强化学习及其应用

Reinforcement Learning and Its Applications

第三章 值估计

Value Evaluation

授课人：周晓飞
zhouxiaofei@iie.ac.cn

2018-6-26

第三章 值估计

第三章 值估计

随机逼近

累计平均逼近 E(x)

$For\ time\ k:$

$$S \leftarrow S + x_k;$$

$$N \leftarrow N + 1;$$

$$u_k \leftarrow S/N;$$

$$N \rightarrow \infty, \qquad u \rightarrow E(x)$$

随机逼近

增量均值逼近 E(x)

For time k:

$$N \leftarrow N+1;$$

$$u_k \leftarrow u_{k-1} + (1/N)\,(x_k - u_{k-1});$$

相当于均值的更新：

$$\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) \\
&= \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)
\end{aligned}$$

随机逼近

Robbins-Monro 逼近 E(x)

For time k:

$$N \leftarrow N+1;$$

$$u_k \leftarrow u_{k-1} + a\,(x_k - u_{k-1});$$

相当于权重比例更新：

$$u_k \leftarrow (1-a)u_{k-1} + a x_k$$

本课程常用 Robbins-Monro 随机逼近公式

第三章 值估计

蒙特卡洛值估计

问题描述

- Goal: learn v_π from episodes of experience under policy π

$$S_1, A_1, R_2, ..., S_k \sim \pi$$

- Recall that the *return* is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$$

- Recall that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi [G_t \mid S_t = s]$$

- Monte-Carlo policy evaluation uses *empirical mean* return instead of *expected* return

蒙特卡洛值估计

First-Visit MC Evaluation

- To evaluate state s
- The first time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

只对 episode 的起始状态进行统计。

蒙特卡洛值估计

Every-Visit MC Evaluation

- To evaluate state s
- Every time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- Again, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

episode 的每个状态进行统计。

蒙特卡洛值估计

Incremental MC Evaluation

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, ..., S_T$
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

蒙特卡洛值估计

Incremental MC Evaluation

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, ..., S_T$
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

- In non-stationary problems, it can be useful to track a running mean, i.e. forget old episodes.

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

第三章 值估计

3.1 随机逼近

3.2 蒙特卡洛值估计

3.3 时序差分值估计

3.4 算法总结

时序差分值估计

问题描述

采用不完整的 episodes，估计V值。

- TD methods learn directly from episodes of experience
- TD is *model-free*: no knowledge of MDP transitions / rewards
- TD learns from *incomplete* episodes, by *bootstrapping*
- TD updates a guess towards a guess

时序差分值估计

TD (0)

- Update value $V(S_t)$ toward *estimated* return $\underline{R_{t+1} + \gamma V(S_{t+1})}$

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$

Bellman 迭代的随机形式

- $R_{t+1} + \gamma V(S_{t+1})$ is called the *TD target*
- $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the *TD error*

时序差分值估计

TD (0)

- **V.S. MC**
  - Incremental every-visit Monte-Carlo
    - Update value $V(S_t)$ toward *actual* return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

时序差分值估计

TD (0)

- TD can learn *before* knowing the final outcome
  - TD can learn online after every step
  - MC must wait until end of episode before return is known
- TD can learn *without* the final outcome
  - TD can learn from incomplete sequences
  - MC can only learn from complete sequences
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments

时序差分值估计

TD (0)

- Return $G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$ is *unbiased* estimate of $v_\pi(S_t)$
- True TD target $R_{t+1} + \gamma v_\pi(S_{t+1})$ is *unbiased* estimate of $v_\pi(S_t)$
- TD target $R_{t+1} + \gamma V(S_{t+1})$ is *biased* estimate of $v_\pi(S_t)$
- TD target is much lower variance than the return:
  - Return depends on *many* random actions, transitions, rewards
  - TD target depends on *one* random action, transition, reward

时序差分值估计

TD (0)

- MC has high variance, zero bias
  - Good convergence properties
  - (even with function approximation)
  - Not very sensitive to initial value
  - Very simple to understand and use
- TD has low variance, some bias
  - Usually more efficient than MC
  - TD(0) converges to $v_\pi(s)$
  - (but not always with function approximation)
  - More sensitive to initial value

时序差分值估计

TD (0)

- TD exploits Markov property
  - Usually more efficient in Markov environments
- MC does not exploit Markov property
  - Usually more effective in non-Markov environments

时序差分值估计

TD (0)

AB Example:

Two states A, B; no discounting; 8 episodes of experience

$\mathrm{A}, 0, \mathrm{B}, 0$
$\mathrm{B}, 1$
$\mathrm{B}, 1$
$\mathrm{B}, 1$
$\mathrm{B}, 1$
$\mathrm{B}, 1$
$\mathrm{B}, 1$
$\mathrm{B}, 0$

What is $V(A), V(B)$?

