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IMPLICIT CROWDS:

OPTIMIZATION INTEGRATOR FOR ROBUST CROWD SIMULATION

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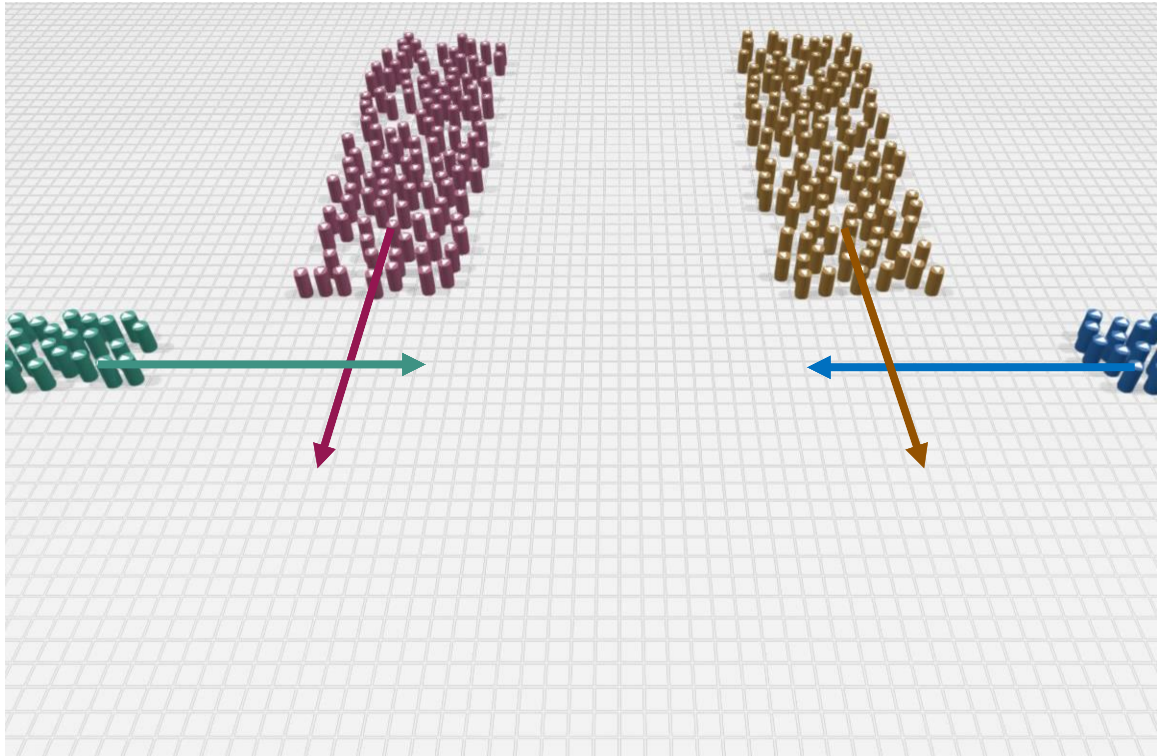
COLLISION AVOIDANCE IN CROWDS

Given desired velocities, how should agents navigate around each other?



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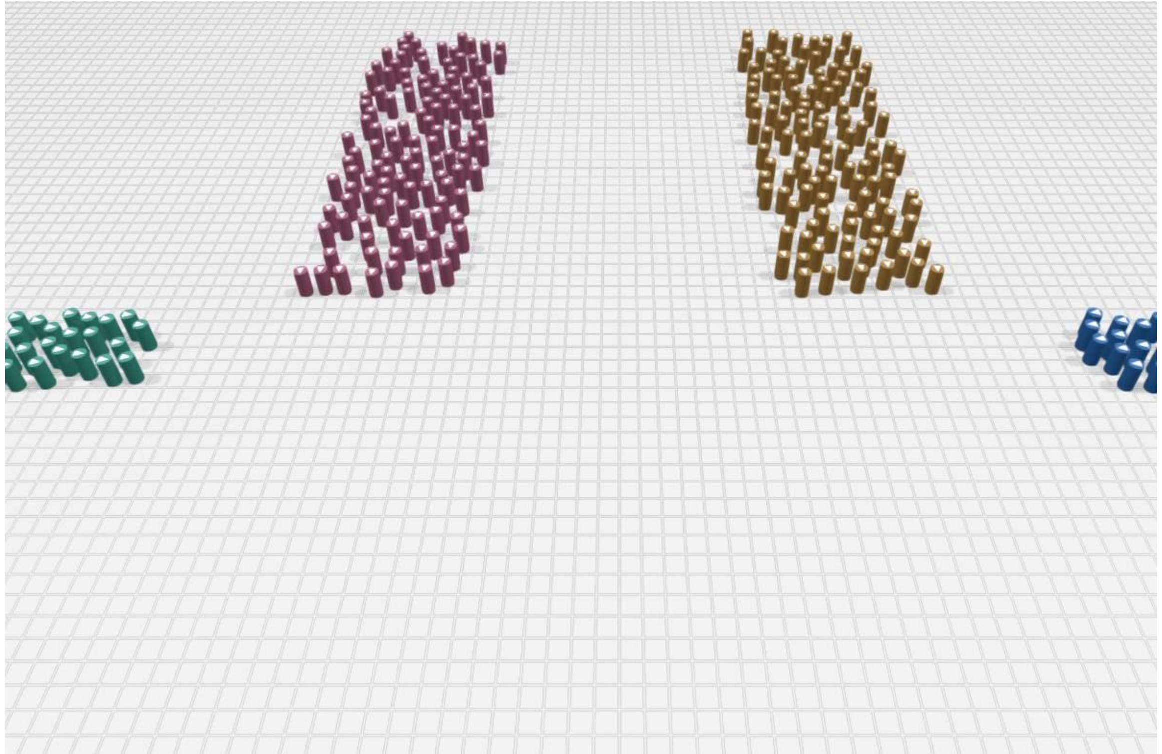
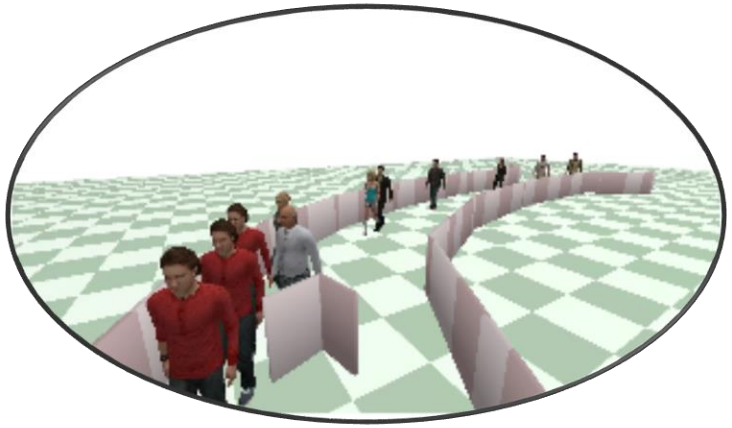


LOCAL COLLISION AVOIDANCE

Adding Realism

- Vision-based approaches [Ondřej et al. 2010, Kapadia et al. 2012, Hughes et al. 2015, Dutra et al. 2017]
- Probabilistic approaches [Wolinski et al. 2016]
- …..

