power in any receiver test is actually only a measure of the energy *available* to the receiver. It may be expressed either in terms of voltage which is applied in series with the source impedance, or in terms of the maximum power available, i.e., that power which would be delivered to the receiver if it were carefully matched to the source impedance. The relation between applied voltage and available power is the following (see Fig. 1):

$$P_{\max} = E_s^2/4R_s \qquad (8)$$

where

$P_{\max}$ = available power

E_s = root-mean-square voltage from signal generator

R_s = resistive component of source impedance (including both dummy antenna and signal generator).[12]

In order to illustrate the relative magnitudes of corresponding power and voltage figures, Table I is presented for the familiar broadcast situation.

The first factor pointing to the desirability of a shift from voltage to power definitions, as the higher frequencies are reached, is the types of measuring instruments available. Signal generators up to 50 megacycles may use vacuum-tube voltmeters to measure the radio-frequency voltage, but such voltmeters are generally not usable at higher frequencies. The next step is a crystal-type voltmeter, which has been used in signal generators up to 1000 megacycles. Beyond 1000 megacycles, however, it has proved more expedient to use devices such as thermistors and bolometers, which are basically power devices since their operation depends upon a heating phenomenon. Another factor is that power is the natural measure for noise. If the equation for thermal noise in (1) is converted to express available power, it becomes

$$P_n = E_n^2/4R_s = kT\Delta f. \qquad (9)$$

It is thus seen that the available noise power is independent of the impedance level, Δf being the prime consideration. Finally, high-frequency propagation and transmission are calculated more easily on a power basis. With high-gain antennas, particularly those using reflectors, it is more convenient to talk of intercept areas, rather than effective heights, and thus to talk in terms of watts per square meter, rather than in volts per meter.

TABLE I

	Root-Mean-Square Voltage in Series with Antenna[13] (E_s)	Available Power[14] ($P_{\max}$)
Distant signal	50×10^{-6} volts (50 microvolts)	25×10^{-12} watts (25 micromicrowatts)
Mean signa	5×10^{-3} volts (5 millivolts)	0.25×10^{-6} watts (0.25 microwatts)
Local signal	100×10^{-3} volts (100 millivolts)	100×10^{-6} watts (100 microwatts)
Strong signal	2 volts	40×10^{-3} watts (40 milliwatts)

[12] Any reactance which may be in the signal source does not enter in this power equation.

[13] Listed in "Standards on Radio Receivers," 1938, p. 14. Institute of Radio Engineers.

[14] For a frequency of 1 megacycle where R_s for the standard dummy antenna is about 25 ohms.

Balanced Amplifiers*

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Summary—**Push-pull impedance-coupled amplifiers have wide applicability in electronic instrumentation. Four gain factors are required to completely describe their performance. Of these factors, one should be large and the others as small as possible. These characteristics are obtained by in-phase feed-back, applied properly. A number of circuits, suitable for various applications, are described.**

1. Introduction

BALANCED push-pull impedance-coupled[1] amplifiers have a wide applicability in electronic instrumentation. Among their uses are: differential input amplifiers with balanced output for bioelectric and other measurements; differential input stages for single-ended amplifiers; and phase inverters. In addition, such amplifiers frequently have advantages over single-ended amplifiers for low-frequency applications, since cathode and screen-grid by-pass capacitors may be eliminated, and power-supply impedance does not affect the frequency response.

The amplification of a single-ended amplifier is given by a single parameter μ, the ratio of output to input voltage. Generally, μ is complex and a function of frequency. In contrast, four gain factors are required to completely describe the performance of a push-pull amplifier. Of these, the first factor, the conventional differential gain, should be high, while the other factors, the in-phase gain, the inversion gain, and the differential unbalance, should be as low as possible. It will be shown that the desired characteristics are obtained by the use of in-phase feedback.

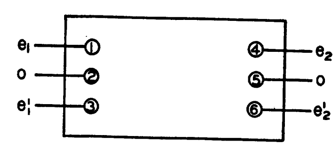

Fig. 1—Generalized push-pull amplifier.

