ME131 HW1 - Trey Fortmuller, 26037758

1) **Answer**

E	C
F	D
B	G

note: associated matlab script at github.com/treyfortmuller/me131

Work

Need to plot the unit step response of several linear ODEs in matlab to match the eqn. to their graph of response.

A) $\dot{y} + y = u$

step 1: take the Laplace transform to obtain a transfer function

step 2: throw the transfer function into matlab, sys.

step 3: step(sys) to plot the step response.

$$\mathcal{L}\{\dot{y} + y = u\} \Rightarrow sY(s) + Y(s) = U(s)$$

transfer function $\equiv$ output to input ratio in the Laplace domain

$$(s+1)\,Y(s) = U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+1} \quad \checkmark$$

sys = tf(1, [1, 1])

step(sys) $\longleftarrow$ no graph matches

- you can also have MATLAB take care of the Laplace transform w/ the "tf" command

$\underline{\text{Ex}}$

$$5y^{(3)} - 4y' + 10y = 20u' + 4u$$

$D = [5 \;\; 0 \;\; -4 \;\; 10]$

$N = [20 \;\; 4]$

then $H = tf(N, D)$

B) $\ddot{y} + 1.5\dot{y} + 16y = 16u$

$$s^2 Y(s) + 1.5 s Y(s) + 16 Y(s) = 16 U(s)$$

$$Y(s)\left(s^2 + 1.5 s + 16\right) = 16 U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{16}{s^2 + 1.5s + 16}$$

note:
we can also just compute the roots of the characteristic polynomial of each diff. eq, then the time constant for the step responce will be determined by the "slowest" root.

$$T = \frac{1}{\min|\text{Real}(R_i)|}$$

we can use these to determine the matching step response plots.

example w/ (b) $\quad y = e^{rt}$, $\dot{y} = re^{rt}$, $\ddot{y} = r^2 e^{rt}$

$$e^{rt}\underbrace{\left(r^2 + 1.5r + 16\right)}_{\text{characteristic polynomial}} = 0$$

$$r = \frac{-1.5 \pm \sqrt{2.25 - 4(16)}}{2}$$

$$r = -0.75 \pm 3.929i$$

$$\Rightarrow T = \frac{1}{|-0.75|} = 1.33s \rightarrow \text{graph in lower left}$$

c) $\ddot{y} + \dot{y} + 4y = \dot{u} + 4u$

$\mathcal{L} \rightarrow \quad s^2 Y(s) + sY(s) + 4Y(s) = sU(s) + 4U(s)$

$Y(s)\left(s^2 + s + 4\right) = U(s)(s+4)$

T.F. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s+4}{s^2 + s + 4}$

D) $\ddot{y} + 2.4\dot{y} + 9y = 5\dot{u} - 9u$

$\mathcal{L} \rightarrow \quad s^2 Y(s) + 2.4sY(s) + 9Y(s) =$

$5sU(s) - 9U(s)$

$Y(s)\left(s^2 + 2.4s + 9\right) = U(s)(5s - 9)$

T.F. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{5s - 9}{s^2 + 2.4s + 9}$

E) $\ddot{y} + 1.5\dot{y} + 9y = -2\dot{u} - 9u$

$$\mathcal{L} \rightarrow s^2 Y(s) + 1.5s Y(s) + 9Y(s) =$$

$$-2s\,U(s) - 9U(s)$$

$$Y(s)\left(s^2 + 1.5s + 9\right) = U(s)\left(-2s - 9\right)$$

$$\text{T.F.} \rightarrow H(s) = \frac{Y(s)}{U(s)} = \frac{-2s - 9}{s^2 + 1.5s + 9}$$

F) $\ddot{y} + 4\dot{y} + 2y = -3\dot{u} + 2u$

$$\mathcal{L} \rightarrow s^2 Y(s) + 4s Y(s) + 2Y(s) =$$

$$-3s\,U(s) + 2U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{-3s + 2}{s^2 + 4s + 2}$$

G) $y^{(3)} + 305\ddot{y} + 21600\dot{y} + 100{,}000y = 100{,}000u$

$$s^3 Y(s) + 305s^2 Y(s) + 21600s\,Y(s) + 100{,}000Y(s)$$

$$= 100{,}000\,U(s)$$

$$Y(s)\left(s^3 + 305s^2 + 21600s + 100{,}000\right) = 100{,}000\,U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{100{,}000}{s^3 + 305s^2 + 21600s + 100{,}000}$$