[Yu et al. 2012]



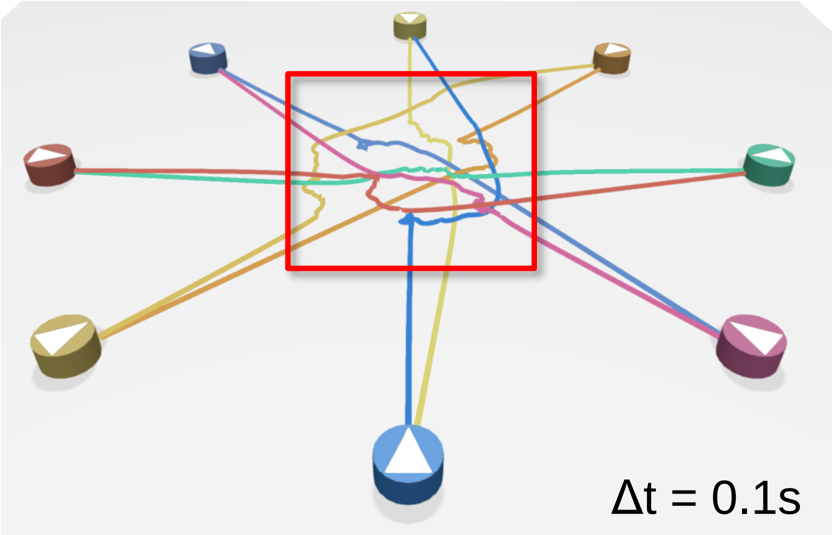
[Wolinski et al. 2016]

[Kapadia et al. 2012]

LOCAL COLLISION AVOIDANCE

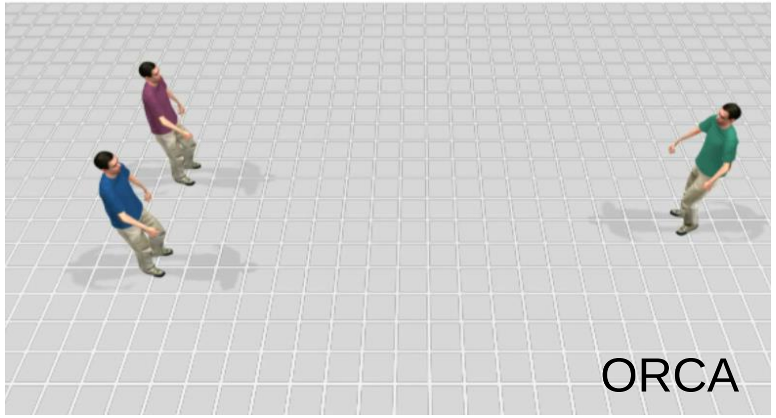
Force-based methods [Reynolds 1987, 1999; Helbing et al. 2000; Pelechano et al. 2007, …]

- Require very small time steps for stability



Velocity-based methods [van den Berg et al. 2008, 2011; Pettré et al 2011, …]

- Overly conservative behavior



DESIDERATA

We seek a generic technique for multi-agent navigation that

- guarantees collision-free motion
- is robust to variations in scenario, density, time step
- exhibits high-fidelity behavior
- can update at footstep rates (0.3-0.5 s)

OUR CONTRIBUTIONS

1. General form of collision avoidance behaviors

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{\partial R(\mathbf{x}, \mathbf{v})}{\partial \mathbf{v}}$$

   supporting optimization-based implicit integration

2. Application to state-of-the-art power law model

$$R(\mathbf{x}, \mathbf{v}) \propto \tau(\mathbf{x}, \mathbf{v})^{-p}$$

   for practical crowd simulations

I. OPTIMIZATION INTEGRATOR FOR CROWDS

IMPLICIT INTEGRATION

- $\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \mathbf{v}\end{bmatrix} = \begin{bmatrix}\mathbf{v}\\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x},\mathbf{v})\end{bmatrix}$

[Baraff and Witkin, 1998]

IMPLICIT INTEGRATION

- $$\frac{d}{dt}\begin{bmatrix}\mathbf{x}\\ \mathbf{v}\end{bmatrix} = \begin{bmatrix}\mathbf{v}\\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x},\mathbf{v})\end{bmatrix} \Leftrightarrow$$

$$\begin{aligned}\mathbf{x}^{n+1} - \mathbf{x}^{n} &= \mathbf{v}^{n+1}\Delta t,\\ \mathbf{M}(\mathbf{v}^{n+1} - \mathbf{v}^{n}) &= \mathbf{f}^{n+1}\Delta t\end{aligned}$$

- Unconditionally stable, but
- Need to solve a non linear system
- Slow (but we can use large time steps)

[Kaufman et al. 2014]

OPTIMIZATION INTEGRATORS

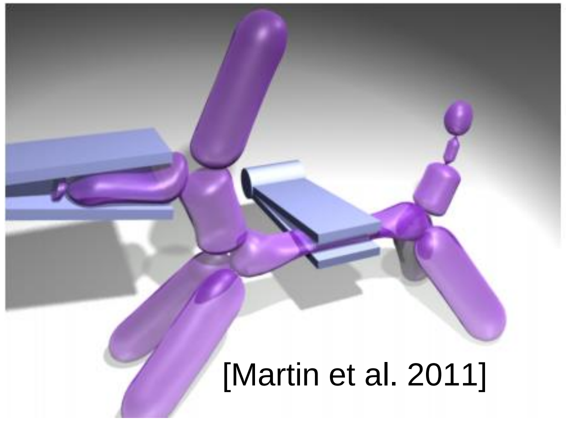
As long as forces are *conservative*:

$$\mathbf{f}(\mathbf{x}) = -\frac{\mathrm{d}U(\mathbf{x})}{\mathrm{d}\mathbf{x}},$$

we can express backward Euler in optimization form [Martin et al. 2011; Gast et al. 2015]:

$$\mathbf{x}^{n+1} = \arg\min_{\mathbf{x}} \left(\frac{1}{2\Delta t^2} \|\mathbf{x} - \tilde{\mathbf{x}}\|_{\mathbf{M}}^2 + U(\mathbf{x}) \right)$$

[Bouaziz et al. 2014]



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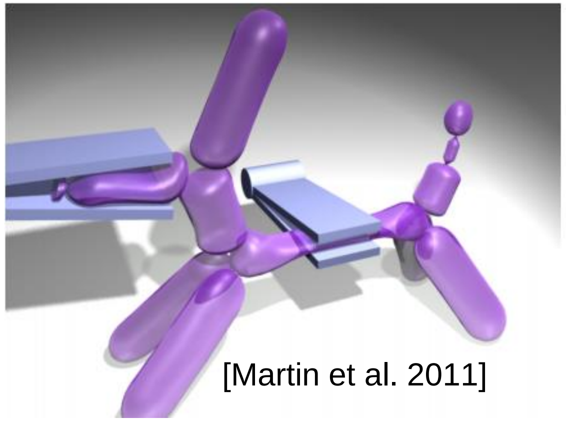
Interpretation: tradeoff between *maintaining velocity* and *reducing potential energy*

[Bouaziz et al. 2014]



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Why this is good:

- Simple and fast algorithms (e.g. gradient descent, Gauss-Seidel) can be given guarantees
- Highly nonlinear forces can be used without linearization
- Lots of recent advances in optimization for data mining, machine learning, image processing, …

[Fratarcangeli et al. 2016]

NON-CONSERVATIVE FORCES

Conservative potentials $U(\mathbf{x})$ can only model *position-dependent forces*

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- Humans anticipate
  - People anticipate future trajectories of others [Cutting et al. 2005, Olivier et al. 2012; Karamouzas et al. 2014]
  - Brains have special neurons for estimating collisions [Gabbiani 2002]

Crowd forces depend both on positions and velocities!

ANTICIPATORY FORCES

Hypothesis: Multi-agent interactions can be expressed as

$$\mathbf{f}(\mathbf{x}, \mathbf{v}) = -\frac{\partial R(\mathbf{x}, \mathbf{v})}{\partial \mathbf{v}}$$

where R is an *anticipatory potential* that drives agents away from high-cost velocities

(This is analogous to dissipation potentials [Goldstein 1980] in classical mechanics)

OPTIMIZATION INTEGRATOR FOR NON-CONSERVATIVE FORCES

Anticipatory forces

$$\mathbf{v}^{n+1} = \arg\min_{\mathbf{v}} \frac{1}{2} \|\mathbf{v} - \tilde{\mathbf{v}}\|_{\mathbf{M}}^2 + R(\mathbf{x} + \mathbf{v}\Delta t, \mathbf{v})\Delta t$$

OPTIMIZATION INTEGRATOR FOR NON-CONSERVATIVE FORCES

Anticipatory + conservative forces

$$\mathbf{v}^{n+1} = \arg\min_{\mathbf{v}} \frac{1}{2} \|\mathbf{v} - \tilde{\mathbf{v}}\|_{\mathbf{M}}^2 + U(\mathbf{x} + \mathbf{v}\Delta t) + R(\mathbf{x} + \mathbf{v}\Delta t, \mathbf{v})\Delta t$$

- First-order accurate
- $U(\cdot) + R(\cdot)\Delta t$ is analogous to the "effective interaction potential" in symplectic integrators [Kane et al. 2000; Kharevych et al. 2006]
- Simple interpretation: tradeoff between maintaining velocity, reducing U, and reducing R

