Ajay Kumar Garg Engineering College, Ghaziabad
Department of AS & Hum.
Sessional Test-2
SOLUTION

Course: B.Tech
Session: 2017-18
Subject: Eng. Physics-I
Max. Marks: 50

Semester: I
Sections: CS-1,2,3, EC-1,2,3, ME-1,2,3, EN-1,2, IT-1,2, CE-1,2, EI
Sub. Code: RAS-101
Time: 2 hour

Section-A

A. Attempt all parts. (5x2=10)

1. Calculate the wavelength associated with (i) 1 MeV photon (ii) 1 MeV electron

for photon, rest mass is zero. The energy of photon

$E = \frac{hc}{\lambda}$, $\lambda = \frac{hc}{E}$, $\lambda = \frac{6.64\times10^{-34}\times3\times10^{8}}{1\times10^{6}\times1.6\times10^{-19}}$

$\boxed{\lambda = 1.24\times10^{-2}\ \text{Å}}$

for electron, rest mass energy $m_oc^2 = 9.1\times10^{-31}\times(3\times10^{8})^2$

$m_oc^2 = 81.9\times10^{-15}\,J$, $m_oc^2 = \frac{81.9\times10^{-15}}{1.6\times10^{-19}}$,

$m_oc^2 = 0.51\,MeV$

$E = K + m_oc^2$, $K = 1\,MeV$, $E = K + m_oc^2$

$\lambda = \frac{h}{\sqrt{2m(K+m_oc^2)}} = 8.75\times10^{-13}\,m.$ $\boxed{= 8.75\times10^{-3}\ \text{Å}}$

(2.) Derive the relation between group velocity and phase velocity in dispersive medium.

Ans. Group velocity $V_g = \frac{d\omega}{dk}$, and $\omega = k v_p$

$$V_g = \frac{d}{dk}(k v_p) = v_p + k\frac{dv_p}{dk} \quad , k = \frac{2\pi}{\lambda}, \; dk = -\frac{2\pi}{\lambda^2} d\lambda$$

$$V_g = v_p + \frac{2\pi}{\lambda} \frac{dv_p}{\left(-\frac{2\pi}{\lambda^2}\right) d\lambda}$$

$$\boxed{v_g = v_p - \lambda \frac{dv_p}{d\lambda}} \rightarrow \text{dispersive medium.}$$

(3.) Show that ultra thin film appears dark in reflected light.

Ans. The ultra thin film appears dark in reflected because the path difference $2t + \frac{\lambda}{2} \rightarrow$ reduces to only $\frac{\lambda}{2}$ as t becomes extremely small.

(4.) Write down the two differences between matter waves and electromagnetic waves.

Ans. The difference between matter waves and electromagnetic waves is

- electromagnetic waves are generated by charged particle, whereas deBroglie matter waves are uncharged particles.
- The electromagnetic waves have a constant velocity, whereas, de Broglie matter waves travel with velocity depending on the medium.

(5.) Define coherent and incoherent light sources. (3)

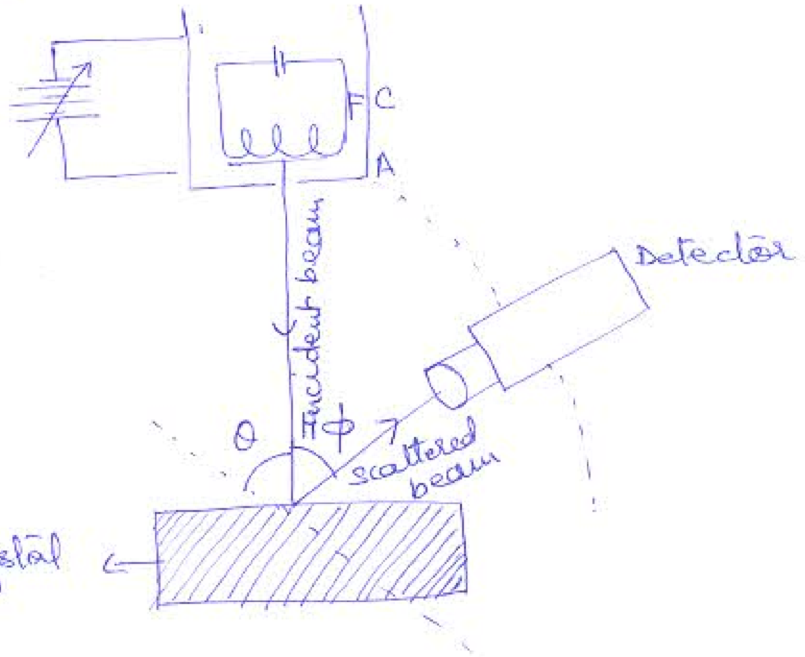
Ans. Two sources are said to be coherent if they emit light which have always a constant phase difference between them.
The sources of light which emit beams whose phase change with time in a random way, are known as Incoherent sources.

