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陈雪琳 2017[illegible]

中国科学院大学

University of Chinese Academy of Sciences

1. ① $[l_1, l_3, end]\ (y = n*n*n - n)$

$= [l_1, l_3]\ (3y = n*n*n - n)$

$= (x<n) \rightarrow (3y = n*n*n - n)$

②. 证明 $\models \{(x \le n) \wedge 3y = x*x*x - x\}\ (l_1, l_3, end)\ \{3y = n*n*n - n\}$

假设 $(l_1, \sigma_1)(l_3, \sigma_2)(end, \sigma_3)$ 且 $\sigma_1 \models (x \le n) \wedge (3y = x^3 - x)$

须证明 $\sigma_3 \models 3y = n^3 - n$

$\sigma_1(x) \le n \ \wedge\ 3\sigma_1(y) = \sigma_1^3(x) - \sigma_1(x)$ $[a_0]$

$(l_1, \sigma_1) \rightarrow (l_3, \sigma_2)$: $\sigma_2 = \sigma_1 \wedge \neg(\sigma_1(x) < n)$ $[a_1]$

$(l_3, \sigma_2) \rightarrow (end, \sigma_3)$: $\sigma_3 = \sigma_2[y / \sigma_2(y)]$ $[a_2]$

须证明 $x_1.\ 3\sigma_3(y) = \sigma_3^3(n) - \sigma_3(n)$

$x_2.\ 3\sigma_2(y) = \sigma_2^3(n) - \sigma_2(n).$

$x_3.\ 3\sigma_1(y) = \sigma_1^3(n) - \sigma_1(n)$

$\because$ $[a_0]$ $\sigma_1(x) \le n$. 且 $[a_1]$. $\neg(\sigma_1(x) < n)$ 为真

$\therefore$ $\sigma_1(x) = n$ $[a_4]$.

$\therefore$ $3\sigma_1(y) = \sigma_1^3(n) - \sigma_1(n)$ 成立 由 $[a_0]$, $[a_3]$ 可得.

$\therefore$ 得证.

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2. (1) 证 $\{n \geq 0\}$ [T] $\{y = n^3 - n\}$ 部分正确. 不太…

即证. 对所有完全路径 $a \models \{n \geq 0\} \to [a] \{y = n^3 - n\}$

随择 $C = \{l_1\}$ $\quad q_{l_1} = x < n \wedge y = \frac{x^3 - x}{3}$ $x \leq n$ 成立

找到路径 (l_1, l_2, l_1) 证明其正确性 $\models IVC(q_{l_1}, (l_1, l_2, l_1), q_{l_1})$.

设 $(l_1, \sigma_1)(l_2, \sigma_2)(l_1 \sigma_3)$ 且 $\sigma_1 \models x < n \wedge 3y = x^3 - x$

须证明. $\sigma_3 \models x < n \wedge 3y = x^3 - x$.

$\sigma_1(x) < n \wedge 3\sigma_1(y) = \sigma_1^3(x) - \sigma_1(x)$ [a0]

$(l_1, \sigma_1) \to (l_2, \sigma_2)$: $\sigma_2 = \sigma_1$ [a1]

$(l_2, \sigma_2) \to (l_1, \sigma_3)$: $\sigma_3 = \sigma_2 \wedge \sigma_3[y \mid \sigma_2(y) + \sigma_2(x) \cdot (\sigma_2(x)+1)]$

$\wedge\ \sigma_3[x \mid \sigma_2(x)+1]$. [a2]

须证明. $\sigma_3(x) < n \wedge \sigma_3(y) = \frac{\sigma_3^3(x) - \sigma_3(x)}{3}$.

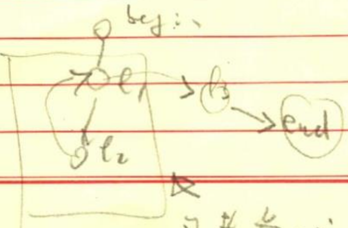
$\sigma_3(x) < n \Rightarrow \sigma_1(x)+1 < n$ ~~$\sigma_3(x) < n$ 只要~~ $\sigma_1(x) < n$ 成立 ~~$\sigma_1(x) < n+1$ 即可~~ 条件. 由 [a2] 可得

$\sigma_2(y) + \sigma_2(x) \cdot (\sigma_2(x)+1) = \frac{(\sigma_2(x)+1)^3 - (\sigma_2(x)+1)}{3}$

由 [a1] [a0] 可得 $\frac{\sigma_1^3(x) - \sigma_1(x)}{3} + \sigma_1(x)(\sigma_1(x)+1) = \frac{(\sigma_1(x)+1)^3 - (\sigma_1(x)+1)}{3}$

化简可得等式两边相等. ∴ 得证.

∴ $\{n \geq 0\}$ [T] $\{y = n^3 - n\}$ 部分正确.



这个证明不完整.