时序差分值估计

TD (0)

- MC converges to solution with minimum mean-squared error
  - Best fit to the observed returns

$$\sum_{k=1}^{K}\sum_{t=1}^{T_k}\left(G_t^k - V(s_t^k)\right)^2$$

  - In the AB example, $V(A) = 0$
- TD(0) converges to solution of max likelihood Markov model
  - Solution to the MDP $\langle \mathcal{S}, \mathcal{A}, \hat{\mathcal{P}}, \hat{\mathcal{R}}, \gamma \rangle$ that best fits the data

$$\hat{\mathcal{P}}_{s,s'}^{a} = \frac{1}{N(s,a)} \sum_{k=1}^{K}\sum_{t=1}^{T_k} \mathbf{1}(s_t^k, a_t^k, s_{t+1}^k = s, a, s')$$

$$\hat{\mathcal{R}}_{s}^{a} = \frac{1}{N(s,a)} \sum_{k=1}^{K}\sum_{t=1}^{T_k} \mathbf{1}(s_t^k, a_t^k = s, a) r_t^k$$

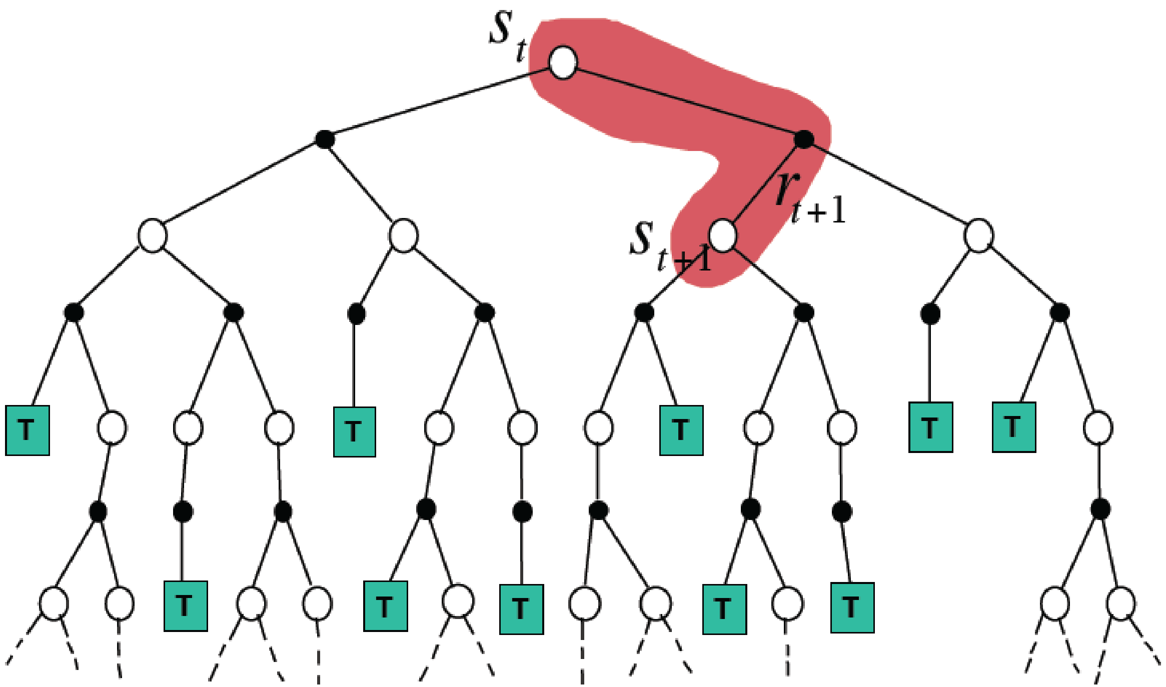
  - In the AB example, $V(A) = 0.75$

时序差分值估计

TD (0)