SOME EXISTING MODELS

Alignment behavior in boids [Reynolds 1987]:

$$\mathbf{f}_{ij} = -w\left(\left\|\mathbf{x}_{ij}\right\|\right)\mathbf{v}_{ij} \Leftrightarrow R_{ij} = w\left(\left\|\mathbf{x}_{ij}\right\|\right)\left\|\mathbf{v}_{ij}\right\|^2$$

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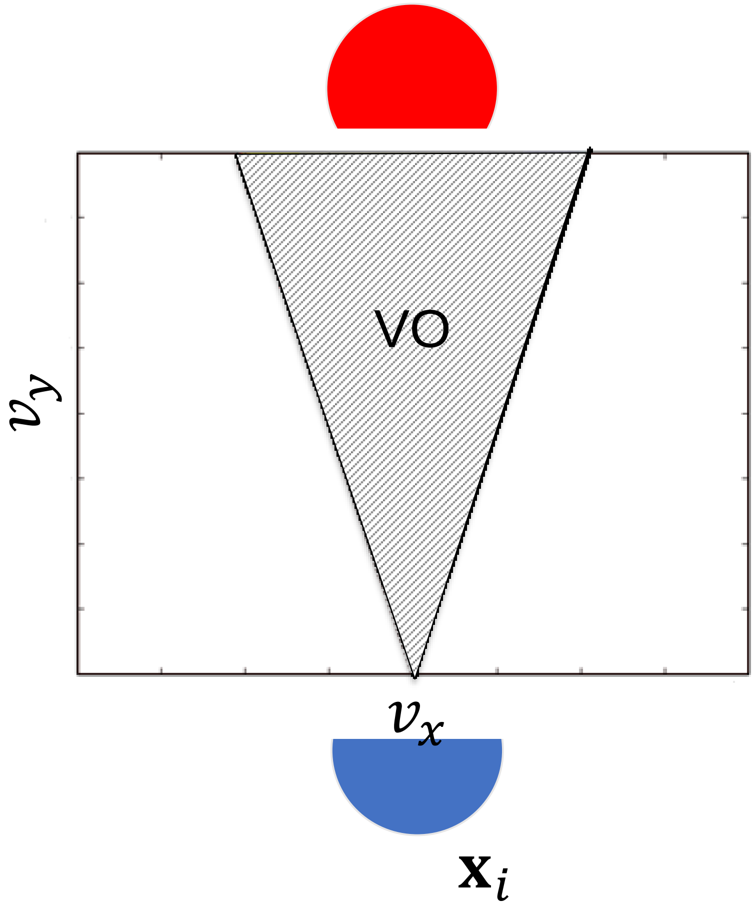
Velocity obstacles [Fiorini and Shiller 1998]:

$$\mathbf{v}_{ij} \notin \mathrm{VO}(\mathbf{x}_{ij}) \Leftrightarrow R_{ij} = \begin{cases} \infty \text{ if } \mathbf{v}_{ij} \in \mathrm{VO}(\mathbf{x}_{ij}) \\ 0 \text{ otherwise} \end{cases}$$



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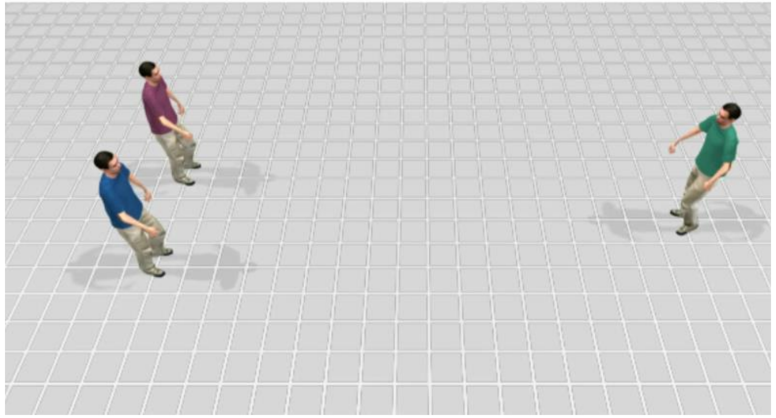

What is R_{ij} for humans?

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II. IMPLICIT CROWDS USING THE POWER-LAW MODEL

POWER-LAW MODEL [Karamouzas et al. 2014]

For each pair of agents:

- Compute time to collision $\tau(\mathbf{x}, \mathbf{v})$

Collisions occurs when

$$\left\|\mathbf{x}_{ij} + \mathbf{v}_{ij}\tau\right\| = \mathrm{r}_i + \mathrm{r}_j$$

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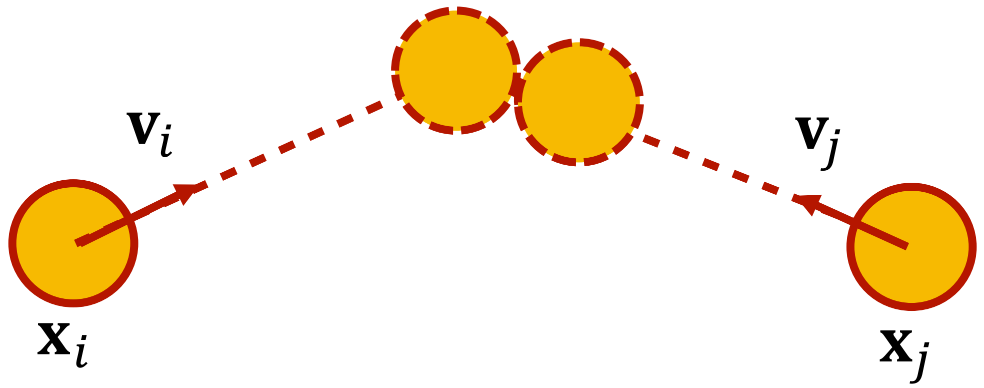

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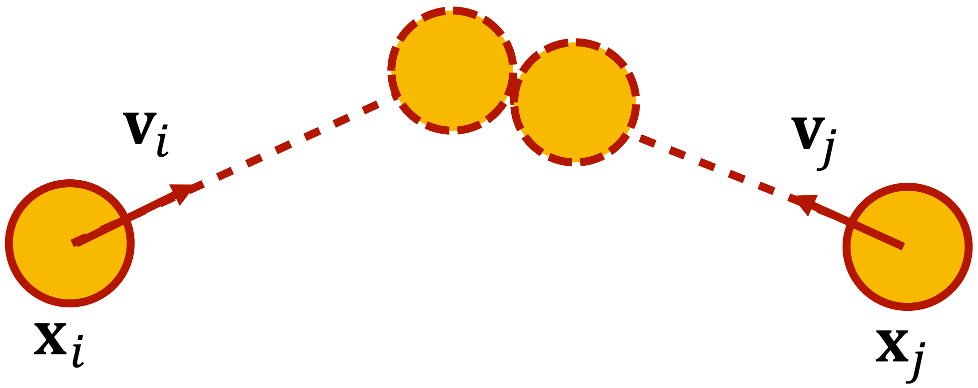

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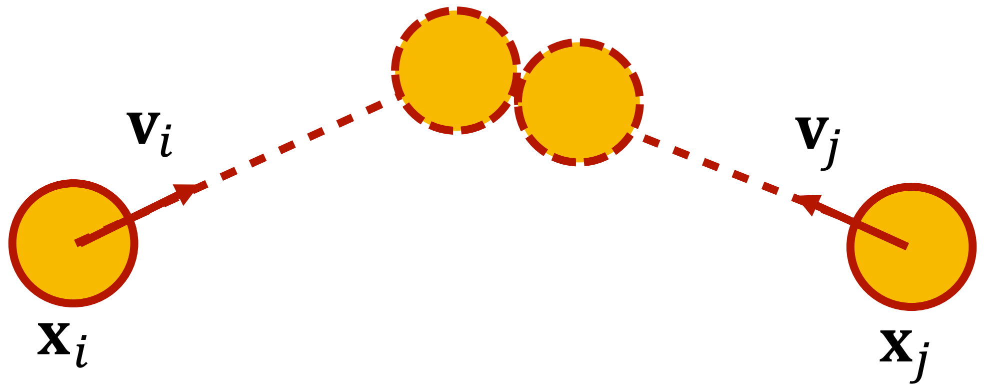
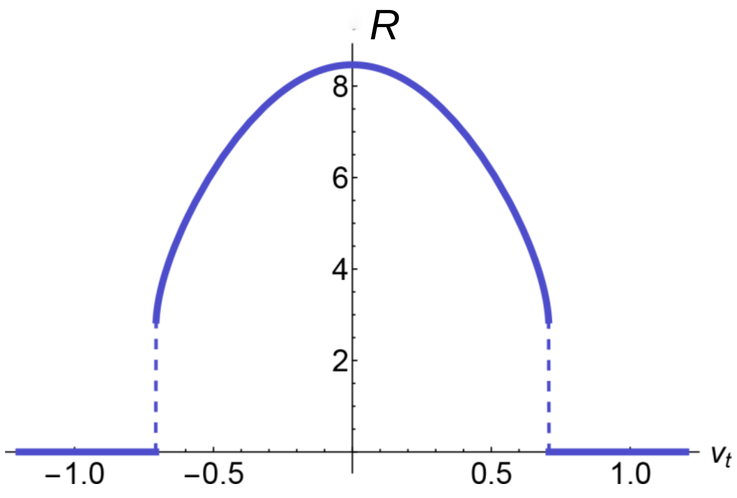


