CS2040 Data Structures and Algorithms Lecture Note #5

List ADT – Array Lists and Linked Lists

Outline

- 1. Use of a List (Motivation)
 - List ADT
- 2. List ADT Implementation via Array
 - Adding and removing elements in an array
 - Time and space efficiency
- 3. List ADT Implementation via Linked Lists
 - Linked list approach
 - ListNode class: forming a linked list with ListNode
 - BasicLinkedList
- 4. More Linked Lists
 - EnhancedLinkedList, TailedLinkedList
- 5. Other Variants
 - CircularLinkedList, DoublyLinkedList
- 6. Java API: LinkedList class
- 7. Summary

1 Use of a List

Motivation

Motivation

- List is one of the most basic types of data collection
 - For example, list of groceries, list of modules, list of friends, etc.
 - In general, we keep items of the same type (class) in one list
- Typical Operations on a data collection
 - Add data
 - Remove data
 - Query data
 - The details of the operations vary from application to application. The overall theme is the management of data





ADT of a List (1/3)

- A list ADT is a dynamic linear data structure
 - A collection of data items, accessible one after another starting from the beginning (head) of the list
- Examples of List ADT operations:
 - Create an empty list
 - Determine whether a list is empty
 - Determine number of items in the list
 - Add an item at a given position
 - Remove an item at a position
 - Remove all items
 - Read an item from the list at a position
 - ...

ADT of a List (2/3)

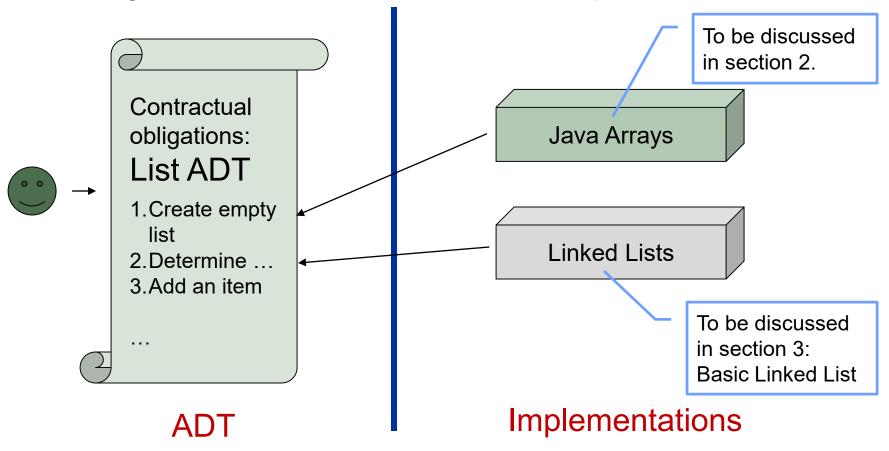
```
import java.util.*;
                                            ListInterface.java
// list interface for a list of integers
// Note: 1st item at index 0 & last item at index N-1
//
        (where N is number of items in the list)
public interface ListInterface {
  public boolean isEmpty();
  public int size();
  public int indexOf(int item);
  public boolean contains(int item);
  public int getFirst();
  public int getLast();
             addAtIndex(int index, int item);
 public void
  public void addFront(int item);
  public void addBack(int item);
  public int
               removeAtIndex(int index);
  public int
               removeFront();
  public int      removeBack();
  public void
              print();
```

ADT of a List (2/3)

- □ The ListInterface defines the operations (methods) we would like to have in a List ADT
- The operations shown here are just a small sample. An actual List ADT usually contains more operations.
- Here we assume that the List ADT only contains integer elements
- Using indices to access the elements in the list, 1st element is at index 0 and last element is at index N-1 (where N is the number of elements in the list)

ADT of a List (3/3)

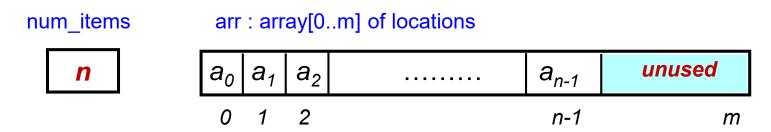
■ We will examine 2 implementations of list ADT, both using the ListInterface shown in the previous slide



2 List Implementation via Array

List Implementation: Array

- This is a straight-forward approach
 - ☐ Use Java array of a sequence of *n* items

