浙江工业大学2019/2020（一）期末试卷
《复变函数与积分变换》

《Final Exam for ‘Functions of a Complex Variable and Integral Transforms’》

Class:__________ Name (in Chinese):____________ Student ID:____________

Please write in English! Good luck!

Question	1	2	3	4	5	6	7	8	Total
Score									

1. Choose only one correct answer to each question （3 points each, total 15 points）

(1) If $z = \frac{1}{\sqrt{2}}(1 - i)$, then $z^{100} =$ (   )

(A) -i (B) i (C) 1 (D) -1

(2) Let $f(t) = te^{iw_0 t}$, then $\mathscr{F}[f(t)] =$ (   )

(A) $2\pi\delta'(w - w_0)$ (B) $2\pi\delta'(w + w_0)$ (C) $2\pi i\delta'(w - w_0)$ (D) $2\pi i\delta'(w + w_0)$

(3) If $C : |z - 1| = \frac{1}{2}$ in counterclockwise oriented, then $\oint_C \frac{\cos z}{z^2} dz =$ (   )

(A) $2\pi i$ (B) 0 (C) πi (D) 1

(4) Let $F(s) = \frac{5}{(s+1)^2+25}$, then $f(t) = \mathscr{L}^{-1}[F(s)] =$ (   )

(A) $e^{-t}\cos 5t$ (B) $e^{-t}\sin 5t$ (C) $e^{t}\cos 5t$ (D) $e^{t}\sin 5t$

(5) Which of the followings is not correct? (   )

(A) $\delta(-t) = \delta(t)$ (B) $\mathscr{F}[u(t)] = \frac{1}{w} + \pi\delta(w)$

(C) $\mathscr{F}[e^{-at}] = \dfrac{1}{a + iw}$ (D) $\mathscr{F}[sgnt] = \frac{2}{iw}$

2. Fill in each blank (3 points each, total 30 points)

(1) $\sqrt[3]{-2 + 2i} =$ ___________.

(2) $\lim\limits_{z\to 1+i} \frac{\bar{z}}{z} =$ ___________.

(3) Let $C : |z| = 1$ in counterclockwise oriented, then when $|a| < 1$, $\oint_C \frac{\sin z}{(z - a)^2} dz =$_____________;

when $|a| > 1$, $\oint_C \frac{\sin z}{(z - a)^2} dz =$_____________.

(4) The radius of the convergence of $\sum\limits_{n=0}^{\infty} \frac{n}{2^n} z^n$ is _____________.

(5) The Taylor series of $\dfrac{1}{1+z}$ around $z = 0$ is ______________.

(6) Let $f(z) = \dfrac{z^2}{1+z^2}$, then $\mathrm{Res}[f(z), -i] =$___________, $\mathrm{Res}[f(z), i] =$___________.

(7) What kind of isolated singular point of $\frac{\sin z - z}{z^3}$ at $z = 0$?______________.

What kind of isolated singular point of $f(z) = \dfrac{\sin z}{z^3}$ at $z = 0$?______________.

(Please fill in removable singularity, pole and its order, essential singularity, or not isolated singularity).

3. (7 points)

Given a function $v(x, y) = \arctan \dfrac{y}{x}$ $(x > 0)$, find an analytic function $f(z) = u(x, y) + iv(x, y)$.

4. (4 points each)

Compute Laurent series of function $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the assigned annulus:

(1) $0 < |z-1| < 1$

(2) $1 < |z-1| < +\infty$

(3) $0 < |z-2| < 1$

5. (5 points each)

Find Laplace transform of the following $f(t)$:

(1) $f(t) = e^{-at} \sin kt$

(2) $f(t) = t \cos t$.

6. (6 points each)

Evaluate the following integrals:

(1) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2}$

(2) $\displaystyle\int_{0}^{2\pi} \frac{d\theta}{2 + \cos\theta}$.

7. (7 points)

Solve the following differential equation using Laplace transform:
$y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$.

8. (7 points) Suppose that $f(z)$ and $g(z)$ are analytic at $z = a$ and $f(a) \neq 0$. If $g(z)$ has a zero of order 2 at $z = a$, then prove that $\text{Res}[\frac{f(z)}{g(z)}, a] = \dfrac{6f'(a)g''(a) - 2f(a)g'''(a)}{3[g''(a)]^2}$.