ME131 HW1 – Trey Fortmuller, 26037758

1) Answer

E	C
F	D
B	G

note: associated matlab script at github.com/treyfortmuller/me131

Work

Need to plot the unit step response of several linear ODEs in matlab to match the eqn. to their graph of response.

A) $\dot{y} + y = u$

step 1: take the Laplace transform to obtain a transfer function

step 2: throw the transfer function into matlab, sys.

step 3: step(sys) to plot the step response.

$$\mathcal{L}\{\dot{y} + y = u\} \Rightarrow sY(s) + Y(s) = U(s)$$

transfer function $\equiv$ output to input ratio in the Laplace domain

$$(s+1)\,Y(s) = U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+1} \checkmark$$

```
sys = tf(1, [1, 1])

step(sys)
```

$\leftarrow$ no graph matches

- you can also have MATLAB take care of the Laplace transform w/ the "tf" command

$\underline{\text{Ex}}$

$$5y^{(3)} - 4y' + 10y = 20u' + 4u$$

$$D = [5 \quad 0 \quad -4 \quad 10]$$

$$N = [20 \quad 4]$$

then $H = tf(N, D)$

B) $\ddot{y} + 1.5\dot{y} + 16y = 16u$

$$s^2 Y(s) + 1.5 s Y(s) + 16 Y(s) = 16 U(s)$$

$$Y(s)\left(s^2 + 1.5 s + 16\right) = 16 U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{16}{s^2 + 1.5s + 16}$$

note:
we can also just compute the roots of the characteristic polynomial of each diff. eq, then the time constant for the step responce will be determined by the "slowest" root.

$$T = \frac{1}{\min|\mathrm{Real}(R_i)|}$$

we can use these to determine the matching step response plots.

example w/ (b) $\quad y = e^{rt},\ \dot{y} = re^{rt},\ \ddot{y} = r^2 e^{rt}$

$$e^{rt}\underbrace{\left(r^2 + 1.5r + 16\right)}_{\text{characteristic polynomial}} = 0$$

$$r = \frac{-1.5 \pm \sqrt{2.25 - 4(16)}}{2}$$

$$r = -0.75 \pm 3.929\,i$$

$$\Rightarrow \quad T = \frac{1}{|-0.75|} = 1.33\,s \rightarrow \text{graph in lower left}$$

c) $\ddot{y} + \dot{y} + 4y = \dot{u} + 4u$

$\mathcal{L} \rightarrow \quad s^2 Y(s) + sY(s) + 4Y(s) = sU(s) + 4U(s)$

$Y(s)\left(s^2 + s + 4\right) = U(s)\left(s+4\right)$

T.F. $\quad H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s+4}{s^2 + s + 4}$

D) $\ddot{y} + 2.4\dot{y} + 9y = 5\dot{u} - 9u$

$\mathcal{L} \rightarrow \quad s^2 Y(s) + 2.4sY(s) + 9Y(s) =$

$5sU(s) - 9U(s)$

$Y(s)\left(s^2 + 2.4s + 9\right) = U(s)\left(5s - 9\right)$

T.F. $\quad H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{5s - 9}{s^2 + 2.4s + 9}$

E) $\ddot{y} + 1.5\dot{y} + 9y = -2\dot{u} - 9u$

$\mathcal{L} \to s^2 Y(s) + 1.5s Y(s) + 9Y(s) =$

$-2s U(s) - 9U(s)$

$Y(s)\left(s^2 + 1.5s + 9\right) = U(s)\left(-2s - 9\right)$

T.F. $\to H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{-2s - 9}{s^2 + 1.5s + 9}$