Section - B

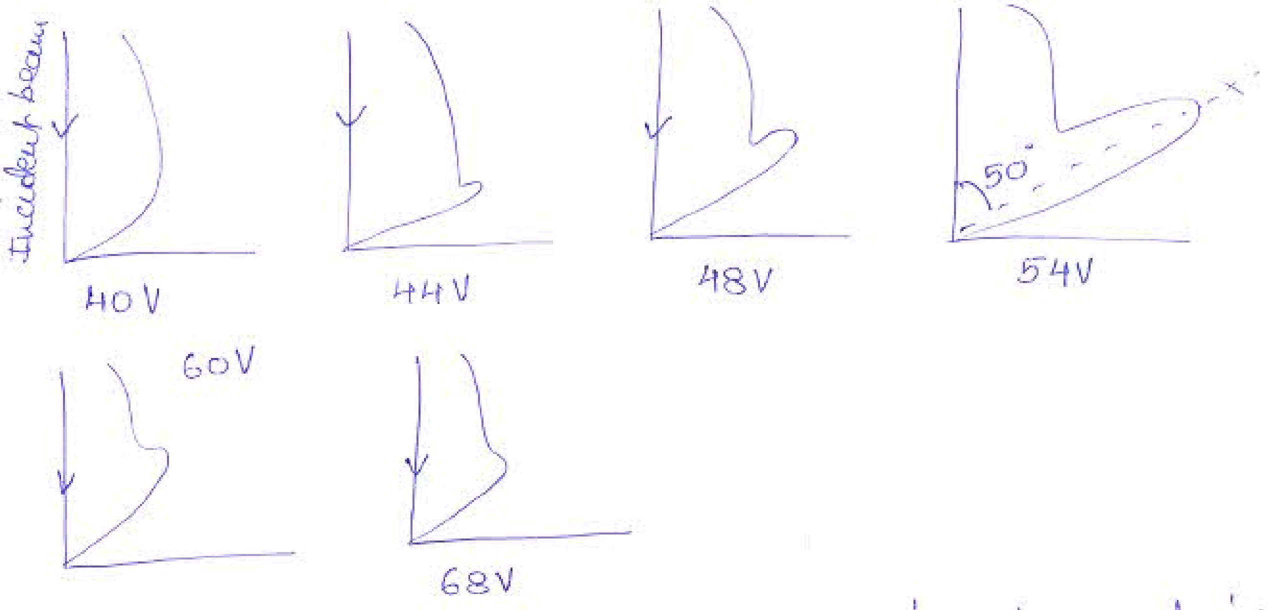
B. Attempt all parts. (5x5=25)

(6.) Describe construction and findings of the Davisson and Germer's experiment.

Ans. In 1927, Clinton Davisson and Lester Germer conducted an experiment of diffraction electrons to confirm deBroglie's hypothesis.



A beam of electrons is incident on the crystal of nickel. The scattered beam is collected on the detector which can slide on a cylinder so that scattering from all possible directions can be collected. The scattered beam and the scattering angle are measured and studied in terms of graphs.



Amongst these graphs, the prominent peak is observed for $\phi = 50°$ and $V = 54$ volts. This shows the waves are associated with electrons, which interfere constructively.

By diffraction $2d\sin\theta = n\lambda$

for nickel, $n=1$, $d = 0.91°A$, $\theta = 65°$

$$\boxed{\lambda = 1.65°A}$$

by de Broglie method $\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times54}}$

$$\boxed{\lambda = 1.66°A}$$

Thus Davisson-Germer experiment directly verifies deBroglies hypothesis.

7. explain and state the Heisenberg's uncertainty principle. Using this principle prove non existence of electron inside nucleus.