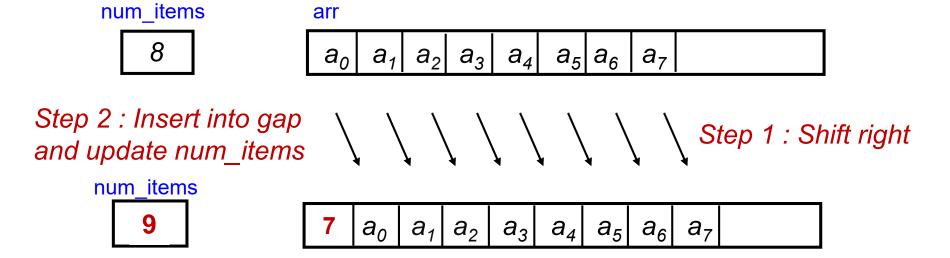


□ Since it is an array, items occupy a contiguous block of memory

Basic operations: Insert an item

- To insert an item at position/index i
 - Shift last item to item at index i to the right by 1 to create a "gap"
 - 2. Insert the new item in the "gap" created

Example: inserting 7 at index 0



Basic operations: Insert an item

- What happens if the array is already filled when inserting an item?
 - Enlarge it by creating a new array (usually doubling the size of original array) and copy original array over
 - Insert new item as per normal

 This makes the array a dynamic array (can be re-sized) instead of a static array (fixed size) Insert method with array resizing

```
ListUsingArray.java
import java.util.*;
class ListUsingArray implements ListInterface {
  public int capacity = 1000; // size of the array
  public int num items;  // number of items in the array
  public int[] arr = new int[capacity];
  // helper non-interface methods
 public void insert(int index, int item) {
    if (num items+1 > capacity) // array is full, enlarge it
     enlargeArr();
    for (int i=num items-1; i >= index; i--) // create gap
      arr[i+1] = arr[i];
    arr[index] = item; // insert item in gap
   num items++;
```

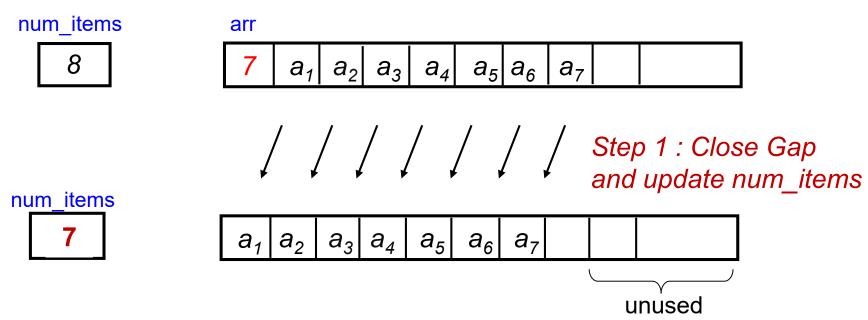
Insert method with array resizing

```
ListUsingArray.java
public void enlargeArr() {
  int newSize = capacity * 2; // double the size
  int[] temp = new int[newSize];
  if (temp == null) { // not enough memory
    System.out.println("run out of memory!");
    System.exit(1);
  // copy the original array to the new array
  for (int j=0; j < num items; j++)</pre>
    temp[j] = arr[j];
  arr = temp; // point arr to the new array
  capacity = newSize;
```

Basic operations: Remove an item

- To remove/delete an item at position/index i
 - Shift current items from index i+1 and onwards to the left by 1 to delete the item and close the gap

Example: remove first item



Remove method

```
ListUsingArray.java
import java.util.*;
class ListUsingArray implements ListInterface {
  // remove the item at index and return it
 public int remove(int index) {
    int item = arr[index];
    // shift item from index+1 onwards to the left to close the gap
    for (int i=index+1; i < num items; i++)</pre>
      arr[i-1] = arr[i];
    num items--;
    return item;
```

List operations using array implementation

- Useful operations
 - empty() return true if num_items is 0, false otherwise
 - size() return num_items

- Operations to check if item exist in the list
 - indexOf(int item) scan through the array and return the index of item if it is found otherwise return -1
 - contains(int item) if indexOf(item) == -1 return false
 else return true

List operations using array implementation

- Operations to retrieve/access item at index i in the list
 - getItemAtIndex(int i) if i within the bounds of the array return arr[i]
 - getFirst() return getItemAtIndex(0)
 - getLast() return getItemAtIndex(num_items-1)
- Operations to add item at index i in the list
 - *(i = num_items means add as last item in the list)
 - addAtIndex(int i, int item) if 0 <= i <= num_items call insert(i,item)
 - addFront(int item) call addAtIndex(0,item)
 - addBack(int item) call addAtIndex(num_items,item)

List operations using array implementation

- Operations to remove an item at index i in the list and return it
 - removeItemAtIndex(int i) if i is within the bounds of the array return remove(i) otherwise do nothing
 - removeFront() return removeItemAtIndex(0)
 - removeBack() return removeItemAtIndex(num_items-1)
- Refer to ListUsingArray.java for the full program

