浙江工业大学 2020/2021(一) 期末试卷
《复变函数与积分变换》
《Final Exam for 'Functions of a Complex Variable and Integral Transforms'》

Class: ______________ Name (in Chinese): ______________ Student ID: ______________

Please write in English! Good luck!

1. Choose only one correct answer to each question (3 points each, total 15 points)

(1) If $C: |z| = 1$ in counterclockwise oriented, then $\oint_C \bar{z} dz =$ ______________

(A) $4\pi i$ (B) $-4\pi i$ (C) $2\pi i$ (D) $-2\pi i$

(2) $z = 1$ is a zero of order m of an analytic function $f(z)$, then $\text{Res}\left[\frac{f'(z)}{f(z)}, 1\right] =$ ______________

(A) m (B) $-m$ (C) $m-1$ (D) $-(m-1)$

(3) Let $f(t) = e^{-\beta|t|} (\beta > 0)$, then $\mathscr{F}[f(t)] =$ ______________

(A) $\dfrac{2\omega}{\beta^2+\omega^2}$ (B) $\dfrac{2\beta}{\beta^2+\omega^2}$ (C) $\dfrac{2\omega}{\beta^2-\omega^2}$ (D) $\dfrac{2\beta}{\beta^2-\omega^2}$

(4) The radius of the convergence of series $\sum_{n=1}^{+\infty} \frac{n^n}{2^n n!}(z-2)^2$ is ______________

(A) 2 (B) $\dfrac{1}{2}$ (C) $\dfrac{e}{2}$ (D) $\dfrac{2}{e}$

(5) If $f(z) = a\ln(x^2+y^2) + i\arctan\frac{y}{x}$ is analytic at $x > 0$, then $a =$ ______________

(A) $\dfrac{1}{2}$ (B) $-\dfrac{1}{2}$ (C) 2 (D) -2

2. Fill in each blank (3 points each, total 30 points)

(1) $\left(\frac{1-\sqrt{3}i}{2}\right)^3 =$ ______________.

(2) Let $z = \frac{\sqrt{2}}{2}(1+i)$, then $\sqrt{z} =$ ______________.

(3) Let $C: |z| = 1$ in counterclockwise oriented, then when $|a| < 1$, $\oint_C \frac{e^z}{(z-a)^2} dz =$ ______________;

when $|a| > 1$, $\oint_C \frac{e^z}{(z-a)^2} dz =$ ______________.

(4) Let $z = e^{i\theta}$, then $|z-1| =$ ______________.

(5) Let $f(z) = \frac{1}{z^2 - z^4}$, then $\text{Res}[f(z), 0] =$ ______________, $\text{Res}[f(z), -1] =$ ______________.

(6) The Fourier transform of $f(t) = \delta(t)e^{3t+1}$ is ______________.

(7) Let $f_1(t) = \cos t$ and $f_2(t) = \delta(t+1)$, then $f_1 * f_2 =$ ______________.

(8) The Taylor expansion of $f(z)=\dfrac{1}{z^2}$ around $z_0=-1$ is ________________.

3. (12 points) Compute the integral $\oint_C \dfrac{dz}{(z-2)^2 z}$, where

(1) $C: |z-3|=2$

(2) $C: |z+1|=2$

(3) $C: |z-1|=3$

4. (12 points) Compute Laurent series of function $f(z)=\dfrac{-1}{(z-1)(z-2)}$ in the assigned annulus:

(1) $|z|<1$

(2) $1<|z|<2$

(3) $2<|z|<+\infty$

5. (6 points) Evaluate $f*g$ where

$$f(t)=\begin{cases} e^{-t}, & t\geq 0 \\ 0, & t<0 \end{cases}, g(t)=\begin{cases} e^{-3t}, & t\geq 0 \\ 0, & t<0 \end{cases}$$

6. (12 points) Evaluate the following integrals:

(1) $\displaystyle\int_0^{\pi} \frac{d\theta}{2\cos\theta+3}$

(2) $\displaystyle\int_0^{+\infty} \frac{x\sin x}{1+x^2}dx$

7. (8 points) Solve the following differential equation using Laplace transform:

$$y''+4y'+3y=0 \text{ for } y(t), t\geq 0, y'(0)=1, y(0)=0.$$

8. (5 points) Let $f(z)=\cos\left(z+\dfrac{1}{z}\right)$, its Laurent expansion at $z=0$ is $\displaystyle\sum_{n=-\infty}^{+\infty} c_n z^n$.

Prove that $c_n=\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\cos(2\cos\theta)\cos n\theta d\theta$.