Model Paper

Subject :- Basic Signals & Systems   Subject Code :- REE 303

Prepared by :- Aditya Chaudhary & Apoorv Vats.

Section - A.

Q1. Differentiate between Fourier Series and Fourier Transform.

Sol.

Fourier Series	Fourier Transform.
1) It is the representation of non-sinusoidal periodic signals in terms of complex exponential or in terms of sine & cosine terms.	1) It is the representation of non-periodic signals in frequency domain.
2) Trigonometric Fourier Series :- $x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$	2) Fourier Transform $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.$

Q2. Write Dirichlet Conditions for existence of Fourier series.

Sol. Dirichlet Conditions are :-

(i) If $x(t)$ is discontinuous, there are a finite number of discontinuities in the period T.

(ii) $x(t)$ has a finite average value over the period T.

(iii) $x(t)$ has a finite number of positive and negative maxima in the period T.

Q3. Find the Fourier Transform of $e^{-at} u(t)$.

Sol.
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.$$

$$x(t) = e^{-at} u(t)$$

$$\therefore X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$$

$$\therefore\ X(\omega) = \int_0^{\infty} e^{-(a+j\omega)t}\, dt.$$

$$= \frac{-1}{a+j\omega}\left[e^{-(a+j\omega t)}\right]_0^{\infty}$$

$$= \frac{1}{a+j\omega}.$$

$$\Rightarrow \quad e^{-at}u(t) \xleftrightarrow{F} \frac{1}{a+j\omega}$$

Q4. Find the laplace transform of $(1+0.5\sin(t))\sin(1000t)u(t)$

Sol. $x(t) = (1+0.5\sin(t))\sin(1000t)u(t)$

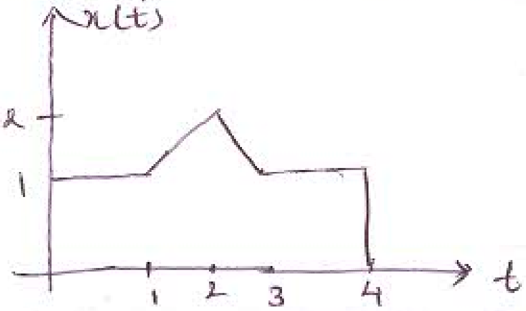
$$= \sin(1000t)u(t) + 0.5\sin(t)\sin(1000t)u(t)$$

$$= \sin(1000t)u(t) + \frac{1}{4}(\cos(999t) - \cos(1001t))u(t)$$

$$= \sin(1000t)u(t) + \frac{1}{4}\cos(999t)u(t) - \frac{1}{4}\cos(1001t)u(t)$$

$$\therefore\ L[x(t)] = \frac{1000}{s^2+1000^2} + \frac{1}{4}\frac{s}{s^2+999^2} - \frac{1}{4}\frac{s}{s^2+1001}$$

Q5. Find the laplace transform of the signal shown in figure



Sol. $x(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) - u(t-4)$

$$L[x(t)] = \frac{1}{s} + \frac{1}{s^2}e^{-s} - \frac{2}{s^2}e^{-2s} + \frac{1}{s^2}e^{-3s} - \frac{1}{s}e^{-4s}$$

$$= \left(\frac{1-e^{-4s}}{s}\right) + \left(\frac{e^{-s} - 2e^{-2s} + e^{-3s}}{s^2}\right)$$

Section - B.

Q6. Prove the frequency shifting property and time scaling property of Fourier Series.

Sol. Frequency Shifting Property :-

if $x(t) \longleftrightarrow X_n$ then $y(t) = e^{jm\omega_0 t} x(t)$

has fourier series co-efficient as $Y_n = X_{n-m}$

Proof :- $Y_n = \frac{1}{T}\int_0^T y(t)\, e^{-jn\omega_0 t}\, dt.$

$$= \frac{1}{T}\int_0^T e^{jm\omega_0 t}\, x(t)\, e^{-jn\omega_0 t}\, dt.$$

$$= \frac{1}{T}\int_0^T x(t)\, e^{-j(n-m)\omega_0 t}\, dt$$

$$= X_{n-m}$$

Time Scaling Property :-

if $x(n) \longleftrightarrow X_n$, then $y(t) = x(at) \longleftrightarrow Y_n = X_n.$

