

ML/DL for Everyone with PYTORCH

Lecture 3: Gradient Descent

Sung Kim <hunkim+ml@gmail.com> HKUST

Code: <https://github.com/hunkim/PyTorchZeroToAll>

Slides: <http://bit.ly/PyTorchZeroAll>



Call for Comments

Please feel free to add comments directly on these slides.

Other slides: <http://bit.ly/PyTorchZeroAll>



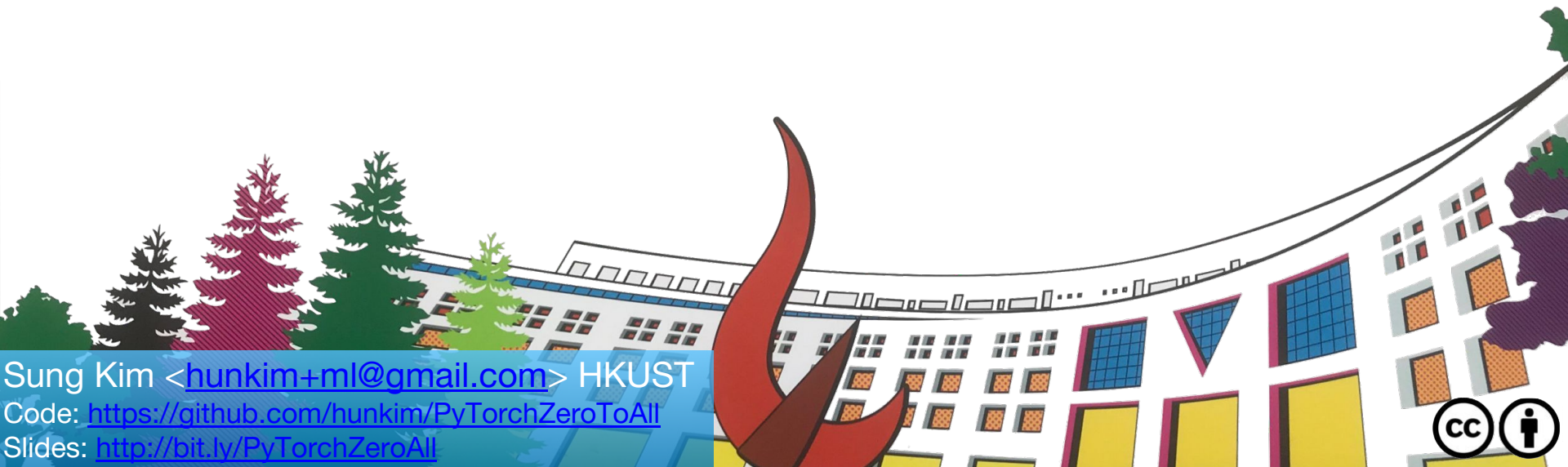
ML/DL for Everyone with PYTORCH

Lecture 3: Gradient Descent

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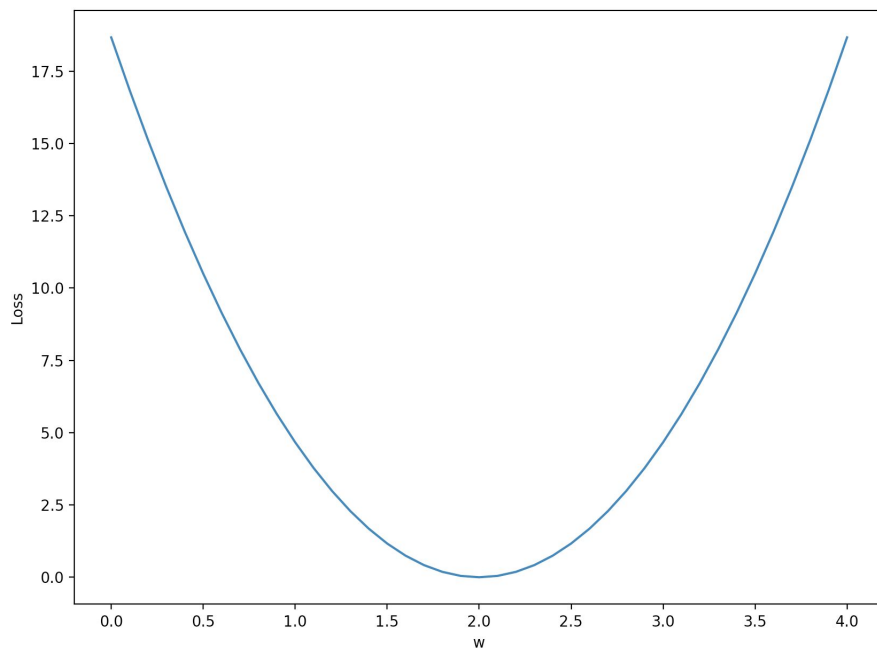
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Loss graph

