is the difference between transient response and steady state response of a control system?

Ans) Time response is divided into two parts

a) Transient response $[y_t(t)]$

b) Steady state response $[y_{ss}(t)]$

$$y(t) = [y_t(t)] + [y_{ss}(t)]$$

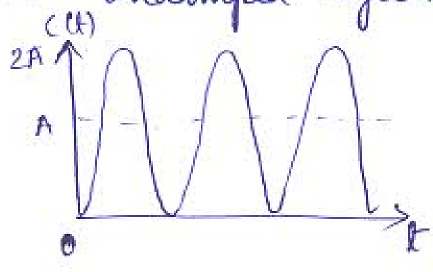


★ Transient response reveals the nature of response (i.e oscillatory or overdamped) and also indicates about its speed.

★ Steady state response reveals the accuracy of a control system. Steady state error shows that the output does not exactly match with the input.

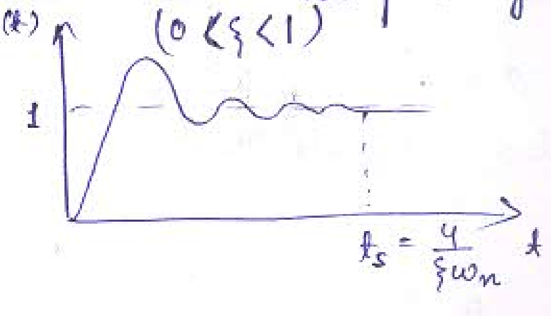
Question 4) Mention the nature of transient response of second order control system for different types of damping ratios.

Ans) Case-1 Undamped System ($\xi = 0$)

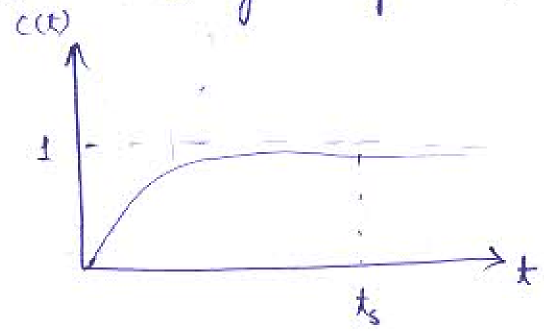


$t_s = \infty$

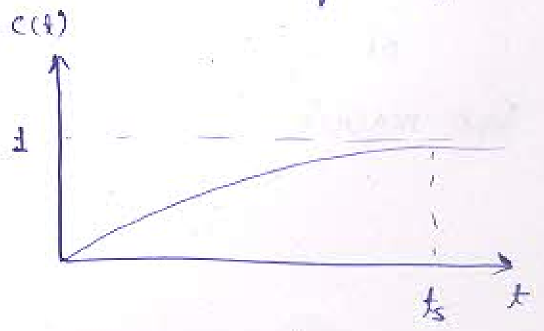
Case-2 Underdamped System ($0 < \xi < 1$)



Case-3 Critically damped ($\xi = 1$)



Case-4 Overdamped System ($\xi > 1$)



Question 5) Find and explain the type of open loop transfer function for the system

$$G(s)H(s) = \frac{10(1+5s)}{s^3(1+6s)}$$

Ans) The given system is type three.

Type of open loop transfer function defines the presence of number of open loop poles at the origin.

for eg. $$G(s)H(s) = \frac{(s+T_1)(s+T_2)\cdots(s+T_z)}{s^N(s+T_a)(s+T_b)\cdots(s+T_p)}$$

The given system as 'N' no. of poles at origin.

Section B

Question 6) What is State Transition Matrix? Give Laplace transform method of computing the State Transition matrix. Also, give the properties of state transition matrix with proof.