2. Generalized Theory

A push-pull impedance-coupled amplifier is a six-terminal network, with three input and three output terminals (Fig. 1). The input signal voltages e_1 and e_1' are applied between terminals 1, 2 and 3, 2, respectively. The output voltages e_2 and e_2' are developed between

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[1] Impedance coupling as used here includes resistance, resistance-capacitance, and choke coupling, but not transformer coupling.

terminals 4, 5 and 6, 5. Four gain factors must be considered. These are most directly taken as:

$$\mu = e_2/e_1 \quad \text{for} \quad e_1' = 0 \tag{1a}$$

$$\mu' = e_2'/e_1' \quad \text{for} \quad e_1 = 0 \tag{1b}$$

$$\gamma = -e_2'/e_1 \quad \text{for} \quad e_1' = 0 \tag{1c}$$

$$\gamma' = -e_2/e_1' \quad \text{for} \quad e_1 = 0 \tag{1d}$$

In a truly balanced amplifier, $\mu = \mu'$, and $\gamma = \gamma'$. In any real amplifier, this is never exactly true.

To illustrate the significance of these factors, consider the amplifier of Fig. 2. The amplification of the upper half of the amplifier is μ; that of the lower, μ'. There is no cross-coupling, and a signal applied at 1, 2 will produce no output at 6, 5. Thus $\gamma = \gamma' = 0$.

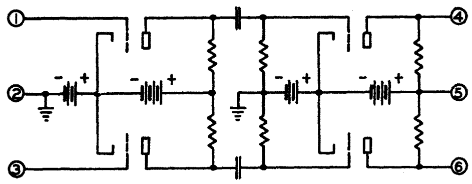

Fig. 2—Push-pull amplifier without cross-coupling.

If in the above amplifier a cathode-resistor bias common to the two tubes of each stage is used, in-phase degeneration will produce cross-coupling, and γ and γ' will no longer be zero.

Assuming a linear amplifier, the principle of superposition gives the total output voltage as

$$e_2 = \mu e_1 - \gamma' e_1' \tag{2}$$

$$e_2' = \mu' e_1' - \gamma e_1. \tag{3}$$

New gain factors derived from those above are more easily interpretable in terms of amplifier performance.

Differential Gain

The differential gain of the amplifier is

$$G_0 = (e_2 - e_2')/(e_1 - e_1') \tag{4}$$

for $e_1' = -e_1$. Substituting (2) and (3),

$$G_0 = \tfrac{1}{2}(\mu + \mu' + \gamma + \gamma'). \tag{5}$$

In-Phase Gain

The in-phase gain of the amplifier is

$$G_c = (e_2 + e_2')/(e_1 + e_1') \tag{6}$$

for $e_1' = e_1$. Substituting,

$$G_c = \tfrac{1}{2}(\mu + \mu' - \gamma - \gamma'). \tag{7}$$

Thus, if $\mu + \mu' = \gamma + \gamma'$, the in-phase gain is zero.

Inversion Gain

In general, if an in-phase signal is applied to the input, a differential output signal will be obtained. This may be termed the ***inversion gain*** G_i of the amplifier:

$$G_i = (e_2 - e_2')/\tfrac{1}{2}(e_1 + e_1') \tag{8}$$

for $e_1 = e_1'$.

$$G_i = \mu - \mu' + \gamma - \gamma'. \tag{9}$$

It is seen, therefore, that even though the in-phase gain is made zero, the inversion gain may be large. This is of importance in many applications of differential amplifiers. It will be shown later, however, that reduction of the in-phase gain by properly applied in-phase feedback will simultaneously reduce the inversion gain.

Differential Unbalance

The fourth gain factor, the differential unbalance G_u, gives the unbalance of the output when a balanced input signal is applied:

$$G_u = (e_2 + e_2')/(e_1 - e_1') \tag{10}$$

for $e_1 = -e_1'$.

$$G_u = \tfrac{1}{2}(\mu - \mu' - \gamma + \gamma'). \tag{11}$$

3. Gain Factors With In-Phase Feedback

In the generalized circuit of Fig. 1, in-phase feedback may be introduced.[2] This is shown in Fig. 3, where voltage feedback from the 4, 6 output terminals

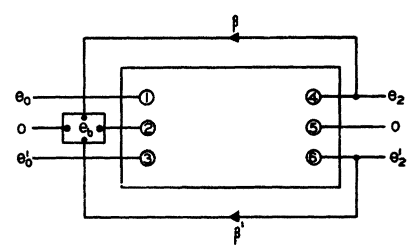

Fig. 3—Generalized push-pull amplifier with in-phase feedback.