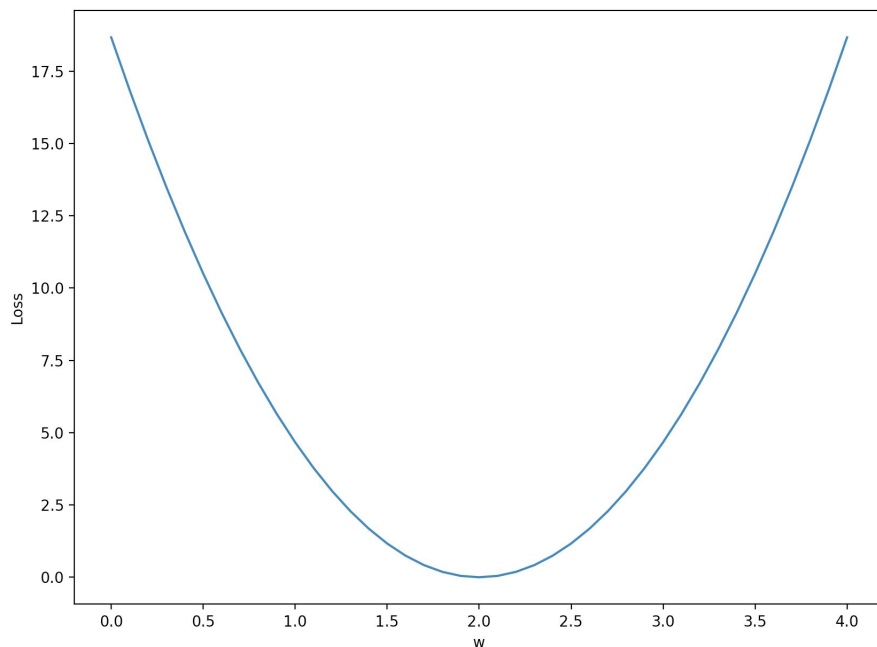
Loss (w=0)	Loss (w=1)	Loss (w=2)	Loss (w=3)	Loss (w=4)
mean=56/3=18.7	mean=14/3=4.7	mean=0	mean=14/3=4.7	mean=56/3=18.7



