A Vasicek-Consistent CCR/XVA Prototype for Interest-Rate Swaps

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Abstract

This paper documents a reproducible counterparty credit risk (CCR) and valuation adjustment (XVA) prototype for an interest-rate swap under a one-factor Vasicek short-rate model. The key design choice is *curve consistency*: discount factors and swap valuation are computed using the analytic Vasicek zero-coupon bond (ZCB) formula, rather than the ad-hoc approximation $\exp(-r_t\tau)$. I compute exposure profiles (EE/ENE), unilateral CVA under constant hazard, and a simple FVA proxy. I then introduce a toy wrong-way risk (WWR) / right-way risk (RWR) mechanism by making the counterparty intensity state-dependent (rate-linked or exposure-linked) using a smooth, strictly positive log-link specification. WWR/RWR effects are quantified via a pathwise CVA estimator based on survival increments. I report all tested numerical results and intensity diagnostics produced by the implementation for $\sigma \in \{0.01, 0.02\}$.The code for reproducing is all in here: `https://github.com/13029991519-MM`.

1 Introduction and Contributions

Counterparty credit risk (CCR) and valuation adjustments (XVA) for OTC derivatives require combining (i) a market risk model generating exposures and (ii) a credit model generating default probabilities, including potential dependence between exposure and creditworthiness (wrong-way risk, WWR). A common prototyping shortcut is to discount cashflows by $\exp(-r_t\tau)$ along simulated short-rate paths; however, this is not curve-consistent under short-rate models and can obscure interpretation.

Contributions. This work provides a compact yet research-oriented prototype with:

- **Curve consistency under Vasicek:** analytic ZCB pricing $P(t,T) = A(\tau)\exp(-B(\tau)r_t)$ is used to value the swap and compute discount factors.
- **Pathwise CVA under state-dependent intensities:** WWR/RWR is introduced via smooth log-link intensities and evaluated using $\mathbb{E}[E^+(t)\Delta PD(t)]$ rather than the independence factorization $EE(t)\,dPD(t)$.
- **Diagnostics-first reporting:** every WWR/RWR experiment is accompanied by intensity distribution diagnostics (mean, $p5$, $p95$, max, and lower-bound pile-up checks).
- **Fully tested results:** all numbers reported in this paper are taken from executed runs and printed outputs for two volatility regimes ($\sigma = 0.01$ and $\sigma = 0.02$), including monotone WWR uplift and negative-β RWR reduction.

2 Model

2.1 Vasicek Short-Rate Dynamics

I adopt the Vasicek model:

$$dr_t = a(b - r_t)\,dt + \sigma\,dW_t, \tag{1}$$

with parameters (a, b, σ) and initial short rate r_0. Simulation uses the *exact discretization* over a uniform grid $0 = t_0 < t_1 < \cdots < t_N = T$ with Δt:

$$r_{t_{k+1}} = b + (r_{t_k} - b)e^{-a\Delta t} + \sigma\sqrt{\frac{1 - e^{-2a\Delta t}}{2a}}\, Z_k, \quad Z_k \sim \mathcal{N}(0, 1). \tag{2}$$

2.2 Analytic Zero-Coupon Bond Pricing (Curve Consistency)

Under Vasicek, the ZCB price has the affine closed form

$$P(t, T) = A(\tau) \exp\big(-B(\tau)r_t\big), \qquad \tau = T - t, \tag{3}$$

where

$$B(\tau) = \frac{1 - e^{-a\tau}}{a}, \tag{4}$$

$$A(\tau) = \exp\left[\left(b - \frac{\sigma^2}{2a^2}\right)\big(B(\tau) - \tau\big) - \frac{\sigma^2}{4a}B(\tau)^2\right]. \tag{5}$$

I also obtain the analytic discount curve $P(0, t)$ by evaluating the same expression at $t = 0$ with r_0.