is introduced in series with the common input lead, in such phase relationship as to reduce the in-phase gain. The feedback voltage e_b is

$$e_b = \beta e_2 + \beta' e_2'.$$

In general, because of variation in components, β is not necessarily equal to β'. Then

$$e_1 = e_0 - e_b = e_0 - \beta e_2 - \beta' e_2' \tag{12a}$$

$$e_1' = e_0' - e_b = e_0' - \beta e_2 - \beta' e_2', \tag{12b}$$

and by (2) and (3),

$$e_2 = \mu(e_0 - e_b) - \gamma'(e_0' - e_b) \tag{13a}$$

$$e_2' = \mu'(e_0' - e_b) - \gamma(e_0 - e_b), \tag{13b}$$

and

$$e_b = \beta[\mu(e_0 - e_b) - \gamma'(e_0' - e_b)] + \beta'[\mu'(e_0' - e_b) - \gamma(e_0 - e_b)]$$

$$e_b = \frac{\beta(\mu e_0 - \gamma' e_0') + \beta'(\mu' e_0' - \gamma e_0)}{1 + \beta(\mu - \gamma') + \beta'(\mu' - \gamma)}. \tag{14}$$

[2] The generalized amplifier of Fig. 1 may already contain in-phase feedback (as in Section 5 below). In this section the effect of adding in-phase feedback around an amplifier is considered irrespective as to whether it already had feedback of this type.

To calculate the gain factors for in-phase signals, put $e_0 = e_0'$:

$$e_b = \frac{\beta(\mu - \gamma') + \beta'(\mu' - \gamma)}{1 + \beta(\mu - \gamma') + \beta'(\mu' - \gamma)} e_0. \tag{15}$$

In-Phase Gain with Feedback

The in-phase gain with feedback, G_c', is obtained by putting the new input voltage $e_0 = e_0'$ for $e_1 = e_1'$ in (6); and substituting e_2 and e_2' from (13):

$$G_c' = (\mu + \mu' - \gamma - \gamma')(e_0 - e_b)/2e_0$$

$$G_c' = G_c/[1 + \beta(\mu - \gamma') + \beta'(\mu' + \gamma)]. \tag{16}$$

For $\beta = \beta'$ this reduces to

$$G_c' = G_c/(1 + 2\beta G_c) \tag{17}$$

which is similar to the familiar feedback equation.

Inversion Gain with In-Phase Feedback

The inversion gain with in-phase feedback, G_i', may be calculated by a similar procedure, using equation (8):

$$G_i' = (\mu - \mu' + \gamma - \gamma')(e_0 - e_b)/e_0$$

$$G_i' = G_i/[1 + \beta(\mu - \gamma') + \beta'(\mu' - \gamma)]. \tag{18}$$

For $\beta = \beta'$, this again reduces to

$$G_i' = G_i/(1 + 2\beta G_c). \tag{19}$$

Differential Gain with Feedback

The differential gain with feedback, G_0', is calculated from (4) by putting $e_0' = -e_0$ for $e_1' = -e_1$; and substituting e_2 and e_2' from (13):

$$G_0' = [(\mu + \gamma)(e_0 - e_b) + (\mu' + \gamma')(e_0 + e_b)]/2e_0$$

$$G_0' = \tfrac{1}{2}(\mu + \mu' + \gamma + \gamma') - \tfrac{1}{2}(\mu - \mu' + \gamma - \gamma')e_b/e_0$$

$$G_0' = G_0 - \tfrac{1}{2}G_i e_b/e_0. \tag{20}$$

Substituting e_b from (14), with $e_0' = -e_0$:

$$G_0' = G_0 - \tfrac{1}{2}G_i[\beta(\mu + \gamma') - \beta'(\mu' + \gamma)] \\ /[1 + \beta(\mu - \gamma') + \beta'(\mu' - \gamma)]. \tag{21}$$

If $\beta = \beta'$, this reduces to

$$G_0' = G_0 - \beta G_u G_i/(1 + 2\beta G_c) \tag{22}$$

and if $\beta G_c \gg 1$, this becomes

$$G_0' = G_0 - \tfrac{1}{2}G_u G_i/G_c. \tag{23}$$

Differential Unbalance with In-Phase Feedback

The differential unbalance with in-phase feedback, G_u', is obtained from (10) in a similar manner:

$$G_u' = [(\mu - \gamma)(e_0 - e_b) - (\mu' - \gamma')(e_0 + e_b)]/2e_0$$

$$= \tfrac{1}{2}(\mu - \mu' - \gamma + \gamma') - \tfrac{1}{2}(\mu + \mu' - \gamma - \gamma')e_b/e_0$$

$$= G_u - G_c e_b/e_0 \tag{24}$$

$$= G_u - G_c[\beta(\mu + \gamma') - \beta'(\mu' + \gamma)] \\ /[1 + \beta(\mu - \gamma') + \beta'(\mu' - \gamma)]. \tag{25}$$