Testing Array Implementation of List

```
TestListUsingArray.java
import java.util.*;
public class TestListUsingArray {
  public static void main(String [] args) {
    ListUsingArray list = new ListUsingArray();
    list.addFront(1);
    list.addFront(2);
                                         List is: 3, 2, 5, 1, 4.
    list.addFront(3);
                                          Testing removal
    list.addBack(4);
                                         List is: 2, 1.
    list.addAtIndex(2,5);
    list.print();
                                         List is: 6, 2, 1.
    System.out.println("Testing removal");
    list.removeFront();
    list.removeBack();
    list.removeAtIndex(1);
    list.print();
    if (list.contains(1))
      list.addFront(6);
    list.print();
```

Analysis of Array Implⁿ of List

- Time complexity of the different list operations
 - Retrieval: getItemAtIndex(int i), getFirst(), getLast()
 - O(1) indexing into an array is constant time due to random access memory of the computer
 - Insertion: addItemAtIndex(int i, int item), addFront(), addBack()
 - Best case = O(1) if adding at the back and no need to enlarge array
 - Worst case = O(n) if adding to the front due to shifting all item to the right or need to enlarge the array so have to perform copying of all n items to new array
 - Average case = O(n) on average need to shift $\frac{1}{2}(n)$ items to the right
 - Deletion: removeItemAtIndex(int i), removeFront(), removeBack()
 - Best case = O(1) if removing from the back
 - Worst case = O(n) if removing from the front due to shifting all items to the left
 - Average case = O(n) on average need to shift $\frac{1}{2}(n)$ items to the left

Analysis of Array Implⁿ of List

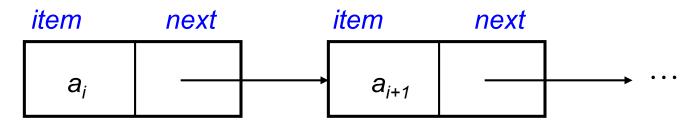
- What about the Space Complexity?
 - In the best case, we use exactly n space for n items
 - In the worst case, we use 2n space for n+1 item where n is multiple of 2
 - Either way, the space complexity is O(n)

3 List Implementation via Linked List



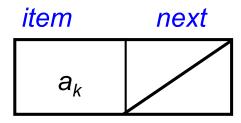
3.1 Linked List Approach

- Idea
 - Each item in the list is stored in a *node*, which also contains a next pointer that references/points to the node to its right (its neighbour)
 - Order the nodes by associating each with its neighbour(s)
 - Allow elements in the list to occupy non-contiguous memory



This is one node of the collection...

... and this one comes after it in the collection (most likely not occupying contiguous memory that is next to the previous node).



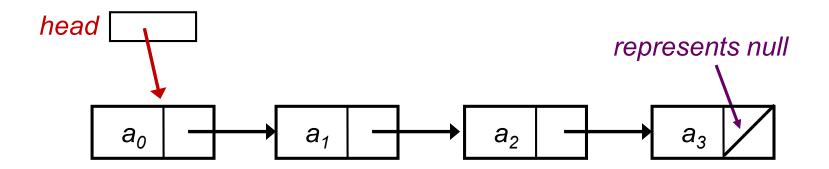
Next pointer of this node is "null", i.e. it has no next neighbour.

3.3 ListNode – contain integer <u>element</u>

```
ListNode.java
class ListNode {
  /* attributes */
  public int item;
  public ListNode next;
  /* constructors */
  public ListNode(int val) { this(val, null); }
  public ListNode(int val, ListNode n) {
    item = val;
                           Mark this slide – You may need to refer to it later
    next = n;
                           when we study the different variants of linked list.
  }
  /* get the next ListNode */
  public ListNode getNext() { return next; }
  /* set the next reference */
  public void setNext(ListNode n) { next = n };
  /* get the item of the ListNode */
  public int getItem() { return item; }
  /* set the item of the ListNode */
  public int setItem(int val) { item = val; }
```

Forming a Linked List

 \square For a sequence of 4 items < a_0 , a_1 , a_2 , a_3 >



We need a *head* to indicate where the first node is. From the *head* we can get to the rest.

