AKGEC/IAP/FM/02

AJAY KUMAR GARG ENGINEERING COLLEGE, GHAZIABAD

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

SESSIONAL TEST-2

Course: B.Tech
Session: 2017-18
Subject: Control System
Max Marks:50

Semester: V
Branch: EN-1,2
Sub. Code: NEE-503
Time: 2 hours

Note: Answer all the sections. Section - A

1. Define absolute stability & relative stability of a system.

Solⁿ: Relative stability means how close is the system is to instability. The degree or extent of the system is called relative stability.
Absolute stability means qualitative assessment, i.e. if a system characteristic equation is given then without obtaining a direct solⁿ of ch. eqⁿ. how one can say whether the system is stable or not.

2. Enlist the limitations of Routh-Hurwitz criteria.

Solⁿ: 1) It is valid only for real coefficients of the characteristic equation.
2) It does not provide exact locations of the closed loop poles in left or right half of s-plane.
3) It does not suggest methods of stabilizing an unstable system.

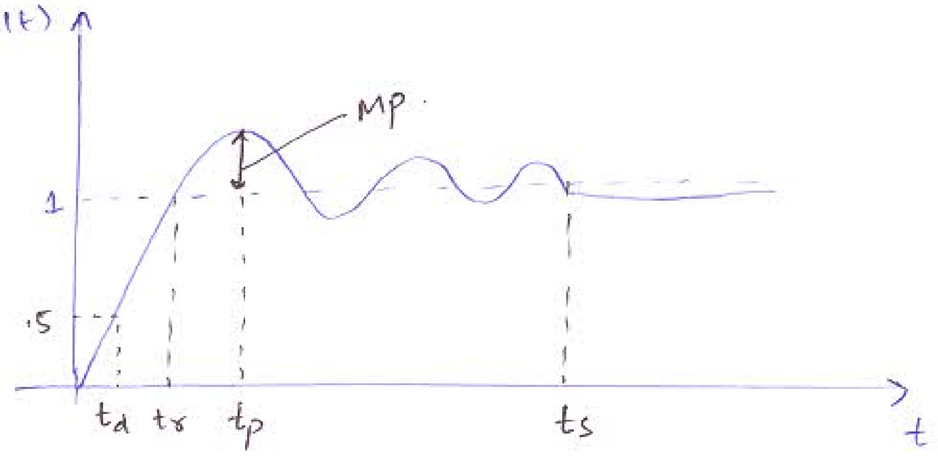
3. What is the effect of adding poles & zeros to the closed loop transfer function.

Solⁿ. The addition of poles to the closed loop transfer function increases the rise time & decreases the overshoot.

* Addition of zeros to closed loop transfer function decreases the rise time & increases the overshoot.

4. Draw time domain step response curve of a second order system & indicate important specifications.

Solⁿ:



5. A unity feedback system has forward path transfer function

$$G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$. find Kp, Kv & Ka for the system.

Solⁿ:

$$K_p = \lim_{s\to 0} G(s)H(s) \Rightarrow \lim_{s\to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)} = \infty$$

$$K_v = \lim_{s\to 0} sG(s)H(s) \Rightarrow \lim_{s\to 0} \frac{s.5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)} = \infty$$

$$K_a = \lim_{s\to 0} s^2G(s)H(s) \Rightarrow \lim_{s\to 0} \frac{\cancel{s^2}.\ 5(s^2+2s+100)}{\cancel{s^2}(s+5)(s^2+3s+10)} = \frac{\cancel{500}^{10}}{50}$$ Ans

Section-B

6. What is the effect of P-I controller on steady state error of a second order system with unit ramp input. Prove your answer mathematically.

Solⁿ: In P-I controller, the actuating signal consist of proportional error signal added to the integral of the error signal. The actuating signal in time domain is given by:

$$e_a(t) = k_p e(t) + k_i \int_0^t e(t)\, dt \quad \text{—①}$$

where k_p & k_i are proportional & integral gains known as controller parameters.