IMPLICIT POWER-LAW CROWDS

Problem: Apply power law potential to optimization-based backward Euler

Easy? Not quite…

- R is discontinuous at boundary of collision cone
- R becomes infinitely steep as agents graze past



Both phenomena cause numerical solvers to “get stuck”

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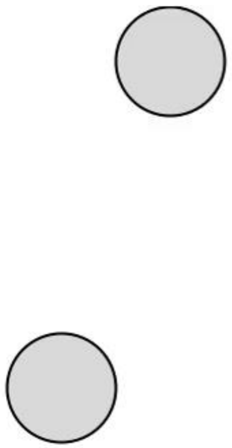

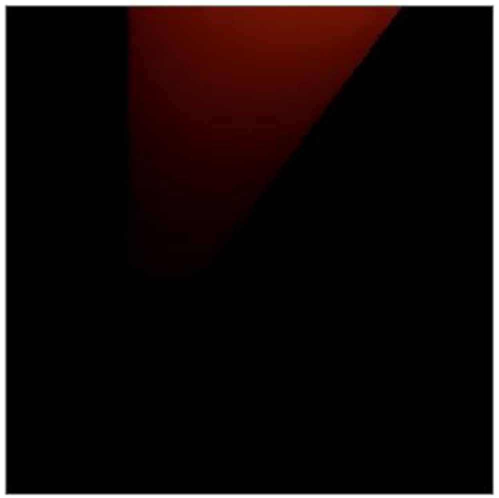

A CONTINUOUS TTC POTENTIAL

Discontinuity due to time to collision (finite if collision predicted, infinite if not)

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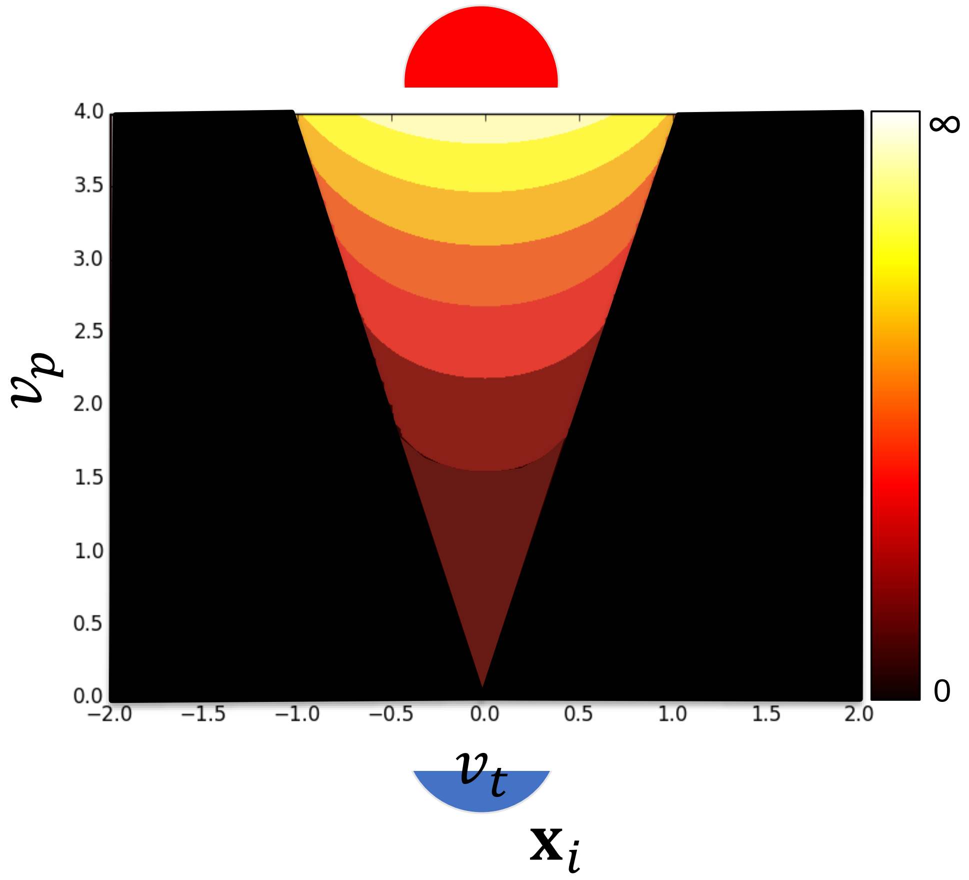

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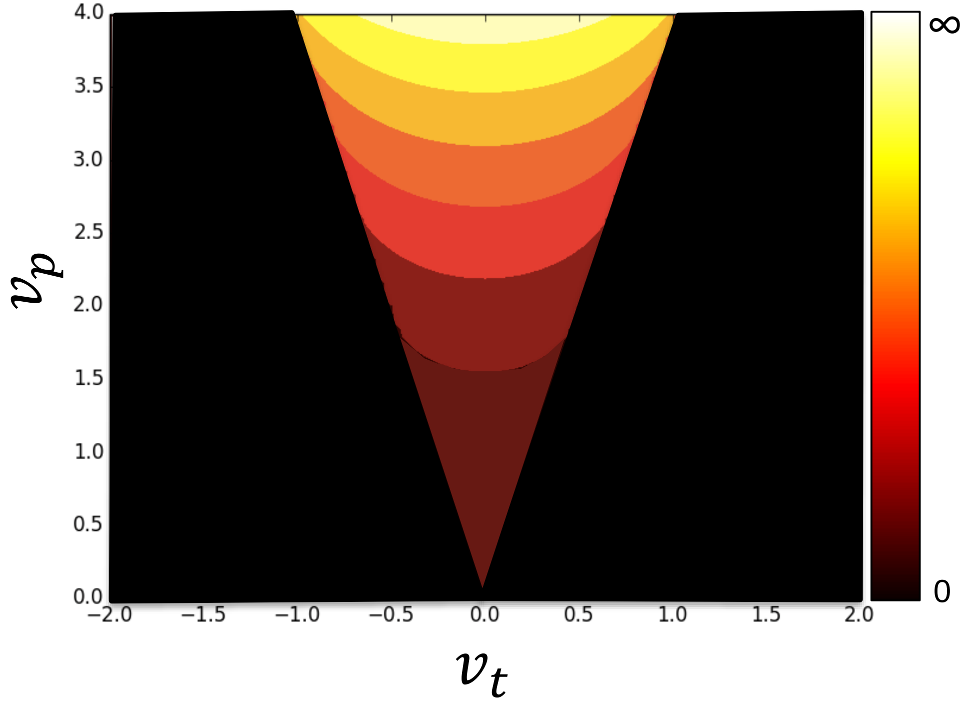
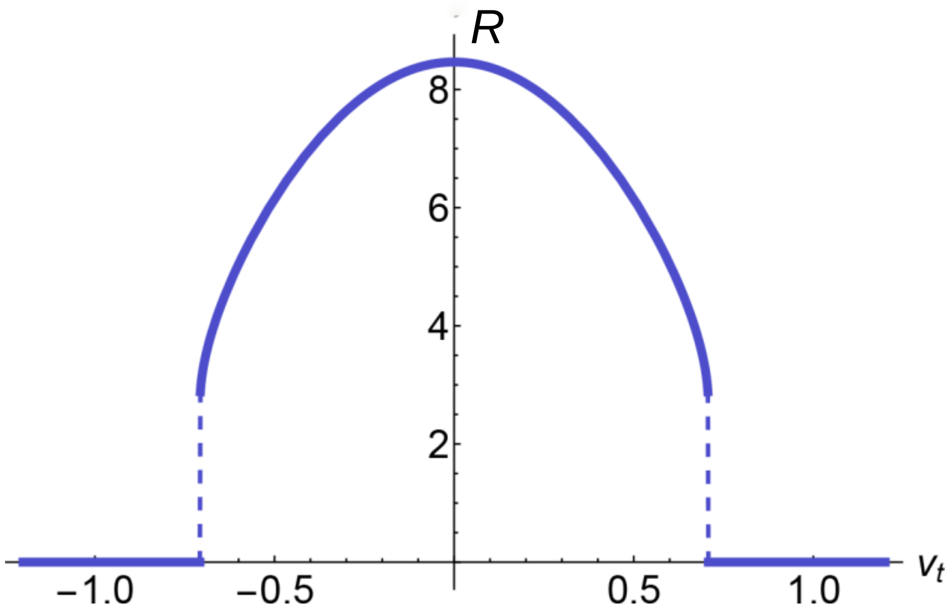