Ans) The state-transition matrix is defined as a matrix that satisfies the linear homogeneous state equation:

$$\frac{dx(t)}{dt} = Ax(t)$$

Let, $\phi(t)$ be the $n \times n$ matrix that represents the state transition matrix; then it must satisfy the equation

$$\frac{d\phi(t)}{dt} = A\phi(t)$$

now, if $x(0)$ denote the initial state at $t = 0$; then $\phi(t)$ is also defined by the matrix equation

$$x(t) = \phi(t)\, x(0)$$

which is the solution of the homogeneous state equation for $t \geq 0$

Laplace Transform method of computing STM

Let $\dot{x} = Ax$ —— ①

$x(0)$ be initial conditions

Taking Laplace transform of eq ①

$$sX(s) - X(0) = AX(s)$$

or $[sI - A]\, X(s) = X(0)$

or $X(s) = [sI - A]^{-1} X(0)$

$\therefore$ $x(t) = L^{-1}[sI - A]^{-1} x(0)$ —— eq ②

$\because$ $x(t) = e^{At} x(0)$

Comparing eq ① & ② we have STM as :-

$$\boxed{\phi(t) = e^{At} = L^{-1}[sI - A]^{-1}}$$

Properties of State Transition Matrix

1) $\phi(0) = I$

Put $t = 0$ in $\phi(t) = e^{At}$

$\Rightarrow$ $\phi(0) = e^{A\times 0} = I$ $\Rightarrow$ $\boxed{\phi(0) = I}$

2) $\phi^{-1}(t) = \phi(-t)$

now, $\phi(t) = e^{At}$

Post multiplying by e^{-At}

$\Rightarrow \phi(t)\, e^{-At} = e^{At}\, e^{-At} = I$

now, pre-multiplying by $\phi^{-1}(t)$

$\phi^{-1}(t)\,\phi(t)\, e^{-At} = \phi^{-1}(t)\, I$

or $\quad e^{-At} = \phi^{-1}(t)$

$\boxed{\phi(-t) = \phi^{-1}(t)}$ $\quad [\because e^{-At} = \phi(-t)]$

3) $\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)$ for any t_0, t_1 and t_2

$\phi(t_2 - t_1)\,\phi(t_1 - t_0) = e^{A(t_2 - t_1)}\, e^{A(t_1 - t_0)} = e^{A(t_2 - t_0)}$

$\Rightarrow \boxed{\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)}$

4) $[\phi(t)]^k = \phi(kt)$

$[\phi(t)]^k = e^{At}\, e^{At} \ldots e^{At}$ (upto k no. of terms)

$[\phi(t)]^k = e^{Akt}$

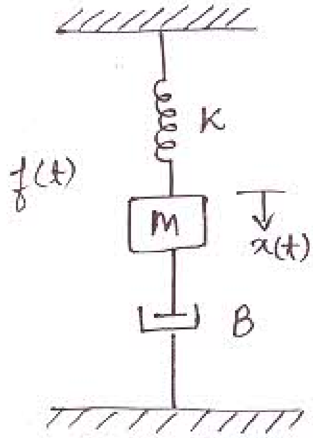
$\boxed{[\phi(t)]^k = \phi(kt)}$

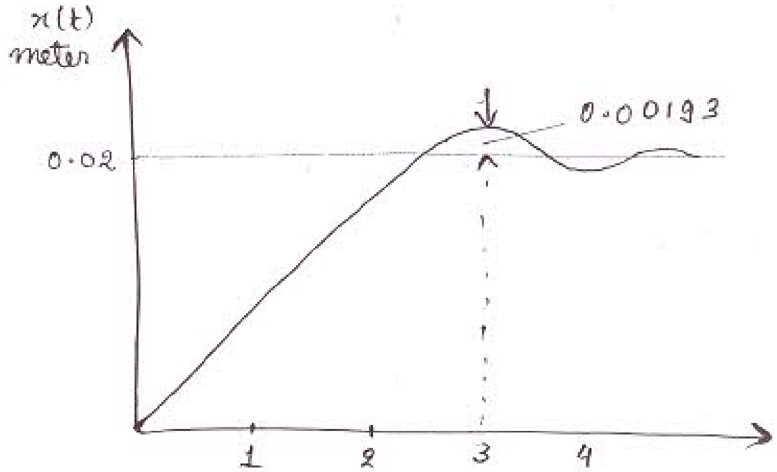
5) $\dot{\phi}(t) = A\,\phi(t)$

$\dot{\phi}(t) = \frac{d}{dt} e^{At} = Ae^{At}$

$\boxed{\dot{\phi}(t) = A\,\phi(t)}$

Question 7 The figure below shows mechanical system and its response when 20 N of force is applied to the system. Calculate the value of M and B.