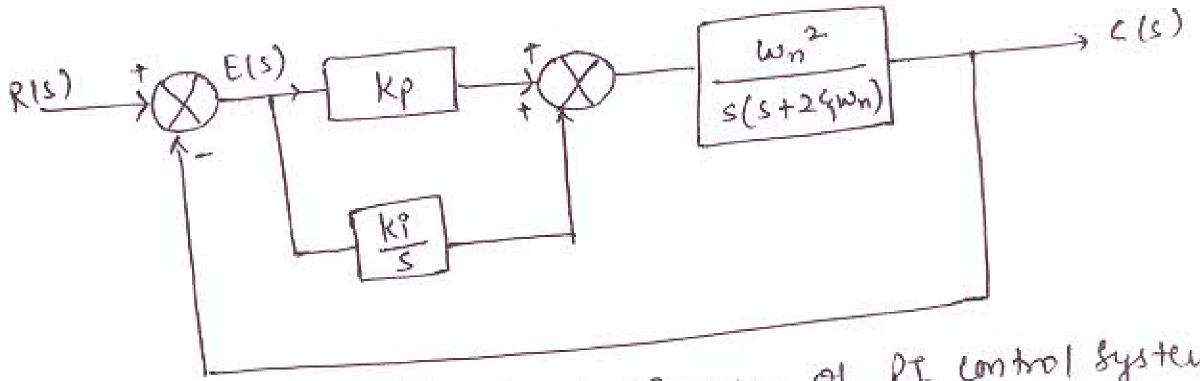


fig: Block diagram of PI control system.

Taking Laplace of eq ①.

$$E_a(s) = k_p E(s) + k_i \frac{E(s)}{s}$$

$$E_a(s) = \left(k_p + \frac{k_i}{s}\right) E(s) \quad \text{—②}$$

from the block diagram, open loop transfer function is

$$G(s) = \frac{C(s)}{E(s)} = \frac{W_n^2 \left(k_p + \frac{k_i}{s}\right)}{s^2 + 2\xi W_n s}$$

or

$$G(s) = \frac{(k_p s + k_i)\, W_n^2}{s(s^2 + 2\xi W_n s)}$$

Therefore, closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\dfrac{(k_p s + k_i) W_n^2}{s(s^2 + 2\xi W_n s)}}{1 + \dfrac{(k_p s + k_i) W_n^2}{s(s^2 + 2\xi W_n s)}}$$

or $\frac{C(s)}{R(s)} = \frac{(K_p s + K_i)\omega_n^2}{s^3 + 2\xi\omega_n s^2 + K_p \omega_n^2 s + K_i \omega_n^2}$

The characteristic equation is

$$s^3 + 2\xi\omega_n s^2 + K_p\omega_n^2 s + K_i\omega_n^2 = 0.$$

The above equation is of third order,

Thus, a second order system has been changed to a third order system by adding an integral control in the system. Therefore, the effect of PI controller on the system performance is that it increases the order of the system by one, which results in the reduction of the steady state error. The system relatively becomes less stable. Therefore, K_i should be designed properly to maintain stability of the system.

7. Explain construction & working of A.C servomotor. Also, discuss its torque speed characteristics.

Solⁿ: It is a two phase induction motor. The stator has two distributed windings. These windings are displaced from each other by 90° electrical. One winding is called main winding or reference winding. The reference winding is excited by constant a.c voltage. The other winding is called control winding. The rotor of a.c servomotors are of two types a) squirrel cage rotor b) drag cup type rotor. In drag cup type there are two air gaps. For the rotor a cup of non-magnetic conducting material is used. A stationary iron core is placed between the conducting cup to complete the magnetic circuit. The resistance of drag cup type is high & therefore having high starting torque.

