Electronic Stability Control (ESC) (Yaw stability control system)

Review from last lecture:

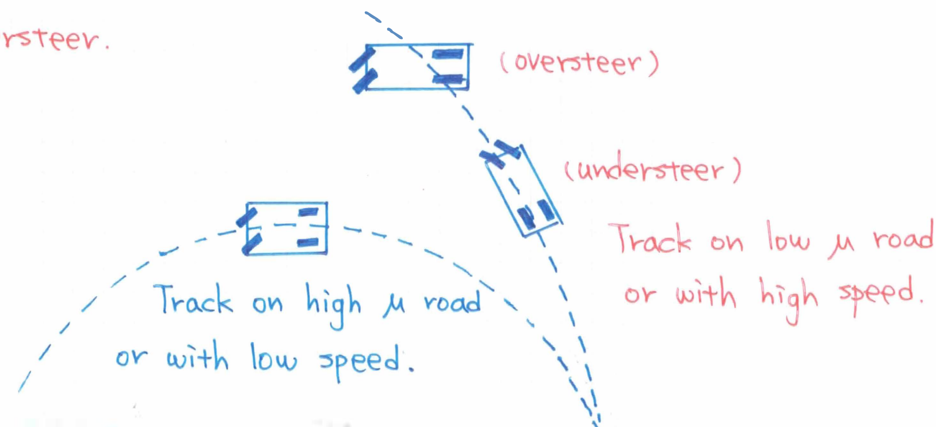
$$\begin{cases} \text{Understeer:} & |\alpha_f| > |\alpha_r| \\ \text{Oversteer:} & |\alpha_r| > |\alpha_f| \end{cases}$$

Note: At the limit of tire adhesion, front tires will be saturated for understeer. On the other hand, rear tire will be saturated first for oversteer.

Aside: The reason of add an absolute sign on the slip angle is that we assume $\dot{\psi}_0 \simeq \frac{V_x}{R} > 0$ to calculate the steady state cornering behavior. This is only for the case of turning left ($\alpha_f, \alpha_r > 0$).

For having a right turn, $\dot{\psi}_0 \cong \frac{V_x}{-R} < 0$, both $\alpha_f, \alpha_r < 0$.

Therefore, the result becomes $\alpha_f < \alpha_r < 0$ in understeer.



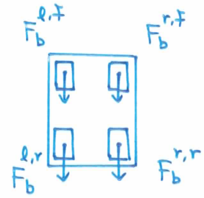
Motivation of ESC:

At the limit of adhesion,

1. The behavior of the vehicle is quite different from its nominal behavior.

2. When the spinning happens, the driver very often reacts in a wrong way.

3. The sensitivity of the yaw moment to change in steering angle reduced.



⇒ Different ways to creat a yaw moment.

☆ 1. Differential Braking (We will focus on this)

2. Steer-by-wire : (modify the driver's input)

3. Active torque distribution for all wheel drive car.

Vehicle model : We are going to use dynamics bicycle model.

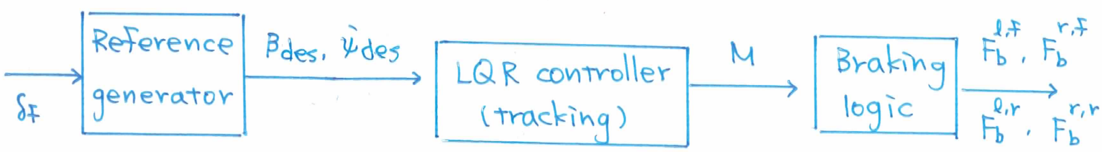
Assume V_x = const. , $\delta_r = 0$, $\beta = \frac{V_y}{V_x}$ and the driver provides a front steering angle as an input.

Note: A and B here are different from the (A,B) matrices of the bicycle model where the state is v_y and psi_dot

$$\begin{bmatrix}\dot{\beta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix}\beta\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix} B \end{bmatrix}\delta_f + \begin{bmatrix}0\\ \frac{1}{I_z}\end{bmatrix} M$$

M is the yaw moment produced by the braking force of the wheels.