Ans) The transfer function of the mechanical system is :

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Given $F(t) = 20N$ $\therefore$ $F(s) = \frac{20}{s}$

$$\therefore \quad X(s) = \frac{F(s)}{Ms^2 + Bs + K} = \frac{20}{s(Ms^2+Bs+K)}$$

steady state value of $X(s)$ can be calculated as :

$$\lim_{s \to 0} sX(s) = \frac{20}{K} = 0.02 \text{ (given)} \quad \Rightarrow \quad K = 1000$$

$$M_p = \frac{0.00193}{0.02} = 9.66\,\%$$

$$\because \quad M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = \frac{9.66}{100} \quad \Rightarrow \quad \xi = 0.6$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$\frac{\pi}{\omega_n \sqrt{1-(0.6)^2}} = 3 \quad \text{(given)}$$

$$\omega_n = 1.31 \text{ rad/sec}$$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{s^2 + \frac{Bs}{M} + \frac{K}{M}}$$

$$\therefore \quad \omega_n^2 = \frac{K}{M}$$

$$\text{or} \quad M = \frac{1000}{\omega_n^2} = \frac{1000}{(1.31)^2} = 582.7 \text{ kgs}$$

$$\boxed{M = 582.7 \text{ kg}}$$

$$\therefore \quad \frac{B}{M} = 2\xi\omega_n = 2 \times 0.6 \times 1.31$$

$$\boxed{B = 916 \text{ N/m/sec}}$$

Question 8) The open loop transfer function of a unity feedback is. $G(s) = \dfrac{K}{s(1+0.025s)}$ and damping ratio, $\xi = 0.4$.

Determine the K and the steady state error for ramp i/p.

Ans)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(1+0.025s)}}{1 + \frac{K}{s(1+0.025s)}} \quad [\because H(s)=1]$$

$$\frac{C(s)}{R(s)} = \frac{K}{0.025s^2 + s + K} = \frac{40K}{s^2 + 40s + 40K} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{40K}$$

$$\text{\&} \quad 2\xi\omega_n = 40 \quad \Rightarrow \quad 2\xi\sqrt{40K} = 40$$

$$2 \times 0.4 \times \sqrt{40K} = 40$$

$$\Rightarrow \quad \boxed{K = 62.5}$$

Steady state error for ramp input

$$K_v = \lim_{s\to 0} s\,G(s)\,H(s) = \lim_{s\to 0} \cancel{s} \frac{K}{\cancel{s}(1+0.25s)}$$

$$K_v = K = 62.5$$

Now, $e_{ss} = \frac{A}{K_v} = \frac{1}{62.5}$

$$\boxed{e_{ss} = 0.016}$$

Question 9) What is transfer function matrix? Obtain the transfer function $Y(s)/U(s)$ for the state space representation

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t)$$

Ans) Transfer function matrix $\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D$

$$[sI - A] = \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s+3 & -1 \\ 3 & 4 & s+5 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s^3 + 10s^2 + 35s + 41} \begin{bmatrix} s^2 + 8s + 19 & s+5 & 1 \\ -3 & s^2 + 7s + 10 & s+2 \\ -3s-9 & -(4s+11) & s^2+5s+6 \end{bmatrix}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} [\sin\phi \cos\omega_d t + \cos\phi \sin\omega_d t]$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)}$$

where, $\omega_d = \omega_n \sqrt{1-\xi^2}$ $\qquad \phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

i) $0 < \xi < 1$

$$c(t) = 1 - \frac{e^{\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

ii) $\xi = 0$

$$c(t) = 1 - \frac{e^0}{\sqrt{1}} \sin(\omega_n \sqrt{1}\, t + \frac{\pi}{2})$$

$$\boxed{c(t) = 1 - \cos\omega_n t}$$

Section C

Question 11) Give time domain specifications. Derive the expressions for rise time, peak time and peak overshoot for a second order system subjected to unit step input.