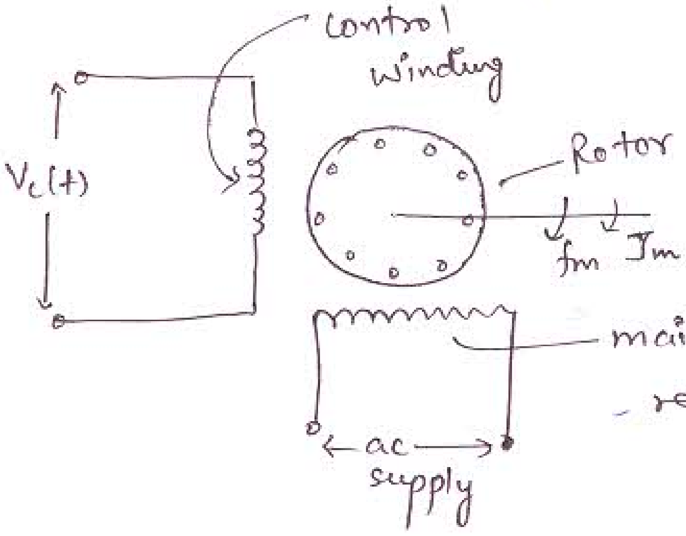


fig: a.c servomotor

If T_0 is stalling torque & w_0 is no load speed. The parameters m & K are determined in terms of T_0 & w_0 as follows:

i) When the speed is zero, the torque T_0 is proportional to V_c.

$$\therefore T_0 = KV_c$$

$$K = (T_0/V_c) \text{ Nm/V}$$

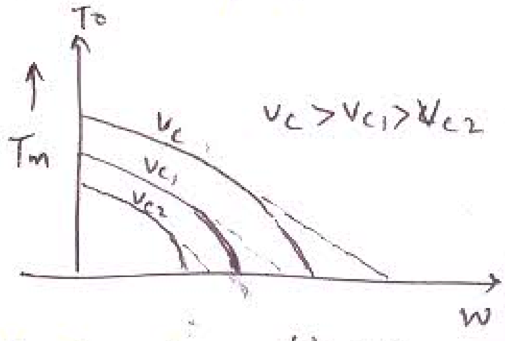


fig: τ-N chara.

ii) The slope of τ-N characteristic is

$$m = \frac{-T_0}{W_0} \text{ Nm/rad/sec.}$$

$$\because W_m = \frac{d\theta_m}{dt} \quad \& \quad T_m = mW_m + KV_c$$

$$\therefore T_m = m\frac{d\theta m}{dt} + KV_c$$

$$T_m = J_m\frac{d^2\theta m}{dt^2} + f_m\frac{d\theta m}{dt}.$$

8. A system has the following characteristic equation

$F(s) = s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0.$

Examine stability of a system.

Soln:

$$\begin{array}{c|cccc} s^6 & 1 & 4 & 5 & 2 \\ s^5 & 3 & 6 & 3 & \\ s^4 & 2 & 4 & 2 & \\ s^3 & 0 & 0 & 0 & \\ s^2 & & & & \\ s^1 & & & & \\ s^0 & & & & \end{array}$$

s^3 row: 0 0 0 $\longrightarrow$ special case (Rows of zeros)

Let $A(s) = 2s^4 + 4s^2 + 2 = 0.$

$$\frac{dA(s)}{ds} = 8s^3 + 8s$$

$\Rightarrow s_{1,2,3,4} = \pm j.$

$$\begin{array}{c|cc} s^3 & 8 & 8 \\ s^2 & 2 & 2 \\ s^1 & 0 & 0 \end{array}$$

s^1 row: 0 0 $\longrightarrow$ Row of zeros (special case)

Let $A'(s) = 2s^2 + 2 = 0 \Rightarrow s_{1,2} = \pm j.$

$$\frac{dA'(s)}{ds} = 4s + 0$$

$$\therefore \begin{array}{c|cc} s^1 & 4 & 0 \\ s^0 & 2 & \end{array}$$

As there are repeated roots on imaginary axis, system is 'unstable'.

