Lateral Control: Model Based

In this week's lab, we are going to introduce two different models:

1. Kinematic Bicycle model
2. Dynamic Bicycle model

for vehicle controller design. Since each model is simplified under different assumptions from a higher fidelity vehicle model, it is necessary to know how well the two models are able to predict a vehicles future states for different purposes in controller design. In this lab, we are going to use these two models in open-loop to reconstruct the path of a vehicle by using experimental data (i.e. severe Figure-8 maneuver). We will examine how well the two models performed in open-loop predictions by comparing the estimate of global position and heading angle $(x,\ y,\ \psi)$ to the measured ones from the differential GPS (dGPS).

Task 1 Kinematic Bicycle Model

In autonomous vehicles, it is not only important to control the speed of the vehicle but also the steering angle to turn on roads and highways. Model-free lateral control, especially at high speeds, can be dangerous when the car overreacts to a desired heading angle and it may cause the car to drive in a sinosoidal motion. Model-based lateral control such as Model Predictive Control (MPC) predicts the optimal trajectory based minimization of a cost function and its performance depends on how well the model matches the plant.

In this section of the lab, we are going to use the kinematics bicycle model to open-loop reconstruct the path of the vehicle $(x,\ y,\ \psi)$. Kinematics is the study of motion provides a mathematical description of motion without considering the forces. Its equation of the motion are based purely on geometric relationships governing by the system. In the kinematic bicycle model, we assume the velocity of the front and rear wheels are pointing in their own directions and there is no slip on each of the tires. Recall the simplified kinematic bicycle model shown in Figure 1:
The state space representation for the system dynamics is given below:

$$\begin{aligned}
\dot{x} &= v\cos(\psi+\beta) \\
\dot{y} &= v\sin(\psi+\beta) \\
\dot{\psi} &= \frac{v\cos(\beta)}{l_r+l_f}\tan(\delta_f)
\end{aligned} \tag{1}$$

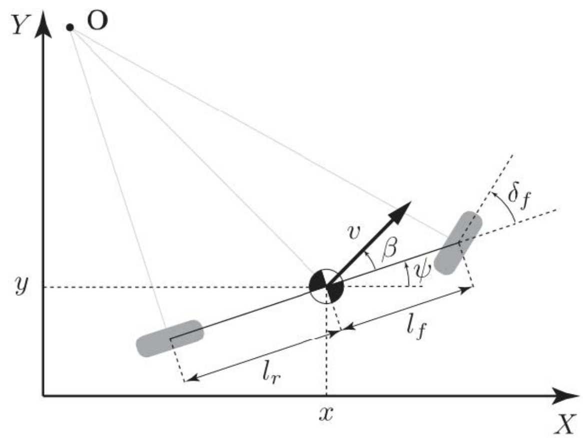

Figure 1: Simplified Kinematic Bicycle Model

with states $z = \begin{bmatrix} x, & y, & \psi \end{bmatrix}^T$ and inputs $u = \begin{bmatrix} v_x, & \delta_f \end{bmatrix}^T$ where

$$
\begin{aligned}
x &= \text{inertial X coordinate of CoM} \\
y &= \text{inertial Y coordinate of CoM} \\
\psi &= \text{global heading angle} \\
v &= \text{velocity of the vehicle at CoM, } v = \sqrt{v_x^2 + v_y^2} \\
v_x &= \text{longitudinal velocity of the vehicle at CoM} \\
v_y &= \text{lateral velocity of the vehicle at CoM} \\
\delta_f &= \text{steering angle of the front wheels with respect to the longitudinal axis of the car} \\
\beta &= \text{side slip angle at CoM, } \beta = \tan^{-1}\left(\frac{l_r}{l_r + l_f}\tan(\delta_f)\right) \approx \frac{v_y}{v_x} \\
l_r &= \text{distance from the CoM to the rear axle} \\
l_f &= \text{distance from the CoM to the front axle}
\end{aligned}
$$

Given the longitudinal velocity provided from the dGPS and the front steering angle provided from the Steering Angle Sensor(SAS), construct the open-loop estimated trajectory of the vehicle with the kinematic bicycle model.