Proof :- if $x(t)$ is periodic with period T, then $x(at)$ will have a period as T/a & $\omega_0 \longrightarrow a\omega_0$

$$Y_n = \frac{1}{T/a}\int_0^{T/a} x(at)\, e^{-jn\omega_0 t}\, dt.$$

Let $at = \tau$ & $d\tau = a\,dt.$

$$Y_n = \frac{1}{T}\int_0^T x(\tau) e^{-jn\omega_0 \tau}\, d\tau.$$

$$Y_n = X_n.$$

Q7. State and prove the duality property of fourier transform. Using duality property, find the fourier transform of $g(t) = \frac{1}{1+jt}$.

Sol. (a) Duality property :- if $x(t) \longleftrightarrow X(\omega)$

then $X(t) \longleftrightarrow 2\pi x(-\omega)$

Proof :- By Fourier Transform definition

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega.$$

$$2\pi x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega.$$

Replacing t by $-t$ $\Rightarrow$ $2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega) e^{-j\omega t}\, d\omega$

Interchanging the variables t & ω

$$\Rightarrow \quad 2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t) e^{-j\omega t}\, dt.$$

$$\Rightarrow \quad F[X(t)] = 2\pi x(-\omega)$$

(b) $F[e^{-at} u(t)] = \frac{1}{a+j\omega}$

For $a=1$; $F[e^{-t} u(t)] = \frac{1}{1+j\omega}$

$$e^{-t} u(t) \longleftrightarrow \frac{1}{1+j\omega}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$x(t) = e^{-t} u(t) \quad \& \quad X(\omega) = \frac{1}{1+j\omega}$$

$$x(-\omega) = e^{\omega} u(-\omega) \quad \& \quad X(t) = \frac{1}{1+jt}.$$

By duality property ; $F[X(t)] = 2\pi x(-\omega)$

$$F\left[\frac{1}{1+jt}\right] = 2\pi e^{\omega} u(-\omega)$$

Q8. Using laplace transform solve the following differential eq. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = 2x(t)$; $x(t) = e^{-2t} u(t)$ & $y(0^-) = -2$ & $\frac{dy(0^-)}{dt} = 1$

Sol: $L[y''(t)] = s^2 Y(s) - s(y(0^-)) - y'(0)$

$L[y(t)] = sY(s) - y(0^-)$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = 2x(t)$$

Taking laplace both the sides.

$$s^2Y(s) - s(-2) - (-1) + 5[sY(s) - (-2)] + 4Y(s) = 2X(s)$$

$$Y(s)[s^2+5s+4] + 2s+11 = 2X(s)$$

$$Y(s) = \frac{2X(s)}{s^2+5s+4} - \frac{2s+11}{s^2+5s+4}$$

$$x(t) = e^{-2t}u(t)$$

$$\therefore X(s) = \frac{1}{s+2}$$

$$\Rightarrow Y(s) = \frac{2}{(s+2)(s^2+5s+4)} - \frac{2s+11}{(s^2+5s+4)}$$

$$= \frac{2}{(s+2)(s+1)(s+4)} - \frac{(2s+11)}{(s+1)(s+4)}$$

$$= 2\left[\frac{1/3}{(s+1)} - \frac{1/2}{(s+2)} + \frac{1/6}{(s+4)}\right] - 2\left[\frac{3/2}{(s+1)} - \frac{1/2}{(s+4)}\right]$$

Taking inverse laplace transform.

$$y(t) = \left[\frac{2}{3}e^{-t} - e^{-2t} + \frac{1}{3}e^{-4t}\right] + \left[-3e^{-t} + e^{-4t}\right]$$

$$y(t) = \left(\frac{4}{3}e^{-4t} - \frac{7}{3}e^{-t} - e^{-2t}\right)u(t)$$

Q9. The output of a linear system is $y(t) = 10e^{-t}\cos(4t)u(t)$ when the input is $x(t) = e^{-t}u(t)$. Find the transfer function of the system and its impulse response.