Ans. In 1927, German physicist Werner Hesienberg stated a very important principle, known as uncertainty principle. It states,

It is impossible to determine the exact position and momentum of a particle simultaneously.

$$\Delta x \cdot \Delta p \geq \hbar \left(= \frac{h}{2\pi}\right)$$

Δx is the uncertainty of position and Δp is the uncertainty of momentum.

<u>Non-existence of electrons in the Nucleus</u>. The size of nucleus is of the order $\sim 10^{-14}$ m. If the electron exists inside the nucleus, the uncertainty in the position of nucleus electron is $\Delta x = 2 \times 10^{-14}$ m

The uncertainty in momentum $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}}$

$$\Delta p \approx 5.275 \times 10^{-21} \text{ kg-m/s.}$$

The energy of electron relativistically is given by

$$E^2 = p^2c^2 + m_0^2c^4$$

$\because$ $m_0c^2 = 0.511$ MeV, (much small)

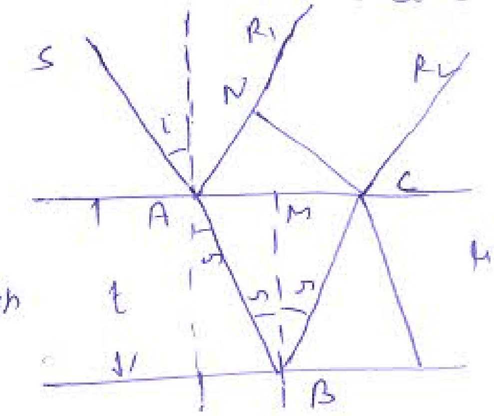
$$\therefore E = pc$$
$$= (5.275) \times 10^{-21} \times 3 \times 10^{8}$$

$$\boxed{E = 10 \text{ MeV}}$$

But experimentally electrons emitted during β-decay have energies 3-4 MeV. Therefore, it is not possible for electrons to be present inside the nucleus.

Q-8(i) Derive the condition for dark bands in reflected light for thin films of uniform thickness.

Ans-8 (i) Interference in thin films for reflected light



When a film of oil spread over the surface of water, or a thin glass plate, is illuminated by light, interference occurs between the light waves reflected the film and also between the light waves transmitted through the film.

Let CN and BM be perpendiculars to AR_1 and AC. As the paths of the rays AR_1 and CR_2 beyond CN are equal the path difference between them is

$$P = \mu(AB+BC) - AN \quad --(1)$$

Now $AB = BC = \dfrac{BM}{\cos r} = \dfrac{t}{\cos r}$

and $AN = AC\sin i = (AM+MC)\sin i$

$AN = (BM\tan r + BM\tan r)\sin i$

$= 2t\tan r \sin i$

$= 2t\dfrac{\sin r}{\cos r}\sin i = 2t\dfrac{\sin^2 r}{\cos r}\dfrac{\sin i}{\sin r} = \dfrac{2\mu t\sin^2 r}{\cos r}$

on substituting these values of AB, BC and AN in (1) we have

$$P = \mu\left(\frac{t}{\cos r}+\frac{t}{\cos r}\right) - \frac{2\mu t\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1-\sin^2 r) = 2\mu t\cos r$$

The ray AR_1 undergoes additional path difference of $\lambda/2$ since it is reflected from the surface of a denser medium

$\therefore$ effective path difference would be

$$2\mu t\cos r - \lambda/2$$

Condition for minima $2\mu t\cos r - \lambda/2 = (2n-1)\lambda/2$

or $2\mu t\cos r = n\lambda$

$n = 0, 1, 2, 3 - - - -$

(7)

Q 8 (ii) white light is incident on a soap film at an angle $\sin^{-1}(4/5)$ and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands corresponds to wavelengths 6.1×10^{-5} cm and 6.0×10^{-5} cm. If the refractive index of film is $4/3$. Calculate the thickness of the film

Ans 8 (ii). we know that the condition for dark band or fringe in the reflected light is

$$2\mu t \cos r = n\lambda$$

If n and $(n+1)$ are the orders of consecutive dark bands for wavelengths λ_1 and λ_2 respectively then

$$2\mu t \cos r = n\lambda_1 = (n+1)\lambda_2.$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$2\mu t \cos r = \frac{\lambda_2 \lambda_1}{\lambda_1 - \lambda_2}$$

$$t = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \frac{1}{2\mu \cos r}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2}$$