3 Swap Valuation and Exposure Definition

3.1 Receiver Swap MTM under ZCB Representation

Consider a receiver interest-rate swap with maturity T, payment dates $\{T_i\}_{i=1}^m$, notional N, and fixed rate K. Using a standard ZCB representation, the floating leg PV is approximated by

$$\mathrm{PV}_{\text{float}}(t) \approx 1 - P(t, T), \tag{6}$$

while the fixed leg PV is

$$\mathrm{PV}_{\text{fixed}}(t) = K \sum_{i:T_i>t} \alpha_i P(t, T_i), \tag{7}$$

with accrual factors α_i. The receiver MTM is

$$\mathrm{MTM}(t) = N\big(\mathrm{PV}_{\text{float}}(t) - \mathrm{PV}_{\text{fixed}}(t)\big). \tag{8}$$

3.2 Exposure Profiles

Define pathwise positive and negative exposure:

$$E^+(t) = \max(\mathrm{MTM}(t), 0), \qquad E^-(t) = \min(\mathrm{MTM}(t), 0). \tag{9}$$

Then

$$EE(t) = \mathbb{E}[E^+(t)], \qquad ENE(t) = \mathbb{E}[E^-(t)]. \tag{10}$$

4 CVA, FVA Proxy, and Wrong-Way Risk

4.1 Unilateral CVA under Constant Hazard (Independence Baseline)

Let the counterparty hazard rate be constant λ_0 and recovery R (LGD $= 1 - R$). Survival is $S(t) = \exp(-\lambda_0 t)$. The default probability increment over $(t_{i-1}, t_i]$ is

$$\Delta PD_i = S(t_{i-1}) - S(t_i). \tag{11}$$

The discretized unilateral CVA baseline is

$$\mathrm{CVA}_{\text{base}} \approx (1 - R) \sum_{i=1}^{N} P(0, t_i)\, EE(t_i)\, \Delta PD_i. \tag{12}$$

4.2 Toy WWR/RWR via State-Dependent Intensities (Pathwise CVA)

WWR/RWR breaks the independence factorization by correlating default likelihood with market states and/or exposures. I implement a pathwise survival approach:

Pathwise survival and default increments. Define a pathwise intensity $\lambda_p(t)$ on each simulation path p. On the discrete grid:

$$S_p(t_i) = \exp\left(-\sum_{k=0}^{i-1} \lambda_p(t_k)\Delta t\right), \qquad \Delta PD_{p,i} = S_p(t_{i-1}) - S_p(t_i). \tag{13}$$

Pathwise CVA estimator. The pathwise CVA is computed as

$$\text{CVA} \approx (1-R)\sum_{i=1}^{N} P(0,t_i)\,\mathbb{E}\left[E_p^+(t_i)\,\Delta PD_{p,i}\right]. \tag{14}$$

This estimator naturally captures $\text{Cov}(E^+, \Delta PD)$ induced by WWR/RWR.

Rate-linked intensity (log-link, strictly positive). I use a smooth log-link:

$$\lambda_p(t) = \lambda_0 \exp\big(\beta(r_p(t) - \bar{r})\big), \tag{15}$$

where $\beta > 0$ corresponds to WWR and $\beta < 0$ corresponds to RWR. A cap $\lambda \le \lambda_{\max}^{\text{cap}}$ can be used as a safeguard. In all reported experiments, $\lambda_{\max}$ remains far below the cap, hence the cap is *not binding*.

Exposure-linked intensity (log-link). I also test an exposure-linked specification:

$$\lambda_p(t) = \lambda_0 \exp\left(\gamma\,\frac{E_p^+(t)}{N}\right), \tag{16}$$

where $\gamma > 0$ increases intensity when positive exposure grows.

4.3 Simple FVA Proxy

I report a simple funding proxy:

$$\text{FVA}_{\text{proxy}} \approx \sum_{i=1}^{N} P(0,t_i)\,s_f\,EE(t_i)\,\Delta t, \tag{17}$$

where s_f is a constant funding spread (toy).