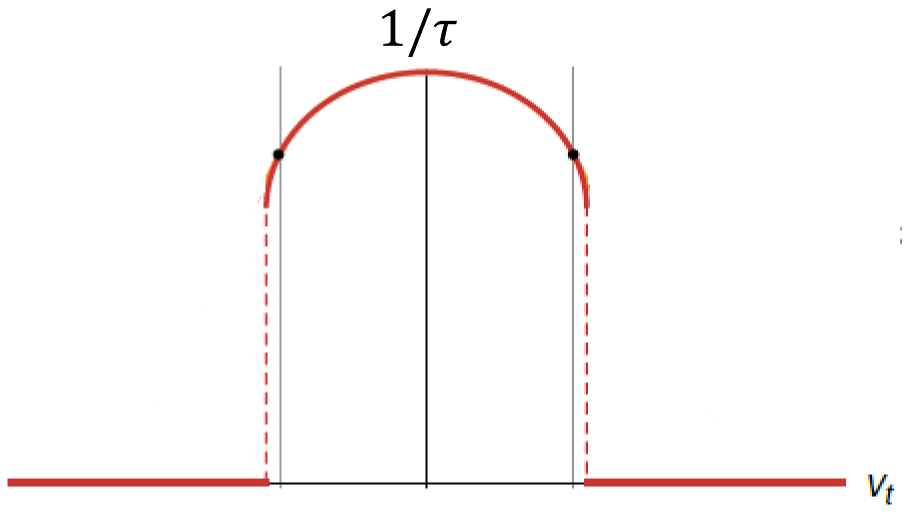


A CONTINUOUS TTC POTENTIAL

Solution:

Let's work with $\frac{1}{\tau}$ (or, "imminence" of collision)

- Replace it with a continuous approximation, e.g., by linear extrapolation



A CONTINUOUS TTC POTENTIAL

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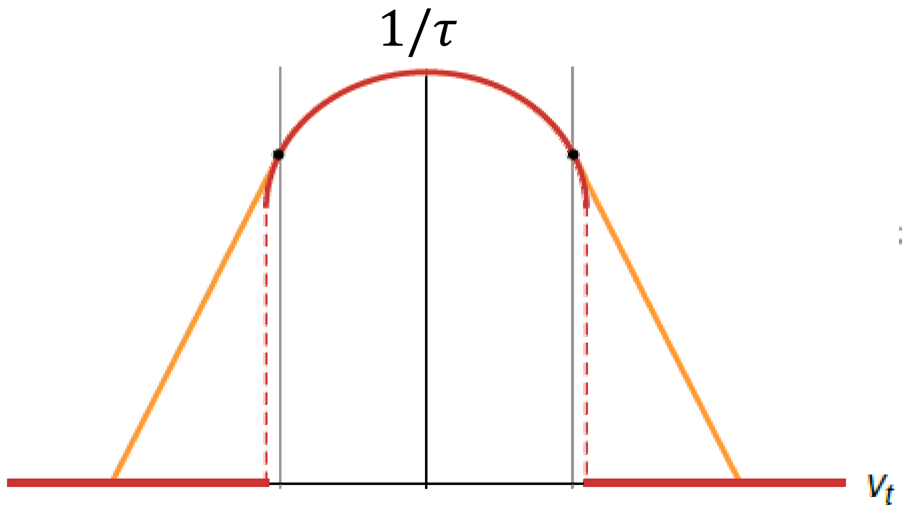
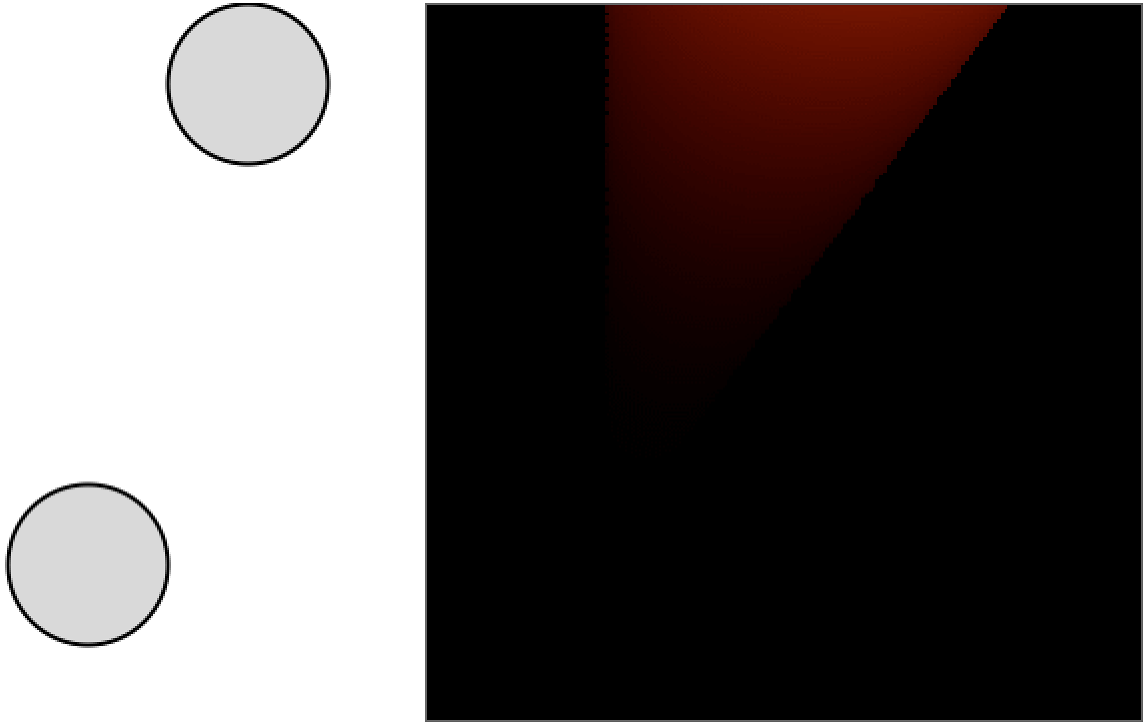
- R becomes C^{p-1}-smooth



MAINTAINING SEPARATION

Grazing trajectories make R badly behaved



MAINTAINING SEPARATION

Grazing trajectories make R badly behaved

- Add some distance-based repulsion $U_{ij} \propto \frac{1}{\|\mathbf{x}_{ij}\| - r_{ij}}$
- Continuous collision detection: replace distance $\|\mathbf{x}_{ij}\|$ with *minimum* distance over the time step

Alternative approach to repulsion: add uncertainty to time-to-collision computation [Forootaninia et al. 2017]

III. ANALYSIS AND RESULTS

THEORETICAL ANALYSIS

Implicit integration + continuous PowerLaw potential

- Guaranteed collision-free motion
- Smooth (C^2-continuous) trajectories

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Collision-free proof

- $\mathbf{v}^{n+1}$ minimizes $\frac{1}{2}\left\|\mathbf{v}^{n+1} - \tilde{\mathbf{v}}\right\|_{\mathbf{M}}^2 + U(\mathbf{x}^{n+1}) + R(\mathbf{x}^{n+1}, \mathbf{v}^{n+1})\Delta t$
- R is infinite for a colliding state. U is infinite for a tunneling step. So these cannot be minima.