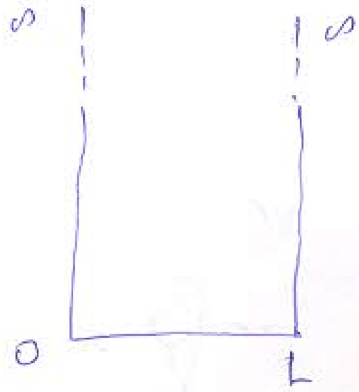
on solving $\cos r = 4/5$

$$\therefore t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5} \times 3 \times 5}{0.1 \times 10^{-5} \times 2 \times 4 \times 4}$$

$$t = .0017 \text{ cm}$$

Q. 9 Find out the energy eigen values and eigen wave functions of a particle enclosed in one dimensional potential box of infinite height.

Sol. 9. Let us consider a particle trapped in infinite potential well.



$V = 0 \qquad 0 < x < L$

$V = \infty \qquad 0 < x$ and $x > L$

The schroedinger eqn for the particle within the box $(V=0)$ is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0$$

Let us suppose $\frac{2mE}{\hbar^2} = k^2$

$$\therefore \frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The General solution of the above eqn is

$$\psi = A\sin kx + B\cos kx$$

on applying the boundary condition we get

$$(\psi)_{x=0} = 0 \qquad 0 = A\sin 0 + B\cos 0$$

$$\Rightarrow B = 0$$

$$\psi = A\sin kx$$

on applying the second boundary condition we have

$$(\psi)_{x=L} \qquad 0 = A\sin kL$$

$$\text{or} \quad \sin kL = \sin n\pi$$

$$kL = \pm n\pi \qquad k = \frac{n\pi}{L}$$

$$\psi = A\sin\frac{n\pi}{L}x.$$

Now by applying the condition of normalization we have

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

$$A^2\int_0^L \sin^2\frac{n\pi x}{L}\, dx = 1$$

$$\text{or } A^2\int_0^L \left(\frac{1-\cos 2\frac{n\pi x}{L}}{2}\right) dx = 1$$

or solving $A = \sqrt{\frac{2}{L}}$

$$\therefore \boxed{\psi = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$

Now $k = \frac{n\pi}{L}$

$\therefore k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{n\pi}{L}$

$\therefore \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$

$$\boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

Q-10 Derive schrodinger's time independent wave equation. what is wavefunction, describe its Physical significance and write down the properties of wave function.

Ans-10: Schrodinger's time independent wave eq^n:

The eq^n of classical wave is given by

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \quad \text{--- (1)}$$

$$\psi(x,t) = \psi(x)\, e^{-i\omega t} \quad \text{--- (2)}$$

$$\frac{d\psi}{dt} = (-i\omega)\,\psi(x)\, e^{-i\omega t}$$

$$\frac{d^2\psi}{dt^2} = (-i\omega)^2\,\psi(x)\, e^{-i\omega t} \quad \text{--- (3)}$$

From eq^n (1) and (3) we have

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

or $$\frac{d^2\psi}{dx^2} + \frac{\omega^2}{v^2}\psi = 0$$

Now $\omega = 2\pi\nu$

$v = \nu\lambda$

$$\therefore \left(\frac{\omega}{v}\right) = \frac{2\pi\cancel{\nu}}{\cancel{\nu}\lambda} = \frac{2\pi}{\lambda}$$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Now $\lambda = h/p$ $\therefore \frac{1}{\lambda} = \frac{p}{h}$ or $\frac{1}{\lambda^2} = \frac{p^2}{h^2}$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{4\pi^2 p^2}{h^2}\psi = 0$$

$E = K + V$ $K = E - V$ or $\frac{p^2}{2m} = (E - V)$

or $p = \sqrt{2m(E-V)}$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{4\pi^2}{h^2} 2m(E-V)\psi = 0$$

$$\text{or } \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

wave function and its significance:

A wave describes the state of quantum mechanical particle, its not having physical significance directly, but when it is multiplied by its complex conjugate it gives the Probability of finding a particle. and

Total Prob $= \int_{-a}^{a} |\psi|^2 dx = \int_{-a}^{a} \psi^* \psi \, dx = 1$. which is known as condition of normalization

Properties of a wave function:

(1) ψ is always a finite number

(2) ψ is always single valued

(3) ψ is continuous

(4) $\frac{d\psi}{dx}$ is also continuous

(5) Value of ψ vanishes at the boundary.

