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Q1:

设 $B = (\{x, y, n, a\}, \{s_0, s_1, s_2, s_3, s_4, 0, 1, 2, 3, +, -, *\}, \{<, =, >\})$
给定迁移系统 (T, Θ) ，其中 Θ 为 $a = s_0$ 且 T 为以下迁移：

$a = s_0$	\longrightarrow	$(x, y, a) := (0, 0, s_1)$
$a = s_1 \wedge x < n$	\longrightarrow	$(a) := (s_2)$
$a = s_2$	\longrightarrow	$(y, x, a) := (y + x * (x + 1), x + 1, s_1)$
$a = s_1 \wedge \neg(x < n)$	\longrightarrow	$(a) := (s_3)$
$a = s_3$	\longrightarrow	$(y, a) := (3 * y, s_4)$

给定 I 为 B 在整数上的正常解释。

计算最弱宽松前断言 $wlp(T, a=s_4)$ 即 $[T](a=s_4)$

并证明 $(a=s_3) \rightarrow X(a=s_4)$ 。

A1:

(1)

t1:a=s0

t2:a=s1 \wedge x<n

t3:a=s2

t4:a=s1 \wedge \neg (x<n)

t5:a=s3

$[t1](a=s_4) \quad a=s_0 \rightarrow (s_1=s_4)$

$[t2](a=s_4) \quad a= s_1 \wedge x<n \rightarrow (s_2=s_4)$

$[t3](a=s_4) \quad a=s_2 \rightarrow (s_1=s_4)$

$$[t4](a=s4) \quad a=s1 \wedge \neg(x<n) \rightarrow (s3=s4)$$

$$[t5](a=s4) \quad a=s3 \rightarrow (s4=s4)$$

$$[T](a=s4) = [t1](a=s4) \wedge [t2](a=s4) \wedge [t3](a=s4) \wedge [t4](a=s4) \wedge [t5](a=s4) =$$

$$a=s0 \rightarrow (s1=s4) \quad \wedge$$

$$a=s1 \wedge x<n \rightarrow (s2=s4) \quad \wedge$$

$$a=s2 \rightarrow (s1=s4) \quad \wedge$$

$$a=s1 \wedge \neg(x<n) \rightarrow (s3=s4) \quad \wedge$$

$$a=s3 \rightarrow (s4=s4)$$

(2)

$$[T^+]\varphi \equiv [T]\varphi \wedge (E(T) \vee \varphi)$$

$$E(T) = (a=s0 \vee (a=s1 \wedge x<n) \vee a=s2 \vee (a=s1 \wedge \neg(x<n)) \vee a=s3)$$

$$\varphi = (a=s4)$$

$$\text{原式} = [T]\varphi \wedge (a=s0 \vee (a=s1 \wedge x<n) \vee a=s2 \vee (a=s1 \wedge \neg(x<n)) \vee a=s3 \vee a=s4)$$

$$= [T]\varphi \wedge (s1=s4 \vee s2=s4 \vee s3=s4 \vee s4=s4)$$

上面部分，暂时不知道应该怎么化简能得到下面的式子

$$a=s3 \rightarrow [T^+](a=s4)$$

可得， $s3 \rightarrow X(a=s4)$

Q2:

设 $B = (\{x, y, n, a\}, \{s_0, s_1, s_2, s_3, s_4, 0, 1, 2, 3, +, -, *\}, \{<, =, >\})$
 给定迁移系统 (T, Θ) , 其中 Θ 为 $a = s_0$ 且 T 为以下迁移:

$a = s_0$	\longrightarrow	$(x, y, a) := (0, 0, s_1)$
$a = s_1 \wedge x < n$	\longrightarrow	$(a) := (s_2)$
$a = s_2$	\longrightarrow	$(y, x, a) := (y + x * (x + 1), x + 1, s_1)$
$a = s_1 \wedge \neg(x < n)$	\longrightarrow	$(a) := (s_3)$
$a = s_3$	\longrightarrow	$(y, a) := (3 * y, s_4)$

给定 I 为 B 在整数上的正常解释。证明以下命题成立:

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- (1) $(T, \Theta) \vdash_I n \geq 0 \rightarrow G(a = s_4 \rightarrow y = n * n * n - n)$
 (2) $(T, \Theta) \vdash_I n \geq 0 \rightarrow F(a = s_4)$
-

A2:

(1)

$$\phi \Rightarrow \varphi'$$

$$\varphi' \Rightarrow [T] \varphi'$$

$$\varphi' \Rightarrow \varphi$$

$$\phi \Rightarrow G\varphi$$

选用上述证明规则

通过证明 $(T, \Theta) \vdash n \geq 0 \wedge a = s_0 \Rightarrow G(a = s_4 \rightarrow y = n * n * n - n)$

然后推导得到我们想要的结果, 即下面的 (4)

$$(1) (T, \Theta) \vdash n \geq 0 \wedge a = s_0 \Rightarrow G(a = s_4 \rightarrow y = n * n * n - n)$$

$$(2) (T, \theta) \vdash n \geq 0 \wedge a = s0 \rightarrow G(a = s4 \rightarrow y = n * n * n - n) \quad (1) \text{ 推得}$$

$$(3) (T, \theta) \vdash a = s0 \quad \theta \text{ 即为 } a = s0$$

$$(4) (T, \theta) \vdash n \geq 0 \Rightarrow G(a = s4 \rightarrow y = n * n * n - n) \quad (2)(3)$$

为了证明 $(T, \theta) \vdash n \geq 0 \wedge a = s0 \Rightarrow G(a = s4 \rightarrow y = n * n * n - n)$

对于证明规则，我们给出以下参数即可

$$\phi = (a = s0 \wedge n \geq 0)$$

$$\phi' = (a = s0 \wedge n \geq 0) \vee$$

$$(a = s1 \wedge 3 * y = (x * x * x - x) \wedge x \leq n) \vee$$

$$(a = s2 \wedge 3 * y = (x * x * x - x) \wedge x < n) \vee$$

$$(a = s3 \wedge 3 * y = (x * x * x - x) \wedge x = n) \vee$$

$$(a = s4 \wedge y = (n * n * n - n))$$

$$\varphi = (a = s4 \rightarrow y = (n * n * n - n))$$

(2)

$$\phi \Rightarrow (\psi \vee \varphi)$$

$$\varphi \Rightarrow (w(t/x) \wedge (E(T) \vee \psi))$$

$$(\varphi \wedge t = v) \Rightarrow [T](\psi \vee (\varphi \wedge t < v))$$

$$\phi \Rightarrow F\psi$$