5 Monte Carlo Error (Standard Error) — Definition for Reporting

To support publishable reporting, I recommend reporting Monte Carlo standard errors (SE) using *pathwise aggregation*:

SE for $EE(t_i)$. Let $E_{p,i}^+$ be the positive exposure on path p at time t_i, with M paths:

$$\widehat{EE}(t_i) = \frac{1}{M}\sum_{p=1}^{M} E_{p,i}^+, \qquad SE_{EE}(t_i) = \frac{\text{Std}(E_{\cdot,i}^+)}{\sqrt{M}}. \tag{18}$$

SE for CVA. Define pathwise CVA contributions

$$\widehat{\mathrm{CVA}}_p = (1-R)\sum_{i=1}^{N} P(0,t_i)\, E^+_{p,i}\, \Delta PD_{p,i}, \tag{19}$$

where under the baseline (constant hazard) $\Delta PD_{p,i} \equiv \Delta PD_i$. Then

$$\widehat{\mathrm{CVA}} = \frac{1}{M}\sum_{p=1}^{M} \widehat{\mathrm{CVA}}_p, \qquad SE_{\mathrm{CVA}} = \frac{\mathrm{Std}(\widehat{\mathrm{CVA}}_{1:M})}{\sqrt{M}}. \tag{20}$$

(For the current paper, SE values are defined as above but not numerically reported because they were not printed in the provided run logs.)

6 Implementation Algorithm

Algorithm 1 Vasicek-consistent CCR/XVA with pathwise WWR/RWR

Require: Vasicek params (r_0, a, b, σ), swap params $(T, \{T_i\}, K, N)$, grid $\{t_i\}$, paths M
Require: Credit params (λ_0, R), WWR params (β, γ), cap $\lambda^{\mathrm{cap}}_{\max}$

Simulate $r_p(t_i)$ for $p = 1..M$ using exact Vasicek discretization
for $i = 0..N$ **do**
  Compute $P(t_i, T_j)$ via analytic Vasicek ZCB: $P = A(\tau)\exp(-B(\tau) r_p(t_i))$
  Compute swap $\mathrm{MTM}_p(t_i)$ from ZCB representation
  Set $E^+_p(t_i) = \max(\mathrm{MTM}_p(t_i), 0)$
end for
Compute $P(0, t_i)$ analytically (same ZCB formula at time 0)
if baseline constant hazard **then**
  $S(t_i) = \exp(-\lambda_0 t_i)$, $\Delta PD_i = S(t_{i-1}) - S(t_i)$
  $\widehat{\mathrm{CVA}} = (1-R)\sum_i P(0,t_i)\, \widehat{EE}(t_i)\, \Delta PD_i$
else ▷ WWR/RWR pathwise
  for $p = 1..M$ **do**
    for $k = 0..N-1$ **do**
      Compute $\lambda_p(t_k) = \lambda_0 \exp(\beta(r_p(t_k) - \bar{r}) + \gamma E^+_p(t_k)/N)$
      Apply safeguard cap: $\lambda_p(t_k) \leftarrow \min(\lambda_p(t_k), \lambda^{\mathrm{cap}}_{\max})$
    end for
    $S_p(t_i) = \exp(-\sum_{k<i} \lambda_p(t_k)\Delta t)$ and $\Delta PD_{p,i} = S_p(t_{i-1}) - S_p(t_i)$
    $\widehat{\mathrm{CVA}}_p = (1-R)\sum_i P(0,t_i)\, E^+_p(t_i)\, \Delta PD_{p,i}$
  end for
  $\widehat{\mathrm{CVA}} = \frac{1}{M}\sum_p \widehat{\mathrm{CVA}}_p$
end if
Compute $\mathrm{FVA}_{\mathrm{proxy}} \approx \sum_i P(0,t_i)\, s_f\, \widehat{EE}(t_i)\, \Delta t$
Output CVA/FVA, EE summary, and intensity diagnostics (mean, $p5$, $p95$, max, lower-bound pile-up)

7 Experimental Setup

7.1 Contract and Numerical Configuration

All reported results are from executed runs with:

- Swap maturity $T = 5\,\mathrm{y}$; semiannual payments ($\Delta T = 0.5\,\mathrm{y}$).