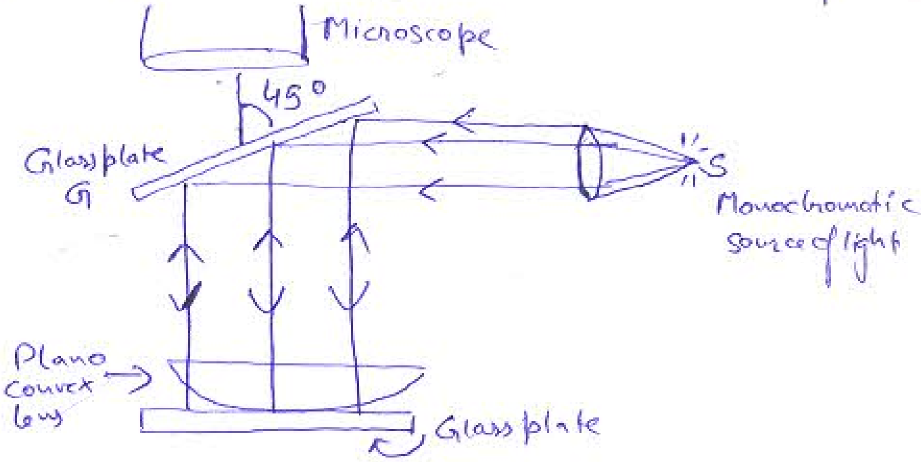
Section C

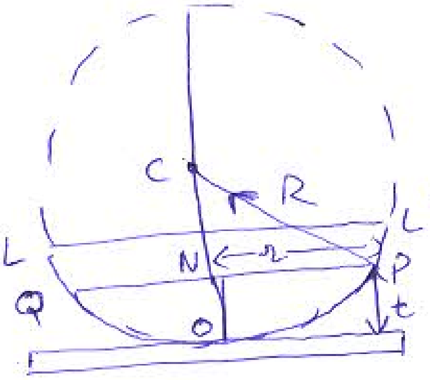
C. Attempt all parts

Q11 (1) Derive the expression for the diameter of the bright and dark ring in reflected light.

(2) In Newton's ring expt. the diameter of 4^{th} and 12^{th} ring are 0.4cm and 0.7cm in reflected light respectively. Find the diameter of 20^{th} dark ring

Ans (i)





In Newton's ring expt, let LOL' be the ~~lens~~ lens placed on the glass plate. The lens is a part of spherical surface centered at C. R be the radius of curvature and r be the radius of the ring corresponding to film thickness t.

We know

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{For bright ring}$$

$$2t = n\lambda \quad \text{For dark ring.}$$

From the Property of circle $\quad NP \times NQ = NO \times ND$

Substituting the value

$$r \times r = t(2R - t)$$

$$= 2Rt - t^2 \approx 2Rt$$

$$\therefore \; r^2 = 2Rt \text{ or } t = \frac{r^2}{2R}.$$

Diameter of Bright ring.

$$\frac{2r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

or $r = \frac{D}{2}$

$$\therefore \; \frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

or $D \propto \sqrt{2n-1}$

Diameter of dark ring

$$\frac{2r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{nR\lambda}$$

or $D \propto \sqrt{n}$.

Ans (ii) We know that

$$(D_{n+p})^2 - (D_n)^2 = 4p\lambda R.$$

$(n+p) = 12$, $n = 4$, $p = 12 - 4 = 8$.

$$\therefore \; D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R. \quad -(1)$$

Also $D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda R \quad -(2) \quad (\because p = 20 - 4 = 16)$

Dividing (2) by (1).

$$\frac{D_{20}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 16 \times \lambda \times R}{4 \times 8 \times \lambda \times R} = 2$$

$$\frac{(D_{20})^2 - 0.16}{(0.7)^2 - (0.4)^2} = 12 .$$

Which gives

$$(D_{20})^2 - 0.16 = 0.66$$

$$\text{or } (D_{20})^2 = 0.82$$

$$D_{20} = 0.906 \text{ cm} .$$

Q 12 (i) Derive Planck's law of radiation. Show that Rayleigh Jeans law and Wien's law are special cases of Planck's radiation law.

(ii) Calculate the average energy of Planck's oscillator having temperature 300K and wavelength 10^{-6} mts.