1. ***Deliverable**: Complete kinematic equations in **subsystem 1** of the ME131_lab8.slx file.
2. ***Deliverable**: Submit a plot of the trajectories generated by the kinematic bicycle model and the dGPS data.
3. ***Deliverable**: What do you observe by comparing the trajectories of open-loop prediction and the one provided from dGPS? Can you use your knowledge to explain which driving conditions the kinematic bicycle model would be suitable for. When does it fail?

Task 2 Dynamic Bicycle Model

In this section of the lab, we are going to use the dynamic bicycle model to open-loop predict the vehicle trajectory. We assume the vehicle is traveling with slow time-vary longitudinal velocity and small front steering angle with linear tire models. The vehicle model is derived by writing out the balance of linear momentum and balance of angular momentum equations. The state space representation for the system dynamics is given below:

$$
\begin{aligned}
\dot{v}_x &= \omega_z v_y + a_x \\
\dot{v}_y &= \frac{C_f}{m}\left(\delta_f - \frac{v_y + l_f\omega_z}{v_x}\right) + \frac{C_r}{m}\left(-\frac{v_y - l_r\omega_z}{v_x}\right) - \omega_z v_x \\
\dot{\omega}_z &= \frac{l_f C_f}{I_z}\left(\delta_f - \frac{v_y + l_f\omega_z}{v_x}\right) + \frac{l_r C_r}{I_z}\left(-\frac{v_y - l_r\omega_z}{v_x}\right) \\
\dot{x} &= v_x\cos(\psi) - v_y\sin(\psi) \\
\dot{y} &= v_x\sin(\psi) + v_y\cos(\psi) \\
\dot{\psi} &= \omega_z
\end{aligned}
\tag{2}
$$

with states $z = \left[v_x,\ v_y,\ \omega_z,\ x,\ y,\ \psi\right]^T$ and inputs $u = \left[a_x,\ \delta_f\right]^T$ where

$$
\begin{aligned}
x &= \text{inertial X coordinate of CoM} \\
y &= \text{inertial Y coordinate of CoM} \\
\psi &= \text{global heading angle} \\
v_x &= \text{longitudinal velocity of the vehicle} \\
\delta_f &= \text{steering angle of the front wheel} \\
l_r &= \text{distance from the CoM to the rear axle} \\
l_f &= \text{distance from the CoM to the front axle} \\
m &= \text{vehicle mass} \\
\omega_z &= \text{yaw rate} \\
C_r &= \text{rear tire cornering stiffness} \\
C_f &= \text{front tire cornering stiffness} \\
I_z &= \text{yaw moment of inertia}
\end{aligned}
$$

We define the following parameter values for our system:

$$
\begin{aligned}
m &= 1830.59[\text{kg}] \\
I_z &= 3477\ [\text{kg m}^2] \\
l_r &= 1.15214\ [\text{m}] \\
l_f &= 1.69286\ [\text{m}] \\
C_r &= 57269\ [\text{N/rad}] \\
C_f &= 48703\ [\text{N/rad}]
\end{aligned}
$$

The dynamic vehicle model utilizes a cornering stiffness factor that is calculated from driving data. Given the longitudinal acceleration provided from the dGPS and the front steering angle provided from SAS, construct the open-loop estimated trajectory of the vehicle with the dynamics bicycle model.

1. ***Deliverable**: Complete dynamics equations in **subsystem 2** of the ME131_lab8.slx file.
2. ***Deliverable**: Submit a plot of the trajectories generated by the dynamic bicycle model and the dGPS data.
3. ***Deliverable**: What do you observe by comparing the trajectories of open-loop prediction and the one provided from dGPS? Can you use your knowledge to explain which driving conditions the dynamic bicycle model would be suitable for.
4. ***Deliverable**: Compare the trajectories of both bicycle models and explain why one model outperforms the other in this case.