$$loss(w) = \frac{1}{N} \sum_{n=1}^N (\hat{y}_n - y_n)^2$$

What is the learning: find w that minimizes the loss

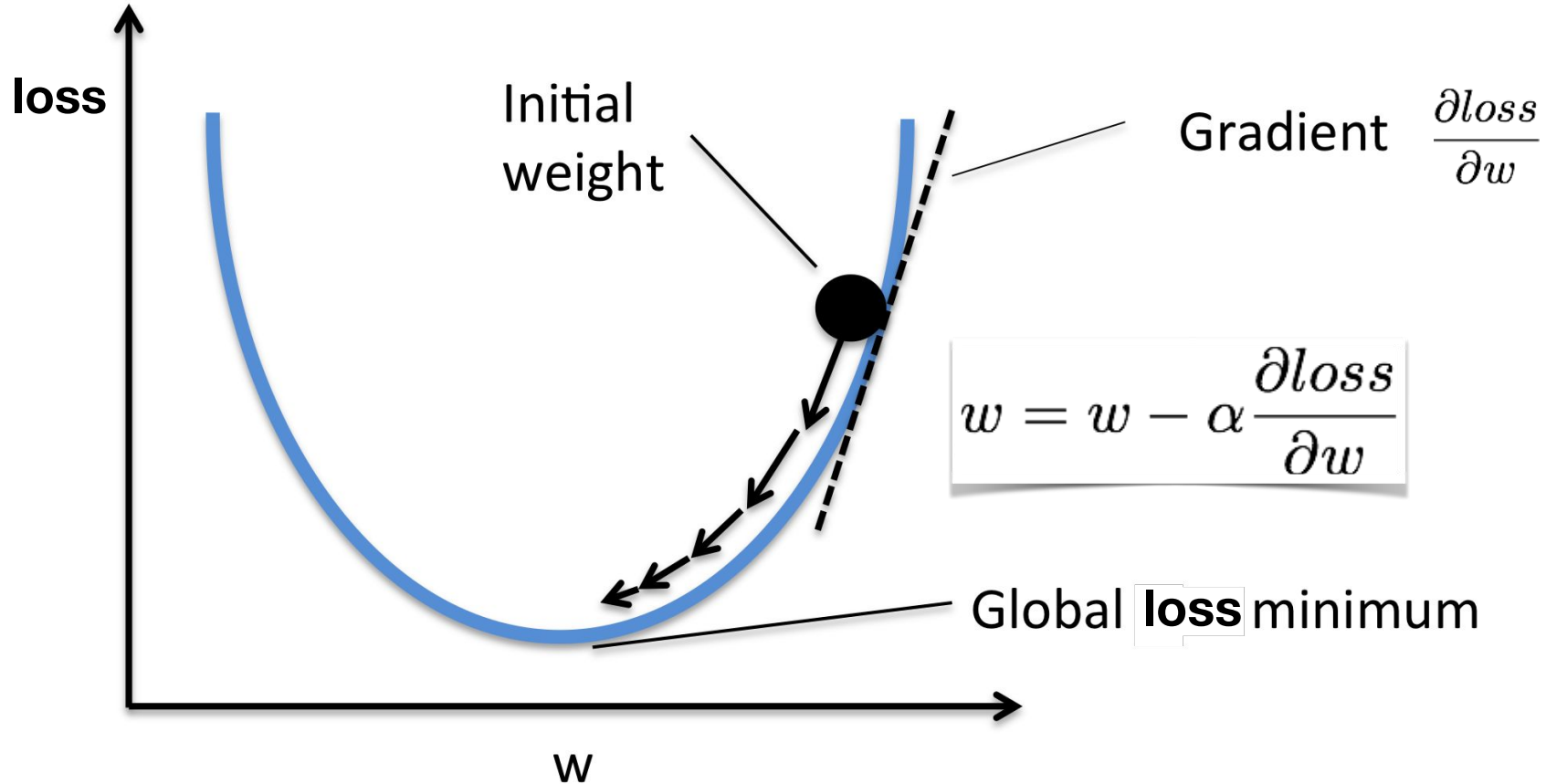
Loss ($w=0$)	Loss ($w=1$)	Loss ($w=2$)	Loss ($w=3$)	Loss ($w=4$)
mean=56/3=18.7	mean=14/3=4.7	mean=0	mean=14/3=4.7	mean=56/3=18.7



