强化学习及其应用

Reinforcement Learning and Its Applications

第五章 值函数逼近

Value Function Approximation

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课件放映 PDF-〉视图-〉全屏模式

第五章 值函数逼近

第五章 值函数逼近

值函数逼近

问题描述

- **Large-Scale Reinforcement Learning**

Prediction 和 Control 问题，都需要值函数估计，S --> V(S)， (S, A) --> Q(S, A)

Large MDPs 状态集大，甚至是连续状态空间。

带来的问题：存储需求大，每个 S 的值都要估计，计算代价大。

- **解决办法：值函数逼近**

$$\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$$

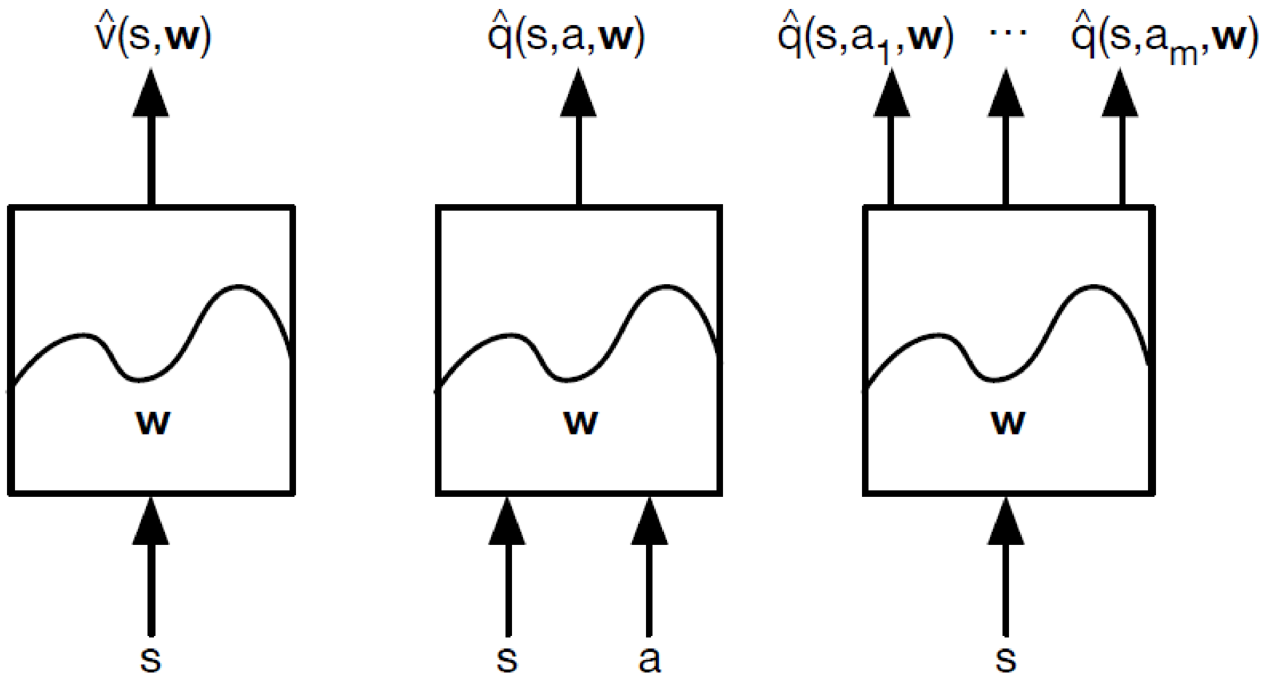
$$\text{or } \hat{q}(s, a, \mathbf{w}) \approx q_\pi(s, a)$$

训练学习值函数，对S的值函数可以通过函数逼近来得到。

值函数逼近

问题描述

- **逼近的值函数**



值函数逼近

问题描述

学习函数

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

第五章 值函数逼近

5.1 值函数逼近

5.2 增量预测与控制

5.3 批量预测与控制

5.4 算法总结

增量预测与控制

增量预测

- **Value Function Approx. By Stochastic Gradient Descent**

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$$

- Gradient descent finds a local minimum

$$\begin{aligned} \Delta \mathbf{w} &= -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \\ &= \alpha \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \right] \end{aligned}$$