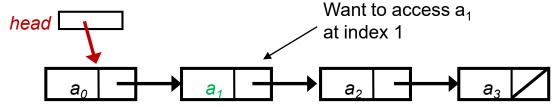
Create a BasicLinkedList class that contains a head attribute and implements the list operations

3.5 Basic Linked List

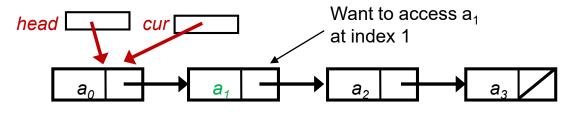
Using ListNode to define BasicLinkedList

Basic Operations: Accessing items in the LL

 To access an item at index i in the LL we need to have a reference pointing to its node (cannot directly access the node like an array)

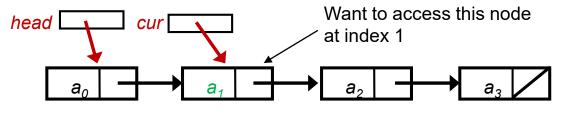


Have a reference cur that start from head



ListNode cur = head;

And "moves" towards node containing a₁



*To access nodes further away, simply use a loop

cur = cur.getNext();

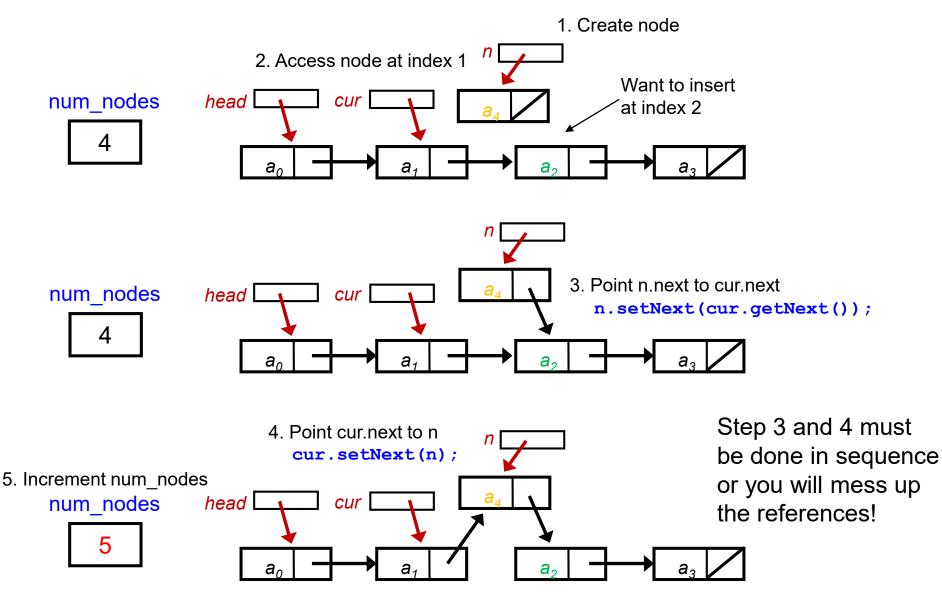
Accessing an item: getItemAtIndex(int i)

```
BasicLinkedList.java
import java.util.*;
class BasicLinkedList implements ListInterface {
                                              We have an equivalent method
  public int getItemAtIndex(int index) { ListNode getNodeAtIndex(int index)
                                              which returns the node instead of item in
    int counter = 0;
    int item = 0;
                                              the node. Refer to BasicLinkedList.java
    if (index < 0 || index > size()-1) {
      System.out.println("invalid index");
      System.exit(1);
    for (ListNode cur = head; cur != null; cur = cur.getNext(),
         counter++) {
      if (counter == index) {
         item = cur.getItem();
                                                     Move through the list
                                 Start from head
        break;
                                             While not moved past
    return item;
                                             last node in the list
```

Basic Operations: Insert an item

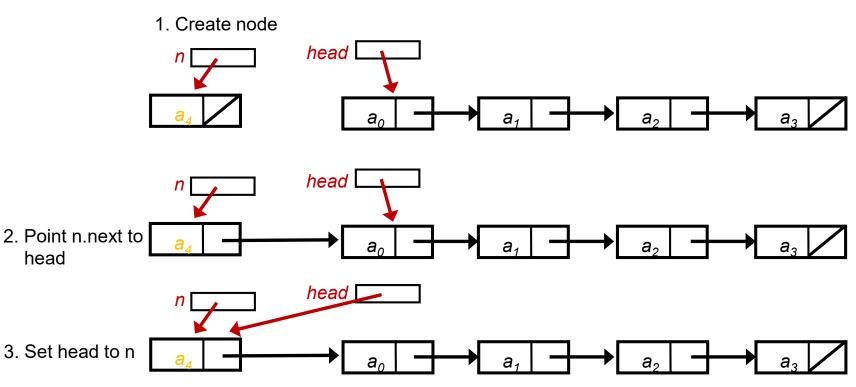
- Insert an item at index i in the linked list
 - Create a new node n containing the item
 - Access the node cur at index i-1
 - Point next reference of n to neighbor of cur
 - 4. Point next reference of cur to n
 - Increment number of nodes

Basic Operations: Insert an item



Insert an item - Special cases?

