

AKGEC/IAP/FM/01

**AJAY KUMAR GARG ENGINEERING COLLEGE, GHAZIABAD**  
**DEPARTMENT OF CIVIL ENGINEERING**  
**Sessional Test-2 (SOLUTION)**

Course: B.Tech.  
 Session: 2017-18  
 Subject: Design of Concrete Structures-I  
 Max Marks: 50

Semester: V  
 Section: CE-1& 2  
 Sub. Code: NCE-505  
 Time: 2 hours

Answer all the questions. Any data if missing may be assumed suitably.

Section - A

QNS 1(a) what do you mean by anchorage length? And also write its maximum value for simply supported beam.

QNS 1.(a) ANS:- "Anchorage length" - This is the additional length of bar. for example - Main bar of the beam in column at beam column junction. This is represented by  $L_o$ . And maximum Anchorage length for simply supported beam is subjected to a maximum of  $12\phi$  or  $d$ .

where  $\phi$  - diameter of main bar.  
 $d$  - effective depth of beam.

QNS 1(b) when the shear reinforcement is necessary in a beam?

QNS 1.(b) ANS:- If the value of  $T_v$  is greater than  $T_c$ . Then shear reinforcement is needed.

where,

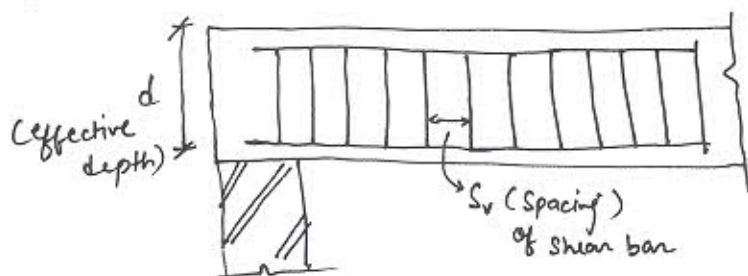
$T_v$  = Nominal shear stress

$T_c$  = Shear strength of concrete.

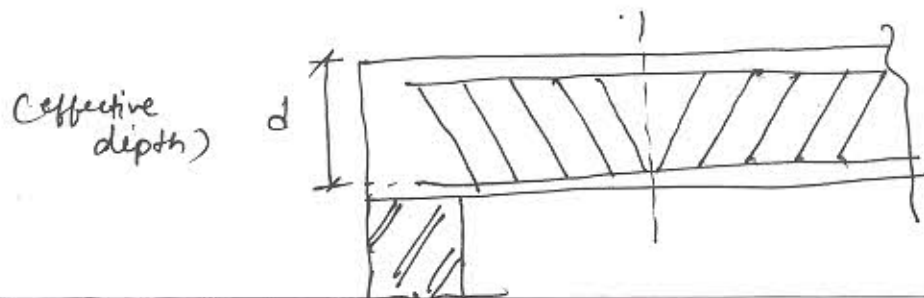
QNS 1(c) Enumerate types of shear reinforcement with neat sketch.