- Stochastic gradient descent *samples* the gradient

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

增量预测

Linear Value Function Approximation

$$\hat{v}(S,\mathbf{w}) = \mathbf{x}(S)^\top \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S)\mathbf{w}_j$$

Represent state by a *feature vector*

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

$$\nabla_{\mathbf{w}}\hat{v}(S,\mathbf{w}) = \mathbf{x}(S)$$

$$\Delta\mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S,\mathbf{w}))\mathbf{x}(S)$$

增量预测算法

MC with Value Function Approximation

- Return G_t is an unbiased, noisy sample of true value $v_\pi(S_t)$
- Can therefore apply supervised learning to "training data":

$$\langle S_1, G_1\rangle, \langle S_2, G_2\rangle, ..., \langle S_T, G_T\rangle$$

- For example, using *linear Monte-Carlo policy evaluation*

$$\begin{aligned}\Delta \mathbf{w} &= \alpha({\color{red}G_t} - \hat{v}(S_t, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(S_t, \mathbf{w}) \\ &= \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)\end{aligned}$$

- Monte-Carlo evaluation converges to a local optimum
- Even when using non-linear value function approximation

增量预测算法

TD(0) with Value Function Approximation

- The TD-target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$ is a *biased* sample of true value $v_\pi(S_t)$
- Can still apply supervised learning to "training data":

$$\langle S_1, R_2 + \gamma \hat{v}(S_2, \mathbf{w}) \rangle, \langle S_2, R_3 + \gamma \hat{v}(S_3, \mathbf{w}) \rangle, \ldots, \langle S_{T-1}, R_T \rangle$$

- For example, using *linear TD(0)*

$$\begin{aligned} \Delta \mathbf{w} &= \alpha (R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha \delta \mathbf{x}(S) \end{aligned}$$

- Linear TD(0) converges (close) to global optimum

增量预测与控制

增量预测算法

TD(λ) with Value Function Approximation

- The λ-return G_t^λ is also a biased sample of true value $v_\pi(s)$
- Can again apply supervised learning to "training data":

$$\left\langle S_1, G_1^\lambda \right\rangle, \left\langle S_2, G_2^\lambda \right\rangle, ..., \left\langle S_{T-1}, G_{T-1}^\lambda \right\rangle$$

- Forward view linear TD(λ)

$$\begin{aligned}\Delta \mathbf{w} &= \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{v}(S_t, \mathbf{w}) \\ &= \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)\end{aligned}$$

- Backward view linear TD(λ)

$$\begin{aligned}\delta_t &= R_{t+1} + \gamma\hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \\ E_t &= \gamma\lambda E_{t-1} + \mathbf{x}(S_t) \\ \Delta \mathbf{w} &= \alpha\delta_t E_t\end{aligned}$$

Forward view and backward view linear TD(λ) are equivalent

增量控制

- **Action-Value Function Approximation by SGD**

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))^2 \right]$$

stochastic gradient descent to find a local minimum

$$-\frac{1}{2}\nabla_{\mathbf{w}} J(\mathbf{w}) = (q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S,A,\mathbf{w})$$

$$\Delta\mathbf{w} = \alpha(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S,A,\mathbf{w})$$

增量控制

Linear Action-Value Function Approximation

$$\hat{q}(S, A, \mathbf{w}) = \mathbf{x}(S, A)^\top \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S, A)\mathbf{w}_j$$

Represent state *and* action by a *feature vector*

$$\mathbf{x}(S, A) = \begin{pmatrix} \mathbf{x}_1(S, A) \\ \vdots \\ \mathbf{x}_n(S, A) \end{pmatrix}$$

Stochastic gradient descent update

$$\nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w}) = \mathbf{x}(S, A)$$

$$\Delta \mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\mathbf{x}(S, A)$$

