浙江工业大学 2018/2019（一）期末试卷
《复变函数与积分变换》

《Final Exam for 'Functions of a Complex Variable and Integral Transforms'》

Class:____________ Name (in Chinese):____________ Student ID:____________

Please write in English! Good luck!

Question	1	2	3	4	5	6	7	8	Total
Score									

1. Choose only one correct answer to each queation （3 points each, total 15 points）

Score	

(1) $f(z) = u(x, y) + iv(x, y)$ is continuous at $z_0 = x_0 + iy_0$ if only if ( )

(A) $u(x, y)$ is continuous at (x_0, y_0)

(B) $u(x, y)$ and $v(x, y)$ are both continuous at (x_0, y_0)

(C) $v(x, y)$ is continuous at (x_0, y_0)

(D) $u(x, y) + v(x, y)$ is continuous at (x_0, y_0)

(2) If $f(z) = u + iv$ is analytic in domain D, which one of the followings is not correct? ( )

(A) $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

(B) $f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$

(C) $f'(z) = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}$

(D) $f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

(3) If $C : |z + 1| = 1$ in counterclockwise oriented, then $\oint_C \frac{dz}{(z-1)^3(z+1)^3} =$ ( )

(A) $\frac{3\pi}{8}i$ (B) $-\frac{3\pi}{8}i$ (C) $\frac{3\pi}{4}i$ (D) $-\frac{3\pi}{4}i$

(4) $z = 2$ is _____ of $e^{\frac{1}{z-2}}$ ( )

(A) analytic point

(B) removable singularity

(C) pole

(D) essential singularity

(5) Let $\mathscr{F}[f(t)] = F(\omega)$, then which of the followings is not correct? ( )

(A) $\mathscr{F}[f(t) * f(t)] = [F(\omega)]^2$

(B) $\delta(-t) = \delta(t)$

(C) $\int_{-\infty}^{+\infty} e^{-i\omega t}dt = 2\pi\delta(\omega)$

(D) $\mathscr{F}[f(at)] = \frac{1}{a}F(\frac{w}{a}), a \neq 0, a \in \mathbb{R}$

2. Fill in each blank (3 points each, total 30 points)

Score	

(1) Determine the real constant $k =$_____ such that $f(z) = e^x(cosky + isinky)$ is analytic.

(2) The trigonometric and exponential forms of $z = \sin\frac{\pi}{5} + i\cos\frac{\pi}{5}$ are __________ and __________.

(3) $\sqrt[4]{1+i} =$ ________________ (in exponential form).

(4) Let C be the upper half of the circle $|z| = 1$ joining 1 to -1, then $\int_C \frac{1}{\sqrt[3]{z}} dz =$_____________.

(5) The power series of $\frac{1}{(1-z)^2}$ is _____________.

(6) Let $f(z) = \frac{e^{\frac{1}{z}}}{1-z}$, then $\mathrm{Res}[f(z), 0] =$_____________.

(7) The Fourier transform of $f(t) = \delta(t-1)(t-2)^2 \cos t$ is _____________.

(8) What kind of isolated singular point of $f(z) = \frac{z^4}{1+z^4}$ at $z = \infty$?_____________.

What kind of isolated singular point of $f(z) = \cot z - \frac{1}{z}$ at $z = 0$?_____________.

(Please fill in removable, pole and its order, essential singularity, or not isolated singularity).

3. (7 points)

Score	

Verify that

$$f(z) = \begin{cases} 0, & z = 0 \\ \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & z \neq 0 \end{cases}$$

satisfies C-R equations at the point $z = 0$, but it is not analytic.

4. (5 points each)

Score	

Compute Laurent series of function $f(z) = \dfrac{1}{(z-1)(z-3)}$ in the assigned annulus:

(1) $0 < |z-1| < 1$

(2) $2 < |z-3| < +\infty$

5. (6 points each)

Score	

Find Laplace transform of the following $f(t)$:

(1) $f(t) = t\sin kt$

(2) $f(t) = \int_0^t \tau e^{-2\tau} \sin 3\tau d\tau$

6. (6 points each)

Score	

Evaluate the following integrals:

(1) $\displaystyle\int_0^{2\pi} \frac{d\theta}{a+\cos\theta} \qquad (a>1).$

(2) $\displaystyle\int_{-\infty}^{+\infty} \frac{(\cos 3x)^2}{x^2+4x+5}dx$

7. (6 points)

Score | |

Solve the following differential equation using Laplace transform:
$y'' + 3y' + y = 3\cos t$, $y(0) = 0$, $y'(0) = 1$.

8. (8 points)

Score | |

Given $\mathscr{F}[e^{-\beta t^2}] = \sqrt{\frac{\pi}{\beta}} e^{-\frac{\omega^2}{4\beta}}, \beta > 0$. Let $f(t) = te^{-t^2}$, then

(1) Find the Fourier transform of $f(t)$.
(2) Prove that $\int_0^{+\infty} \omega e^{-\frac{\omega^2}{4}} \sin \omega t d\omega = 2\sqrt{\pi} t e^{-t^2}$.