TD Backup

$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$

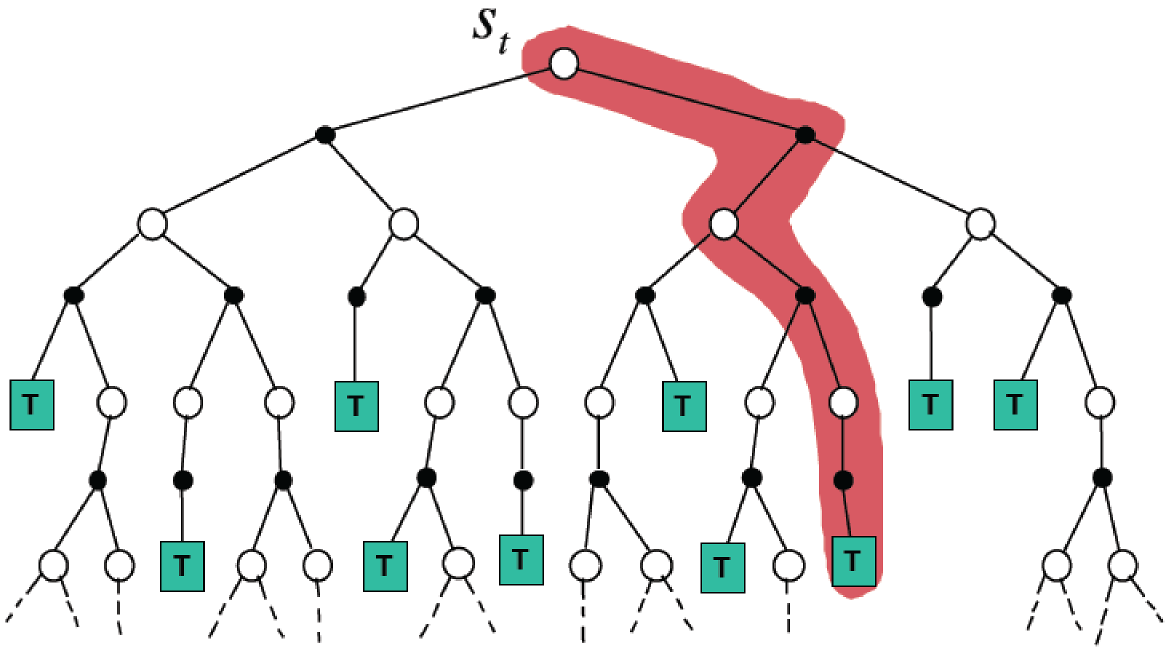


时序差分值估计

TD (0)