If $\beta = \beta'$, this reduces to

$$G_u' = G_u - 2\beta G_c G_u/[1 + 2\beta G_c]. \tag{26}$$

If $\beta G_c \gg 1$, this becomes

$$G_u' = 0$$

and the output is balanced.

4. Discussion of the Feedback Equations

The general equations for in-phase gain and inversion gain with feedback, (16) and (18), show that G_c' and G_i' may be reduced to a low value by sufficient feedback, even though the amplifier and feedback circuit both be nonsymmetrical ($\mu \neq \mu'$; $\gamma \neq \gamma'$; $\beta \neq \beta'$). This is of great importance in the design of differential amplifiers, as it allows excellent input balance to be maintained with components of commercial tolerance. For example, in some bioelectric recording it is necessary to keep the inversion gain less than one-thousandth the differential gain. This can be achieved even though the individual tubes, resistors, etc., differ by ten or twenty per cent.

Referring to (21) and (25) for G_0' and G_u', however, it is seen that the terms in β and β' appear as a difference. Thus, an inequality between β and β' will affect the differential gain and the output balance.

Another possible effect of having β and β' unequal is to make the amplifier unstable. Referring to (16), in an unsymmetrical amplifier γ' could be, for example, greater than μ. Then, if β were large and β' small, the denominator could become zero at some frequency, and the amplifier could oscillate. However, if $\beta = \beta'$ (17) shows that the expected phase relationships hold, the amplifier will be stable. A similar analysis applies to the other gain factors. Thus it is necessary that a satisfactory degree of balance be held in the feedback circuit to insure stability.

5. Characteristics of Ideal Feedback Amplifier

If the amplifier of Fig. 1 has a very large amount of in-phase feedback with $\beta = \beta'$, $G_c = G_i = G_u = 0$. That is,

$$\mu + \mu' - \gamma - \gamma' = 0$$

$$\mu - \mu' + \gamma - \gamma' = 0$$

$$\mu - \mu' - \gamma + \gamma' = 0.$$

Solving simultaneously,

$$\mu = \mu' = \gamma = \gamma'.$$

If a signal is applied to the 1, 2 terminals, by (1a) and (1c), $e_2 = -e_2'$, and the amplifier may be used as a phase inverter.

If a signal e_1 is applied to the 1, 2 terminals, and e_1' to the 3, 2 terminals, and the output is taken from the 4, 5 terminals, by (2) the output is proportional to $e_1 - e_1'$. Thus the amplifier acts as a differential-input amplifier and may be followed by a single-ended amplifier. In the latter example, an equal and opposite output signal also appears at the 6, 5 terminals. The amplifier, therefore, acts as a differential amplifier with balanced output.

6. Illustrative Examples—Conventional Amplifiers

Noncross-Coupled Push-Pull Amplifier

In the amplifier of Fig. 2, $\gamma=\gamma'=0$ as shown above. Due to variation in components, the amplifier will seldom be completely symmetrical, so μ is rarely exactly equal to μ'. The differential gain is then, by (5), the average of the gains of the two sides:

$$G_0 = \tfrac{1}{2}(\mu + \mu'),$$

and the in-phase gain is, by (7), equal to the differential gain:

$$G_c = \tfrac{1}{2}(\mu + \mu').$$

The inversion gain is, by (9),

$$G_i = \mu - \mu'$$

or just the difference in the gains of the two sides; and by (11), the differential unbalance is

$$G_u = \tfrac{1}{2}(\mu - \mu').$$

In a multistage amplifier μ and μ' may differ easily by 50 per cent, so that G_i becomes of the same order as G_0. An amplifier, as shown in Fig. 2, is thus entirely unsuited for differential use where there is any in-phase signal, or where the output must be balanced.

Phase Inverters

The familiar two-tube phase-inverter circuit (Fig. 4) may be considered as a push-pull amplifier in which $\mu'=\gamma'=0$, since the number 3 terminal is not connected.