COLLISION-FREE MOTION

- Comparisons to
  - ORCA [van den Berg et al. 2011] (representative velocity-based approach)
  - PowerLaw [Karamouzas et al. 2014] (non-continuous TTC + forward Euler)

	# Agents	# Obstacles	Roadmap	Density
Hallway	300	2	no	medium
Crossing	400	0	no	high
Random	500	0	no	low
Evacuation	1200	178	yes	very high
Blocks	2000	112	yes	low

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					Maximum collision-free Δt [ms]		
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Crossing	400	0	no	high	20	35	
Random	500	0	no	low	30	140	
Evacuation	1200	178	yes	very high	< 5	25	
Blocks	2000	112	yes	low	30	90	

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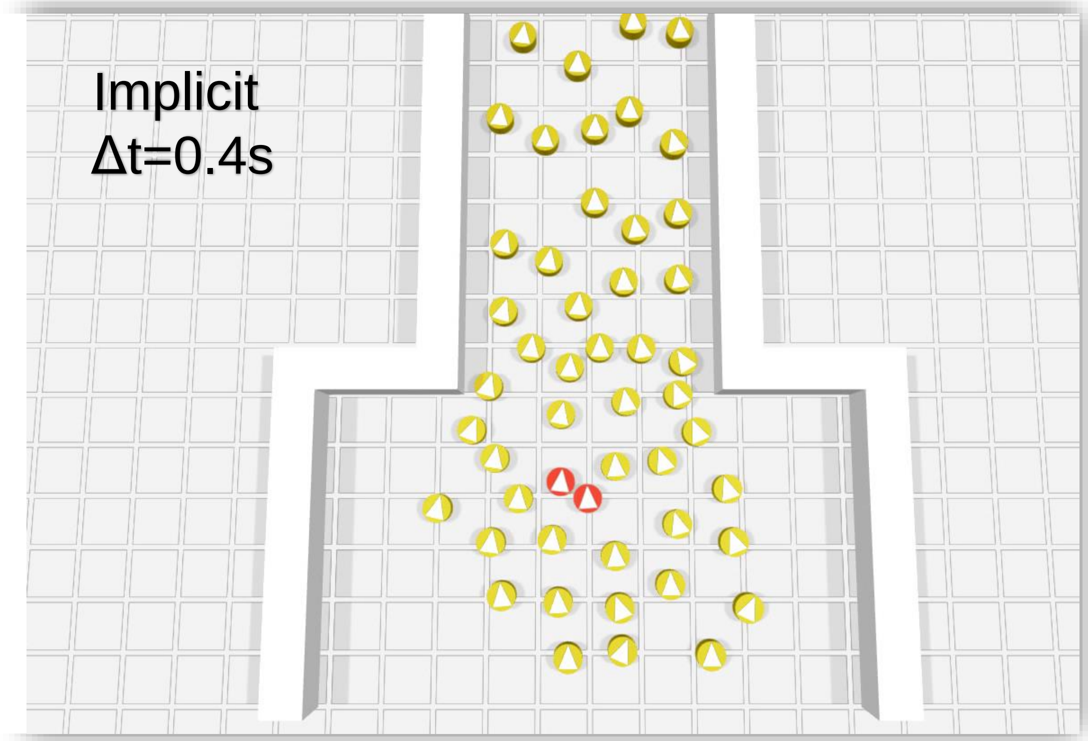
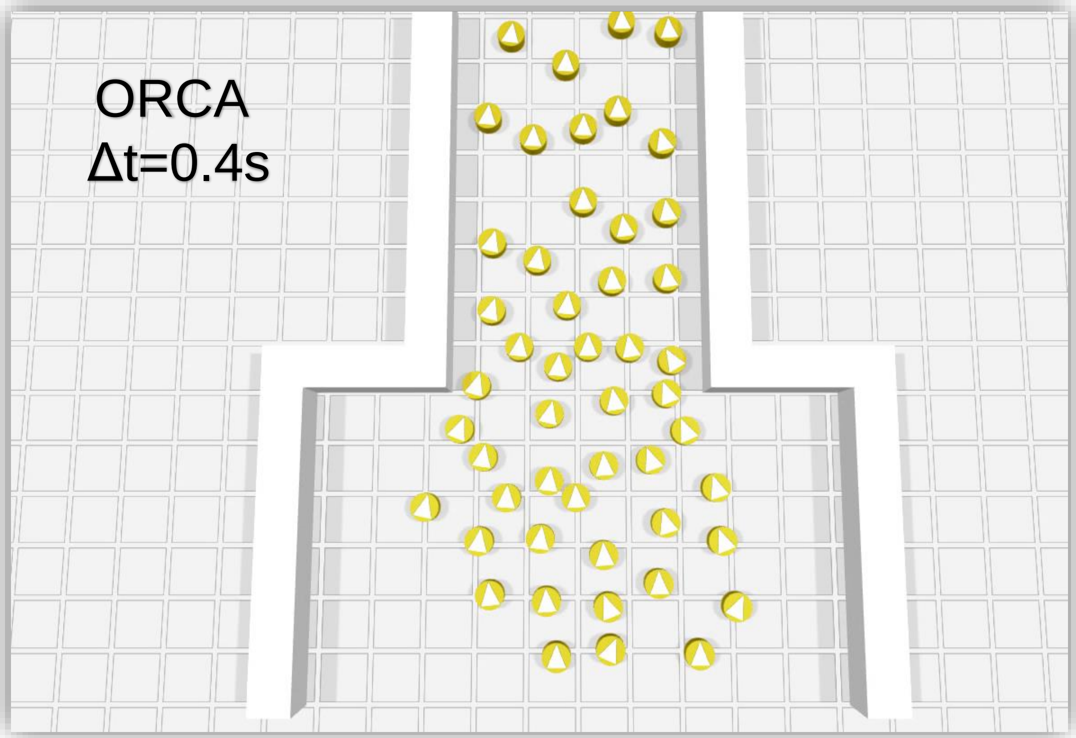
Evacuation	1200	178	yes	very high	< 5	25	> 1000
Blocks	2000	112	yes	low	30	90	> 1000