另外，用语义比较

只考虑了这一部分

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2. (1) 证 $\{n\geq 0\}$ $[P]$ $\{y=n^3-n\}$ 部分正确.　　　不太对

即证. 对所有完全路径 $\alpha \models \{n\geq 0\} \rightarrow [\alpha]\{y=n^3-n\}$

选择 $C=\{l_1\}$　$q_{l_1} = x<n \wedge y=\frac{x^3-x}{3}$　　　$x\leq n$ 才对

找到路径 (l_1, l_2, l_1) 证明其正确性 $\models IVC(q_{l_1}, (l_1, l_2, l_1), q_{l_1})$

设 $(l_1, \sigma_1)(l_2, \sigma_2)(l_1, \sigma_3)$ 且 $\sigma_1 \models x<n \wedge 3y = x^3 - x$

须证明. $\sigma_3 \models x<n \wedge 3y = x^3-x$

$\sigma_1(x)<n \wedge 3\sigma_1(y) = \sigma_1^3(x) - \sigma_1(x)$　$[a_0]$

$\delta(l_1, \sigma_1) \rightarrow (l_2, \sigma_2)$:　$\sigma_2 = \sigma_1$　$[a_1]$

$(l_2, \sigma_2) \rightarrow (l_1, \sigma_3)$:　$\sigma_3 = \sigma_2 \wedge \sigma_3[y \mid \sigma_2(y) + \sigma_2(x)\cdot(\sigma_2(x)+1)]$

$\wedge\ \sigma_3[x \mid \sigma_2(x)+1]$.　$[a_2]$

须证明. $\sigma_3(x)<n \wedge \sigma_3(y) = \frac{\sigma_3^3(x) - \sigma_3(x)}{3}$.

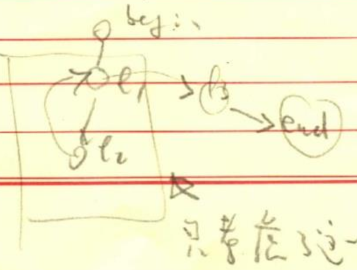
$\sigma_3(x)\leq n \Rightarrow \sigma_1(x)$ 小于 n ~~$\sigma_3(x)<n$ 只要 $\sigma_1(x)<n+1$ 即可~~ $\sigma_1(x)<n$ 成立 条件 由. $[a_2]$ 可得

$\sigma_2(y) + \sigma_2(x)\cdot(\sigma_2(x)+1) = \frac{(\sigma_2(x)+1)^3 - (\sigma_2(x)+1)}{3}$

由 $[a_0]$ $[a_1]$ 可得　$\frac{\sigma_1^3(x) - \sigma_1(x)}{3} + \sigma_1(x)(\sigma_1(x)+1) = \frac{(\sigma_1(x)+1)^3 - (\sigma_1(x)+1)}{3}$

化简可得等式两边相等. ∴ 得证.

∴ $\{n\geq 0\}$ $[P]$ $\{y=n^3-n\}$ 部分正确.



只考虑了这一部分

这个证明不完整.

另外, 用语义比较

繁琐. 参见下一页

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(2). 证明T对于前断言{n>0}可终止.

随择 $C_{\cdot} = \{beg, l_1\}$. $q_{beg} = n > 0$. $q_{l_1} = x < n \wedge y = \frac{x^3 - x}{3}$ ($x=0, y=0$)

找到路径 $(beg, \cancel{\text{exit}}\ l_1)$ (l_1, l_2, l_1)

证明这两条路径正确性 (beg, l_1) 显然成立而 (l_1, l_2, l_1) 的正确性上题已证.

随择 $C' = \{l_1\}$ Select $(W, \leq) = (\{0, 1, 2, \ldots\}, \leq)$.

Select $g_{l_1}: \Sigma \to W$. $g_{l_1}(\sigma) = \cancel{\sigma(x) - \sigma(y)}$ $\sigma(n) - \sigma(x) - i$ if $\sigma(n) - \sigma(x) \geq 0$

0 otherwise

需要证明

找到路径 (l_1, l_2, l_1)

证明 $I(q_{l_1})(\sigma) = n > 0 \wedge (\sigma \xrightarrow{(l_1, l_2, l_1)} \sigma')$

$g_{l_1}(\sigma) \geq 0$

$\to g_{l_1}(\sigma') < g_{l_1}(\sigma)$

$\therefore$ ~~$x < n \wedge \sigma(y) = \frac{\sigma(x)^3 - \sigma(x)}{3}$ $\to$ $(x+1) - \frac{(x+1)^3 - (x+1)}{3} < x - \frac{x^3 - x}{3}$~~ ~~$\sigma'(x) - \sigma'(y)$~~ ~~$\sigma(x) - \sigma(y)$~~

~~即证 $\wedge\ \sigma'(y) = \sigma(y) + \sigma(x) \cdot (\sigma(x) + 1) \wedge \sigma'(x) = \sigma(x+1)$.~~

