ME131 HW2

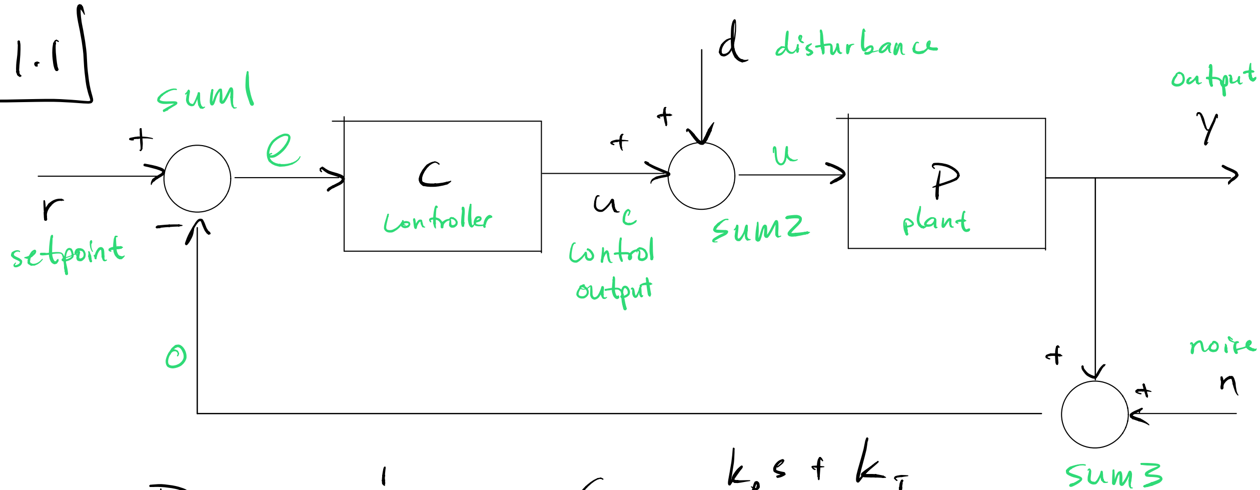
Due: 2/13/18

Trey Fortmuller
26037758

Problem 1: PI Controller Design

(the accompanying MATLAB scripts are available at github.com/treyfortmuller/me131)

1.1



$$P(s) = \frac{1}{s-1} \qquad C(s) = \frac{k_p s + k_I}{s}$$

let $k_p = 2$
$k_I = 1$

- first enter the controller & plant transfer functions in a MATLAB workspace.
- then build the model in Simulink using step, sine, LTI, summation, and scope blocks
- Then set the parameters of input, noise, and disturbance as given & run the simulation.

steady state response of CL system output (w/ $\mu(t)$ the unit step input)

inputs			output y_{ss}
$r(t)$	$d(t)$	$n(t)$	$y_{ss}(t)$
$2\mu(t)$	$\mu(t)$	$3\mu(t)$	$y_{ss}(t) = -1$
$2\sin(5t)$	$\sin(t)$	$3\mu(t)$	$y_{ss}(t)$ is oscillatory about -3 $\forall$ t

(step response scopes and simulink block diagram below).

1.2 Design k_p, k_I s.t. unit step forced response $G_{r \to y}(s)$ satisfies

- $\% OS \approx 10\%$
- 5% settling time $t_s \leq 20$ s
- Rise time $5 \leq t_r \leq 10$s

↳ use 2nd order approximation

Typical second order system: $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

using MATLAB's "feeback" we get the CL transfer function $H(s) = \dfrac{2s+1}{s^2+5+1} = \dfrac{k_p s + k_I}{s^2 + (k_p - 1)s + k_I}$

$H(s) \approx G(s)$ with $k_I = \omega_n^2 \quad k_p - 1 = 2\xi\omega_n$

formulas

% overshoot:

$$\% OS = 100\left(\frac{y_{peak} - y_{ss}}{y_{ss}}\right) \qquad (1)$$

$$\xi = \frac{-\ln(\% OS/100)}{\sqrt{\pi^2 + \ln^2(\% OS/100)}} \qquad (2)$$

settling time: $T_s = -\dfrac{\ln\left(0.05\sqrt{1-\xi^2}\right)}{\xi\,\omega_n}$ (3)

rise time: $T_r \approx \dfrac{0.8 + 1.1\xi + 1.4\xi^2}{\omega_n}$ (4)

we'll solve (2) for a desirable ξ:

$$\xi = \frac{-\ln\left(\frac{1}{10}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{1}{10}\right)}} = \boxed{0.59}$$

next we'll pick a rise time of 7.5s & find ω_n

$$\omega_n \approx \frac{0.8 + 1.1(0.59) + 1.4(0.59)}{7.5} = \boxed{0.303}$$

Now we'll just ensure these parameters satisfy the settling time constraint.

$$T_s = -\frac{\ln\left(0.05\sqrt{1-(0.59)^2}\right)}{(0.59)(0.303)} = 18 \text{ seconds}$$

so the settling time constraint is satisfied.

$$k_I = \omega_n^2 \quad k_p = 1 + 2\xi\omega_n$$

$$\Rightarrow \boxed{\begin{aligned} k_I &= 0.09 \\ k_p &= 1.36 \end{aligned}}$$

1.3) Write a MATLAB function implementing the controller in discrete time.

$$u = \text{controller}(e, \Delta t)$$

(submit the file)