ME131 Vehicle Dynamics and Control

HW5: Longitudinal Dynamics: Adaptive Cruise Control

Assigned: 2/13/2019 Due: 2/20/2019, 11:59pm (On bCourses)

Please submit your homework solutions on bCourses as a single PDF of your solutions. When videos are required, please only submit the link as part of the solution PDF document. Late homeworks will be penalized.

Problem 1 Longitudinal Vehicle Model and Coast Down Test Derivation - No Deliverables (0pt)

As described in class, a simple model for longitudinal vehicle dynamics moving on an inclined road can be expressed according to Newton's law:

$$m\dot{V} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin(\theta) \tag{1}$$

where F_{aero} is the equivalent aerodynamic drag force which can be further represented as

$$F_{aero} = \frac{1}{2}\rho C_d A_F (V + V_{wind})^2 \tag{2}$$

the aerodynamic drag coefficient C_d and rolling resistance R_x can be roughly determined from a coast down test. In a coast down test, the throttle angle (α) is kept at zero and the vehicle is allowed to slow under the effects of aerodynamic drag and rolling resistance. Since there is neither braking nor throttle angle inputs, the longitudinal tire force under these conditions is small and can be assumed to be zero. The road level $\theta = 0$ and the wind velocity (V_{wind}) are both assumed to be zero. Therefore, the vehicle dynamics model is rewritten into:

$$-m\frac{dV}{dt} = \frac{1}{2}\rho V^2 A_F C_d + R_x \tag{3}$$

Integrating eqn. (3) results in the relation:

$$t = \left[\frac{2m^2}{\rho A_F C_d R_x}\right]^{\frac{1}{2}} \left\{ \tan^{-1}\left[V_0\left(\frac{\rho A_F C_d}{2R_x}\right)^{\frac{1}{2}}\right] - \tan^{-1}\left[V(t)\left(\frac{\rho A_F C_d}{2R_x}\right)^{\frac{1}{2}}\right]\right\} \tag{4}$$

and this can be further broken down to:

$$\frac{v(t)}{V_0} = \frac{1}{\beta}\tan\left[(1 - \frac{t}{T})\tan^{-1}\beta\right] \qquad \text{with } \beta = V_0\left(\frac{\rho A_F C_d}{2R_x}\right)^{\frac{1}{2}}. \tag{5}$$

This last equation describes how your vehicle's longitudinal velocity evolves as a function of air drag and rolling resistance.

Problem 2 Coast Down Test on a Passenger Vehicle (10pt)

In the next two questions you are given an experimental data set (coast down test while driving straight) for a the test vehicle with an automatic transmission in neutral. You will use the above equations to estimate β, C_d and R_x.

2.1 (5pt) Define T as the time the vehicle stops (i.e. $V(T) = 0$). Download the `straightLineTest.mat` file from **here**. It includes the measured variables Vx (longitudinal velocity), t (time), m (mass) and rho (mass density of the air). Use the coefficients listed below.

- $\rho = 1.225$ kg/m^3 is the mass density of air,
- Mass (m) is 1755 kg (3,871 lbs).
- $g = 9.81$ m/s^2.

- A_F is frontal area of the vehicle. Here, you can use the empirical formula provided in Raj. & Wong to approximate the true value: $A_F = 1.6 + 0.00056(m - 765)$ m^2.
- $V_0 = \text{Vx}(0)$ m/s is the initial longitudinal vehicle velocity

Load the into MATLAB and submit a plot of $\frac{v(t)}{V_0}$ v.s. $\frac{t}{T}$ with normalized axes.

2.2 (5pt) Implement the curve fitting of the nonlinear equation (5) by using the MATLAB commands `fminunc` or `lsqcurvefit`, and solve for the parameters β, C_d and R_x accordingly. Submit the parameters.

Problem 3 Method of Least Squares (10pt)

The method of least squares is a standard approach in regression analysis to the approximate solution of overdetermined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of each individual equation.