QNS 1.(c) ANS:- There are three types of shear reinforcement

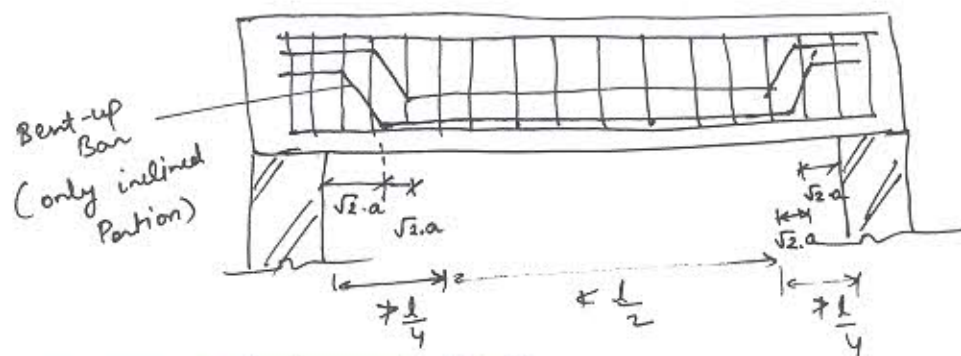
① Vertical Shear Reinforcement



② Inclined Shear Reinforcement



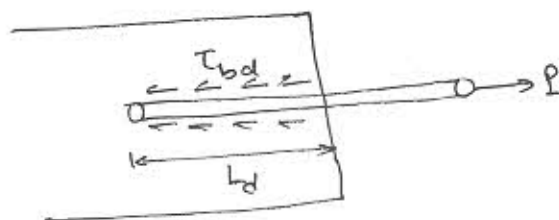
### ③ Bent-up Bars



QNS1(d) Define Development length.

QNS1. (d) ANS:-

"Development length":- This is the minimum length of reinforcement required to be provided inside concrete, so that the strength of bond is not less than strength of reinforcement. This is denoted by  $L_d$ .



$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

where,  $f_y$  - characteristic strength of steel.

$\phi$  - diameter of bar.

$\tau_{bd}$  - bond stress.

$L_d$  - development length.

QNS1(e) Under what circumstances a doubly reinforced beam is designed?

QNS1. (e) ANS:- If the size of beam is restricted (width and depth both)

and if the beam has to sustain a higher value of B.M. more than  $M_R$  of the singly reinforced limiting section, Doubly reinforced beam is required.

### SECTION-B

QNS2:- (a) Determine ultimate moment of capacity of a doubly reinforced beam of  $300\text{mm} \times 600\text{mm}$  overall depth. Area of steel in compression is equal to  $804\text{mm}^2$ , area of steel in tension is equal to  $2060\text{mm}^2$ . If M20 concrete & Fe-415 Steel are used. Take the value of effective cover  $50\text{mm}$ .

QNS(2) (a)

ANS:-

Step ① limiting depth of Neutral Axis (N.A):-  
( $x_{u,lim}$ )

$$= 0.48d$$

$$= 0.48 \times 550 = 264 \text{ mm}$$

Step ② Actual depth of N.A :- ( $x_u$ )

$$C_1 + C_2 = T$$

$$0.36 f_{ck} B x_u + (f_{sc} - 0.45 f_{ck}) \cdot A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + (f_{sc} - 0.45 \times 20) \times 804 = 0.87 \times 415 \times 2060$$

$$2160 x_u + 804 f_{sc} = 750999 \quad \text{--- (1)}$$

Trial ①

Consider  $f_{sc} = 350 \frac{\text{N}}{\text{mm}^2}$

$$x_u = 217.4$$

Strain in compressive  
Steel,

$$E_{sc} = \frac{x_u - d_c}{x_u} \times 0.0035$$

$$= \frac{217.4 - 50}{217.4} \times 0.0035$$

$$E_{sc} = 0.00269$$

By Interpolation Rule,

$$f_{sc} = 342 + \frac{351 - 342}{0.00276 - 0.00241} \times (0.00269 - 0.00241)$$

$$= \underline{349.2}$$

$$x_u = 217.6$$

$\therefore x_u < x_{u,lim}$  { It is a under reinforced section }

Step ③ Moment of Resistance ( $M_{Ru}$ ):

$$M_{Ru} = 0.36 f_{ck} B x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) \cdot A_{sc} (d - d_c)$$

