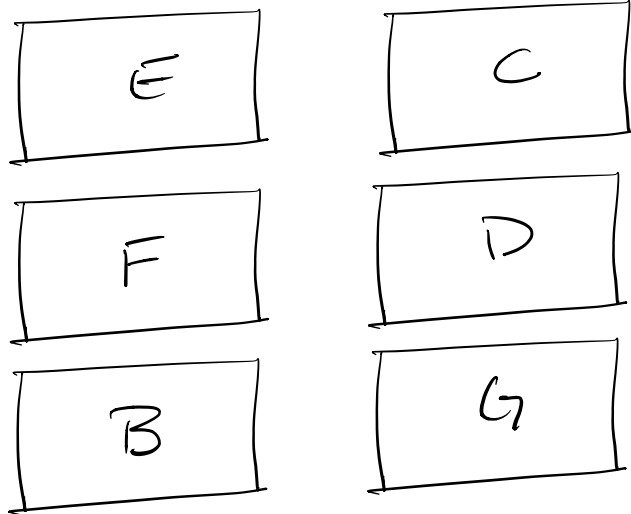


ME131 HW1 - Trey Fortmuller, 26037758

1) Answer



note: associated matlab script at
github.com/treyfortmuller/me131

Work

Need to plot the unit step response of several linear ODEs in matlab to match the eqn. to their graph of response.

A) $\dot{y} + y = u$

step 1: take the Laplace transform to obtain a transfer function

step 2: throw the transfer function into matlab, sys.

step 3: `step(sys)` to plot the step response.

$$\mathcal{L}\{\dot{y} + y = u\}$$

$$sY(s) + Y(s) = U(s)$$

transfer function = output to input ratio
is the Laplace domain

$$(s+1)Y(s) = U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+1} \quad \checkmark$$

$$\text{sys} = \text{tf}(1, [1, 1])$$

what is this doing
math-wise? can
I plot this by hand?
is it a numerical
solve?

$$\text{step}(\text{sys})$$

looks like this is the
extra diff. eq, no
matches.

→ you can also have MATLAB take care of
the Laplace transform w/ the "tf" command

Ex $5y^{(3)} - 4y' + 10y = 20u' + 4u$

$$D = [5 \ 0 \ -4 \ 10]$$

$$N = [20 \ 4]$$

then $H = \text{tf}(N, D)$

$$b) \quad \ddot{y} + 1.5 \dot{y} + 16y = 16u$$

$$s^2 Y(s) + 1.5s Y(s) + 16 Y(s) = 16 U(s)$$

$$Y(s) (s^2 + 1.5s + 16) = 16 U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{16}{s^2 + 1.5s + 16}$$

$$c) \quad \ddot{y} + \dot{y} + 4y = \ddot{u} + 4u$$

$$\mathcal{L} \rightarrow s^2 Y(s) + s Y(s) + 4 Y(s) = s U(s) + 4 U(s)$$

$$Y(s) (s^2 + s + 4) = U(s) (s + 4)$$

$$\text{T.F. } H(s) = \frac{Y(s)}{U(s)} = \frac{s + 4}{s^2 + s + 4}$$

$$d) \quad \ddot{y} + 2.4 \dot{y} + 9y = 5\ddot{u} - 9u$$

$$\mathcal{L} \rightarrow s^2 Y(s) + 2.4s Y(s) + 9 Y(s) = 5s U(s) - 9 U(s)$$

$$Y(s) (s^2 + 2.4s + 9) = U(s) (5s - 9)$$

$$\text{T.F. } H(s) = \frac{Y(s)}{U(s)} = \frac{5s - 9}{s^2 + 2.4s + 9}$$

$$E) \quad \ddot{y} + 1.5 \dot{y} + 9y = -2\ddot{u} - 9u$$

$$\mathcal{L} \rightarrow s^2 Y(s) + 1.5s Y(s) + 9Y(s) = -2s U(s) - 9U(s)$$

$$Y(s) (s^2 + 1.5s + 9) = U(s) (-2s - 9)$$

$$\text{T.F.} \rightarrow H(s) = \frac{Y(s)}{U(s)} = \frac{-2s - 9}{s^2 + 1.5s + 9}$$

$$F) \quad \ddot{y} + 4\dot{y} + 2y = -3\ddot{u} + 2u$$

$$\mathcal{L} \rightarrow s^2 Y(s) + 4s Y(s) + 2Y(s) = -3s U(s) + 2U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{-3s + 2}{s^2 + 4s + 2}$$

$$G) \quad y^{(3)} + 305\ddot{y} + 21500\dot{y} + 100,000y = 100,000u$$

$$s^3 Y(s) + 305s^2 Y(s) + 21500s Y(s) + 100,000Y(s) = 100,000U(s)$$

$$Y(s) (s^3 + 305s^2 + 21500s + 100,000) = 100,000U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{100,000}{s^3 + 305s^2 + 21500s + 100,000}$$