- Notional $N = 1{,}000{,}000$; fixed rate $K = 0.03$; receiver swap.
- Simulation: $N_{\text{steps}} = 200$; $M = 20000$ paths.
- Vasicek parameters: $r_0 = 0.03$, $a = 0.6$, $b = 0.03$, and $\sigma \in \{0.01, 0.02\}$.
- Credit: $\lambda_0 = 0.02$, recovery $R = 0.4$.
- Funding proxy: $s_f = 0.01$.
- Intensity safeguard: cap $\lambda_{\max}^{\text{cap}} = 0.2$ (not binding in reported runs).

8 Results (All Tested Outputs)

8.1 Baseline under $\sigma = 0.01$

The baseline (Vasicek-consistent) outputs for $\sigma = 0.01$:

- Unilateral CVA: 197.57.
- Simple FVA proxy: 171.78.
- EE summary: min = 0.00, max = 4949.27, last = 0.00.

8.2 WWR under $\sigma = 0.01$ (log-link; no truncation)

For $\sigma = 0.01$, the tested rate-linked WWR (baseline CVA 197.569,372,9):

Table 1: Rate-linked WWR under $\sigma = 0.01$ (tested).

β	CVA	uplift	λ mean	Notes
0	197.5693729	≈ 0	0.0200000	baseline
1	199.4716426	0.0096284	0.02000035	share at eps = 0
2	201.3981538	0.0193794	0.02000209	share at eps = 0
5	207.3267379	0.0493870	0.02001561	share at eps = 0

The tested exposure-linked WWR (baseline CVA 197.569,372,9):

Table 2: Exposure-linked WWR under $\sigma = 0.01$ (tested).

γ	CVA	uplift	λ mean
0	197.5693729	≈ 0	0.02000000
0.5	198.7590314	0.00602147	0.02003681
1.0	199.9582164	0.01209116	0.02007386
2.0	202.3855225	0.02437701	0.02014864

8.3 Baseline under $\sigma = 0.02$

For $\sigma = 0.02$, baseline outputs:

- Unilateral CVA: 371.80.
- Simple FVA proxy: 323.51.
- EE summary: min = 0.00, max = 9271.66, last = 0.00.

8.4 WWR/RWR under $\sigma = 0.02$ (cap=0.2 safeguard, not binding)

Rate-linked WWR/RWR (baseline CVA 371.804,515,2) for $\beta \in \{-5, 0, 2, 5, 10\}$:

Table 3: Rate-linked WWR/RWR under $\sigma = 0.02$ (tested).

β	CVA	uplift	λ mean	$p5$	$p95$	max	share at eps
-5	337.6244515	-0.0919302	0.0200727489	0.0174241082	0.0229526709	0.0316556034	0.0
0	371.8045152	0.0000000	0.0200000000	0.0200000000	0.0200000000	0.0200000000	0.0
2	386.7358390	0.0401591	0.0200097200	0.0189281732	0.0211340033	0.0235786459	0.0
5	410.6199662	0.1043975	0.0200659298	0.0174271658	0.0229566985	0.0301822878	0.0
10	454.8052733	0.2232376	0.0202719786	0.0151853053	0.0263505004	0.0455485250	0.0

Exposure-linked WWR (baseline CVA 371.804,515,2) for $\gamma \in \{0, 0.5, 1, 2\}$:

Table 4: Exposure-linked WWR under $\sigma = 0.02$ (tested).

