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Department of ECE

Model Solution SESSIONAL TEST-2

Course:	B.Tech	Semester:	VII
Session:	2017-18	Section:	EI-K
Subject:	Control System -II	Sub. Code:	NIC-701
Max Marks: 50		Time:	2 hour

Section-A

Ques1) State Cayley Hamilton theorem.

Cayley Hamilton theorem is used for matrix analysis which is extremely versatile and useful. It states that every square matrix F satisfies its own characteristics.

Ques2) Explain pulse transfer function.

Ratio of Laplace xform of output to input taking all initial condition zero is known as xfer function.

$$G(s) = \frac{C(s)}{R(s)}$$

The same xfer function is known as pulse xfer function if the input and output is simulated by the ideal sampler.

Ques3) What are the methods of realization of pulse xfer function.

Basically there are three methods of realization of pulse xfer function

i) Direct Realization

ii) Cascade Realization

iii) Parallel realization

Ques 4) What do you mean by Complete state Controllability and Complete output Controllability:

The system is said to Controllable if for any state $x(0) = x^0$ and any other state x^1 there exists a finite positive integer N and input $u(K)$; $K \in [0, N-1]$ that will transfer the state x^0 to the state x^1 at $K = N$

The system is said to be observable if any state x^0 at $K=0$ there exists a finite positive integer N such that the knowledge of the input $u(K)$; $K \in [0, N-1]$ and the output $y(K)$. $K \in [0, N-1]$ is used to determine the state x^0.

Ques 5) Explain Angle and magnitude Condition in root locus

Angle Condition is used for checking whether certain points lie on root locus or not.

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1 + j0 = \pm [2q+1] 180^\circ$$

Magnitude Condition is used for finding the value of system gain at any point on root locus

$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2} = 1$$

Section B

Ques 6) Check the stability for given data system using Liapunov method.

$$x_1(K+1) = -0.5\,x_1(K)$$
$$x_2(K+1) = -0.5\,x_2(K)$$

Let us assign Liapunov function $V(x) = x_1^2(K) + x_2^2(K)$ which is positive for all values of $x_1(K)$ and $x_2(K)$ not equal to zero.

$$\Delta V(x) = V[x(K+1)] - V[x(K)]$$
$$= x_1^2(K+1) + x_2^2(K+1) - x_1^2(K) - x_2^2(K)$$
$$= -0.75\,x_1^2(K) - 0.75\,x_2^2(K)$$

Since $\Delta V(x)$ is negative for all $x(k) \neq 0$, the system is asymptotically stable.

Ques 7) Find inverse Z Xform and check stability for matrix A of state equation.

$$A = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$X(z) = (zI-A)^{-1} z\, x(0)$$

$$(zI-A)^{-1} = \frac{\mathrm{Adj}\,(zI-A)}{|zI-A|}$$

$$X(z) = \frac{\begin{bmatrix} z-0.5 & 0 \\ 0 & z+0.5 \end{bmatrix}}{(z+0.5)(z-0.5)}\, z\, X(0)$$

$$= \begin{bmatrix} \frac{z}{z+0.5} & 0 \\ 0 & \frac{z}{z-0.5} \end{bmatrix} X(0)$$

Inverse Z xform of $X(z)$ is

$$x(K) = \begin{bmatrix} +(0.5)^K & 0 \\ 0 & (0.5)^K \end{bmatrix} x(0)$$

The characteristics equation of system is $|zI-A|=0$

$$(z-0.5)(z+0.5)=0$$

Roots are $z=0.5, -0.5$ These roots are inside unit circle.
Hence system is stable.

Ques- State and prove Controllability test.

The discrete time system is Controllable if and only if rank $(n \times np)$ Controllability matrix U;

$$U = [G|FG|F^2G| \text{———} F^{n-1}G] \text{ is } n \quad \rho(U)=n$$

Proof - $x(K+1) = Fx(K) + GU(K)$

By successive substitution we get

$$x(1) = Fx(0) + GU(0)$$
$$x(2) = Fx(1) + GU(1)$$
$$= F^2x(0) + FGU(0) + GU(1)$$
$$|$$
$$x(N-1) = F^{N-1}x(0) + F^{N-2}GU(0) + \text{———} + FGU(N-3) + GU(N-2)$$
$$x(N) = F^N x(0) + F^{N-1}GU(0) + \text{———} + FGU(N-2) + GU(N-1)$$
$$x^1 - F^N x(0) = F^{N-1}GU(0) + \text{———} + FGU(N-2) + GU(N-1)$$
$$\hat{x} = [G|FG| \text{———} |F^{N-1}G] \begin{bmatrix} U(N-1) \\ | \\ U(0) \end{bmatrix}$$

