Ajay Kumar Garg Engineering College G3b (1)

Department of Applied Science & HU

Model Solution ST2

CBOT

Course: - MCA

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Subject - CBOT

Max Marks 50

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Semester III

Section - MCA-1, MCA-2

Code - RCA-304

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Sec A

A. Attempt all parts

Q1. Define scrap value in replacement problem.

The scrap value in replacement problem is the 'resale value' of the machine i.e. the cost at which machine could be sold having profit or loss.

Q.2. What is inventory system?

Inventory system consist of -

1) Holding or storage cost.
2) Shortage cost
3) Setup cost
4) Demand
5) Dead time.

Q.3. Write the need of Inventory.

Inventory for any business is needed to maintain the degree of setup cost and shortage cost. If the demand of the customer is not fulfilled, it results in the loss of goodwill.

Thus, Inventory is needed for the smooth and efficient running of the business and the economics in transportation.

Q.4. Define Replacement problem.

Replacement problem arises at following situations –

- Replacement of item, which deteriotates with the time. eg – Bus, Machine etc.
- Replacement of item, with complete failure.
- Problem in mortality and staffing.
- Replacement of item, when new machine is invented or arises, otherwise machine will go out of date.

Q.5. What is the Dual of the Dual of a given Problem.

The Dual of the Dual of a given Problem is 'primal itself.'

SecB

B. Attempt all the parts

Question 6 → what is the Dual of the Dual is the given problem

$\min z = x_1 + x_2 + 2x_3$

s.t. $x_1 + 2x_2 \geq 3$

$x_2 + 7x_3 \leq 6$

$x_1 - 3x_2 + 5x_3 = 5$

$x_1, x_2 \geq 0$, x_3 is unristricted

Soluation, Here x_3 is unristricted

Let $x_3 = x_3' - x_3''$

where $x_3', x_3'' \geq 0$

Then the problem become

$\min z = x_1 + x_2 + 2x_3' - 2x_3''$

s.t. $x_1 + 2x_2 \geq 3$ — (1)

$-x_2 - 7x_3 \geq -6$ — (2)

$-x_1 + 3x_2 - 5x_3 \geq -5$ — (3)

$x_1 - 3x_2 + 5x_3 \geq 5$ — (4)

Therefore equation become

$\min z = [1, 1, 2, -2][x_1, x_2, x_3', x_3'']$

in eqn (1) (2) (3) and (4)

$x_1 + 2x_2 \geq 3$

$-x_2 - 7x_3' + 7x_3'' \geq -6$

$x_1 - 3x_2 + 5x_3' - 5x_3'' \geq 5$

$-x_1 + 3x_2 - 5x_3' + 5x_3'' \geq -5$

Therefore

$$\begin{bmatrix} 1 & +2 & 0 & 0 \\ 0 & -1 & -7 & 7 \\ 1 & -3 & 5 & -5 \\ -1 & 3 & -5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3' \\ x_3'' \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 5 \\ -5 \end{bmatrix}$$

$$x_1, x_2, x_3', x_3'' \geq 0$$

Convert in dual

$$\text{Max } Z_D = [3\ -6\ 5\ -5]\,[w_1\ w_2\ w_3'\ w_3'']$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 2 & -1 & -3 & 3 \\ 0 & -7 & 5 & -5 \\ 0 & 7 & -5 & 5 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3' \\ w_3'' \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\text{Max } Z_D = 3w_1 - 6w_2 + 5w_3' - 5w_3''$$

$$w_1 + 0w_2 + w_3' - w_3'' \leq 1$$

$$2w_1 - w_2 - 3w_3' + 3w_3'' \leq 1$$

$$0w_1 - 7w_2 + 5w_3' - 5w_3'' \leq 2$$

$$0w_1 + 7w_2 - 5w_3' + 5w_3'' \leq -2$$

again Dual

$$\min Z_D' = [1\ 1\ 2\ -2]\,[w_1\ w_2\ w_3'\ w_3'']$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -2 & -7 & 7 \\ 1 & -3 & 5 & -5 \\ -1 & 3 & -5 & 5 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3' \\ w_3'' \end{bmatrix} \geq \begin{bmatrix} 3 \\ -6 \\ 5 \\ -5 \end{bmatrix}$$