Q: What is steady state error? Discuss positional, Velocity & acceleration error constants for type-0,1, & type-2 systems.

Soln: Steady state error is the difference between the actual output & the desired output.

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)H(s)}}$$

Type '0' system

for type '0' system G(s) H(s) is given by

$$G(s)H(s) = \frac{K(1+T_1s)(1+T_2s)\ldots}{(1+T_as)(1+T_bs)\ldots}$$

for step input,

$$K_p = \lim_{s\to 0} G(s)H(s) \Rightarrow \boxed{K_p = K}$$

$$\therefore e_{ss} = \frac{A}{1+K_p}$$

$$\Rightarrow \boxed{e_{ss} = \frac{A}{1+K}}$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$\therefore K_v = \lim_{s\to 0} s \cdot \frac{K(1+T_1s)(1+T_2s)\ldots}{(1+T_as)(1+T_bs)\ldots}$$

$$\therefore \boxed{K_v = 0}$$

& $e_{ss} = \frac{A}{K_v}$

$$\Rightarrow \boxed{e_{ss} = 0}$$

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$K_a = 0$$

$$\therefore \quad e_{ss} = \frac{A}{K_a}$$

$$\Rightarrow \quad \boxed{e_{ss} = \infty}$$

Type '1' System

for type '1' system $G(s)H(s)$ is given by

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{s(1+sT_a)(1+sT_b)\ldots}$$

$$\therefore \quad K_p = \lim_{s \to 0} G(s)H(s) \Rightarrow \boxed{\infty = K_p}$$

$$\therefore \quad e_{ss} = \frac{A}{1+K_p}$$

$$\boxed{e_{ss} = 0}$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \frac{\cancel{s}K(1+sT_1)(1+sT_2)\ldots}{\cancel{s}(1+sT_a)(1+sT_b)\ldots}$$

$$\therefore \quad \boxed{K_v = K}$$

hence $\boxed{e_{ss} = \frac{A}{K}}$

$$\& \quad K_a = \lim_{s \to 0} s^2 G(s)H(s) \Rightarrow \boxed{K_a = 0}$$

$$\boxed{e_{ss} = \frac{A}{K_a} = \infty}$$

Type '2' system

for 'type 2' $G(s)H(s)$ is given by:

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{s^2(1+sT_a)(1+sT_b)\ldots}$$

for, $K_p = \lim_{s\to 0} G(s)H(s) \Rightarrow \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\ldots}{s^2(1+sT_a)(1+sT_b)\ldots}$

$\therefore$ $\boxed{K_p = \infty}$

hence $\boxed{e_{ss} = 0}$

$K_v = \lim_{s\to 0} sG(s)H(s) \Rightarrow \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\ldots}{s^2(1+sT_a)(1+sT_b)\ldots}$

Thus, $\boxed{K_v = \infty}$

$\therefore$ $\boxed{e_{ss} = 0}$

$K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} \cancel{s^2} \cdot \frac{K(1+sT_1)(1+sT_2)\ldots}{\cancel{s^2}(1+sT_a)(1+sT_b)\ldots}$

$\boxed{K_a = K}$

$\therefore$ $\boxed{e_{ss} = \frac{A}{K}}$ Ans

10 Consider the system shown in fig 1. Determine the value of K such that the damping ratio is 0.5. Also, obtain the rise time (t_r), peak time (t_p), maximum overshoot (M_p), settling time (t_s) & time response of the system to a unit step input.