Sol. $y(t) = 10e^{-t}\cos(4t)u(t)$

$$Y(s) = 10 \cdot \frac{(s+1)}{(s+1)^2+16} = \frac{10(s+1)}{s^2+2s+17}$$

$$x(t) = e^{-t} u(t)$$

$$X(s) = \frac{1}{s+1}$$

$\therefore$ Transfer function, $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{10(s+1)/(s^2+2s+17)}{1/(s+1)}$

$$H(s) = \frac{10(s+1)^2}{s^2+2s+17}$$

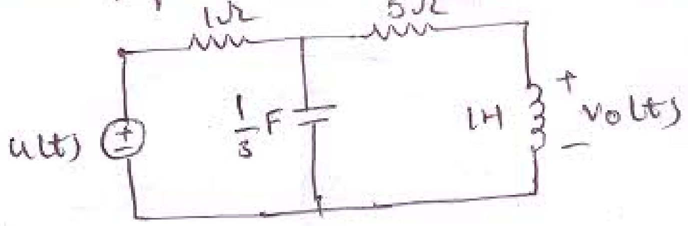
Impulse response, $h(t) = L^{-1}[H(s)]$

$$= L^{-1}\left[\frac{10(s+1)^2}{(s+1)^2+4^2}\right] = L^{-1}\left[10 - 40 \cdot \frac{4}{s^2+2s+17}\right]$$

$$= L^{-1}\left[10 - \frac{40 \cdot 4}{(s+1)^2+4^2}\right]$$

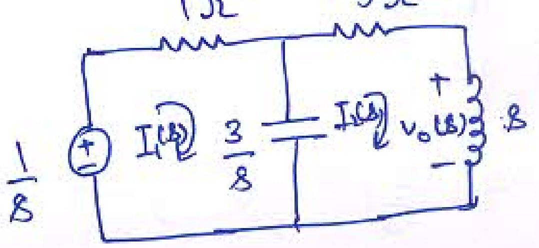
$$h(t) = 10\delta(t) - 40 e^{-t} \sin(4t) u(t)$$

Q10. Assuming zero initial conditions, find the vo(t) in the circuit of figure 2.



Sol.

Taking Laplace Transform of the given circuit :-



From loop 1:-

$$\frac{1}{s} = (1+3/s) I_1(s) - \frac{3}{s} I_2(s) \quad -(1)$$

From loop 2:-

$$0 = -\frac{3}{s} I_1(s) + \left(5 + \frac{3}{s} + s\right) I_2(s) \quad -(2)$$

From (1) & (2)

$$I_2 = \frac{\begin{vmatrix} (1+3/s) & 1/s \\ (-3/s) & 0 \end{vmatrix}}{\begin{vmatrix} (1+3/s) & -3/s \\ (-3/s) & s+3/s+s \end{vmatrix}}$$

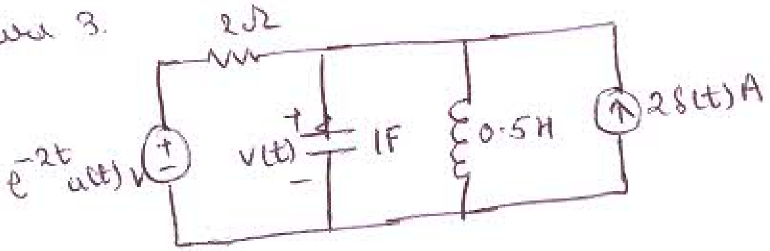
$$\therefore \; I_2 = \frac{3}{s^3+8s^2+18s}$$

$$\therefore \; V_o(s) = s I_2 = \frac{3}{s^2+8s+18} = \frac{3}{\sqrt{2}} \, \frac{\sqrt{2}}{(s+4)^2+(\sqrt{2})^2}$$

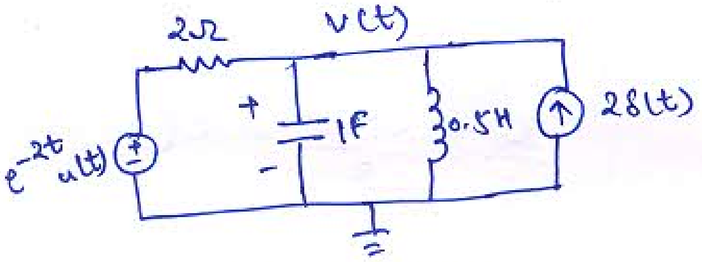
$$\therefore \; v_o(t) = \left(\frac{3}{\sqrt{2}} e^{-4t} \sin \sqrt{2}t\right) V.$$

Section C.