$$\min Z_D' = w_1 + w_2 + 2w_3' - 2w_3''$$

$w_1 + 2w_2 + 0w_3' + 0w_3'' \geq 3$

$0w_1 - 2w_2 - 7w_3' + 7w_3'' \geq -6$

$w_1 - 3w_2 + 5w_3' - 5w_3'' \geq 5$

$-w_1 + 3w_2 - 5w_3' + 5w_3'' \geq -5$

w_3 is unristructed while $w_1, w_2 \geq 0$ 4

$\longleftrightarrow$

$w_1 + 2w_2 + 0w_3 \geq 3$

$0w_1 - 2w_2 - 7(w_3) \geq -6$

$\left.\begin{array}{l} w_1 - 3w_2 + 5(w_3' - w_3'') \geq 5 \\ -w_1 + 3w_2 - 5(w_3' - w_3'') \geq -5 \end{array}\right\}$

this gives

$\left.\begin{array}{l} w_1 + 2w_2 + 0w_3 \geq 3 \\ 0w_1 + 2w_2 + 7w_3 \leq 6 \\ w_1 - 3w_3 + 5w_3 = 5 \end{array}\right\}$

$w_1, w_2 \geq 0$ and w_3 is unrestricted.

Q-7 A company uses annually 24000 units of raw material which cost rs. 1.25/unit. Placing each order cost rs 22.5 and carrying cost is rs 5.4%/yr. of the avg. inventory. find the economic lot size & total inventory cost including the cost of material.

Sol. :— It is given $r = 24000$ units/yr.

$C_1 = 5.4\%$ of avg I/P per year

$$C_1 = \frac{5.4}{100} \times 1.25 = 0.675 \text{ units/yr.}$$

$C_3 = $ Rs 22.5

$$q^* = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 22.5 \times 24000}{0.675}}$$

$= 4000$ units

$$t^* = \frac{q^*}{r} = \frac{4000}{24000} = \frac{1}{6} \times 12 = 2 \text{ month}$$

Total inventory cost will be

$= \sqrt{2C_3C_1R} +$ purchasing cost per year

$= \sqrt{2C_3C_1R} + (1.25) \times 24000$

$= \sqrt{2 \times 0.675 \times 22.5 * 24000} + (1.25) \times 24000$

$= 270 + 30000$

$= 30270$ rs.

total Inventory Cost. $= 30270$

Question: Solve the following LPP by simplex method

$$\max z = 6x_1 + 8x_2$$
$$\text{s.t. } 5x_1 + 10x_2 \le 60$$
$$4x_1 + 4x_2 \le 40$$
$$x_1, x_2 \geq 0$$

Solution: Step 1: Problem is already in maximization.

Step 2: All b_i's are already positive.

Step 3. Change inequality into equality by using slack/surplus variable:

$$5x_1 + 10x_2 + x_3 = 60$$
$$4x_1 + 4x_2 + x_4 = 40$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Step 4: Write equation in matrix form

$$AX = B$$

$$\begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array}$$
$$\begin{bmatrix} 5 & 10 & 1 & 0 \\ 4 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

Step 5: y_3 and y_4 are the element in initial basis

$$\max z = 6x_1 + 8x_2 + 0x_3 + 0x_4$$

Step 6: Construct the simplex table as—

		Δj	6	8	0	0	
B	C_B	X_B	y_1	y_2	y_3	y_4	min Ratio X_B/y_k (y_2)
y_3	0	60	5	$\boxed{10}$	1	0	6 →
y_4	0	40	4	4	0	1	10
		ΔJ	6	8 ↑	0	0	
y_2	8	6	1/2	1	1/10	0	12
y_4	0	16	$\boxed{2}$	0	-2/5	1	8 →
		ΔJ	1.5 ↑	0	-4/5	0	
y_2	8	2	0	1	1/5	-1/4	
y_1	6	8	1	0	-1/5	1/2	
		ΔJ	0	0	-1/2	-15/20	

$y_2 = 2 = x_2$

$y_1 = 8 = x_1$

$\max Z = 6x_1 + 8x_2 + 0x_3 + 0x_4$

$\Rightarrow 6 \times 8 + 8 \times 2$

$\Rightarrow 48 + 16$

$\Rightarrow 64$ Ans

Ques 9 - Prove that the Dual of the Dual is the primal itself.