V.S. MC

$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t - V(S_t)\right)$$

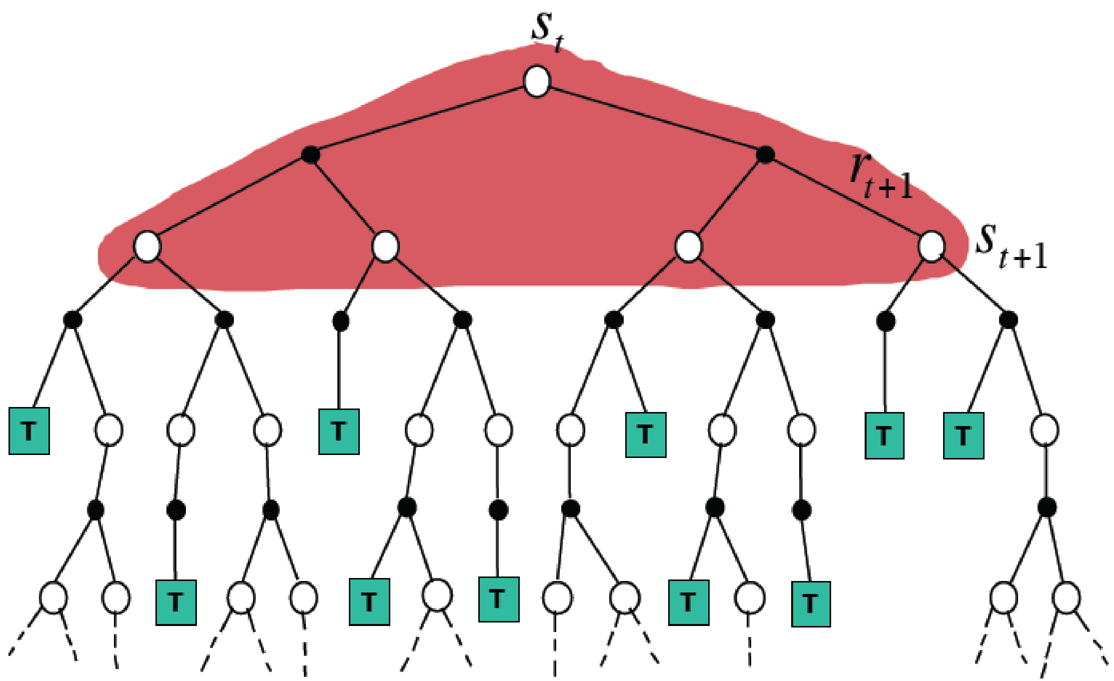


时序差分值估计

TD (0)

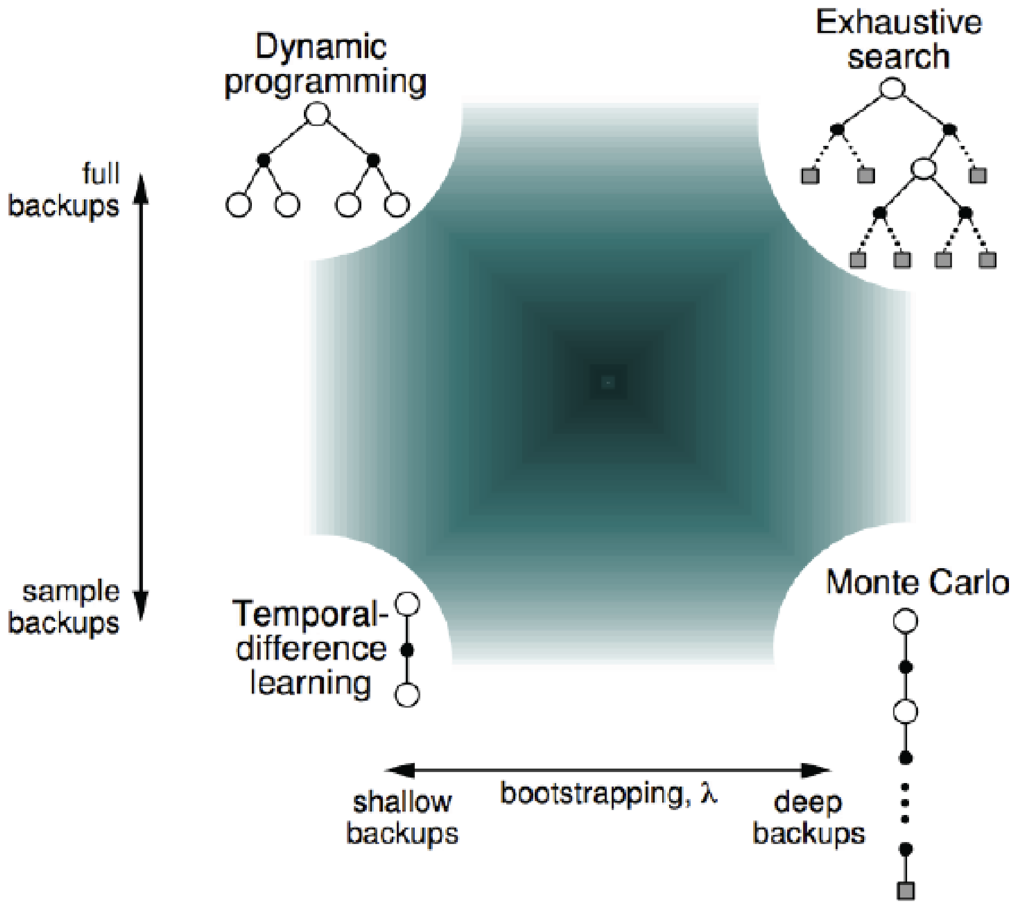
V.S. DP

$$V(S_t) \leftarrow \mathbb{E}_\pi \left[R_{t+1} + \gamma V(S_{t+1})\right]$$



时序差分值估计

TD (0)


