Reconstructing Motion from IMU Data

In Labs 5 and 6, you have implemented controllers for longitudinal motion. This purpose of this lab is to give you more insight about lateral motion and using data collected from an IMU to reconstruct the path taken by a vehicle. This path is then compared to GPS data.

Task 1 Rigid Body Rotation

Rigid body transformations are important for changing the description of a point between two different reference frames. Consider the case where an IMU is mounted at a different angle from ISO convention (right-hand rule) at the center of mass. Linear acceleration in the body frame (of the IMU) must be converted into the inertial frame to find how the vehicle is truly moving. For example, Fig. 1 shows a rotation in yaw which changes the body axes to inertial axes.

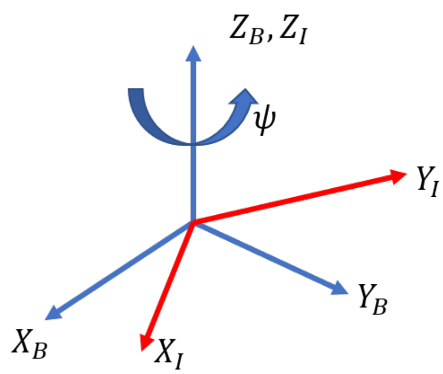

Figure 1: Yaw rotation

The following elementary rotation matrices describe yaw, pitch and roll motions:

$$
\begin{aligned}
R_{yaw}(\psi) &= \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
R_{pitch}(\theta) &= \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{bmatrix} \\
R_{roll}(\phi) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & sin(\phi) & cos(\phi) \end{bmatrix}
\end{aligned}
\tag{1}
$$

where $[\psi, \theta, \phi]$ are the yaw, pitch, and roll angles respectively. By combining the elementary rotation matrices, you can obtain the rotation matrix of the body frame with respect to the inertial frame, $Q_{B/\mathbb{I}}$. Thus, multiply a point in the body frame, **p**, by the rotation matrix to convert it to the inertial frame.

$$
\begin{aligned}
Q_{B/\mathbb{I}}(\psi,\theta,\phi) &= R_{yaw}(\psi)R_{pitch}(\theta)R_{roll}(\phi) \\
\mathbf{p}_{\mathbb{I}} &= Q_{B/\mathbb{I}}\mathbf{p}_B
\end{aligned} \tag{2}
$$

To convert back, you can simply take the inverse of the rotation matrix.

$$
\begin{aligned}
Q_{\mathbb{I}/B} &= Q_{B/\mathbb{I}}^{-1} \\
Q_{\mathbb{I}/B}\mathbf{p}_{\mathbb{I}} &= \mathbf{p}_B
\end{aligned} \tag{3}
$$

1. *****Deliverable**: Complete rotation matrices in **me131_lab7_script.m** (Line 17-25).

Task 2 Rigid Body Dynamics

Recall the rigid body equations of motion:

$$
\begin{aligned}
\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix} &= Q_{B/\mathbb{I}}(\psi,\theta,\phi)\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix} \\
\begin{bmatrix}\dot{\psi}\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix} &= W^{-1}\begin{bmatrix}w_x\\ w_y\\ w_z\end{bmatrix} \\
\begin{bmatrix}\dot{v_x}\\ \dot{v_y}\\ \dot{v_z}\end{bmatrix} &= \frac{1}{m}\left(\begin{bmatrix}F_x\\ F_y\\ F_z\end{bmatrix}\right) - \begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix} \\
I_B\begin{bmatrix}\dot{\omega_x}\\ \dot{\omega_y}\\ \dot{\omega_z}\end{bmatrix} &= \begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix} - \begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix} I_B\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}
\end{aligned} \tag{4}
$$

where $[X, Y, Z]$ are the X-Y-Z coordinates in the inertial frame, $[\psi, \theta, \phi]$ are the yaw, pitch, and roll angles respectively, $[v_x, v_y, v_z]$ are the linear velocities in the body frame, and $[\omega_x, \omega_y, \omega_z]$ are the angular velocities in the body frame. The inverse Wronskian matrix transforms angular velocities to euler angle rates:

$$
W^{-1} = \frac{1}{\cos(\theta)}\begin{bmatrix}0 & \sin(\phi) & \cos(\phi)\\ 0 & \cos(\phi)\cos(\theta) & -\sin(\phi)\cos(\theta)\\ \cos(\theta) & \sin(\phi)\sin(\theta) & \cos(\phi)\sin(\theta)\end{bmatrix} \tag{5}
$$

1. *****Deliverable**: Complete Wronskian and dynamics equations in **odeEuler.m** (Line 17-25) to obtain the Euler angles. Submit the plot of simulated and measured Euler angles vs time.

2. *****Deliverable**: Complete dynamics equations in **odevB.m** to obtain linear velocities in the body frame. Don't forget to compensate for gravity when using acceleration!

3. ***Deliverable**: Submit the plot of simulated and GPS linear velocities. Explain the discrepancies between the two.

4. ***Deliverable**: Complete dynamics equations in **odeIB.m** to obtain the X-Y-Z coordinates in the inertial frame. Submit the plot of simulated and GPS coordinates.