- When adding to the front of the linked list (i.e index 0) → In this case cur == null (no node before index 0)
 - Create new node n
 - Point next of n to head
 - 3. Set head to n && increment num_nodes (not shown due to space constraint)



Insert an item - Special cases?

- 2. When the list is empty
 - New item is added to the front of the list therefore this is same case as 1.
- 3. When adding to back of the list (i.e index num_nodes)
 - No need special handling as cur will be pointing to the last node (which is at index num_nodes-1)

Insert an item: insert(ListNode cur, ListNode n)

 Code for inserting a new node n at index i given reference cur to node at index i-1

```
BasicLinkedList.java
import java.util.*;
class BasicLinkedList implements ListInterface {
  public void insert(ListNode cur, ListNode n) {
    if (cur == null) { // insert in front of list
      n.setNext(head);
      head = n; // update head
    else { // insert anywhere else
                                                    Time complexity of
      n.setNext(cur.getNext());
                                                    insert is O(1) (no loops,
      cur.setNext(n);
                                                    only a fixed number of
                                                    statements)
    num nodes++; // important !
```

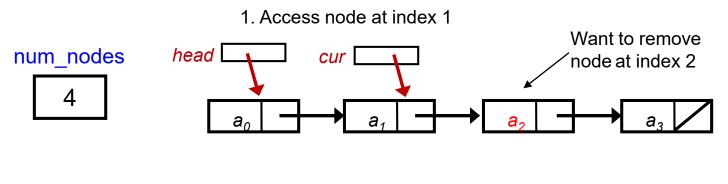
Implement addAtIndex using insert

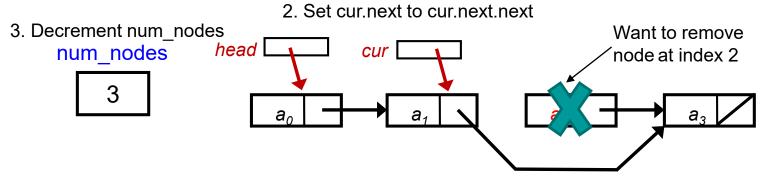
List operation addAtIndex can now be easily implemented using insert

```
BasicLinkedList.java
public void addAtIndex(int index, int item) {
  ListNode cur;
  ListNode newNode = new ListNode(item);
  if (index >= 0 && index <= size()) {</pre>
    if (index == 0) // insert in front
      insert(null,newNode);
    else {
      cur = getNodeAtIndex(index-1); // access node at index-1
      insert(cur,newNode);
  else { // index out of bounds
   System.out.println("invalid index");
   System.exit(1);
```

Basic Operation: Remove an item

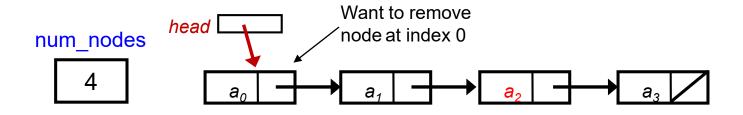
- Remove an item at index i in the linked list
 - Access the node cur at index i-1
 - 2. Point next reference of cur to neighbor of neighbor of cur
 - 3. Decrement number of nodes

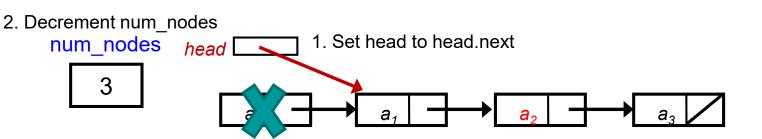




Remove an item - Special case

- When removing from the front of the linked list (i.e index
 - 0) → In this case cur == null (no node before index 0)
 - Point head to head.next
 - decrement num_nodes





Remove an item: remove(ListNode cur)

 Code for removing a node at index i given reference cur to node at index i-1

```
BasicLinkedList.java
import java.util.*;
class BasicLinkedList implements ListInterface {
 public int remove(ListNode cur) {
    int value;
    if (cur == null) { // remove 1st node
      value = head.getItem();
      head = head.getNext(); // update head
    else { // remove any other node
                                                   Time complexity of
      value = cur.getNext().getItem();
                                                   remove is O(1) (no loops,
      cur.setNext(cur.getNext().getNext());
                                                   only a fixed number of
    num nodes--; // important !
                                                   statements)
    return value;
```

Implement removeAtIndex using remove

 List operation removeAtIndex can now be easily implemented using remove

```
BasicLinkedList.java
public int removeAtIndex(int index) {
  ListNode cur;
  int item = 0;
  // index within bounds and list is not empty
  if (index >= 0 \&\& index < size()) {
    if (index == 0) // remove 1st item
      item = remove(null);
    else {
      cur = getNodeAtIndex(index-1); // access node at index-1
      item = remove(cur);
  else { // index out of bounds
    System.out.println("invalid index or list is empty");
    System.exit(1);
  return item;
```