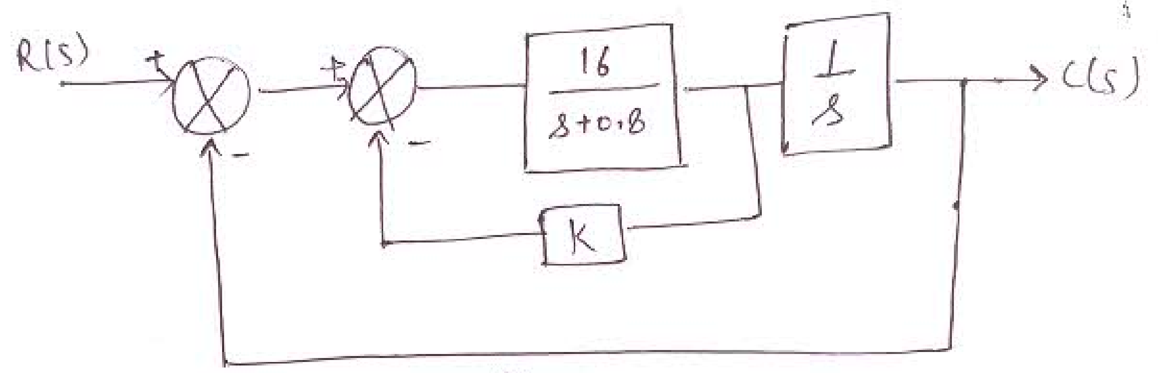


fig. 1

Sol^n: from fig. 1, by block reduction method we get

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+(.8+16K)s+16}$$

Here, characteristic equation is $[s^2+(.8+16K)s+16]$, comparing it with standard equation i.e

$$s^2+2\xi w_n s+w_n^2=0.$$

we get

$$w_n^2=16 \ \& \ 2\xi w_n=(.8+16K)$$

$$\therefore w_n=4 \text{ rad/sec.}$$

& hence $\cancel{2\xi w_n}$ $2\times.5\times4=(.8+16K)$

$$\therefore \boxed{K=0.2} \qquad \{\because \xi=.5 \text{ given}\}$$

$$t_r=\frac{\pi-\phi}{w_n\sqrt{1-\xi^2}} = .605 \text{ sec} \qquad \left\{\phi=\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right\}$$

$$t_p=\frac{\pi}{w_n\sqrt{1-\xi^2}} = .906 \text{ sec}$$

$$M_p=e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}\times100 = e^{\frac{-\pi\times.5}{\sqrt{1-.5^2}}}\times100 = 16.3\%.$$

$$t_s = \frac{4}{\xi w_n} = \frac{4}{.5 \times 4} = 2\,sec \quad \underline{Ans}$$

$$\therefore\ c(t) = 1 - \frac{e^{-\pi \xi w_n t}}{\sqrt{1-\xi^2}}\left[\sin(w_d t + \phi)\right]$$

$$c(t) = 1 - \frac{e^{-2\pi t}}{\sqrt{1-.5^2}} \sin\left(4\sqrt{1-.5^2}\cdot t + \tan^{-1}\frac{\sqrt{1-.5^2}}{.5}\right) \quad Ans$$

Section-C

11. Sketch the complete root locus for the system having

$$G(s)H(s) = \frac{K}{s(s+3)(s^2+3s+11.25)}$$

Soln: Step 1:

$P = 4$, $z = 0$ $\Rightarrow$ $P = 0, -3, -1.5 \pm j3$

$N = 4$, (branches of root locus)

Centroid $\sigma = \dfrac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeros}}{P - Z}$

$$\sigma = \frac{0 - 3 - 1.5 - 1.5}{4} = -1.5.$$

Asymptotes $\phi = \dfrac{(2q+1)180°}{P - Z}$ $\quad q = 0, 1, 2, 3$

$\therefore\ \phi_1 = 45°$, $\phi_2 = 135°$, $\phi_3 = 225°$, $\phi_4 = 315°$.