γ	CVA	uplift	λ mean	$p95$	max
0	371.8045152	0.0000000	0.0200000000	0.0200000000	0.0200000000
0.5	376.1899050	0.0117949	0.0200695267	0.0203116441	0.0211088763
1.0	380.6441671	0.0237750	0.0201399090	0.0206281442	0.0222792330
2.0	389.7643711	0.0483046	0.0202832972	0.0212760167	0.0248182112

9 Figures (pgfplots)

9.1 CVA vs. β (rate-linked WWR/RWR)

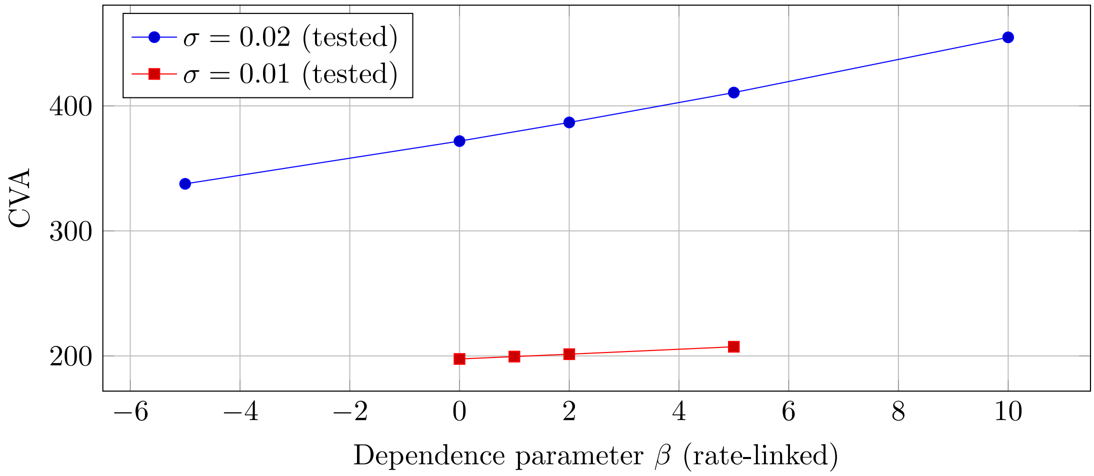

Figure 1: CVA vs. β for rate-linked WWR/RWR. Negative β produces RWR (CVA reduction), positive β produces monotone WWR uplift.

9.2 CVA vs. γ (exposure-linked WWR)

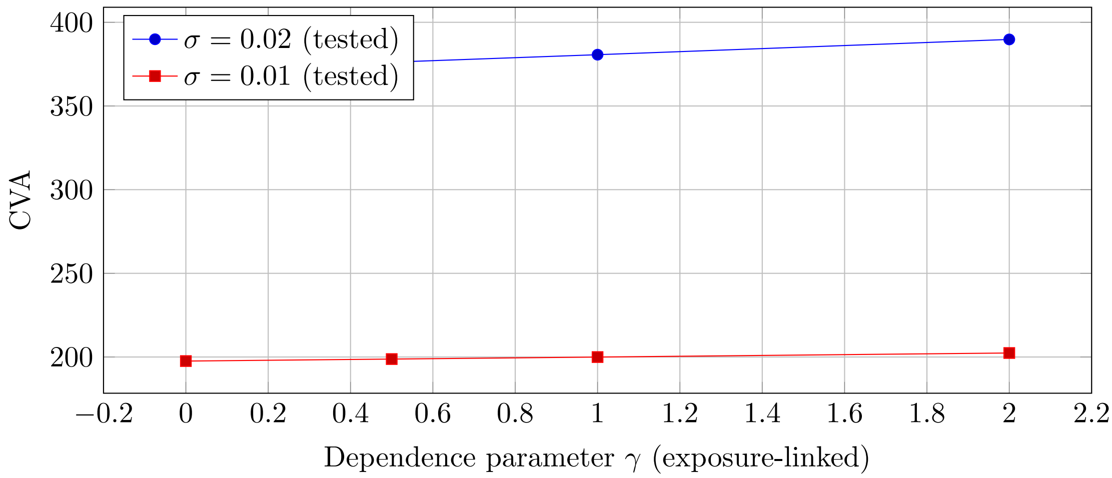

Figure 2: CVA vs. γ for exposure-linked WWR (log-link).