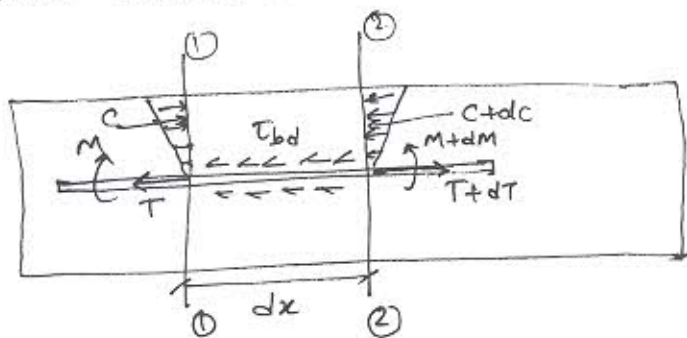
$$= [0.36 \times 20 \times 300 \times 217.6 (550 - 0.42 \times 217.6) + (349.2 - 0.45 \times 20) \times 804 \times (550 - 50)] \times \frac{1}{10^6}$$

$$M_{Ru} = 352.32 \text{ kN-m}$$

QNS:- 2(b) What is the bond strength of concrete? Derive the expression for bond stress in reinforced concrete.



QNS:- 2 (b) ANS:- Bond strength is the measure of effectiveness of the grip between concrete and steel. and



Consider two cross-section 1-1 and 2-2 very close to each other at  $dx$  distance.

In reinforced there will be a  $(dT)$  unbalanced force that creates bond stress between steel and concrete. This bond stress is also called longitudinal shear stress.

At section 1-1  $M = T \cdot jd$  or  $C \cdot jd$  — (1)

At section 2-2  $M + dM = (T + dT) \cdot jd$  — (2)

Eqs (2) - Eqs (1)

$$dM = dT \cdot jd$$

$$\boxed{dT = \frac{dM}{jd}} \quad \text{--- (3)}$$

Resisting Bond force

$$dT = \tau_{bd} \cdot n \pi \phi \cdot dx \quad \text{--- (4)}$$

Equating (3) & (4)

$$dT = \frac{dM}{jd} = n \pi \phi \cdot dx \cdot \tau_{bd}$$

$$\tau_{bd} = \frac{dM}{jd \cdot n \pi \phi \cdot dx} = \frac{V}{jd \cdot n \pi \phi}$$

$$\left\{ \begin{array}{l} \frac{dM}{dx} = V \\ \text{S.F} \end{array} \right.$$

$$\boxed{\tau_{bd} = \frac{V}{jd \Sigma \phi}}$$

$$\Sigma \phi = \text{sum of perimeter all reinforcement} = n \pi \phi$$

QNS:- 2 (c) A simply supported beam is 25 cm by 50 cm and has 2-20 mm HYSD bars going into the support if the shear force at the centre of support is 110 kN at working loads, determine the anchorage length. Assume M-20 mix and Fe-415 steel.

QNS 2:- (C) Ans:-

$$\text{factored shear force} = 1.5 \times 110 = 165 \text{ kN}$$

Assuming 25 mm clear cover to the longitudinal bars

$$d = 500 - 25 - \frac{20}{2} = 465 \text{ mm}$$

$$\sigma_y = 415 \text{ N/mm}^2.$$

moment of resistance ( $M_1$ )

$$= 0.87 \sigma_y A_{st} (d - 0.42 x_u)$$

$$x_u = \frac{0.87 \sigma_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250}$$

$$= 126 \text{ mm} < x_m [0.47]$$

$$M_1 = 93.45 \times 10^6 \text{ N.mm}$$

$$\tau_{bd} = 1.2 \text{ N/mm}^2 \text{ for M20 mix}$$

it can be increased by 60% in case of HYSD Bar

$$L_d = \frac{\phi f_y \times 0.87}{4 \tau_{bd}}$$

$$= \frac{0.87 \times 415 \times \phi}{4 \times 1.6 \times 1.2}$$

$$L_d = 47 \phi$$

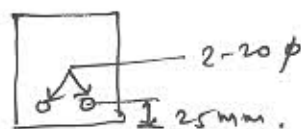
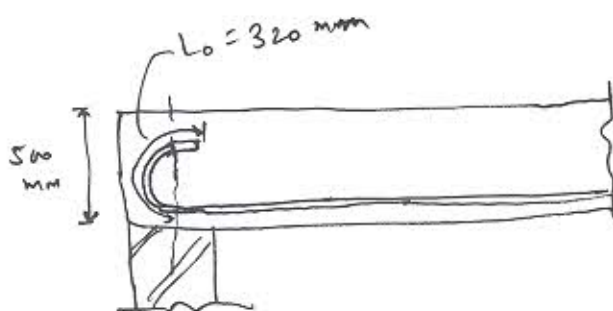
if we provided U-Bend at the centre of support its anchorage value.