Solution:- Let a Linear Programming problem, where

$$\left.\begin{aligned} &\text{Max } z = Cx \\ &\text{St} \quad Ax \leq b \\ &\quad x \geq 0 \end{aligned}\right\} \quad \text{---} (1)$$

Dual of the given LPP is

$$\left.\begin{aligned} &\text{Min } z_D = b'w \\ &\text{St} \quad A'w \geq C' \\ &\quad w \geq 0 \end{aligned}\right\} \quad \text{—} (2)$$

Now again the dual of the given eq is

$$\left.\begin{aligned} &\text{Max } z = (C')'v \\ &\text{St} \quad (A')'v \leq (b')' \\ &\quad v \geq 0 \end{aligned}\right\} \quad \text{—} (3)$$

It can be written as

$$\left.\begin{aligned} &\text{Max } z = Cv \\ &\text{St} \quad Av \leq b \\ &\quad v \geq 0 \end{aligned}\right\} \quad \text{—} (4)$$

Equation (1) and (4) Identical. Hence after dual of a dual is a primal itself.

This proves the theoram.

Q.10 Using dual simplex method, solve the following (10)
L.P.P.

$$\text{Min } Z = 3x_1 + x_2$$
$$\text{st } x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$
$$\text{and } x_1, x_2 \geq 0$$

Ans:- 1. Convert problem into Maximization.

$$\text{Max } Z' = -3x_1 - x_2$$
$$\text{st. } -x_1 - x_2 \leq -1$$
$$-2x_1 - 3x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

2. Change inequality into equality

$$-x_1 - x_2 + x_3 = -1$$
$$-2x_1 - 3x_2 + x_4 = -2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

3. Write the Matrix form $AX = B$

$$\begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array}$$
$$\begin{bmatrix} -1 & -1 & 1 & 0 \\ -2 & -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

y_3 and y_4 are the elements in initial bases.

then $\text{Max } Z' = -3x_1 - x_2 + 0x_3 + 0x_4$

4. Construct simplex table as

B	C_B	X_B	y_1	y_2	y_3	y_4	
y_3	0	-1	-1	-1	1	0	
y_4	0	-2	-2	-3	0	1	→
		$\Delta j =$	-3	-1	0	0	

To determine leaving vector

$$\text{Min } [X_{Bi}] = \text{Min } [-1, -2] = -2$$

(11)

Entering vector $\min\left[\frac{-3}{-3}, \frac{-1}{-3}, 0, 0\right] = \frac{1}{3}$

y_3 is incoming and y_4 is outgoing.

B	C_B	X_B	y_1	y_2	y_3	y_4
y_3	0	$-1/3$	$-1/3$	0	1	$\boxed{-1/3}$
y_2	-1	$2/3$	$2/3$	1	0	$-1/3$
		$\Delta j =$	$-7/3$	0	0	$-1/3$

To determine leaving vector $-\min\left(\frac{-1}{3}, \frac{2}{3}\right) = \frac{-1}{3}$

Entering vector $-\min\left[\frac{\frac{-7}{3}}{-1/3}, \frac{-1/3}{-1/3}\right] = 1$

y_4 is entering vector & y_3 is leaving.

B	C_B	X_B	y_1	y_2	y_3	y_4
y_4	0	1	1	0	-3	1
y_2	-1	1	1	1	-1	0
		$\Delta j =$	-2	0	-1	0

All $x_{Bi} \geq 0$, soln is optimal.

$\therefore$ $y_4 = 1 = x_4$

$y_2 = 1 = x_2$ $\quad x_1 = 0$

$\text{Max } Z' = -3x_1 - x_2 + 0x_3 + 0x_4$

$= -3 \times 0 - 1 + 0 + 0$

$= -1$

$\text{Min } Z = 1$

Sec C

C. Attempt all the parts

8

Q.11 Solve the following LPP by using Big M method:- (12)

$$\text{Max } Z = 5x_1 + 6x_2$$
$$\text{st. } 2x_1 + 5x_2 \le 1500$$
$$3x_1 + x_2 \ge 1200$$
$$x_1, x_2 \ge 0.$$

Soln:- 1. Problem is already for maximisation.