增量控制算法

- For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

- For TD(0), the target is the TD target $R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

- For forward-view TD(λ), target is the action-value λ-return

$$\Delta \mathbf{w} = \alpha(q_t^{\lambda} - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

- For backward-view TD(λ), equivalent update is

$$\begin{aligned}
\delta_t &= R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}) \\
E_t &= \gamma\lambda E_{t-1} + \nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w}) \\
\Delta \mathbf{w} &= \alpha\delta_t E_t
\end{aligned}$$

第五章 值函数逼近

批量预测与控制

批量预测问题

- **Value Function Approx.**

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$$

- **Least Squares Prediction (LSP)**

$$\begin{aligned} LS(\mathbf{w}) &= \sum_{t=1}^{T} (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2 \\ &= \mathbb{E}_\mathcal{D} \left[(v^\pi - \hat{v}(s, \mathbf{w}))^2 \right] \end{aligned}$$

SGD with Experience Replay

Given experience consisting of $\langle state, value \rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, ..., \langle s_T, v_T^\pi \rangle\}$$

Repeat:

1. Sample state, value from experience

$$\langle s, v^\pi \rangle \sim \mathcal{D}$$

2. Apply stochastic gradient descent update

$$\Delta \mathbf{w} = \alpha(v^\pi - \hat{v}(s, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(s, \mathbf{w})$$

Converges to least squares solution

$$\mathbf{w}^\pi = \underset{\mathbf{w}}{\operatorname{argmin}}\ LS(\mathbf{w})$$

批量预测与控制

MSE 预测

LSP 最优解

We can solve the least squares solution directly

$$\mathbb{E}_{\mathcal{D}}\left[\Delta \mathbf{w}\right] = 0$$

Linear LSP 最优解

Using *linear* value function approximation $\hat{v}(s, \mathbf{w}) = \mathbf{x}(s)^\top \mathbf{w}$

$$\alpha \sum_{t=1}^{T} \mathbf{x}(s_t)(v_t^\pi - \mathbf{x}(s_t)^\top \mathbf{w}) = 0$$

批量预测与控制

MSE 预测

Linear LSP 算法

LSMC	$0 = \sum_{t=1}^{T} \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)$
	$\mathbf{w} = \left(\sum_{t=1}^{T} \mathbf{x}(S_t)\mathbf{x}(S_t)^\top\right)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t)G_t$
LSTD	$0 = \sum_{t=1}^{T} \alpha(R_{t+1} + \gamma\hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t)$
	$\mathbf{w} = \left(\sum_{t=1}^{T} \mathbf{x}(S_t)(\mathbf{x}(S_t) - \gamma\mathbf{x}(S_{t+1}))^\top\right)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t)R_{t+1}$
LSTD(λ)	$0 = \sum_{t=1}^{T} \alpha\delta_t E_t$
	$\mathbf{w} = \left(\sum_{t=1}^{T} E_t(\mathbf{x}(S_t) - \gamma\mathbf{x}(S_{t+1}))^\top\right)^{-1} \sum_{t=1}^{T} E_t R_{t+1}$