$$L_0 = 16 \phi = 16 \times 20 = 320 \text{ mm}$$

$$L_d \leq 1.3 \frac{M_1}{V} + L_0$$

$$47 \phi \leq \left[ \frac{1.3 \times 93.45 \times 10^6}{165 \times 1000} \right] + 320$$

$$\phi \leq 22.47 \text{ mm.}$$



QNS:- 2(d) write difference between LSM and WSM.

QNS:- 2 (d) ANS:-

WSM

(Working stress method)

① It is based on deterministic approach.

② Concrete

① stress factor of safety = 3.0

② load factor of safety = 1.0

③ Net factor of safety = 3.0

③ Steel

stress factor of safety = 1.8

load factor of safety = 1.0

net F.O.S = 1.8

LSM

(Limit state method)

① It is based on probabilistic approach.

② Concrete

① stress factor of safety = 2.22

② load factor of safety = 1.5

③ Net factor of safety =  $\frac{3.33}{\text{more than that of WSM}}$

③ Steel

stress factor of safety = 1.15

load F.O.S = 1.5

net F.O.S =  $1.15 \times 1.5 = 1.725$

less than that of WSM

QNS 2 (e) A simply supported beam of 5m effective span is subjected to 24 kN/m line load. Size of beam 250 mm wide and 400 mm overall depth. Design reinforced beam. Use M-20 & Fe-415.

QNS:- 2 (e) ANS:-

step ① Load/B.M

Line load = 24 kN/m

Self wt. =  $0.25 \times 0.40 \times 1.0 \times 25 = 2.5 \text{ kN/m}$

Total W = 26.5 kN/m

factored  $w_u = 26.5 \times 1.5$   
 $= 39.75 \text{ kN/m}$

Max<sup>m</sup> B.M  $(B.M)_u = \frac{w_u l^2}{8} = \frac{39.75 \times 5^2}{8} = 124.21 \text{ kN-m}$

step ② MR of limiting (SR) section:-

$M_{u1} = MR_{lim} = \alpha B d^2$

$= 0.138 f_{ue} B d^2$

$= 0.138 \times 20 \times 250 \times \frac{350^2}{10^6}$

$= 84.525 \text{ kN-m}$

SECTION- C

QNS 3 (a) A simply supported T-beam of span 6.0m in reinforced concrete has following dimensions:  
width of flange = 1600 mm, Depth of flange = 100 mm.  
Overall depth of beam = 850 mm, width of web = 400 mm  
Effective cover to reinforcement = 50 mm, Use Fe-500 steel & M-30 concrete.



Design the beam for a factored B.M of 1250 kN-m.

QNS (3) (a) Ans:-

① Calculate effective width of flange

$$B_f = \frac{l_0}{\left(\frac{l_0}{B} + 4\right)} + b_w = \frac{6000}{\left(\frac{6000}{1600} + 4\right)} + 400$$
$$= 1174.2 \text{ mm}$$

② Calculate M.R ( $M_{u1}$ )

when  $x_u = d_f$

$$M_{u1} = 0.36 f_{ck} B_f x_u (d - 0.42 x_u)$$
$$= 0.36 \times 30 \times 1174.2 \times 100 \times (800 - 0.42 \times 100) \times \frac{1}{10^6}$$
$$= 961.25 \text{ kN-m}$$