3.5 Test Basic Linked List

```
TestBasicLinkedList.java
import java.util.*;
public class TestBasicLinkedList {
  public static void main(String [] args) {
    BasicLinkedList list = new BasicLinkedList();
    list.addFront(1);
    list.addFront(2);
                                          List is: 3, 2, 5, 1, 4.
    list.addFront(3);
                                          Testing removal
    list.addBack(4);
                                          List is: 2, 1.
    list.addAtIndex(2,5);
    list.print();
                                          List is: 6, 2, 1.
    System.out.println("Testing removal");
    list.removeFront();
    list.removeBack();
    list.removeAtIndex(1);
    list.print();
    if (list.contains(1))
      list.addFront(6);
    list.print();
```

Analysis of Linked List Implementation of List

- Time complexity of the different list operations
 - Retrieval: getItemAtIndex(int i), getFirst(), getLast()
 - Best case = O(1) accessing the first node, return the head
 - Worst case = O(n) accessing the last node, since you need to move all the way to the back from the head (n moves)
 - Average case = O(n) need to move about half way through the list to access any node on average so ½(n) iterations of the for loop
 - Insertion: addItemAtIndex(int i, int item), addFront(), addBack()
 - Best case = O(1) if adding at the front (don't have to worry about enlarging the list unlike array)
 - Worst case = O(n) if adding to the back due to having to move all the way to the back from the head (n moves)
 - Average case = O(n) on average need to make $\frac{1}{2}(n)$ moves

Analysis of Linked List Implementation of List

- Deletion: removeItemAtIndex(int i), removeFront(), removeBack()
 - Best case = O(1) if removing from the front
 - Worst case = O(n) if removing from the back, again due to moving all the way to the back from the head
 - Average case = O(n) − on average need to make ½(n) moves

- What about the Space Complexity?
 - We use as much space as there are nodes in the list so exactly O(n)
 (plus some constant overhead to store each node which requires more space then a simple integer that is stored in an array)

When to use Array vs LL implementation of List

Depends on the problem

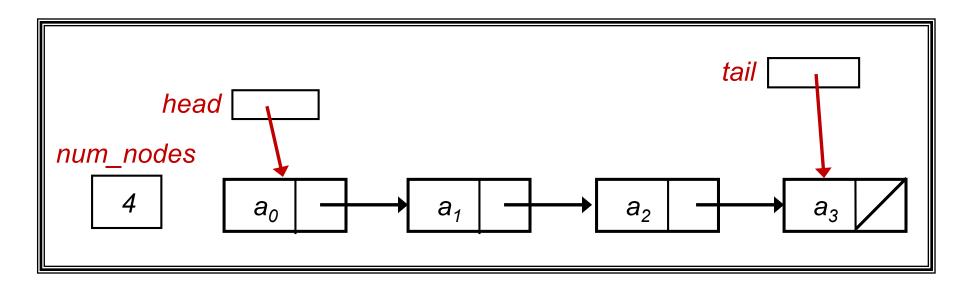
- □ Only need to add/remove to front of the list → Use LL
- □ Only need to add/remove to back of the list → Use Array, especially if we know maximum num. of items in advance, just create a large enough array from start
- If we need to add/remove anywhere in the list
 - Equal chance of adding to any index → both LL or Array is the same
 - If we need to keep adding/removing to a particular index i in the list → Use LL, maintain a reference to the node at index i-1, all insertions/deletions thereafter is only O(1) time
- □ If we have few insertions/deletions but a lot of accesses
 → Use Array since accessing any item is O(1) time

4 More Linked Lists

Exploring variants of linked list

Tailed Linked List

- We further improve on our Basic Linked List
 - To address the issue that adding to the end is slow (need to move all the way to the back)
 - Add an extra attribute called tail that points to the last node in the list
 - Extra attributes means extra maintenance too no free lunch!
- Difficulty: Learn to take care of ALL cases of updating...