$$loss(w) = \frac{1}{N} \sum_{n=1}^N (\hat{y}_n - y_n)^2$$

$$\arg \min_w loss(w)$$

Gradient descent algorithm



Derivative

$$loss = (\hat{y} - y)^2 = (x * w - y)^2$$

$$w = w - \alpha \frac{\partial loss}{\partial w}$$

Derivative

$$loss = (\hat{y} - y)^2 = (x * w - y)^2$$

$$w = w - \alpha \frac{\partial loss}{\partial w}$$

$$\frac{\partial loss}{\partial w} = ?$$

Derivative $loss = (\hat{y} - y)^2 = (x * w - y)^2$

$$\frac{\partial loss}{\partial w} = ?$$

Derivative

$$loss = (\hat{y} - y)^2 = (x * w - y)^2$$

$$\frac{\partial loss}{\partial w} = ?$$

YOUR INPUT:
 $f(w) =$

$(xw - y)^2$

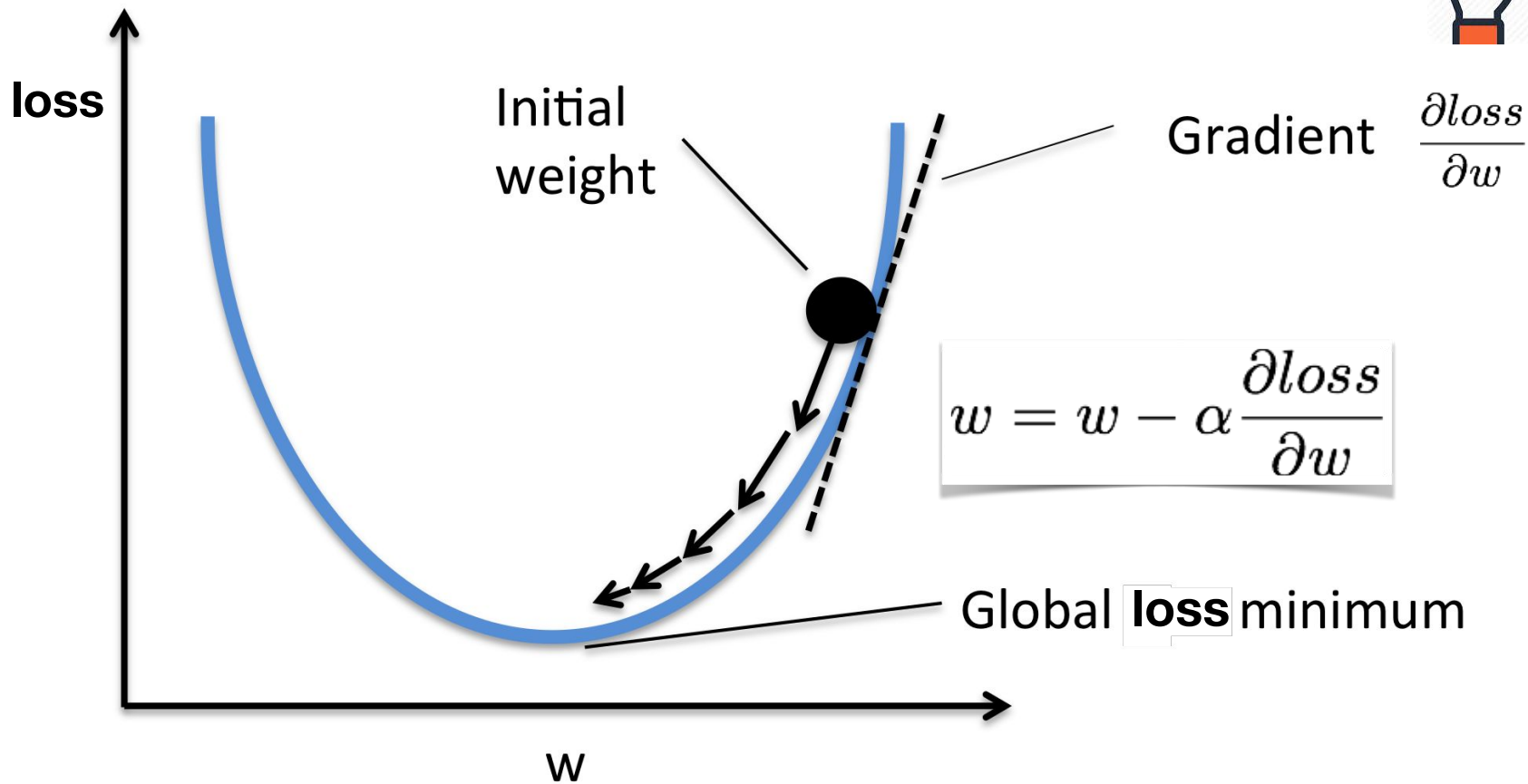
Simplify Roots/zeros

FIRST DERIVATIVE:
 $\frac{d}{dw}[f(w)] = f'(w) =$

The steps of calculation are displayed.
Move the mouse over a derivative $\frac{d}{dw}[\dots]$ or tap it in order to show its calculation.

$$\begin{aligned} & \frac{d}{dw}[(xw - y)^2] \\ &= 2(xw - y) \cdot \frac{d}{dw}[xw - y] \\ &= 2\left(x \cdot \frac{d}{dw}[w] + \frac{d}{dw}[-y]\right)(xw - y) \\ &= 2(1x + 0)(xw - y) \\ &= 2x(xw - y) \end{aligned}$$

Let's implement!