③ Calculate M.R ( $M_{u2}$ )

when  $x_u = \frac{7}{3} d_f$

$$x_u = \frac{7}{3} \times 100 = 233.33 \text{ mm (df case)}$$

$$M_{u2} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) \cdot d_f \cdot \left(d - \frac{d_f}{2}\right)$$
$$= 0.36 \times 30 \times 400 \times 233.33 \times (800 - 0.42 \times 233.33)$$
$$+ 0.45 \times 30 \times [1174.2 - 400] \times 100 \times \left(800 - \frac{100}{2}\right)$$

$$M_{u2} = 1491.48 \text{ kN-m}$$

④ Calculate M.R at  $x_u = x_{u,lim}$

$$x_u = 0.48 \times 800 = 384 \text{ mm}$$

$$\boxed{d_f < \frac{3}{7} x_u} \quad (d_f \text{ case})$$

$$M_{u,lim} = 0.36 \times 30 \times 400 \times 384 \times (800 - 0.42 \times 384) + 0.45 \times 30 \times (1174.2 - 400) \times 100 \times \left(800 - \frac{100}{2}\right)$$

$$M_{u,lim} = 1843.42 \text{ kN-m}$$

$$(BM)_U > MR_{SR\lim}$$

we need a doubly Reinforced section.

Step 3  $A_{st1}$  for  $(M_{U1})$

$$\begin{aligned} A_{st1} &= \frac{M_{U1}}{0.87 f_y (d - 0.42 x_{u\lim})} \\ &= \frac{84.525 \times 10^6}{0.87 \times 415 (350 - 0.42 \times 0.48 \times 350)} \\ &= 837.78 \text{ mm}^2 \end{aligned}$$

Step 4 Remaining BM.

$$\begin{aligned} M_{U2} &= BM_U - M_{U1} \\ &= 124.21 - 84.525 \\ &= 39.685 \approx 36.69 \text{ kN-m} \end{aligned}$$

$$\begin{aligned} \text{Step 5 } A_{st2} &= \frac{M_{U2}}{0.87 f_y (d - d_c)} \\ &= \frac{36.69 \times 10^6}{0.87 \times 415 (350 - 50)} \\ &= 366.43 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Step 6 Total } A_{st} &= A_{st1} + A_{st2} \\ &= 837.78 + 366.43 \\ &= 1204.21 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Step 7 } A_{sc} &= \frac{M_{U2}}{(f_{sc} - 0.45 f_{ck}) (d - d_c)} \\ \text{Considering } f_{sc} &= 350 \text{ N/mm}^2 \\ &= \frac{39.69 \times 10^6}{(350 - 0.45 \times 20) (350 - 50)} \end{aligned}$$

$$\begin{aligned} \text{Area of Compressive Steel } A_{sc} &= 387.97 \text{ mm}^2 \end{aligned}$$

QNS:- 3 (b) A simply supported beam 300 mm wide and 500 mm effective depth carries a uniformly distributed load 50 kN/m over an effective span of 6 m. Design the shear reinforcement in the form of vertical stirrups. Assume that the beam contain 0.78% reinforcement throughout the length. The concrete is of M-20 grade & steel for stirrups is of Fe-250 grade. Take width of support as 400 mm.



QNS 3 (b) Ans:-

In case of simply supported beam the critical section for shear is taken at a distance 'd' from the face of the support we have.

$$W_u = 1.5 \times \text{working load} \\ = 1.5 \times 50 = 75 \text{ kN.}$$

$$V_{u \max} = \frac{W_u \cdot l}{2} = \frac{75 \times 6}{2} = 225 \text{ kN.}$$

As per IS:456 the critical section for shear lies at a distance  $d = 500 \text{ mm}$  from the face of support.

i.e. at a distance  $= 500 + \frac{400}{2} = 700 \text{ mm}$  from centre of support.