批量预测与控制

批量控制问题

- **Action-Value Function Approx.**

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \right]$$

- **Least Squares Control**

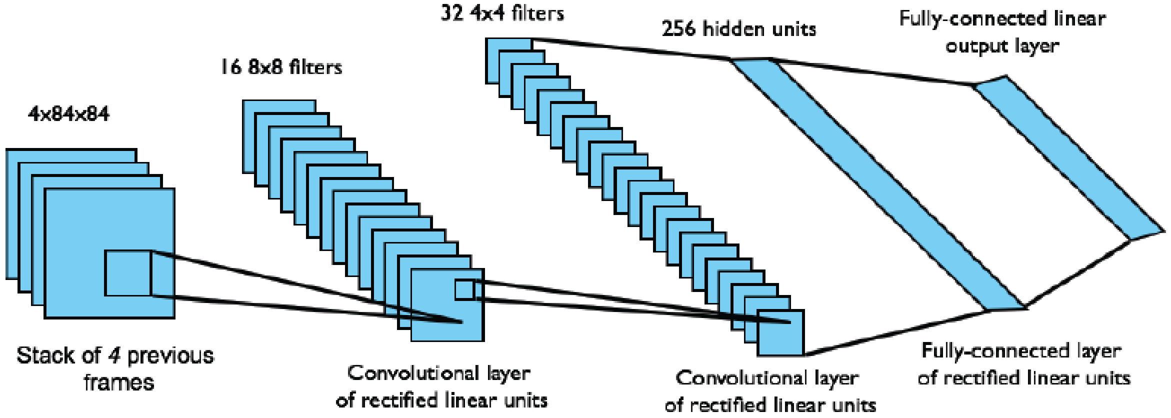
$$\begin{aligned} LS(\mathbf{w}) &= \mathbb{E}_\mathcal{D} \left[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \right] \\ &= \sum_{t=1}^{T} (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \end{aligned}$$

批量预测与控制

Experience Replay in Deep Q-Networks (DQN)

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i}\left[\left(r + \gamma \max_{a'} Q(s', a'; w_i^-) - Q(s, a; w_i)\right)^2\right]$$

- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step



Least Squares Q-Learning

- **Using Linear Action-State Function**

LSTDQ algorithm: solve for total update = zero

$$0 = \sum_{t=1}^{T} \alpha (R_{t+1} + \gamma \hat{q}(S_{t+1}, \pi(S_{t+1}), \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\mathbf{x}(S_t, A_t)$$

$$\mathbf{w} = \left(\sum_{t=1}^{T} \mathbf{x}(S_t, A_t)(\mathbf{x}(S_t, A_t) - \gamma \mathbf{x}(S_{t+1}, \pi(S_{t+1})))^\top \right)^{-1} \sum_{t=1}^{T} \mathbf{x}(S_t, A_t) R_{t+1}$$

Least Squares Q-Learning

- The following pseudocode uses LSTDQ for policy evaluation
- It repeatedly re-evaluates experience $\mathcal{D}$ with different policies

function LSPI-TD($\mathcal{D}, \pi_0$)
　$\pi' \leftarrow \pi_0$
　repeat
　　$\pi \leftarrow \pi'$
　　$Q \leftarrow$ LSTDQ($\pi, \mathcal{D}$)
　　for all $s \in \mathcal{S}$ **do**
　　　$\pi'(s) \leftarrow \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, Q(s, a)$
　　end for
　until $(\pi \approx \pi')$
　return π
end function

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算法总结

Convergence of Prediction Algorithms

On/Off-Policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	TD(0)	✓	✓	✗
	TD(λ)	✓	✓	✗
Off-Policy	MC	✓	✓	✓
	TD(0)	✓	✗	✗
	TD(λ)	✓	✗	✗

算法总结

Gradient Temporal-Difference Learning

On/Off-Policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	TD	✓	✓	✗
	Gradient TD	✓	✓	✓
Off-Policy	MC	✓	✓	✓
	TD	✓	✗	✗
	Gradient TD	✓	✓	✓

算法总结

Convergence of Linear Least Squares Prediction Algorithms

On/Off-Policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	LSMC	✓	✓	-
	TD	✓	✓	✗
	LSTD	✓	✓	-
Off-Policy	MC	✓	✓	✓
	LSMC	✓	✓	-
	TD	✓	✗	✗
	LSTD	✓	✓	-

算法总结

Convergence of Control Algorithms

Algorithm	Table Lookup	Linear	Non-Linear
Monte-Carlo Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-learning	✓	✗	✗
Gradient Q-learning	✓	✓	✗

(✓) = chatters around near-optimal value function

Algorithm	Table Lookup	Linear	Non-Linear
Monte-Carlo Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-learning	✓	✗	✗
LSPI	✓	(✓)	-

(✓) = chatters around near-optimal value function

本讲参考文献

1. Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. (Second edition, in progress，draft).
2. David Silver，Slides@《Reinforcement Learning:An Introduction》,2016.