F) $\ddot{y} + 4\dot{y} + 2y = -3\dot{u} + 2u$

$\mathcal{L} \to s^2 Y(s) + 4s Y(s) + 2Y(s) =$

$-3s U(s) + 2U(s)$

$H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{-3s + 2}{s^2 + 4s + 2}$

G) $y^{(3)} + 305\ddot{y} + 21500\dot{y} + 100{,}000y = 100{,}000u$

$s^3 Y(s) + 305s^2 Y(s) + 21500s Y(s) + 100{,}000 Y(s)$

$= 100{,}000 U(s)$

$Y(s)\left(s^3 + 305s^2 + 21500s + 100{,}000\right) = 100{,}000 U(s)$

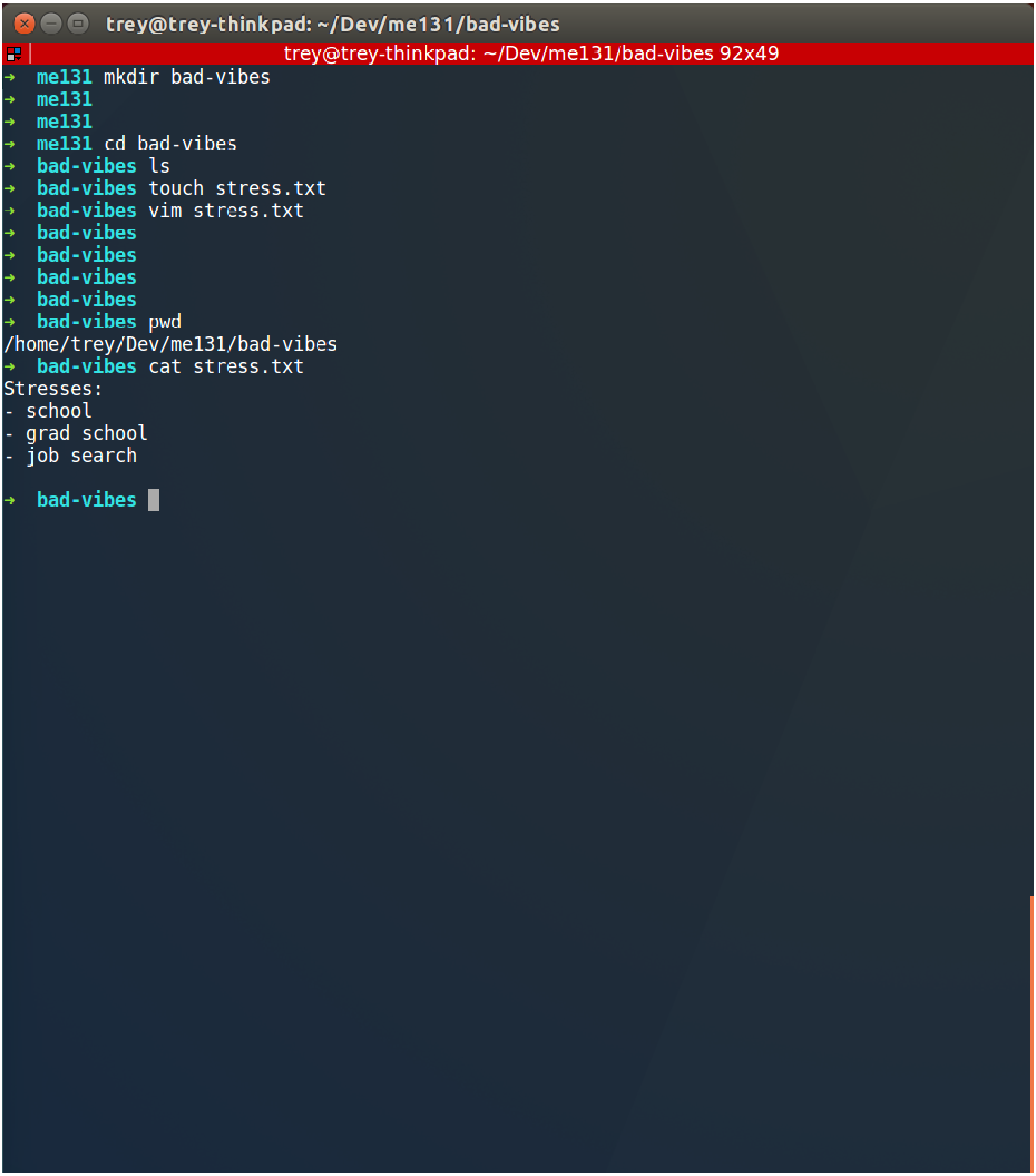
$H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{100{,}000}{s^3 + 305s^2 + 21500s + 100{,}000}$