Step 4: Breakaway point

Characteristic equation $1+G(s)H(s)=0$.

$$\therefore\ 1+\frac{K}{s(s+3)(s^2+3s+11.25)}=0$$

$$\therefore\ s^4+6s^3+20.25s^2+33.75s+K=0$$

$$K=-s^4-6s^3-20.25s^2-33.75s.$$

$$\frac{dK}{ds}=-4s^3-18s^2-40.5s-33.75=0.$$

$$\therefore\ 4s^3+18s^2+40.5s+33.75=0.$$

$$\therefore\ s=-1.5\ \&\ -1.5\pm j1.83.$$

At $s=-1.5$, $K=+20.25$

So $s=-1.5$ is a valid breakaway point.

Step 5: Intersection with imaginary axis

$$s^4+6s^3+20.25s^2+33.75s+K=0.$$

s^4	1	20.25	K
s^3	6	33.75	
s^2	14.62	K	
s^1	$\frac{493.59-6K}{14.62}$	0	
s^0	K		

$\therefore$ $K>0$ & $493.59-6K>0$

$\therefore$ $\boxed{K_{max}=82.26}$

Step 6: Angle of departure

at point $(-1.5+j3)$

$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{3}{1.5}\right) = 116.56$

$\phi_{P_2} = 90^\circ$

$\phi_{P_3} = \tan^{-1}\left(\frac{3}{1.5}\right) = 63.43^\circ$

$\Sigma\phi_P = 270^\circ, \quad \Sigma\phi = 0$

$\therefore \phi = 270^\circ.$

At $(-1.5+j3)$, $\phi d = 180^\circ - \phi = -90^\circ$

at $(-1.5-j3)$, $\phi d = 180^\circ + \phi = 90^\circ.$

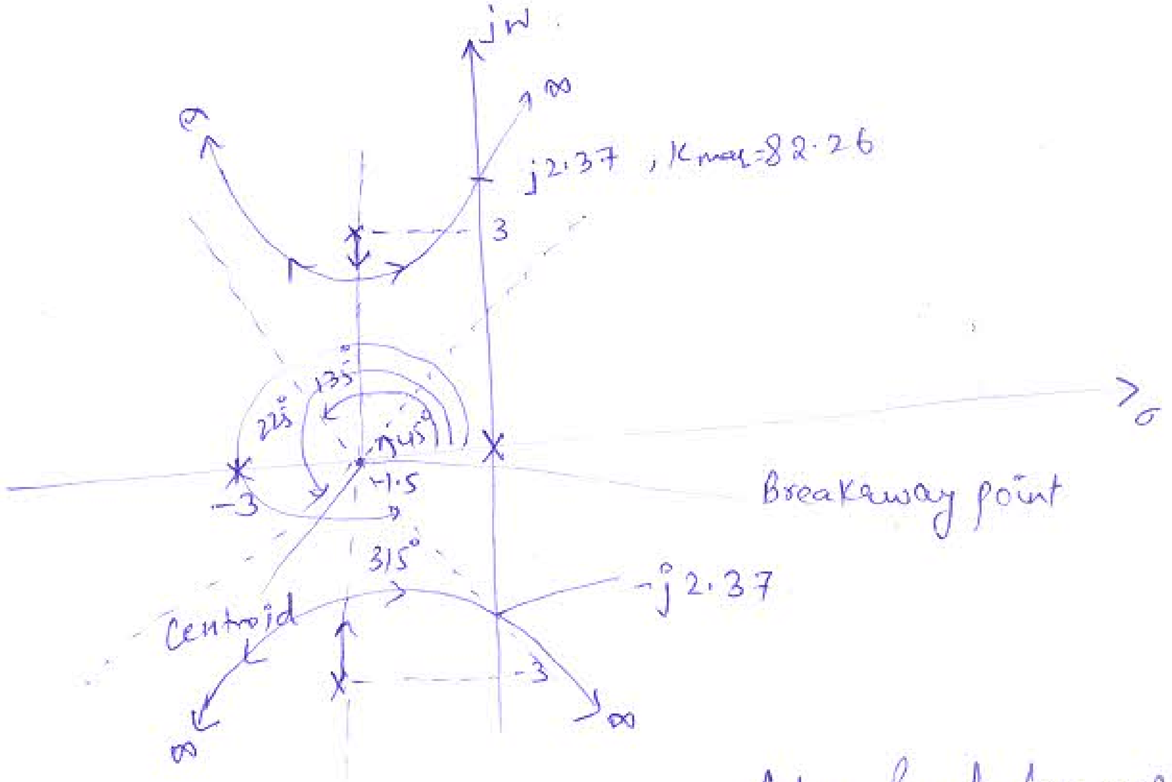


fig: Root locus plot.

$\therefore$ for $0 < K < 82.26$, system is stable.

At $K = 82.26$ system is marginally stable.

$K > 82.26$ system is unstable.

12. Derive the expressions for rise time & maximum peak overshoot of second order system in time domain. Also, define the settling time & rise time for second order system.

Soln: The rise time is the time needed for the response to reach from 10% to 90% or 0% to 100% of the desired value of the output at the very first instant.

Now,

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi]$$

At the first instant when time response reaches 100% of the desired value, i.e $c(t) = 1$, time is t_r, therefore, substituting $c(t) = 1$ in above equation.

$$1 = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi].$$

or

$$\frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi] = 0.$$

As $\frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}}$ is finite

$$\sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi] = 0$$

Above eq Soln is

$$(\omega_n\sqrt{1-\xi^2})t_r + \phi = \pi$$