Control Strategy :



• Reference Generator :

From the last lecture, we obtain the steering angle of the steady state cornering as

$$\delta_{ss} = \frac{L}{R} + \left(\frac{m l_r C_\alpha^r - m l_f C_\alpha^f}{2 C_\alpha^r C_\alpha^f L}\right)\frac{V_x^2}{R}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{\delta_{ss}}{L + \dfrac{m V_x^2 (l_r C_\alpha^r - l_f C_\alpha^f)}{2 C_\alpha^r C_\alpha^f L}}$$

The steady state of the yaw rate then can be obtained as

$$\dot{\psi}_{ss} \simeq \frac{V_x}{R} = \frac{V_x}{L + \dfrac{mV_x^2(l_r C_\alpha^r - l_f C_\alpha^f)}{2C_\alpha^f C_\alpha^r L}} \delta_{ss} = f_1(\delta_{ss}, V_x) \quad \text{—— ①}$$

Substitute δ_{ss} and $\dot{\psi}_{ss}$ into equation (12) under steady state in "Vehicle Dynamics Model" slide, we obtain

$$0 = \dot{V}_y = a_{11} V_x \beta_{ss} + a_{12} \dot{\psi}_{ss} + b_{11} \delta_{ss}$$

$$\Rightarrow \beta_{ss} = \frac{l_r}{R} - \frac{l_f m V_x^2}{2C_\alpha^r L R} = \frac{l_r - \dfrac{l_f m V_x^2}{2C_\alpha^r L}}{L + \dfrac{mV_x^2(l_r C_\alpha^r - l_f C_\alpha^f)}{2C_\alpha^f C_\alpha^r L}} \delta_{ss} = f_2(\delta_{ss}, V_x) \quad \text{—— ②}$$

With the result of ① and ②, we then choose

$$\dot{\psi}_{des} = f_1(\delta_f, V_x)$$

$$\beta_{des} = f_2(\delta_f, V_x) \quad *$$

Next, we need to bound $\dot{\psi}_{des}$ and β_{des} to gaurantee resonable values.

From 3D rigid body dynamics, we know the lateral acceleration at C.G

$$a_y = \dot{V}_y + \dot{\psi} V_x$$

should be bounded by the tire-road friction coefficient

$$\Rightarrow a_y \approx \mu g$$

Assume the lateral velocity and its derivative are small. Then, $\dot{\psi} V_x$ will dominate.

$$\Rightarrow \dot{\psi}_{des,\,bound} = 0.85 \frac{\mu g}{V_x} \operatorname{sgn}(\dot{\psi}_{des})$$

The factor 0.85 allow $\dot{V}_y$ to contribute 15% to the total lateral acceleration.

For the upperbound of β_{des}, we have an emperical relation

$$\beta_{des,\,bound} = \tan^{-1}(0.02\,\mu g)\ \mathrm{sgn}(\beta_{des})$$

which yields $\beta_{des,\,bound} \doteq \pm 10^\circ$ for $\mu = 0.9$

$\beta_{des,\,bound} = \pm 4^\circ$ for $\mu = 0.35$ ※

Note: Some researchers have used $\beta_{des} = 0$, since $\dot{\psi}_{des} \simeq \frac{V_x}{R}$ already assume $V_y = 0$. However the equation ①, ② yield a better approximation.

- LQR controller design:

Bicycle model:
$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \delta_f + \begin{bmatrix} B_1 \end{bmatrix} M$$

The steady state reference provides

$$0 = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \beta_{des} \\ \dot{\psi}_{des} \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \delta_f$$

Define $e_\beta = \beta - \beta_{des}$ and $e_{\dot{\psi}} = \dot{\psi} - \dot{\psi}_{des}$, we obtain an error dynamics as

$$\begin{bmatrix} \dot{e}_\beta \\ \dot{e}_{\dot{\psi}} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} e_\beta \\ e_{\dot{\psi}} \end{bmatrix} + \begin{bmatrix} B_1 \end{bmatrix} M$$

Then, we can apply LQR control law for M.