$$\hat{x} = [U_N]\,[U]$$

To satisfy for x^0 and x^1 it is necessary that $\rho(U_N) = \rho(U_N|\hat{x})$

A state can be transferred to some other state in atmost n steps if and only if

$$\rho(U) = \rho[G|FG| \text{———} F^{N-1}G] = n$$

The theorem gives not only necessary and sufficient conditions for Controllability but also a method to compute the Control.

Ques 9) Evaluate $f(F) = F^k$ for $F = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$

i) find the eigen values for F $\lambda_1, \lambda_2 = -1$

ii) since F is second order polynomial $g(\lambda)$ will be of the form $g(\lambda) = \beta_0 + \beta_1 \lambda$

$$f(\lambda_1) = g(\lambda_1) \longleftrightarrow (-1)^k = \beta_0 - \beta_1$$

$$\frac{d}{d\lambda} f(\lambda)\Big|_{\lambda_2=\lambda_1} = \frac{d\,g(\lambda)}{d\lambda}\Big|_{\lambda_2=\lambda_1} = \beta_1$$

$$\frac{d}{d\lambda}\lambda^k\Big|_{\lambda_2=\lambda_1} = K(-1)^{K-1} = \beta_1$$

The result is $\beta_0 = (-1)^K + \beta_1$

$$= (-1)^K + K(-1)^{K-1} = (1-K)(-1)^K$$

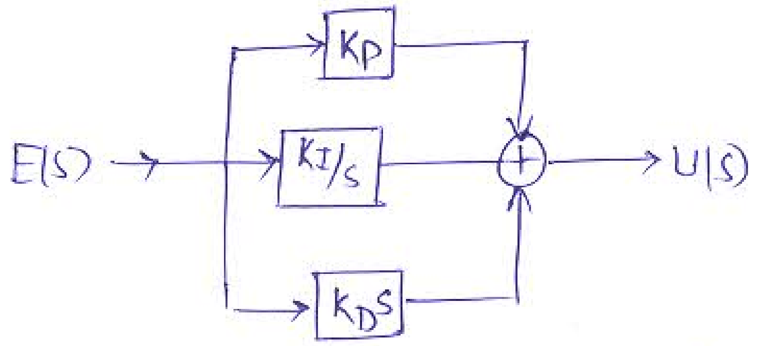
$$\beta_1 = -K(-1)^K$$

$$f(F) = F^K = \beta_0 I + \beta_1 F$$

$$= (1-K)(-1)^K I - K(-1)^K F$$

$$= (-1)^K \begin{bmatrix} 1-K & -K \\ K & 1+K \end{bmatrix}$$

Ques 10) Explain PID Controller.



The transfer function of the Controller is $\frac{U(S)}{E(S)} = D(S) = K_P + \frac{K_I}{S} + K_D S$

Transfer function of PI Controller is

$$D(S) = K_P + \frac{K_I}{S} = \frac{K_P S + K_I}{S}$$

$$= K_I \frac{1+S\tau}{S} \quad ; \quad \tau = \frac{K_P}{K_I}$$

This is phase lag Compensator with pole placed at $s=0$

Transfer function of PD Controller $D(S) = K_P + K_D S$

$$= K_P[1+\tau S] \qquad \tau = \frac{K_D}{K_P}$$

This is phase Lead Compensator with pole placed at $s=\infty$

let $m(t)$ be the integral of $e(t)$ then the value of integral at $t=(K+1)T$ is equal to the value at KT plus the area added from KT to $(K+1)T$.