Dynamic
programming
Exhaustive
search
full
backups
sample
backups
Temporal-
difference
learning
Monte Carlo
shallow
backups
bootstrapping, λ
deep
backups

时序差分值估计

TD (0)

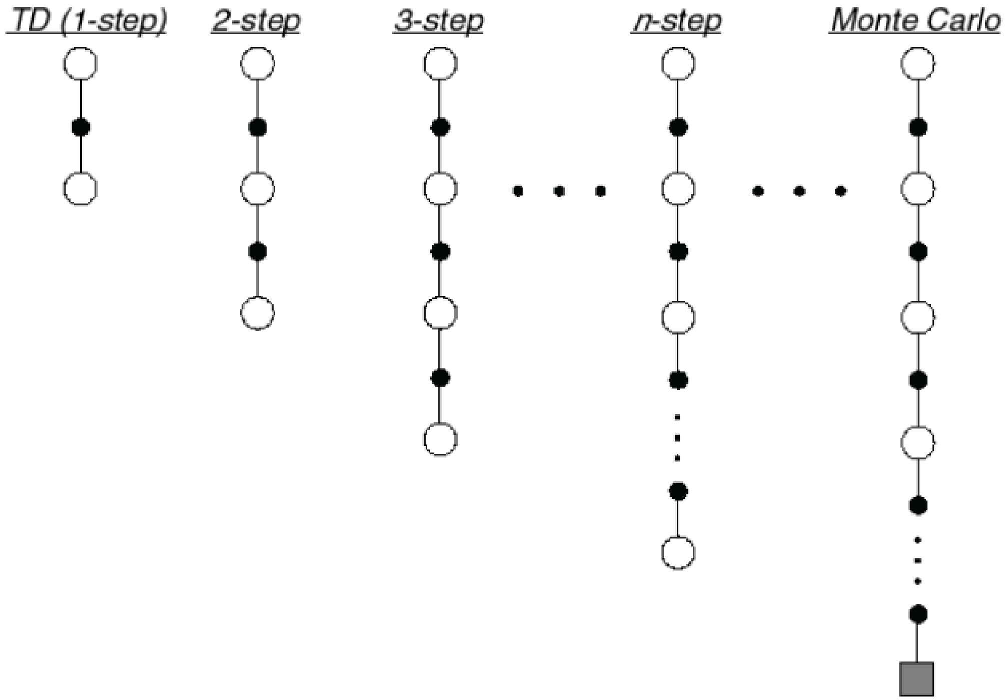
Bootstrapping and Sampling

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update samples an expectation
  - MC samples
  - DP does not sample
  - TD samples

TD(λ)

n-step TD

- Let TD target look n steps into the future



TD(λ)

- Consider the following n-step returns for $n = 1, 2, \infty$:

$$
\begin{array}{lll}
n = 1 & (TD) & G_t^{(1)} = R_{t+1} + \gamma V(S_{t+1}) \\
n = 2 & & G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V(S_{t+2}) \\
\vdots & & \vdots \\
n = \infty & (MC) & G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T
\end{array}
$$