$$V_{ud} = \text{Design Shear at the critical section} \\ = 225 - 75 \times 0.700 = 172.5 \text{ kN}$$

$$\text{Now, } \tau_v = \frac{V_u}{bd} = \frac{172.5 \times 10^3}{300 \times 500} = 1.15 \text{ N/mm}^2$$

$$p_t = \frac{100 A_{st}}{bd} = 0.75\% \text{ (given)}$$

Corresponding to this value of longitudinal reinforcement, & for grade of concrete, we have

$$\tau_c = 0.56 \text{ N/mm}^2 \quad \text{from table (19)}$$

$$\text{Also, } \tau_{\max} = 2.8 \text{ N/mm}^2 \quad \text{(from table 20)}$$

$$\tau_c < \tau_v < \tau_{\max}$$

Hence shear reinforcement is needed.

Design of shear reinforcement.

$$V_{us} = V_{ud} - \tau_c \cdot bd \\ = 172.5 \times 10^3 - 0.56 \times 300 \times 500$$

$$V_{us} = 88.5 \text{ kN}$$

⑥ Design for  $B.M_{u2} = 1250 \text{ kN-m}$

$$M_{u1} < B.M_{u2} < M_{u2}$$

$$d_f < x_{u2} < \frac{7}{3} d_f$$

$$d_f > \frac{3}{7} x_{u2}$$

So, it is a  $y_f$  case

$$y_f = 0.15 x_{u2} + 0.65 d_f$$
$$= 0.15 x_{u2} + 0.65 \times 100$$

$$y_f = 0.15 x_{u2} + 65$$

$$B.M_{u2} = 0.36 f_{ck} b_w x_{u2} (d - 0.42 x_{u2})$$
$$+ 0.45 f_{ck} (B_f - b_w) y_f \cdot (d - \frac{y_f}{2})$$

$$1250 \times 10^6 = 0.36 \times 30 \times 400 x_{u2} (800 - 0.42 x_{u2})$$
$$+ 0.45 \times 30 \times (1174.2 - 400) \times (0.15 x_{u2} + 65)$$
$$\times (800 - \frac{0.15 x_{u2} + 65}{2})$$

$$1569.1 x_{u2}^2 + 4608299.92 x_{u2} + 728590816.2 = 0$$

$$x_{u2} = \underline{170.26 \text{ mm}}$$

$A_{st}$

$$A_{st1} = \frac{0.36 f_{ck} b_w x_{u2}}{0.87 f_y} = \frac{0.36 \times 30 \times 400 \times 170}{0.87 \times 415}$$
$$= 2037.15 \text{ mm}^2$$

$$A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$$
$$= \frac{0.45 \times 30 \times (1170 - 400) (0.15 \times 170.26 + 65)}{0.87 \times 415}$$
$$= 2620.93 \text{ mm}^2$$

total

$$A_{st} = A_{st1} + A_{st2}$$

$$\boxed{A_{st} = 4658.11 \text{ mm}^2}$$

Note

use

6 mm  $\phi$   
8 mm  $\phi$   
10 mm  $\phi$   
12 mm  $\phi$  } 2-legged for  $A_{sv}$

Using 10 mm - 2 legged vertical stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times (10)^2 = 157.1 \text{ mm}^2$$

Now the spacing of the vertical stirrups

$$S_v = \frac{0.87 \sigma_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times 157.1 \times 500}{88500}$$

$$S_v = 193 \text{ mm}$$

Check for spacing

$$\begin{aligned} \text{maximum spacing} &\neq 0.75 d \\ &= 0.75 \times 500 = 375 \text{ mm} \\ &\neq 300 \text{ [O.K]} \end{aligned}$$

Minimum Area of Shear Reinforcement

$$A_{sv} = \frac{0.4 b S_v}{0.87 \sigma_y} = 106.48 \text{ mm}^2 < 157.1 \text{ mm}^2 \text{ [O.K]}$$

Hence provide 10 mm  $\phi$  - 2 legged vertical stirrups @ 190 mm Centre to Centre.