Data, Model, Loss, and Gradient



```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]

w = 1.0 # any random value
```

```
# our model forward pass
```

```
def forward(x):
    return x*w
```

```
# Loss function
```

```
def loss(x, y):
    y_pred = forward(x)
    return (y_pred-y)*(y_pred-y)
```

```
# compute gradient
```

```
def gradient(x, y): # d_loss/d_w
    return 2*x*(x*w-y)
```

Training: updating weight



```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]
```

```
w = 1.0 # any random value
```

```
# our model forward pass
```

```
def forward(x):
    return x*w
```

```
# Loss function
```

```
def loss(x, y):
    y_pred = forward(x)
    return (y_pred-y)*(y_pred-y)
```

```
# compute gradient
```

```
def gradient(x, y): # d_loss/d_w
    return 2*x*(x*w-y)
```

```
# Before training
```

```
print("predict (before training)", 4, forward(4))
```

```
# Training loop
```

```
for epoch in range(10):
    for x, y in zip(x_data, y_data):
        grad = gradient(x, y)
        w = w - 0.01 * grad
        print("\tgrad: ", x, y, grad)
        l = loss(x, y)
```

```
print("progress:", epoch, l)
```

```
# After training
```

```
print("predict (after training)", 4, forward(4))
```

```

predict (before training) 4 4.0
grad: 1.0 2.0 -2.0
grad: 2.0 4.0 -7.84
grad: 3.0 6.0 -16.2288
progress: 0 4.919240100095999
grad: 1.0 2.0 -1.478624
grad: 2.0 4.0 -5.796206079999999
grad: 3.0 6.0 -11.998146585599997
progress: 1 2.688769240265834
grad: 1.0 2.0 -1.093164466688
grad: 2.0 4.0 -4.285204709416961
grad: 3.0 6.0 -8.87037374849311
progress: 2 1.4696334962911515
grad: 1.0 2.0 -0.8081896081960389
grad: 2.0 4.0 -3.1681032641284723
grad: 3.0 6.0 -6.557973756745939
progress: 3 0.8032755585999681
grad: 1.0 2.0 -0.59750427561463
grad: 2.0 4.0 -2.3422167604093502
grad: 3.0 6.0 -4.848388694047353
progress: 4 0.43905614881022015
grad: 1.0 2.0 -0.44174208101320334
grad: 2.0 4.0 -1.7316289575717576
grad: 3.0 6.0 -3.584471942173538
progress: 5 0.2399802903801062
grad: 1.0 2.0 -0.3265852213980338
grad: 2.0 4.0 -1.2802140678802925
grad: 3.0 6.0 -2.650043120512205
progress: 6 0.1311689630744999
grad: 1.0 2.0 -0.241448373202223
grad: 2.0 4.0 -0.946477622952715
grad: 3.0 6.0 -1.9592086795121197
progress: 7 0.07169462478267678
grad: 1.0 2.0 -0.17850567968888198
grad: 2.0 4.0 -0.6997422643804168
grad: 3.0 6.0 -1.4484664872674653
progress: 8 0.03918700813247573
grad: 1.0 2.0 -0.13197139106214673
grad: 2.0 4.0 -0.5173278529636143
grad: 3.0 6.0 -1.0708686556346834
progress: 9 0.021418922423117836
predict (after training) 4 7.804863933862125

```

Output

(from gradient numeric computation)



```

# Before training
print("predict (before training)", 4, forward(4))

# Training loop
for epoch in range(10):
    for x, y in zip(x_data, y_data):
        grad = gradient(x, y)
        w = w - 0.01 * grad
        print("\tgrad: ", x, y, grad)
        l = loss(x, y)

    print("progress:", epoch, l)

# After training
print("predict (after training)", 4, forward(4))

```

Exercise 3-1: compute gradient

$$\hat{y} = x^2 w_2 + x w_1 + b$$

$$loss = (\hat{y} - y)^2$$

$$\frac{\partial loss}{\partial w_1} = ?$$

$$\frac{\partial loss}{\partial w_2} = ?$$

Exercise 3-2: implement

$$\hat{y} = x^2 w_2 + x w_1 + b$$

$$loss = (\hat{y} - y)^2$$

$$\frac{\partial loss}{\partial w_1} = ?$$

$$\frac{\partial loss}{\partial w_2} = ?$$



Lecture 4: Back-propagation