2. All b_i's are +ve.

3. change inequality into equality by adding slack & surplus variable.

$$2x_1 + 5x_2 + x_3 = 1500$$
$$3x_1 + x_2 - x_4 = 1200$$
$$x_1, x_2, x_3, x_4 \ge 0$$

4. Convert Matrix form $AX = B$

$$\begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & A_1 \end{array}$$
$$\begin{bmatrix} 2 & 5 & 1 & 0 & 0 \\ 3 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1200 \end{bmatrix}$$

y_3 and A_1 are on initial basis.

$$\text{Max } Z = 5x_1 + 6x_2 + 0x_3 + 0x_4 + (-MA_1)$$

5. Construct Simplex table -

B	C_B	X_B	y_1	y_2	y_3	y_4	A_1	M.R.
x_3	0	1500	2	5	1	0	0	750
A_1	-M	1200	$\boxed{3}$	1	0	-1	1	400 →
		$\Delta_j =$	5+3M ↑	6+M	0	-M	0	

A_1 is leaving vector & y_1 is entering vector.

B	C_B	X_B	y_1	y_2	y_3	y_4	A_1	M.R.
y_3	0	700	0	$\boxed{13/3}$	1	2/3	-2/3	2100/13 →
y_1	5	400	1	1/3	0	-1/3	1/3	1200
		$\Delta j =$	0	13/3 ↑	0	5/3	$\frac{-5-3M}{3}$	

y_3 is leaving vector and y_2 is entering.

B	C_B	X_B	y_1	y_2	y_3	y_4	M.R.
y_2	6	2100/13	0	1	3/13	$\boxed{2/13}$	4550 →
y_1	5	4500/13	1	0	-1/13	-15/39	-ve
		$\Delta j =$	0	0	-1	1 ↑	

y_2 is leaving vector y_4 is incoming.

B	C_B	X_B	y_1	y_2	y_3	y_4	M.R.
y_4	0	4050	0	13/2	3/2	1	
y_1	5	750	1	5/2	-17/26	0	
		$\Delta j =$	0	-13/2	-5/2	0	

$y_4 = 4050 = x_4$

$y_1 = 750 = x_1$

$\text{Max } z = 5x_1 + 6x_2 + 0x_3 + 0x_4$

$= 5 \times 750$

$= 3750$

Ques:-12. let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every year whereas machine B is replaced after every six year, determine which machine should be purchased.

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Year	Machine A	Machine B
1	1000	1700
2	200	100
3	400	200
4	—	300
5	—	400
6	—	500

Here $v = \frac{100}{100+10} = \frac{100}{110} = \frac{10}{11}$

Total expenditure of Machine A in 3 year

$\Rightarrow 1000 + 200v + 400v^2$

$\Rightarrow 1000 + 200 \times \frac{10}{11} + 400 \times \left(\frac{10}{11}\right)^2$

$\Rightarrow 1512$ (approx)

Average of Machine A in 3 years

$= 1512/3 = 504$ Rs.

For Machine B

Total expenditure of Machine B in 6 years

$= 1700 + 100v + 200v^2 + 300v^3 + 400v^4 + 500v^5$

$= 2765$ (approx)

(15) Average cost of Machine B $= \frac{2765}{6} = 460.83$

Machine B looks like less costly then Machine A but we shall calculate our expenditure of Machine A of 6 years.

Now, expenditure of Machine A in 6 years.

$= 1000 + 200v + 400v^2 + 1000v^3 + 200v^4 + 400v^5$

$= 1000 + 200 \times \frac{10}{11} + 400\left(\frac{10}{11}\right)^2 + 1000\left(\frac{10}{11}\right)^3 + 200\left(\frac{10}{11}\right)^4 + 400 \times \left(\frac{10}{11}\right)^5$

$= 2648$ (approx)

Avg. cost $= \frac{2648}{6} = 441.30$

Total expenditure in 6 years of Machine A is less than Machine B, thus the Machine A is less costly, Machine A should be purchased by B.

$\longleftrightarrow$