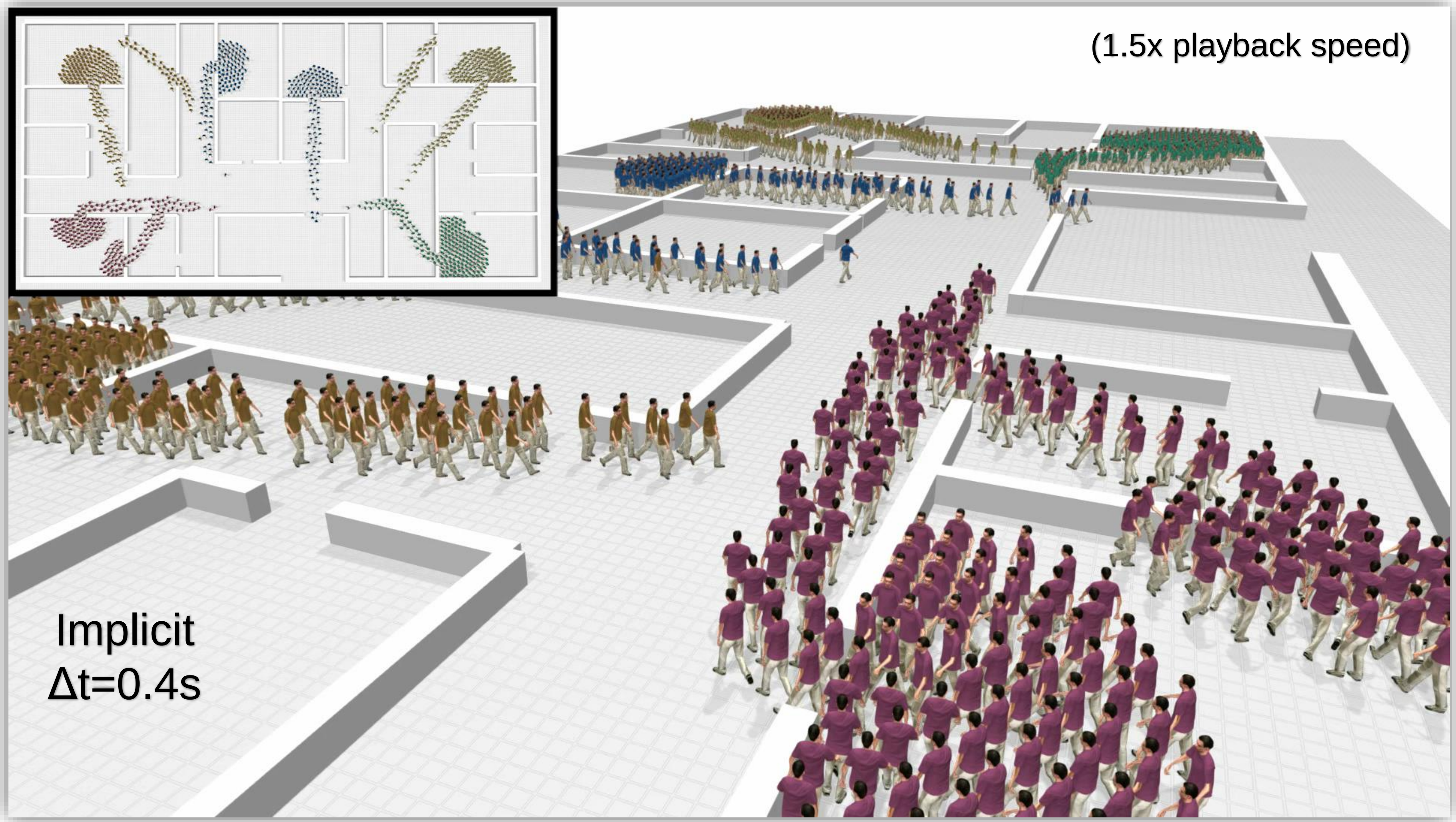
FIDELITY TO HUMAN DATA

Comparison to human crowds [Charalambous et al. 2014]



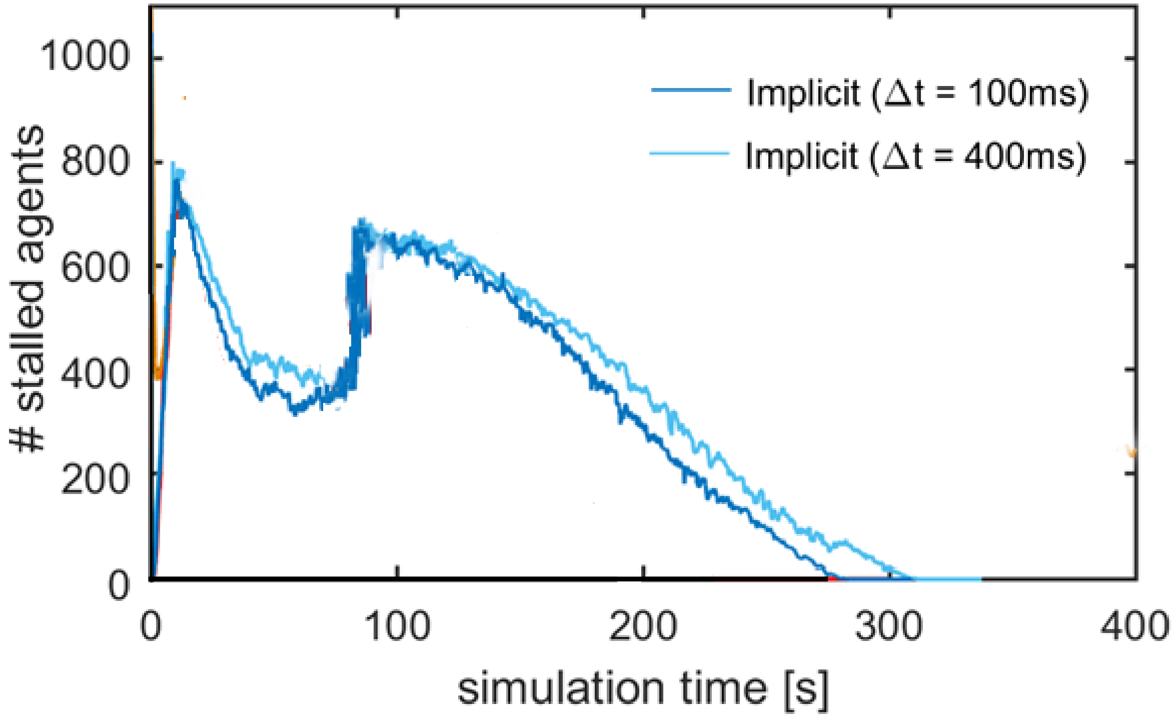


TIME STEP STABILITY

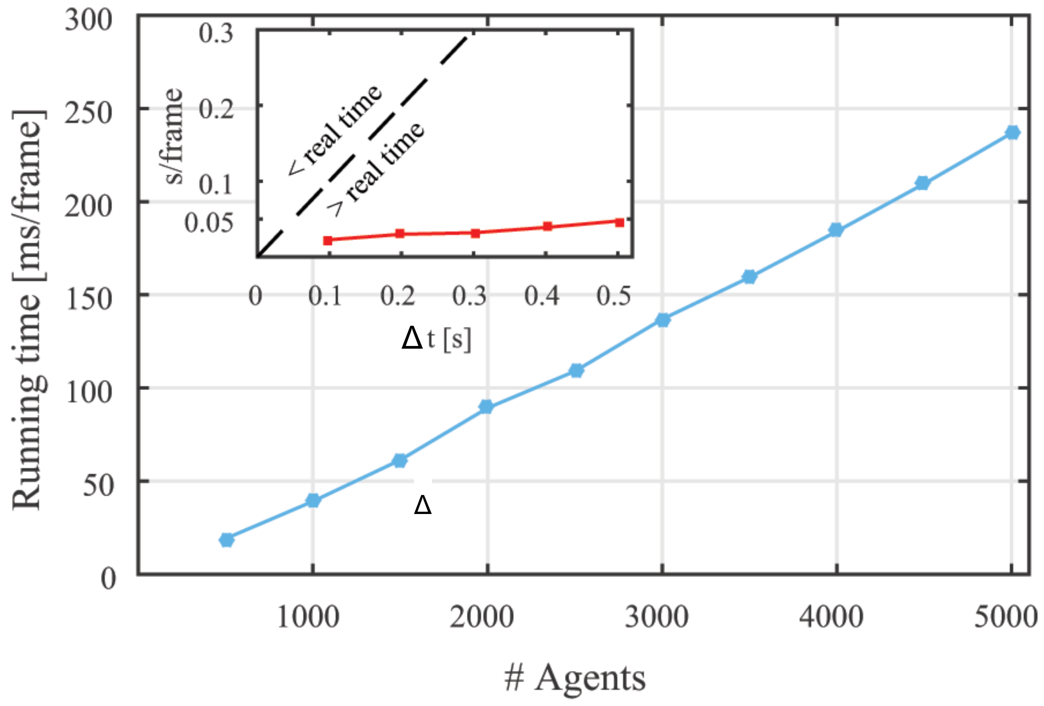


TIME STEP STABILITY

Motion doesn’t change significantly with time step



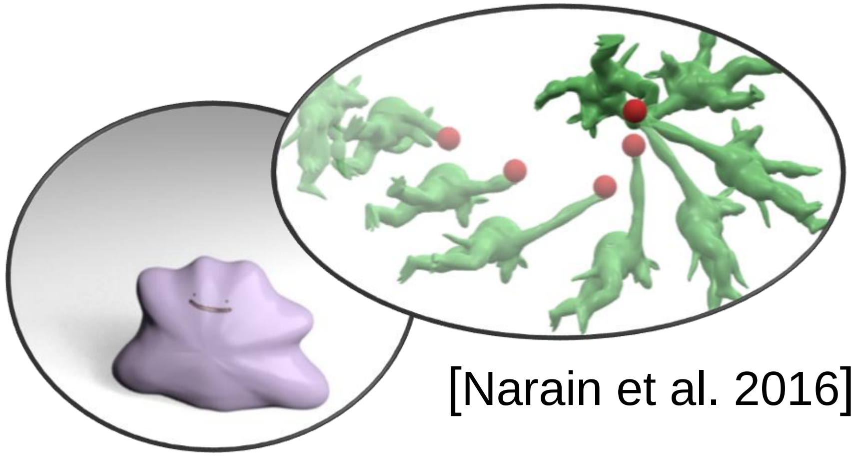
PERFORMANCE



Cost is linear in number of agents, increases slowly with time step size

(Still, 2x-10x slower than ORCA or PowerLaw on a 6-core Intel Xeon E5-1650)