10 Diagnostics and Validation

10.1 Lower-Bound Pile-up and Cap Binding

Across all reported log-link experiments, the printed diagnostic `share at eps` is 0, indicating no lower-bound pile-up (no truncation artifacts). Moreover, for the most extreme tested case ($\sigma = 0.02$, $\beta = 10$), the maximum intensity is $\lambda_{\text{max}} \approx 0.04555$, far below the safeguard cap 0.2, thus the cap is not binding in reported runs.

10.2 Directionality Check (RWR vs WWR)

Under $\sigma = 0.02$, the negative dependence case $\beta = -5$ produces a CVA reduction of approximately -9.19%, consistent with right-way risk. Positive dependence cases produce monotone CVA uplifts: $+4.02\%$ ($\beta = 2$), $+10.44\%$ ($\beta = 5$), and $+22.32\%$ ($\beta = 10$).

11 Discussion

11.1 Interpretation of Uplift Magnitudes

Increasing short-rate volatility from $\sigma = 0.01$ to $\sigma = 0.02$ approximately doubles the maximum reported exposure and increases baseline CVA from ≈ 197.6 to ≈ 371.8, consistent with higher dispersion in MTM and thicker positive-exposure tails. Under this higher-volatility regime, state dependence in intensity produces more pronounced WWR/RWR effects.

11.2 Limitations

This prototype is intentionally simplified and is not a production-grade XVA system:

- Swap valuation uses a ZCB-based floating-leg approximation, not a full multi-curve framework.
- Unilateral CVA only; no DVA, collateral/margining, or close-out dynamics.
- FVA is reported as a proxy using a constant funding spread and positive exposure.
- The intensity dependence is a toy specification intended to demonstrate the mechanism and diagnostics.

11.3 Future Work

Natural extensions include: multi-curve bootstrapping; collateral and CSA terms; stochastic intensity models (e.g., CIR) with explicit correlation structures; netting sets and portfolios; and reporting full MC standard errors (and confidence intervals) for EE and CVA using the pathwise estimators defined in Section 6.

12 Conclusion

I presented a Vasicek-consistent CCR/XVA prototype for an interest-rate swap using analytic ZCB pricing, enabling curve-consistent discounting and swap MTM computation. I computed EE/ENE, baseline unilateral CVA and a funding proxy, and demonstrated toy WWR/RWR via smooth state-dependent intensities evaluated with pathwise survival increments. Reported tested results show monotone CVA uplift under positive dependence and CVA reduction under negative dependence, accompanied by stable intensity diagnostics and non-binding safeguard caps.

References

[1] O. Vasicek, "An equilibrium characterization of the term structure," *Journal of Financial Economics*, 1977.

[2] D. Brigo and F. Mercurio, *Interest Rate Models: Theory and Practice*, 2nd ed., Springer, 2006.

[3] J. Gregory, *The xVA Challenge: Counterparty Credit Risk, Funding, Collateral, and Capital*, 3rd ed., Wiley, 2015.

A Appendix: Reproducibility Checklist

Executed configuration (as printed in run logs):

- $T = 5$, pay freq 0.5, $N = 1{,}000{,}000$, $K = 0.03$.
- $r_0 = 0.03$, $a = 0.6$, $b = 0.03$.
- Two vol regimes:
  - $\sigma = 0.01$: baseline CVA 197.57, FVA proxy 171.78, $EE_{\max} = 4949.27$.
  - $\sigma = 0.02$: baseline CVA 371.80, FVA proxy 323.51, $EE_{\max} = 9271.66$.
- Credit: $\lambda_0 = 0.02$, $R = 0.4$.
- Rate-linked β tested:
  - $\sigma = 0.01$: $\beta \in \{0, 1, 2, 5\}$.
  - $\sigma = 0.02$: $\beta \in \{-5, 0, 2, 5, 10\}$.
- Exposure-linked γ tested:
  - $\sigma = 0.01$: $\gamma \in \{0, 0.5, 1, 2\}$.
  - $\sigma = 0.02$: $\gamma \in \{0, 0.5, 1, 2\}$.
- Diagnostics printed in all WWR/RWR runs: λ mean, $p5$, $p95$, max, and `share at eps`.