- Define the n-step return

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

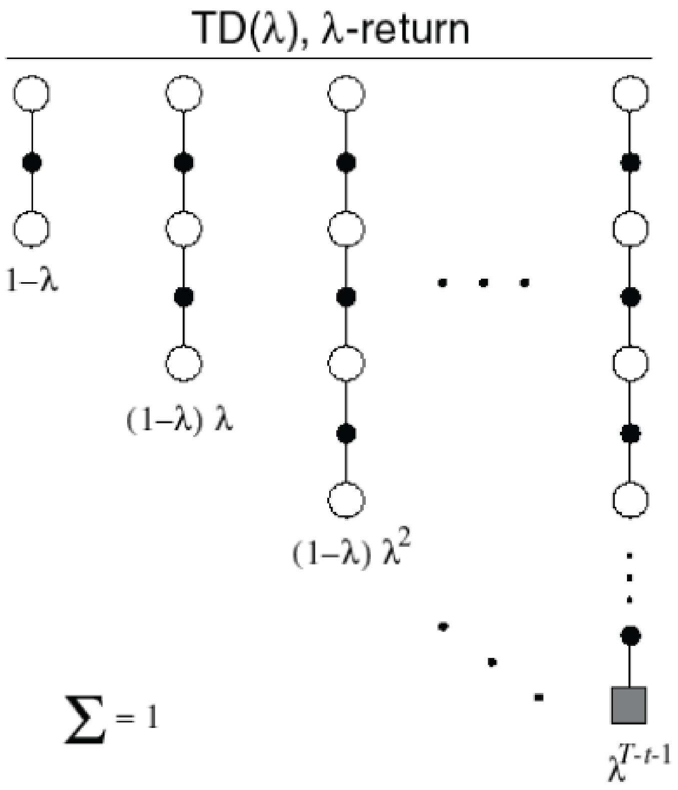
- n-step temporal-difference learning

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{(n)} - V(S_t) \right)$$

时序差分值估计

TD(λ)

Forward Returns and TD(λ)



- The λ-*return* G_t^λ combines all n-step returns $G_t^{(n)}$
- Using weight $(1-\lambda)\lambda^{n-1}$

$$G_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty}\lambda^{n-1}G_t^{(n)}$$

- Forward-view TD(λ)

$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t^\lambda - V(S_t)\right)$$

时序差分值估计

TD(λ)

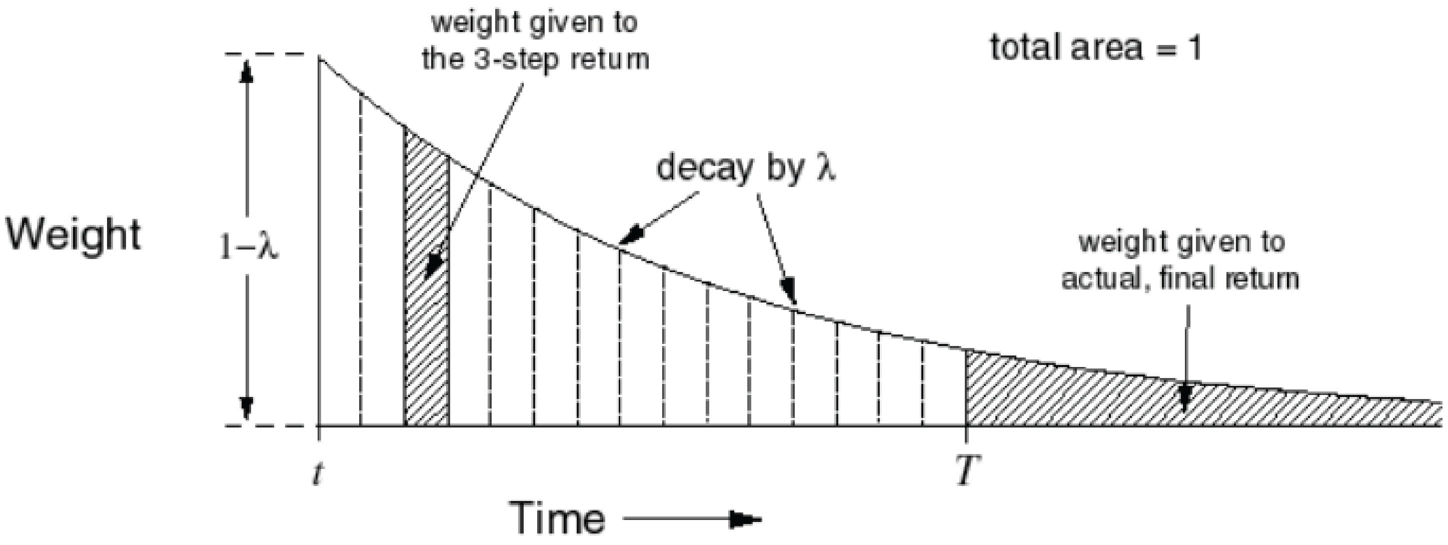
$$\alpha\left(G_k^{\lambda} - V(S_k)\right) = \alpha\sum_{t=k}^{T}(\gamma\lambda)^{t-k}\delta_t$$