ME131 Lab 1 Deliverables

Trey Fortmuller, SID: 26037758

Navigating the CLI

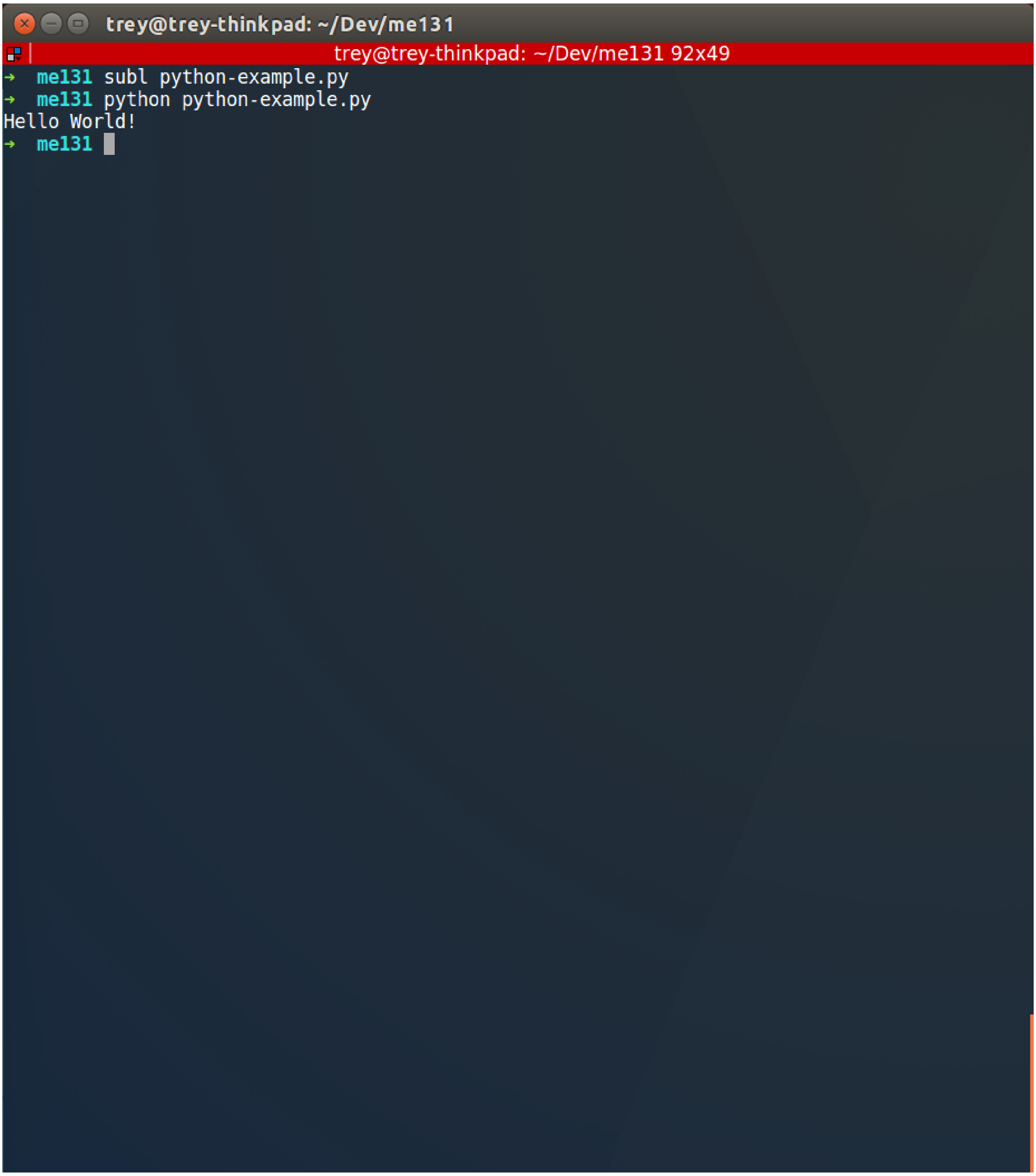
1. Bad vibes directory and stresses.txt



Using the Sublime Text Editor