4.1 Tailed Linked List

We create a new class TailedLinkedList:

```
import java.util.*;

public class TailedLinkedList implements ListInterface {
    ...
    ListNode tail; // additional attribute

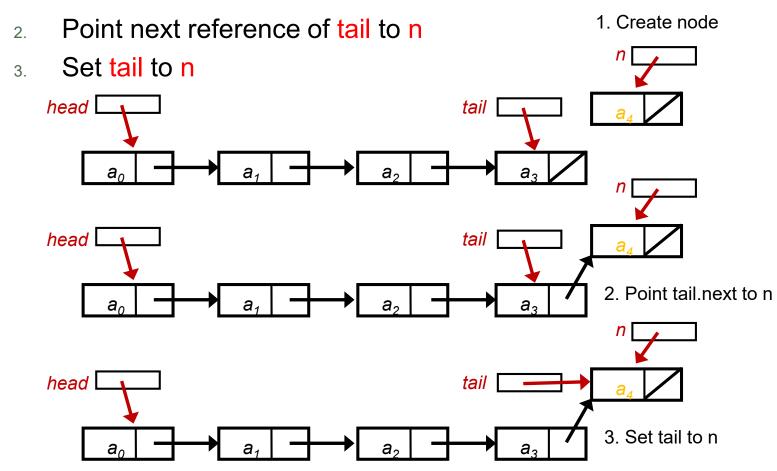
...
    public ListNode getTail(); // method to return tail
}
```

Accessing an item: getItemAtIndex(int i)

```
TailedLinkedList.java
class TailedLinkedList implements ListInterface {
  public int getItemAtIndex(int index) {
    int counter = 0;
    int item = 0;
    if (index < 0 || index > size()-1) {
      System.out.println("invalid index");
      System.exit(1);
                                           Simply return item in tail if
                                           accessing last item, thus
    if (index == size()-1)
                                           getLast() is O(1) time
      item = tail.getItem();
    else {
      for (ListNode cur = head; cur != null; cur = cur.getNext()
           counter++) {
        if (counter == index) {
          item = cur.getItem();
          break;
    return item;
                                                Otherwise same as for
                                                BasicLinkedList
```

Inserting in a tailed linked list

- Inserting at index == size(), Immediately insert using the tail without having to start from head and moving to the back.
 - Create the node n to be inserted



No Free lunch - Extra book keeping in insert()

- In the insert method need to update tail
 - 1. When inserting to tail (illustrated in previous slide)
 - 2. When inserting into empty list (tail == null)

```
TailedLinkedList.java
public void insert(ListNode cur, ListNode n) {
  if (cur == null) { // insert in front of list
    n.setNext(head);
    head = n; // update head
    if (tail == null) // update tail if list originally empty
      tail = head;
                                        Case 2
  else { // insert anywhere else
    n.setNext(cur.getNext());
    cur.setNext(n);
    if (cur == tail)
                              // update tail if insert new last item
      tail = tail.getNext();
                                               Case 1
  num nodes++;
```

Simple modification of addAtIndex

 List operation addAtIndex can now be easily modified to support fast insertion to end of the <u>Tailed Linked List</u>

TailedLinkedList.java

```
public void addAtIndex(int index, int item) {
  ListNode cur;
  ListNode newNode = new ListNode(item);
  if (index \geq= 0 && index \leq= size()) {
    if (index == 0) // insert in front
                                             Use tail as the cur reference
      insert(null,newNode);
                                             when inserting to the back.
    else if (index == size())
                                             Don't have to move all the
      insert(tail,newNode);
                                             way to the back to insert
    else {...} // insert anywhere else
  else {...} // index out of bounds
```

Removing from a tailed linked list

 Need to update tail reference whenever last item and only item is removed (NO FREE LUNCH!)

```
class TailedLinkedList implements ListInterface {
 public int remove(ListNode cur) {
    int value;
    if (cur == null) { // remove 1st node
      value = head.getItem();
      head = head.getNext(); // update head
      if (num nodes == 1)
                                          Update tail to null if only
        tail = null;
                                          item in list is removed
    else { // remove any other node
      value = cur.getNext().getItem();
      cur.setNext(cur.getNext().getNext());
      if (cur.getNext() == null)
                                              Update tail to cur if last item
        tail = cur;
                                              in list is removed
    num nodes--;
    return value;
```

4.2 Test Tailed Linked List (8/10)

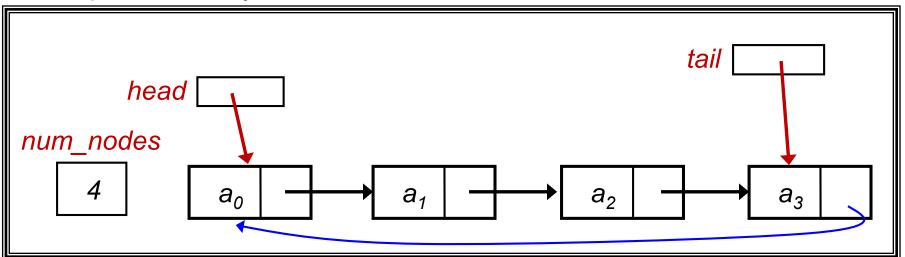
```
TestTailedLinkedList.java
import java.util.*;
public class TestTailedLinkedList {
  public static void main(String [] args) {
    TailedLinkedList list = new TailedLinkedList();
    list.addFront(1);
    list.addFront(2);
                                          List is: 3, 2, 5, 1, 4.
    list.addFront(3);
                                          Testing removal
    list.addBack(4);
                                          List is: 2, 1.
    list.addAtIndex(2,5);
    list.print();
                                          List is: 6, 2, 1.
    System.out.println("Testing removal");
    list.removeFront();
    list.removeBack();
    list.removeAtIndex(1);
    list.print();
    if (list.contains(1))
      list.addFront(6);
    list.print();
```