Q11. Using the Fourier transform, find $v(t)$ in the circuit shown in figure 3.



Sol.



Applying Nodal Analysis

$$\frac{v(t) - e^{-2t}u(t)}{2} + \frac{v(t)}{1/j\omega} + \frac{v(t)}{j\omega 0.5} - 2\delta(t) = 0.$$

$$v(t) - e^{-2t}u(t) + 2j\omega\, v(t) + \frac{v(t)}{j\omega} = 4\delta(t)$$

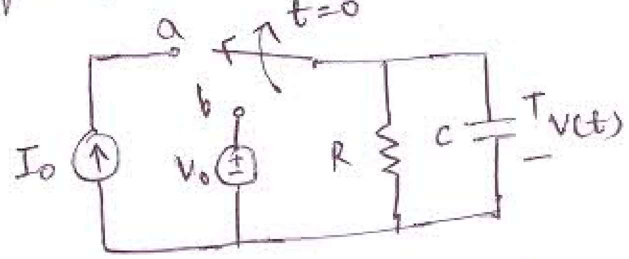
Taking Fourier Transform,

$$V(j\omega) - \frac{1}{2+j\omega} + \left(2j\omega + \frac{1}{j\omega}\right) V(j\omega) = 4$$

$$V(j\omega) = \frac{j\omega\,(4j\omega + 9)}{(2+j\omega)(4 - 2\omega^2 + j\omega)}$$

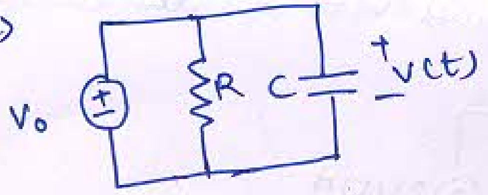
$$V(j\omega) = \frac{2j\omega\,(4.5 + 2j\omega)}{(2+j\omega)(4 - 2\omega^2 + j\omega)}$$

Q12. The switch in figure 4 has been in position b for a long time. It is moved to position a at t=0. Using laplace transform, determine the value of v(t) for t>0.



Sol. For $t<0$

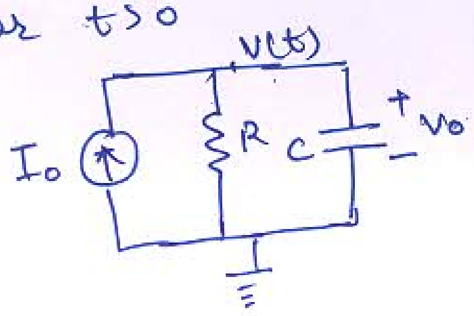
=>



Initial voltage at Capacitor

$v(0^-) = V_0$

For $t>0$



$$\Rightarrow \quad I_0 = \frac{v(t)}{R} + C\frac{dv(t)}{dt}$$

Taking laplace transform

$$\frac{I_0}{s} = \frac{V(s)}{R} + C[sV(s) - V_0]$$

$$\frac{I_0}{s} + CV_0 = \left[\frac{1}{R} + sC\right]V(s)$$

$$\Rightarrow \quad V(s) = \frac{I_0}{s\left[\frac{1}{R}+sC\right]} + \frac{CV_0}{\frac{1}{R}+sC}$$

$$= \frac{I_0}{sC\left[\frac{1}{RC}+s\right]} + \frac{V_0}{\left[\frac{1}{RC}+s\right]}$$

$$V(s) = I_0R\left[\frac{1}{s} - \frac{1}{s+1/RC}\right] + \frac{V_0}{s+1/RC}$$

Taking inverse laplace transform.

$$v(t) = L^{-1}[V(s)]$$

$$= I_0R(1-e^{-t/RC}) + V_0e^{-t/RC}$$

$$v(t) = (V_0 - I_0R)e^{-t/RC} + I_0R$$

—— x —— x ——