2. Python Hello World output

   Python code at https://github.com/treyfortmuller/me131



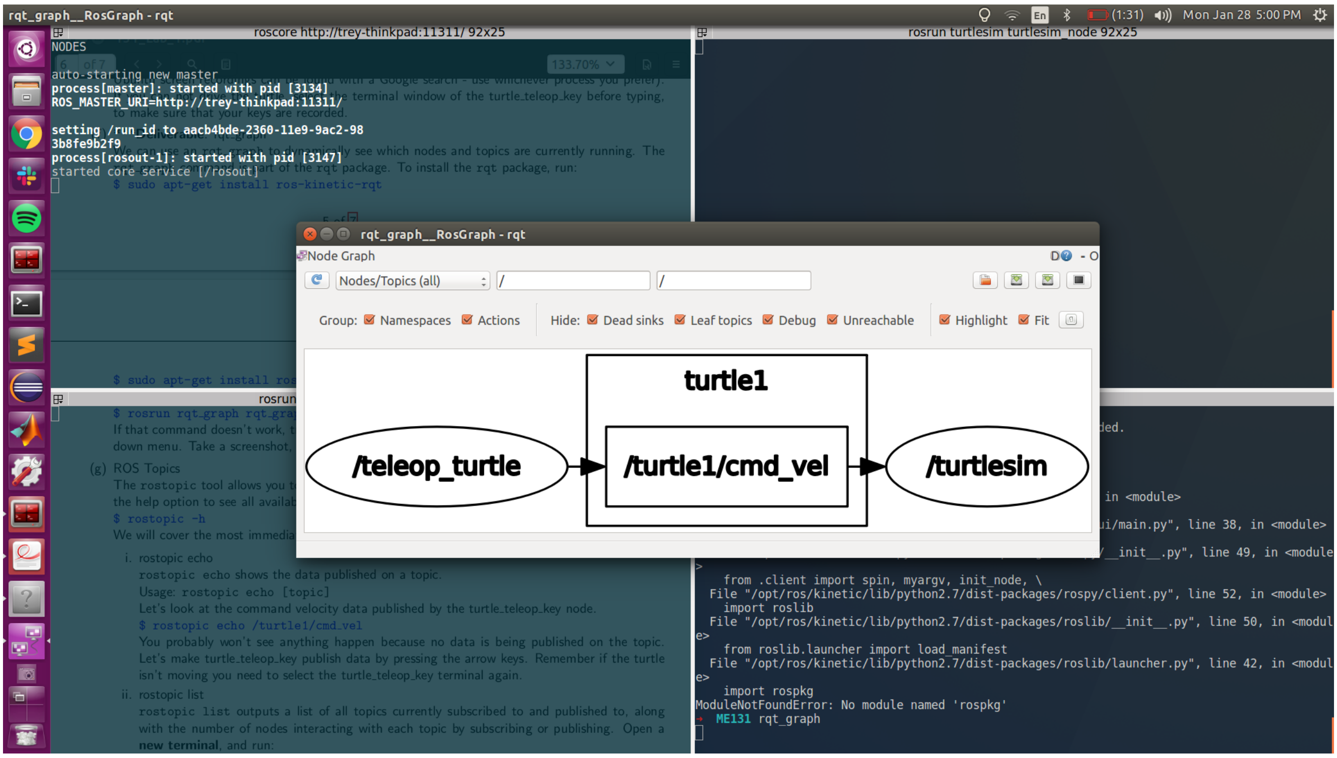
Turtlesim

3. Turtle keyboard teleoperation

   Link to video:

https://drive.google.com/open?id=1Z2C-CCBdfcOmCmpeeHc1dNffXDeRylG7

4. rqt_graph



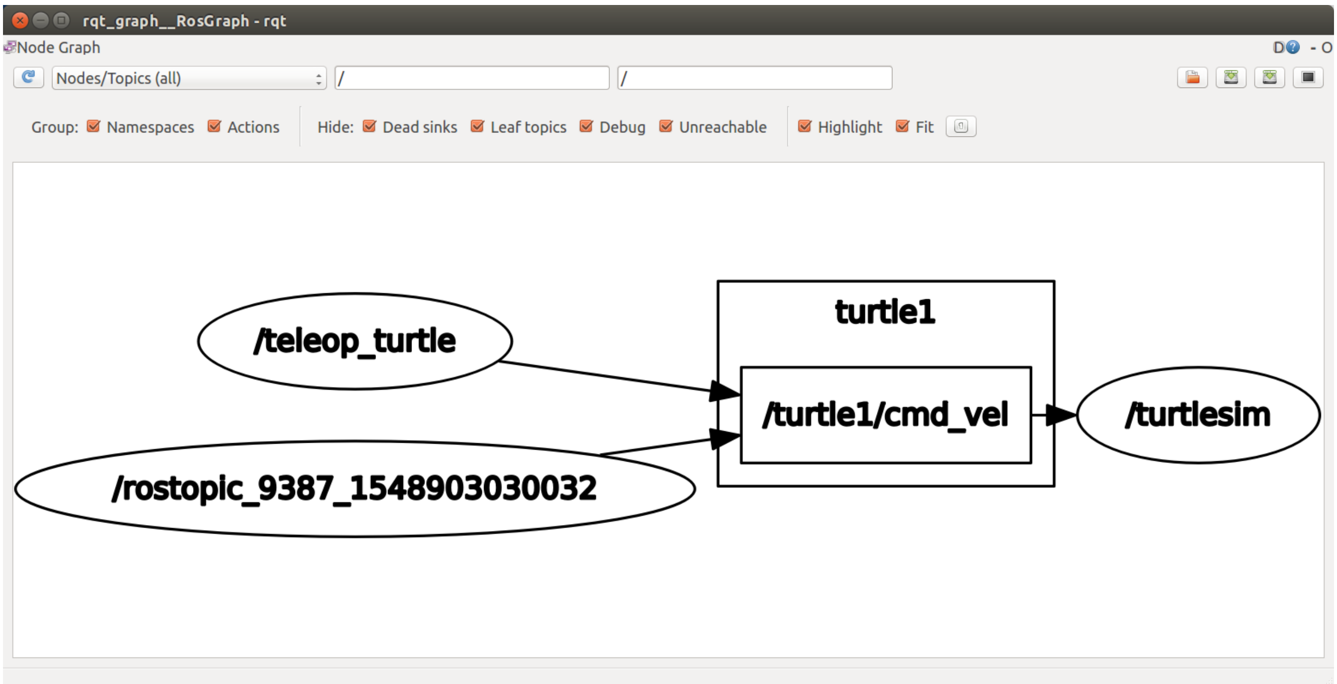
The `rqt_graph` shows that a node called `/teleop_turtle` is publishing to the `/turtle1/cmd_vel` topic, which the `turtlesim` node is subscribing to.

5. rostopic pub

Link to video:

https://drive.google.com/open?id=1iUvn4PzbReiJmPpkvCDxcJCUKXeikCXW

6. Circle movement graph



The graph shows that the `teleop_turtle` node is publishing to the `/turtle1/cmd_vel` node (because I never terminated that process). The new node `/rostopic_938...` is given an arbitrary identifier because it doesn't belong to a named process and is also publishing to `/turtle1/cmd_vel`. This node is publishing the circular `geometry_msgs/Twist` commands. Finally, the `/turtlesim` node is still subscribed to the `/turtle1/cmd_vel`, thus the visualized turtle is shown moving in a circle.

7. rqt_plot

   Link to video:

https://drive.google.com/open?id=1IBcwL6C2qRjT2s07Ca9KKTE4YudiSQ8r

These x and y position values make sense because the projection of the turtle's position onto the x and y axis as a function of time should look like two sinusoids (cosine and sine for x and y respectively) oscillating 90 degrees out of phase, which can we see in the plot. The sinusoid's position along the y axis of the plot will be determined by the coordinates of the turtle in its window. It appears that the origin of the world frame for the turtle resides in the lower left corner of the rectangular blue window.