Ans) Time domain specifications :-

1) Settling time :- It is defined as the time at which output will achieve 98% of the desired output [t_s].

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \frac{1}{s^3+10s^2+35s+41} \begin{bmatrix} s^2+8s+19 & s+5 & 1 \\ -3 & s^2+7s+10 & s+2 \\ -3s-9 & -(4s+11) & s^2+5s+6 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3+10s^2+35s+41} \begin{bmatrix} -3 & s^2+7s+10 & s+2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{s+2}{s^3+10s^2+35s+41}}$$

<u>Question 10)</u> Derive the second order system response subjected to unit step input for the following value of damping ratio

(i) $0 < \xi < 1$ (ii) $\xi = 0$

<u>Ans)</u> Transfer function of 2nd order closed loop system:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \Rightarrow C(s) = \frac{R(s)\ \omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{[(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)]} = \frac{A}{s} + \frac{Bs+C}{[(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)]}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{[(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)]}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right]$$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

now substituting $t_p = \frac{\pi}{\omega_d}$ in response $c(t)$

$$c(t)_{max} = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\because \quad M_p = c(t)_{max} - 1$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

Question 12) Explain state, state variable and state vector. What are the advantages of state space techniques? Find the state equation and output equation for the system given by

$$\frac{Y(s)}{U(s)} = \frac{s^3 + 5s^2 + 6s + 1}{s^3 + 4s^2 + 3s + 3}$$

Ans) State :- The state of a dynamic system is the smallest set of state variable such that the knowledge of these variable at $t = t_0$, together with the knowledge of the i/p for $t \geq 0$, completely determines the behaviour of the system for any time $t \geq 0$

State variable :- The smallest set of variables, which determine the state of a dynamic system are called state variable,

State vector :- The smallest set of variables, which determine the state of a dynamic system are called state variables.

2) **Rise time** :- It is the time needed for the response to reach from 10% to 90% of the desired value at the very first instant [t_r]

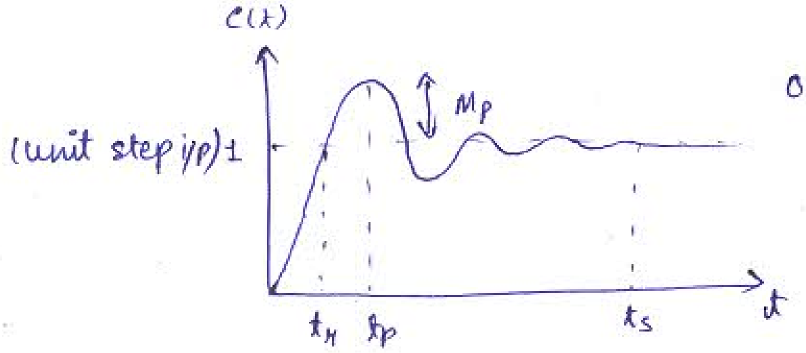
For underdamped system :- 0% to 100%

For over damped system :- 10% to 90%

3) **Peak time** :- It is the time needed for a system to reach the maximum overshoot. It is denoted as [t_p]

4) **Maximum overshoot** :- $M_p = c(t)_{max} - 1$

$$\% M_p = \frac{c(t)_{max} - 1}{1} \times 100$$



$0 < \xi < 1$

Expression for peak time and peak overshoot

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[(\omega_n\sqrt{1-\xi^2}\, t + \phi)\right]$$

$$\frac{d\,c(t)}{dt} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\, \omega_n\sqrt{1-\xi^2} \cos\left[\omega_n\sqrt{1-\xi^2}\, t + \phi\right]$$

$$- \frac{(-\xi\omega_n)\, e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t + \phi\right] = 0$$

Advantages of state space techniques :-

1) Approach is more accurate as it considers zero input response.