5 Other Variants

Other variants of linked lists

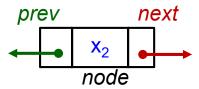
5.1 Circular Linked List

- There are many other possible enhancements of linked list
- Example: Circular Linked List
 - To allow cycling through the list repeatedly, e.g. in a round robin system to assign shared resource
 - Add a link from tail node of the TailedLinkedList to point back to head node
 - Difference in linking need different maintenance no free lunch!
- Difficulty: Learn to take care of ALL cases of updating, such as inserting/deleting the first/last node in a Circular Linked List
- Explore this on your own; write a class CircularLinkedList



Doubly Linked List

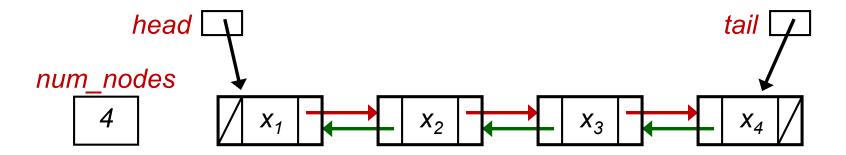
- In the preceding discussion, we have a "next" pointer to move forward
- Often, we need to move backward as well
- Use a "prev" pointer to allow backward traversal
- Once again, no free lunch need to maintain "prev" in all updating methods
- Instead of ListNode class, need to create a DListNode class that includes the additional "prev" pointer



Study DListNode.java for how to implement a DListNode

Doubly Linked List

An example of a doubly linked list



Usefulness of Doubly Linked List

- Reduce moving of references
 - When adding/removing/accessing a particular index, you can start from the end (front or back) that is closer to the index
 - Does not change time complexity of the List operations but improves the constant which is still important in practice! (especially in beating time limit constraints of lab assignments ...)
- Write a class DoublyLinkedList to implement the various linked list operations for a doubly linked list.

6 Java API: ArrayList class and LinkedList class

Using the LinkedList class

6 Java Classes: ArrayList and LinkedList

- These 2 are classes provided by Java library which implements the List ADT/interface
- ArrayList is the array implementation while LinkedList is the linked list implementation
- They have many more methods than what we have discussed so far of our versions of linked lists. On the other hand, we created some methods not available in the Java library class too.
- Please do not confuse this library class from our class illustrated here. In a way, we open up the Java library to show you the inside working.
- For purposes of labs or exam, you can use ArrayList or LinkedList if it is not stated that you have to use your own List implementation

6 Java Classes: ArrayList and LinkedList

- Both ArrayList and LinkedList have the same set of methods as they both implement the List interface
- Thus we can use them interchangeably
 - However note that the time complexity of some methods will differ due to the difference in the underlying data structure used (array vs linked list)
- Also both ArrayList and LinkedList classes are generic containers meaning they can contain objects of any class (but not primitive types)

Eg Using ArrayList

- Declaring an ArrayList variable/reference
 - To declare an ArrayList reference, specify the type of the object it contains. For example,

```
ArrayList<Integer> list;
```

list is a reference to an Arraylist containing Integer objects

- Creating an ArrayList object
 - Same as declaring an ArrayList reference need to specify the type of object it contains. For example,

```
list = new ArrayList<Integer>();
```

list now points to an ArrayList object that contains Integers

- Make sure the reference type and object type is the same/compatible
- LinkedList is used in the same manner

Why "reinvent the wheel"?

- In a data structures course, students are often asked to implement well-known data structures.
- A question we sometimes hear from students: "Since there is the API, why do we need to learn to write our own code to implement a data structure like linked list?"
- Writing the code allows you to gain an in-depth understanding of the data structures and their operations
- The understanding will allow you to appreciate their time complexity analysis and use the API effectively

7 Summary

- We learn to create our own data structure to implement the List ADT
 - Need to be careful with boundary cases
 - Manipulation of references (The sequence of statements is important! With the wrong sequence, the result will be wrong.)
 - Re-use of codes (Think in terms of basic operations and how more complex operations can re-use the basic ones
 → removeLast & removeFirst, addLast & addFirst)
 - Drawings are very helpful in understanding the cases, which then can help in knowing what can be used/manipulated

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