$$t_r = \frac{\pi - \phi}{W_n\sqrt{1-\xi^2}}$$

Where $\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$.

Maximum overshoot Mp :-

$$Mp = c(t)_{max} - 1$$

$$\%\,Mp = \frac{c(t)_{max} - 1}{1} \times 100.$$

The expression for c(t) is

$$c(t) = 1 - \frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \sin[(W_n\sqrt{1-\xi^2})t + \phi]$$

$$\therefore \frac{dc(t)}{dt} = -\frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \cdot W_n\sqrt{1-\xi^2} \cdot \cos[(W_n\sqrt{1-\xi^2})t + \phi]$$

$$- \frac{(-\xi W_n)e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \sin[(W_n\sqrt{1-\xi^2})t + \phi]$$

Put $\frac{dc(t)}{dt} = 0$

$$\therefore \frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \{-W_n\sqrt{1-\xi^2} \cos[(W_n\sqrt{1-\xi^2}\, t) + \phi] + \xi W_n \sin\{(W_n\sqrt{1-\xi^2})t + \phi\}\} = 0.$$

In above eq^n $\frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}}$ is finite

$$\therefore W_n\sqrt{1-\xi^2} \cos\{(W_n\sqrt{1-\xi^2})t + \phi] = \xi W_n \sin[(W_n\sqrt{1-\xi^2})t + \phi]$$

$$\therefore \tan[(\omega_n\sqrt{1-\xi^2})t+\phi] = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\because \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\therefore \tan\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] = \frac{\sqrt{1-\xi^2}}{\xi}$$

general Sol^n of above eq^n is

$$(\omega_n\sqrt{1-\xi^2})t = n\pi.$$

where $n = 0, 1, 2 \ldots$

The instant of occuring M_p is obtained by $n=1$

$$\therefore t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$c(t)_{max}$ is determined by putting $t = t_p$ in time response expression therefore

$$c(t)_{max} = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin[\omega_n(\sqrt{1-\xi^2})t_p + \phi]$$

$$= 1 - \frac{e^{\frac{-\xi\omega_n\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left[\left(\omega_n\sqrt{1-\xi^2}\cdot\frac{\pi}{\omega_n\sqrt{1-\xi^2}}\right) + \phi\right]$$

$$\therefore c(t)_{max} = 1 + e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\therefore M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\boxed{\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$$