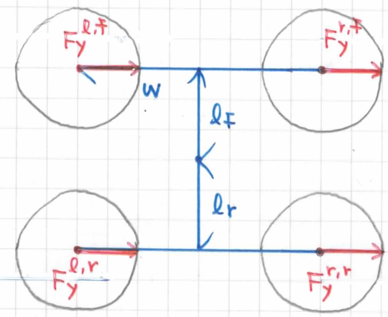
$$\Rightarrow \quad M = K(V_x) \begin{bmatrix} e_\beta \\ e_{\dot{\psi}} \end{bmatrix} \quad ※$$

- Braking logic:

The braking logic is produed by following the below discussion:

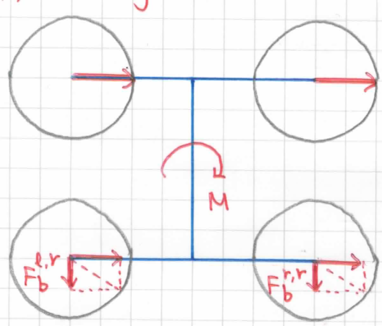
1. Left / right brake distribution is more effective in steering the vehicle than front / rear brake distribution.

Explanation : To produce a clockwise moment,



⇓

apply braking at the rear

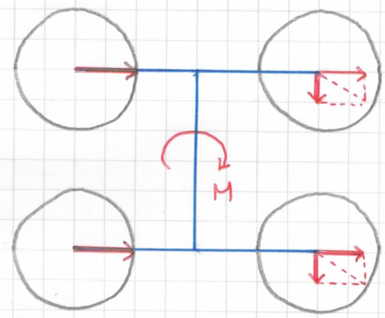


Front / rear distribution

The yaw moment is generated indirectly by utilizing the tire saturation property.

This effect is small except for saturated condition.

apply braking at the right side



Left / right distribution

The yaw moment is generated directly by the braking force.

The effect is significant under a wide turning range.

2. Outside wheel braking induces understeer.

Inside wheel braking induces oversteer.

Assume the car is in understeer condition (we therefore want to induce an oversteer moment)

3. Braking at the rear inside corner is most effective in inducing an oversteer yaw moment.

(We want to introduce an oversteer yaw moment for understeer and for understeer vehicle, front tire will be saturated first. Therefore, apply braking at rear tire is reasonable)

Assume the car is in oversteer condition (we therefore want to induce an understeer moment)

Braking at the front outside corner is most effective in inducing an understeer yaw moment.

Algorithm :

Input : M, α_r, α_f

Output : $F_b^{l,f}$, $F_b^{r,f}$, $F_b^{l,r}$, $F_b^{r,r}$

if $M = 0$, then

$$F_b^{l,f} = F_b^{r,f} = F_b^{l,r} = F_b^{r,r} = 0$$

elseif $M > 0$, then (braking at left side)

if $|\alpha_f| - |\alpha_r| > 0$, then (understeering)

$$F_b^{l,r} = \frac{|M|}{c}, \quad F_b^{l,f} = F_b^{r,f} = F_b^{r,r} = 0$$

else (oversteering)

$$F_b^{l,f} = \frac{|M|}{\sin(\delta_f - \tan^{-1} w/l_f)\sqrt{l_f^2 + w^2}}, \quad F_b^{r,f} = F_b^{l,r} = F_b^{r,r} = 0$$

end

else (M < 0, braking at right side)

if $|\alpha_f| - |\alpha_r| > 0$, then (understeering)

$$F_b^{r,r} = \frac{|M|}{c}, \quad F_b^{l,f} = F_b^{r,f} = F_b^{l,r} = 0$$

else (oversteering)

$$F_b^{r,f} = \frac{|M|}{\sin(\delta_f - \tan^{-1} w/l_f)\sqrt{l_f^2 + w^2}}, \quad F_b^{l,f} = F_b^{l,r} = F_b^{r,r} = 0$$

end.