$$m[(K+1)T] = m(KT) + \int_{KT}^{(K+1)T} e(\tau)\, d\tau$$

$$m(K+1)T = m(KT) + \frac{T}{2}\{e(K+1)T + e(KT)\}$$

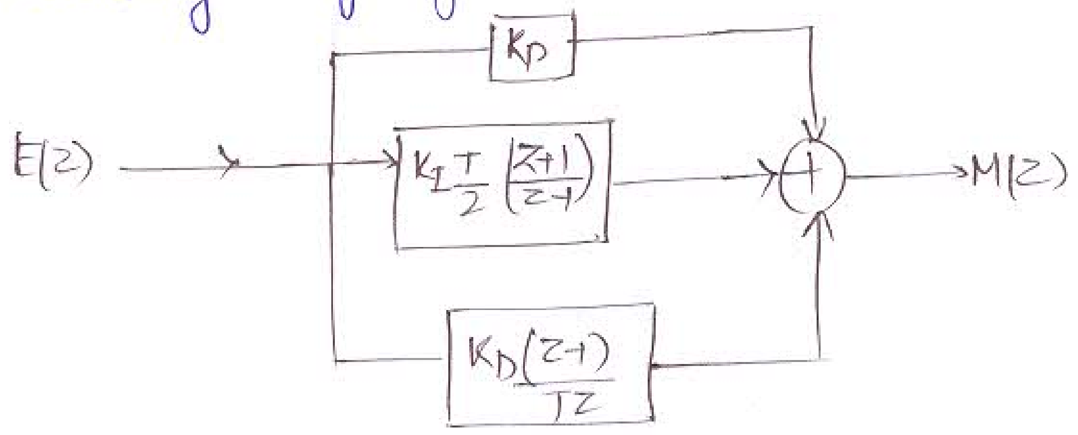
Taking z xform we get

$$\frac{M(z)}{E(z)} = \frac{T}{2}\left[\frac{z+1}{z-1}\right]$$

If derivative of e(t) at $t=KT$ is $m(KT)$ then

$$\frac{M(z)}{E(z)} = \frac{(z-1)}{Tz}$$

The block digram of digital PID Controller is



Section C

Ques 11) Explain Construction rules of root locus.

Rule① Root locus is symmetrical about real axis

$$G(s)H(s) = -1$$

Rule② Let P = No. of open loop poles
Z = No. of open loop zero
P>Z then no. of branches of root locus = P
No. of branches terminating at zeros = Z
No. of branches terminating at infinity = P-Z

Rule③ The point on real axis is said to be on root locus if the sum of open loop poles and zeros to right side of that point is ODD.

Rule④ Angle of asymptote $\theta = \frac{(2q+1)180^\circ}{P-Z}$

Rule⑤ Centroid $\sigma = \frac{\Sigma P - \Sigma Z}{P-Z}$

Rule⑥ Break away/ Break inn point – They are those points where multiple roots of char. eq^n occur:

i) Construct $1+G(s)H(s)=0$

ii) write K in terms of 's'

iii) find $\frac{dk}{ds}=0$

iv) The roots of $\frac{dk}{ds}=0$ will give B.A / B.inn points

v) To test valid B.A / B.inn point substitute in (11) if K is positive then valid B.A / B.inn point.

Rule (7) Roots of Aux equation $A(s)$ at $K = K_{mar}$ from Routh array criteria determines the intersection of root locus with imaginary axis.

Rule (8) Angle of departure / Angle of arrival.

Angle of departure is obtained for complex poles terminating at ∞

$$\phi_D = 180^\circ + \phi$$

$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

Angle of arrival is obtained at complex zero

$$\phi_A = 180^\circ - \phi$$

$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

Ques 12) State and prove Liapunov stability theorem for linear digital system.

Consider a LTI digital system described by difference equation

$$x(K+1) = Ax(K)$$

where $x(K)$ is $n\times 1$, A is $n\times n$ matrix. The equilibrium state $x_e=0$ is asymptotically stable if and only if, give any positive definite real matrix Q, there exists a positive definite real symmetric matrix P such that $A'PA - P = -Q$

then $V(x) = x'(K)Px(K)$ is a Liapunov function for system and further $\Delta V(x) = -x'(K)Qx(K)$

where $\Delta V(x)$ is defined as $\Delta V[x] = V[x(K+1)] - V[x(K)]$

Proof - According to Sylvester's theorem if P is +ve definite matrix then $V(x) = x'Px$ is positive definite

Liapunov function $\Delta V(x) = V[x(K+1)] - V[x(K)]$

$$= x'(K+1)Px(K+1) - x'(K)Px(K)$$

$$= A'x'(K)\ PAx(K) - x'(K)Px(K)$$

$$= x'(K)\left[A'PA - P\right]x(K)$$

$$= -x'(K)Qx(K)$$

Thus if $\Delta V(x)$ is to be negative definite, Q has to be positive definite.