Future work: Improve performance via local-global alternating minimization techniques

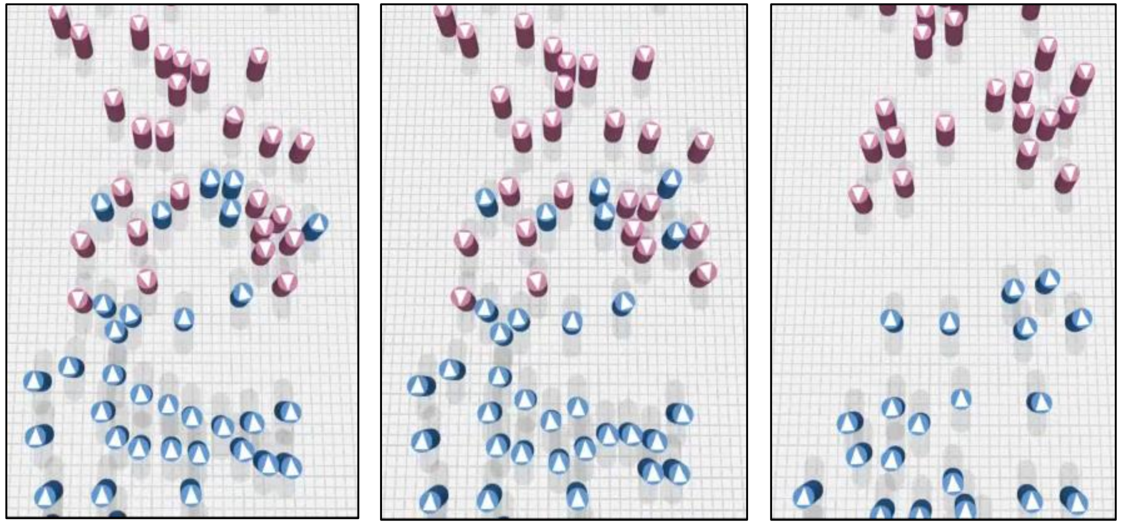


[Narain et al. 2016]

[Liu et al. 2017]

LIMITATIONS AND FUTURE WORK

- Can other recent crowd models be formulated via interaction energies [Wolinksi et al. 2016, Dutra et al. 2017; …]?
- Incorporating asymmetrical interactions, e.g., leader-following behavior
- What is the Δt threshold where quality is maintained?



Δt=0.1s Δt=1s Δt=4s

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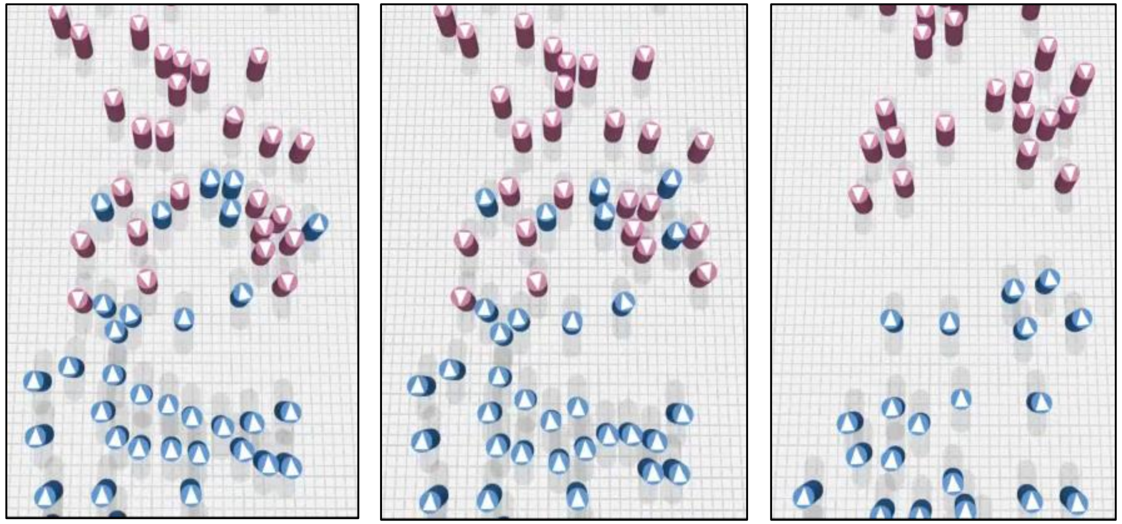

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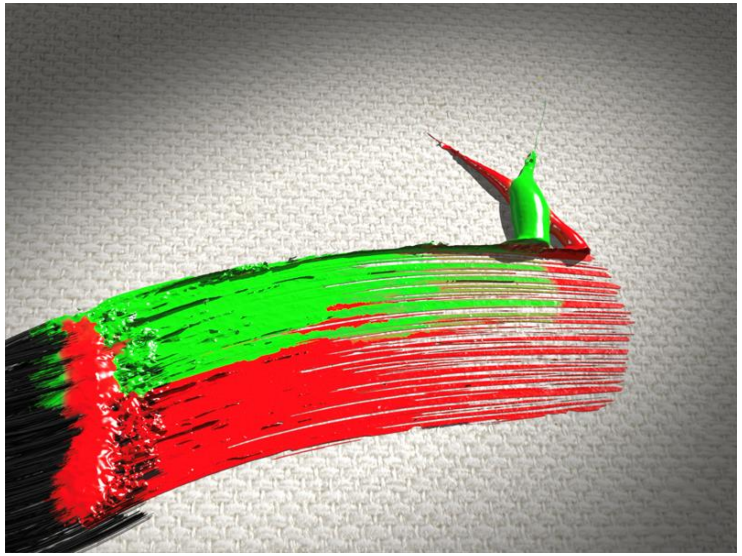
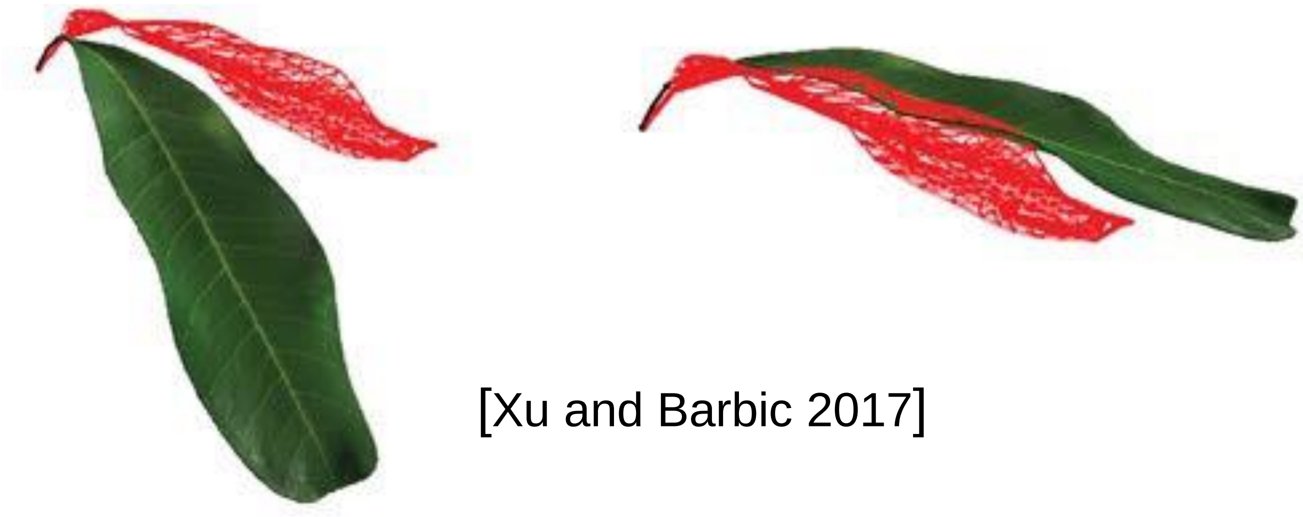
FUTURE WORK

- Applications to LOD systems and footstep-based animation engines
- Applications to nonlinear dissipation forces in physics-based animation



[Zhu et al. 2015]



[Xu and Barbic 2017]

THANK YOU

https://www.cs.clemson.edu/~ioannis/implicit-crowds/