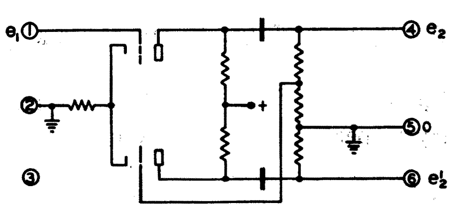

Fig. 4—Conventional phase inverter.

But balanced operation ($e_2=-e_2'$) is obtained, by (2) and (3), when $\mu=\gamma$. By (7), this requires that the in-phase gain $G_c=0$. By (9), the inversion gain is

$$G_i = \mu + \gamma = 2\mu.$$

This illustrates that balanced operation results if the in-phase gain is zero and the amplifier operates by virtue of the inversion gain. G_c is made zero by adjusting the tap on the grid resistor and balance is assisted by the in-phase feedback produced by the unby-passed cathode resistor.

7. Illustrative Examples—In-Phase Feedback

In applying in-phase feedback, it is essential that the feedback be effectively introduced in series with the common input lead, as shown in Fig. 3. Otherwise, inversion gain occurring before the point of introduction of feedback will not be cancelled. Thus, feedback introduced in the second stage of an amplifier loses most of its value.[3]

To insure that the feedback will be effective over the full response range of low-frequency amplifiers, it is desirable to employ only resistors in the feedback mesh. Both of these requirements have been met in the amplifiers described below.

One of the simplest methods of obtaining in-phase feedback is to employ a large cathode resistor in the first stage,[4] as shown in Fig. 5. Here, $\beta=R_k/R_{P1}$; $\beta'=R_k/R_P$.

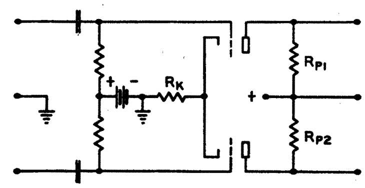

Fig. 5—In-phase feedback from cathode resistor.

With no cathode resistor ($\gamma=\gamma'=0$), $G_c=\frac{1}{2}(\mu+\mu')$. Thus $G_c'=\frac{1}{2}(\mu+\mu')/(1+\mu R_k/R_{P1}+\mu' R_k/R_{P2})$. A typical electroencephalograph amplifier had G_c reduced from 60 to 2 by use of a 60,000-ohm cathode resistor.

Amplified in-phase feedback is used in the vibration amplifier[5] shown in Fig. 6. The input signal is applied to one input grid, and conventional inverse feedback to the other. The in-phase feedback insures symmetrical operation, and the well-known advantages of push-pull amplifiers are obtained.

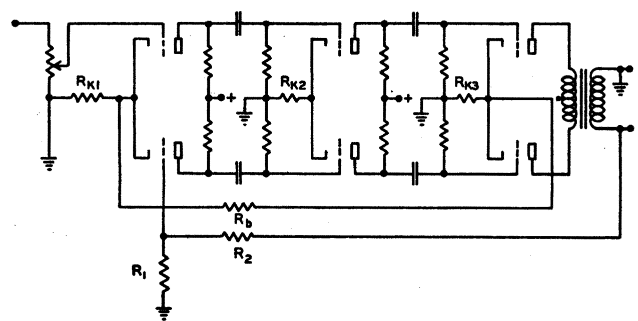

Fig. 6—In-phase feedback over three stages.

The balanced output transformer has low reactance to in-phase signals, and R_{k3} is effectively the load for such signals. The signal across R_{k3} is fed back to the first cathode through R_b. Then $\beta=R_{k1}/2R_b$, and $G_c'=G_c/(1+G_c R_{k1}/R_b)$. G_c was reduced from 200 to 18 with a moderate amount of feedback. Unbalance in the output transformer will make $\beta\neq\beta'$, and the amplifier may oscillate.

[3] Paul Traugott, "Electroencelographic design," *Electronics*, vol. 16, p. 132; August, 1943; and W. M. Rogers and H. O. Parrack, "Electronic apparatus for recording and measuring electrical potentials in nerve and muscle," Proc. I.R.E., vol. 32, p. 738; December, 1944; show in-phase feedback in the second stage only.

[4] S. N. Trevino and Franklin Offner, "An A.C. operated D.C. amplifier with large current output," *Rev. Sci. Instr.*, vol. 11, p. 412; December, 1940.

[5] Offner Electronics Type 134.