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

$$V(s) \leftarrow V(s) + \alpha\sum_{t=k}^{T}(\gamma\lambda)^{t-k}\delta_t$$

时序差分值估计

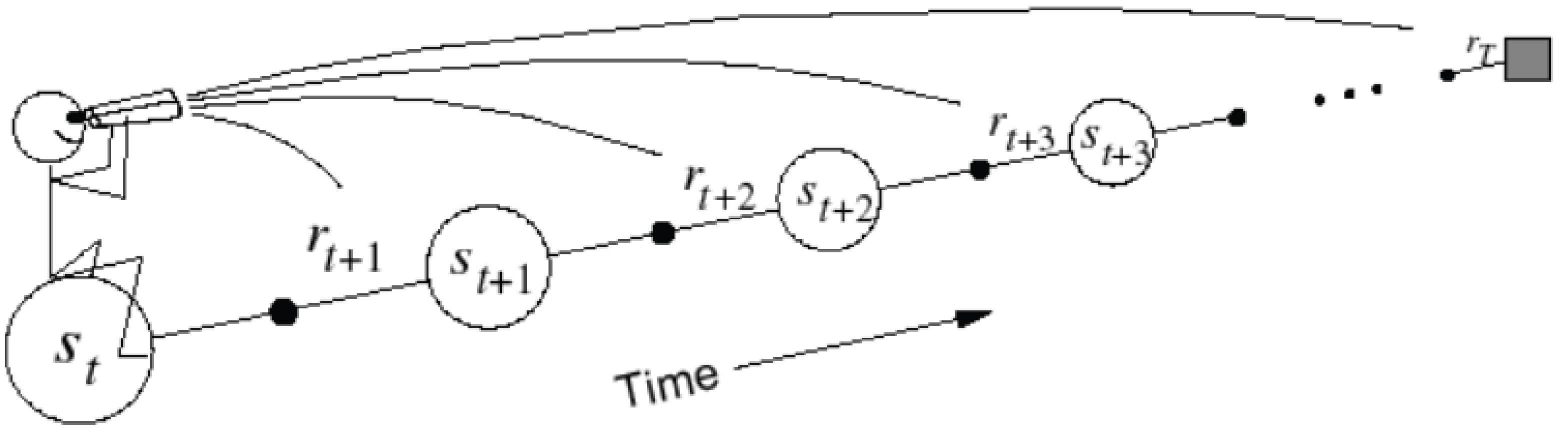
TD(λ)



$$G_t^{\lambda} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

时序差分值估计

TD(λ)



- Update value function towards the λ-return
- Forward-view looks into the future to compute G_t^λ
- Like MC, can only be computed from complete episodes

时序差分值估计

TD(λ)

Backward TD(λ)

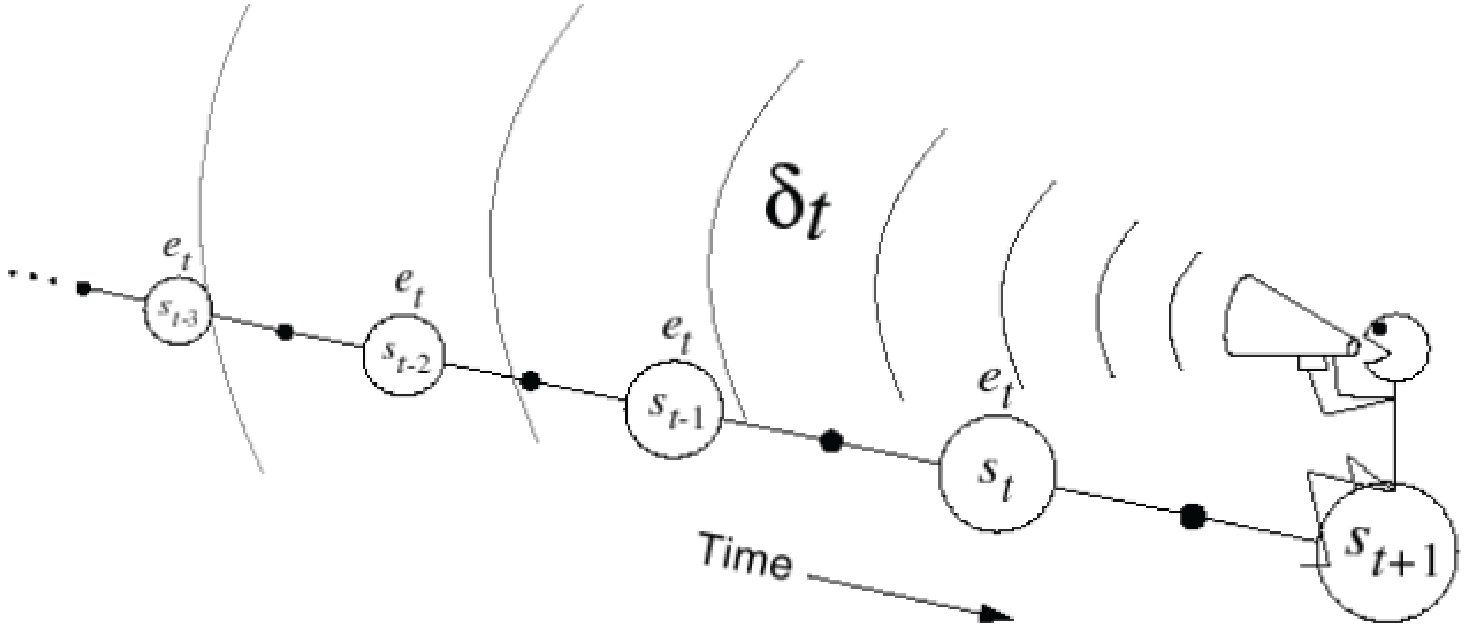
$$E_0(s) = 0$$
$$E_t(s) = \gamma\lambda E_{t-1}(s) + \mathbf{1}(S_t = s)$$