In this problem we will consider a sequence of data sets containing n points (x_i, y_i), where $i = 1, .., n$, $x_i \in \mathbb{R}^d$ is an independent variable and $y_i \in \mathbb{R}$ is a dependent scalar variable. The values of x_i and y_i are both measured. Assume we have a linear model with m parameters, $w \in \mathbb{R}^d$ linking x_i and y_i. Then, the linear model can be written as:

$$\hat{y}_i = x_i^T w \in \mathbb{R} \tag{6}$$

where $\hat{y}_i$ is the predicted value of y_i according to this model. The sum of the squares of the errors made at every prediction step i is:

$$S = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \sum_{i=1}^{n} (x_i^T w - y_i)^2 \tag{7}$$

For convenience, we define $\hat{Y} = Xw$ and $\Delta Y = \hat{Y} - Y$, where Y and $\hat{Y} \in \mathbb{R}^n$ are the vectors collecting of y_i and $\hat{y}_i$ respectively. $X \in \mathbb{R}^{n \times d}$ is vector collecting all x_i.

3.1 (5pt) Represent eqn. (7) in terms of Y, X and w. (Hint: S = $\Delta Y^T \ \Delta Y$)

3.2 (5pt) Derive the solution w, in terms of X and Y, which minimizes the sum of predicted errors S.

Problem 4 Parameter Fitting for Differential Equation (10pt)

Typical drag coefficient values C_d are in the range of (0.19 - 0.38), which is likely very different from the value you calculated in Problem 2. This is because the test roads used for the `straightLineTest.m` data collection were not ideal with zero incline. Therefore, the model of the longitudinal vehicle dynamics should be changed into:

$$m\dot{V} = -\frac{1}{2}\rho C_d A_F V^2 - R_x - mg\sin(\theta) \tag{8}$$

to include the effect of an inclined road. This gives us a linear curve fitting model of:

$$\dot{V} = w_1 V^2 + w_2 - w_3 \tag{9}$$

where $w_1 = -\frac{1}{2m}\rho C_d A_F$, $w_2 = -\frac{R_x}{m}$ and $w_3 = g\sin\theta$. In this problem, you will fit the parameters w_1, w_2 and w_3 for this new longitudinal dynamics model. To do this, you will need a way to distinguish the parameters w_2 and w_3, since the constructed matrix X from Problem 3 is not full rank (their sum could be considered just as one parameter).

For this reason we will provide two sets of test data. One set corresponds to a coast down test with a direction uphill (therefore positive w_3) and another set diving the same road in the opposite direction (downhill, therefore negative w_3). Download the `upNdownTest.mat` file from **here**. Notice we did some data preprocessing for you to smooth out the noise.

- `v1` is the longitudinal velocity for vehicle traveling uphill.
- `v2` is the longitudinal velocity for vehicle traveling downhill.
- $dt = 0.01$ sec is the sampling rate for collecting the data.

You can define all remaining parameters as in Problem 1.

4.1 Since you are only provided velocity data to do the parameter fitting, first obtain the acceleration data by approximating $\dot{V}_i = a_i \simeq \frac{V(i+1)-V(i)}{dt}$ in MATLAB. Construct the matrix Y in problem 3 by using the approximated a_i.

4.2 Construct the matrix X in problem 3 using `v1` and `v2` data in MATLAB. (Hint: X should be a 3-column matrix.)

4.3 (10pt) Find out the least square solution for the parameters C_d, R_x and θ.

Lab Deliverables

1. (5pt) A rqt_graph or drawing of the nodes, topics and messages associated with the BARC's IMU and actuators from Task 6.7.
2. (5pt) Write the command that you used to send a steering angle of 30 degrees to the left and forward velocity from Task 7.1. Include a short description of what each component of that command is doing.
3. (5pt) A rough sketch of the BARC car with the XYZ reference frame of acceleration labeled from Task 8.1. List the approximate constant values of a_x, a_y, and a_z with explanation of the large offset in vertical acceleration of the BARC when it is standing still.
4. (5pt) Label the directions of rotation on the drawing of the acceleration reference frame axes from Task 8.1 when the BARC experiences roll, pitch and yaw changes. Does this follow the right hand rule?