The feedback obtained in the circuit of Fig. 5 can be increased also by passing the screen current from the following stage through the cathode resistor (Fig. 7). It allows amplified in-phase feedback with only two stages, and improves the feedback ratio by about four times.

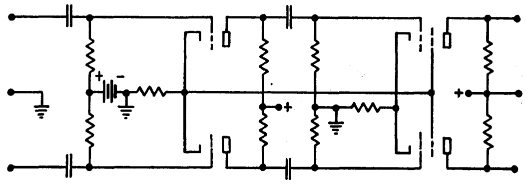

Fig. 7—In-phase feedback using screen current.

Another circuit[6] for amplified in-phase feedback over two stages is shown in Fig. 8. As the source is returned to the second cathode, either the source or power supply must be ungrounded. Therefore, its use in bioelectric work is limited.

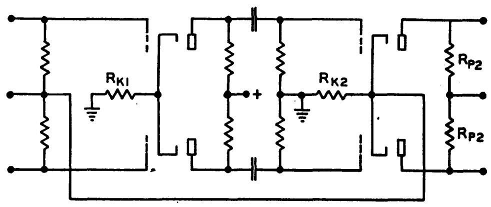

Fig. 8—In-phase feedback over two stages.

In-phase feedback in the above circuits is from indirectly heated cathodes. A portable vibration amplifier with filamentary tubes uses the circuit of Fig. 9. The in-phase feedback is produced by the alternating-current component of the plate current flowing through output-stage filaments. The potential produced is between the first stage filaments and ground, providing in-phase feedback.

[6] Franklin Offner, "Push-pull resistance coupled amplifiers," *Rev. Sci. Instr.*, vol. 8, p. 20; January, 1937.

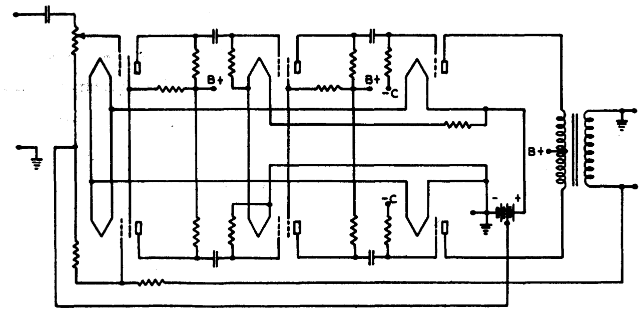

Fig. 9—In-phase feedback using filament tubes.

8. Conclusions

The addition of in-phase feedback gives push-pull amplifiers several desirable characteristics. The use of such amplifiers for special purposes is almost essential. For more conventional applications, where a single ended amplifier would usually be used, the improved performance and simplified design of the push-pull amplifier will frequently make its use desirable. Tubes such as the 12SC7, 12SL7GT, and 12L8GT require the us of but one tube per stage. Thus, a push-pull amplifier will frequently have a smaller total volume of components than a single-ended, and will have superior performance.

Test Equipment and Techniques for Airborne-Radar Field Maintenance*

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***Summary*—The scope of this paper covers the various testing methods, techniques, and equipment that are used for the field maintenance of airborne-radar systems. Techniques used in the measurement of frequency, power, and receiver sensitivity are outlined, as well as measurements of performance characteristics peculiar to airborne-radar equipment. The specially designed instruments required for field maintenance and the unique procedures devised to accomplish the measurement of radar performance characteristics for optimum radar performance are described. Special emphasis is directed to the application of special test equipment, such as echo boxes, directional couplers, and signal generators, for maintenance of microwave radar equipment.**

I. Statement of the Problem

A CONVENTIONAL airborne-radar system consists of an antenna, transmitter, receiver, indicator, modulator, and power supply. To determine whether such a system is not only functioning properly but in most cases at optimum performance, it is necessary to perform the following tests:

(1) Over-all system performance
(2) Transmitter frequency and power output
(3) Receiver sensitivity and local-oscillator frequency and power output
(4) Standing-wave ratio of the transmission line
(5) Frequency pulling of the magnetron
(6) Transmit-receive recovery time and receiver bandwidth
(7) Spectrum analysis of a transmitted pulse
(8) Pulse amplitude, time, and time-delay measurements
(9) Range and rate calibration.

It is evident from the above list that new techniques had to be devised and new instruments developed to make such techniques possible. Too often, and this cannot be too greatly stressed, the problem has been to design field-type instruments which not only possessed intrinsic electrical characteristic comparable

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