- Keep an eligibility trace for every state s
- Update value $V(s)$ for every state s
- In proportion to TD-error δ_t and eligibility trace $E_t(s)$

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$
$$V(s) \leftarrow V(s) + \alpha\delta_t E_t(s)$$

时序差分值估计

TD(λ)



时序差分值估计

TD(λ)

- **Forward and Backward TD**

Theorem

The sum of offline updates is identical for forward-view and backward-view TD(λ)

$$\sum_{t=1}^{T} \alpha \delta_t E_t(s) = \sum_{t=1}^{T} \alpha \left(G_t^{\lambda} - V(S_t) \right) \mathbf{1}(S_t = s)$$

时序差分值估计

TD(λ)

- **TD(0), TD(λ), TD(1)**

TD(0)

$$E_t(s) = \mathbf{1}(S_t = s)$$
$$V(s) \leftarrow V(s) + \alpha \delta_t E_t(s)$$

This is exactly equivalent to TD(0) update

$$V(S_t) \leftarrow V(S_t) + \alpha \delta_t$$

时序差分值估计

TD(λ)

TD(λ)

$$E_t(s) = \gamma\lambda E_{t-1}(s) + \mathbf{1}(S_t = s)$$

$$V(s) \leftarrow V(s) + \alpha\delta_t E_t(s)$$

TD(1)等价于M C

$$\alpha\left(G_k^\lambda - V(S_k)\right) = \alpha\sum_{t=k}^{T}(\gamma\lambda)^{t-k}\delta_t = \boxed{\alpha\sum_{t=k}^{T}\gamma^{t-k}\delta_t = \alpha\left(G_k - V(S_k)\right)}$$

大家请自行证明

第三章 值估计

3.1 随机逼近

3.2 蒙特卡洛值估计

3.3 时序差分值估计

3.4 算法总结

算法总结

Offline updates	$\lambda = 0$	$\lambda \in (0, 1)$	$\lambda = 1$
Backward view	TD(0)	TD(λ)	TD(1)
	$\Vert$	$\Vert$	$\Vert$
Forward view	TD(0)	Forward TD(λ)	MC
Online updates	$\lambda = 0$	$\lambda \in (0, 1)$	$\lambda = 1$
Backward view	TD(0)	TD(λ)	TD(1)
	$\Vert$	$\nparallel$	$\nparallel$
Forward view	TD(0)	Forward TD(λ)	MC
	$\Vert$	$\Vert$	$\Vert$
Exact Online	TD(0)	Exact Online TD(λ)	Exact Online TD(1)

本讲参考文献

1. Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. (Second edition, in progress，draft.
2. David Silver, Slides@《Reinforcement Learning:An Introduction》,2016.
3. Simon Haykin，申富饶等译，神经网络与学习机器，第三版。