Ans(i)

Let N be total no. of Planck's oscillators and E be their total energy

$$\overline{\epsilon} = \frac{E}{N}$$

$N_0, N_1, N_2 \ldots$ are no. of oscillators having energy $0, \epsilon, 2\epsilon, \ldots$

$$N = N_0 + N_1 + N_2 + \ldots$$

$$E = 0 + \epsilon N_1 + 2\epsilon N_2 + 3\epsilon N_3 + \ldots$$

Now $N_n = N_0 \exp(-n\epsilon/kT)$

$$\therefore \quad N = N_0 + N_0 \exp\left(\frac{-\epsilon}{k_BT}\right) + N_0 \exp\left(\frac{-2\epsilon}{k_BT}\right) + \ldots$$

$$= N_0\left(1 + \exp\left(\frac{-\epsilon}{k_BT}\right) + \exp\left(\frac{-2\epsilon}{k_BT}\right) + \ldots\right)$$

$$= \frac{N_0}{1 - \exp\left(\frac{\epsilon}{k_BT}\right)} \qquad \left(\because 1 + x + x^2 + x^3 = \frac{1}{1-x}\right)$$

Also

$$E = N_0 \times 0 + \epsilon N_0 \exp\left(\frac{-\epsilon}{k_BT}\right) + 2\epsilon N_0 \exp\left(\frac{-2\epsilon}{k_BT}\right) + \ldots$$

$$= N_0 \epsilon \exp\left(\frac{-\epsilon}{k_BT}\right)\left[1 + 2\exp\left(\frac{-\epsilon}{k_BT}\right) + 3\exp\left(\frac{-2\epsilon}{k_BT}\right) + \ldots\right]$$

$$= N_0 \epsilon \exp\left(\frac{-\epsilon}{k_B T}\right) \frac{1}{1-\exp\left(-\frac{\epsilon}{k_B T}\right)}$$

$$(\because \ 1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}$$

$\therefore$ Average energy $= \frac{E}{N} = \frac{\epsilon}{\{\exp(\epsilon/kT) - 1\}}$

$$\bar{\epsilon} = \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Now we know
$$f = \frac{8\pi \, d\lambda}{\lambda^4}.$$

$$\therefore E_\lambda d\lambda = \frac{8\pi \, d\lambda}{\lambda^4} \times \left[\frac{h\nu}{e^{h\nu/k_B T} - 1}\right]$$

$$\therefore E_\lambda = \frac{8\pi hc}{\lambda^5\left(\exp\left(\frac{h\nu}{k_B T}\right) - 1\right)}$$

— The above formula is called Planck's radiation law.

Derivation of Wien's law.

For shorter wavelength
$$\exp\left(\frac{h\nu}{k_B T}\right) >> 1.$$

or $\exp\left(\frac{h\nu}{k_B T}\right) - 1 = \exp\left(\frac{h\nu}{k_B T}\right)$

$$\therefore E_\lambda = \frac{8\pi hc}{\lambda^5 \left(\exp\left(\frac{h\nu}{k_B T}\right)\right)}$$

Derivation of Rayleigh Jeans law.

$$\exp\left(\frac{hc}{\lambda k_B T}\right) = 1 + \frac{hc}{\lambda k T} + \frac{h^2 c^2}{\lambda^2 k^2 T^2} + \dots = 1 + \frac{hc}{\lambda k T}$$

For longer wavelengths

$$\therefore E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left(1+\frac{hc}{\lambda kT}-1\right)}$$

$$= \frac{8\pi hc \times \lambda kT}{\lambda^5 hc} d\lambda$$

$$\text{or } E_\lambda = \frac{8\pi kT}{\lambda^4}$$

This is Rayleigh Jeans law.

(ii) Average energy of an oscillator

$$\bar{E} = \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right)-1}$$

$$\nu\lambda = c$$

$$\therefore \nu = \frac{c}{\lambda} = \frac{10^8 \times 3}{10^{-6}}$$

$$\therefore \nu = 3\times 10^{14} \text{ hz}$$

$$\therefore \bar{E} = \frac{6.626\times 10^{-34} \times 3\times 10^{14}}{\exp\left(\frac{6.626\times 10^{-34}\times 3\times 10^{14}}{1.380\times 10^{-23}\times 300}\right)-1}$$

$$= \frac{19.878\times 10^{-20}}{\exp\left(0.048\times 10^{3}\right)-1}$$

$$\bar{E} = \cancel{19.878\times 10^{-20}} \quad 16.32\times 10^{-37} \text{ Joule (Ans)}$$