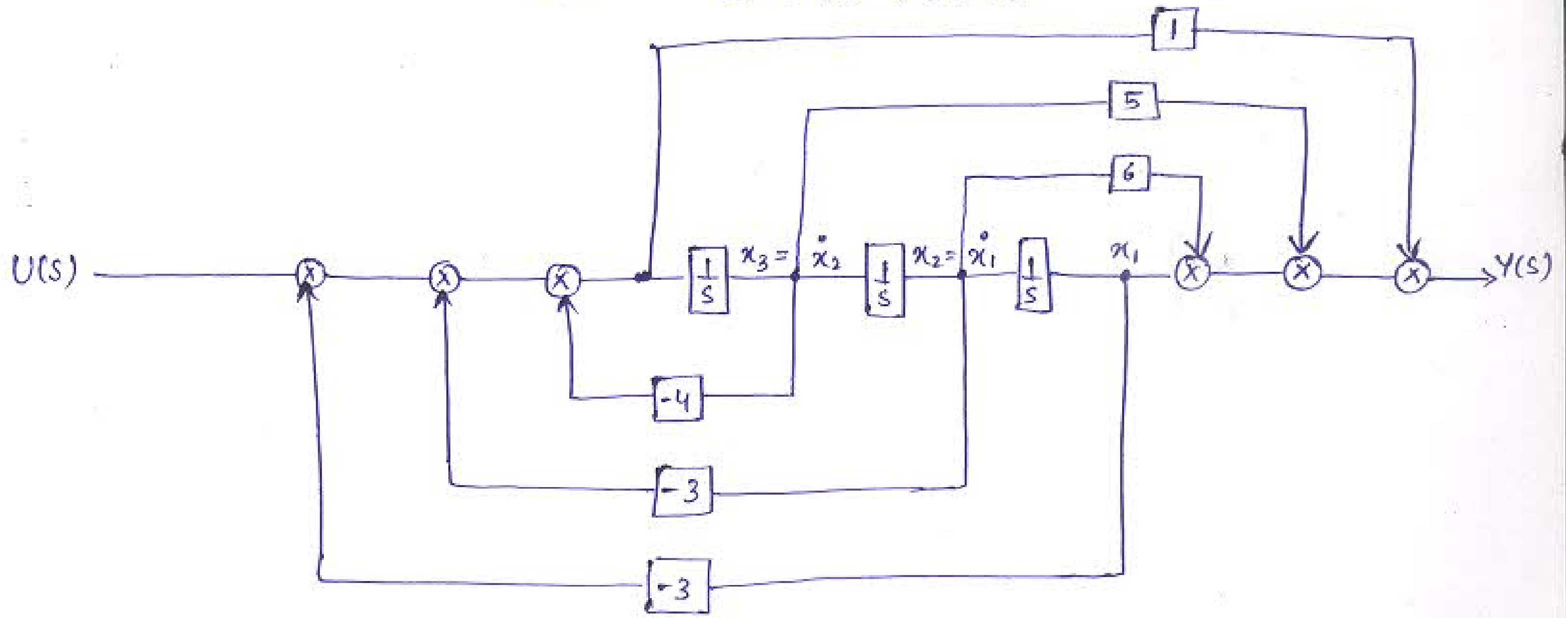
2) Approach can be applied to linear or non-linear, time variant or time invariant systems.

3) It is easier to apply where Laplace transform cannot be applied.

4) N^{th} order differential equation can be expressed as 'n' equation of first order whose solution are easier.

5) This method is suitable for digital computer computation.

$$\frac{Y(s)}{U(s)} = \frac{s^3 + 5s^2 + 6s + 1}{s^3 + 4s^2 + 3s + 3}$$



$$\frac{Y(s)}{U(s)} = \frac{1}{s^3+4s^2+3s+3} + \frac{6s}{s^3+4s^2+3s+3} + \frac{5s^2}{s^3+4s^2+3s+3} + \frac{s^3}{s^3+4s^2+3s+3}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -3x_1 - 3x_2 - 4x_3 + u$$

$$y = -2x_1 + 3x_2 + x_3 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y(s) = \begin{bmatrix} -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [1] u$$