ME131 HW1 - Trey Fortmuller, 26037758

1) Answer

E	C
F	D
B	G

note: associated matlab script at
github.com/treyfortmuller/me131

Work

Need to plot the unit step response of several linear ODEs in matlab to match the eqn. to their graph of response.

A) $\dot{y} + y = u$

step 1: take the Laplace transform to obtain a transfer function

step 2: throw the transfer function into matlab, sys.

step 3: step (sys) to plot the step response.

$\mathcal{L}\{\dot{y} + y = u\}$

$$sY(s) + Y(s) = U(s)$$

transfer function $\equiv$ output to input ratio is the Laplace domain

$$(s+1)\,Y(s) = U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+1} \checkmark$$

what is this doing math-wise? can I plot this by hand? is it a numerical solve?

```
sys = tf(1, [1, 1])

step(sys)
```

looks like this is the extra diff. eq, no matches.

→ you can also have MATLAB take care of the Laplace transform w/ the "tf" command

Ex $5y^{(3)} - 4y' + 10y = 20u' + 4u$

```
D = [5  0  -4  10]
N = [20  4]
```

then `H = tf(N, D)`

B) $\ddot{y} + 1.5\dot{y} + 16y = 16u$

$$s^2 Y(s) + 1.5s\,Y(s) + 16\,Y(s) = 16\,U(s)$$

$$Y(s)\left(s^2 + 1.5s + 16\right) = 16\,U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{16}{s^2 + 1.5s + 16}$$

c) $\ddot{y} + \dot{y} + 4y = \dot{u} + 4u$

$$\mathcal{L} \rightarrow \quad s^2 Y(s) + s\,Y(s) + 4\,Y(s) = s\,U(s) + 4\,U(s)$$

$$Y(s)\left(s^2 + s + 4\right) = U(s)\left(s + 4\right)$$

T.F. $$H(s) = \frac{Y(s)}{U(s)} = \frac{s + 4}{s^2 + s + 4}$$

D) $\ddot{y} + 2.4\dot{y} + 9y = 5\dot{u} - 9u$

$$\mathcal{L} \rightarrow \quad s^2 Y(s) + 2.4s\,Y(s) + 9\,Y(s) =$$

$$5s\,U(s) - 9\,U(s)$$

$$Y(s)\left(s^2 + 2.4s + 9\right) = U(s)\left(5s - 9\right)$$

T.F. $$H(s) = \frac{Y(s)}{U(s)} = \frac{5s - 9}{s^2 + 2.4s + 9}$$

E) $\ddot{y} + 1.5\dot{y} + 9y = -2\dot{u} - 9u$

$\mathcal{L} \to s^2 Y(s) + 1.5s Y(s) + 9Y(s) =$

$-2s\,U(s) - 9U(s)$

$Y(s)\left(s^2 + 1.5s + 9\right) = U(s)\left(-2s - 9\right)$

T.F. $\to H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{-2s - 9}{s^2 + 1.5s + 9}$

F) $\ddot{y} + 4\dot{y} + 2y = -3\dot{u} + 2u$

$\mathcal{L} \to s^2 Y(s) + 4s Y(s) + 2Y(s) =$

$-3s\,U(s) + 2U(s)$

$H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{-3s + 2}{s^2 + 4s + 2}$

G) $y^{(3)} + 305\ddot{y} + 21500\dot{y} + 100{,}000y = 100000u$

$s^3 Y(s) + 305s^2 Y(s) + 21500s\,Y(s) + 100{,}000Y(s)$

$= 100000\,U(s)$

$Y(s)\left(s^3 + 305s^2 + 21500s + 100{,}000\right) = 100{,}000\,U(s)$

$H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{100{,}000}{s^3 + 305s^2 + 21500s + 100{,}000}$