~~即证 $(x+1) - \frac{(x+1)^3 - (x+1)}{3} < x - \frac{x^3 - x}{3}$~~ ✗

$n > 0 \wedge \sigma(x) < n \wedge \sigma'(x) = \sigma(x) + 1 \wedge \sigma'(y) = \frac{\sigma'(x)^3 - \sigma'(x)}{3}$

$\to n - \sigma'(x) - i < n - \sigma(x) - i$

$\Rightarrow \sigma(x) < \sigma'(x) \Rightarrow \sigma(x) < \sigma(x) + 1$

$\therefore$ 得证

第 11 周练习:

11.1

a)

计算正确。

b)

证明过程正确。亦可计算验证条件，然后证明验证条件的正确性。参考如下。

{x<=n∧3y=x*x*x-x }(l1,l3,end){(y=n*n*n-n)}

IFF (x<=n∧3y=x*x*x-x)→ [l1,l3] (3y=n*n*n-n)

IFF (x<=n∧3y=x*x*x-x)→(¬(x<n)→(3y=n*n*n-n))

IFF true

11.2

总体而言，掌握了基本方法，但是在方法的细节上还须仔细琢磨推敲。

a) 尽量用逻辑方法(避开语义的直接应用)来验证，这样计算方便一些。参考如下。

选择 C={beg,l1,end}

选择 q_beg = (n>=0)

q_l1 = (0<=x<=n)∧(3y=x*x*x-x)

q_end = (y=n*n*n-n)

枚举相关路径如下：

(beg,l1), (l1,l2,l1), (l1,l3,end)

证明路径正确性如下：

{0 <=n }(beg,l1){(0<=x<=n)∧(3y=x*x*x-x)}

IFF (0 <=n)[beg,l1]((0<=x<=n)∧(3y=x*x*x-x))

IFF (0<=n→ 0<=n∧0=0)

IFF true

{0<=x<=n ∧3y=x*x*x-x }(l1,l2,l1) {0<=x<=n ∧3y=x*x*x-x }

IFF (0<=x<=n ∧3y=x*x*x-x)→[l1,l2,l1)](0<=x<=n ∧3y=x*x*x-x)

IFF (0<=x<=n∧3y=x*x*x-x)→ [l1,l2] (0<=x+1<=n ∧3(y+x*(x+1)=(x+1)*(x+1)*(x+1)-x-1)

IFF (0<=x<=n∧3y=x*x*x-x)→ (x<y→ (0<=x+1<=n ∧3(y+x*(x+1)=(x+1)*(x+1)*(x+1)-x-1))

IFF true

{0<=x<=n ∧3y=x*x*x-x }(l1,l3,end){(y=n*n*n-n)}

IFF (0<=x<=n ∧3y=x*x*x-x)[l1,l3,end](y=n*n*n-n)

IFF (0<=x<=n∧3y=x*x*x-x)→ [l1,l3] (3y=n*n*n-n)

IFF (0<=x<=n∧3y=x*x*x-x)→(¬(x<n)→(3y=n*n*n-n))

IFF true

b) 同样尽量用逻辑方法来验证。参考如下。

选择 $C=\{beg, l1\}$

选择 $q\_beg =$ $(n>=0)$

$q\_l1 =$ $(0<=x<=n)\wedge(3y=x*x*x-x)$

枚举相关路径如下：

$(beg, l1)$,

$(l1, l2, l1)$

证明路径正确性如下：

$\{0 <= n\}(beg, l1)\{(0<=x<=n)\wedge(3y=x*x*x-x)\}$

IFF $(0 <= n)[beg, l1]((0<=x<=n)\wedge(3y=x*x*x-x))$

IFF $(0<=n \rightarrow 0<=n\wedge 0=0)$

IFF true

$\{0<=x<=n \wedge 3y=x*x*x-x\}(l1, l2, l1)\ \{0<=x<=n \wedge 3y=x*x*x-x\}$

IFF $(0<=x<=n \wedge 3y=x*x*x-x) \rightarrow [l1, l2, l1](0<=x<=n \wedge 3y=x*x*x-x)$

IFF $(0<=x<=n\wedge 3y=x*x*x-x) \rightarrow [l1, l2]\ (0<=x+1<=n \wedge 3(y+x*(x+1)=(x+1)*(x+1)*(x+1)-x-1)$

IFF $(0<=x<=n\wedge 3y=x*x*x-x) \rightarrow (x<n \rightarrow (0<=x+1<=n \wedge 3(y+x*(x+1)=(x+1)*(x+1)*(x+1)-x-1))$

IFF true

选择 $C'=\{l1\}$

选择 $W=NAT$, $w=(x>=0)$.

我们有 $W=\{\sigma(x) \mid I(w)(\sigma)=true\}$

选择 $t\_l1 = (n-x)$

我们有 $q\_l1 \rightarrow (n-x)>=0$.

枚举相关路径如下：$(l1, l2, l1)$

证明路径正确性如下：

$vc(0<=x<=n\wedge(n-x=v), (l1, l2, l1), (n-x<v))$

IFF $(0<=x<=n\wedge(n-x=v)) \rightarrow (x<n \rightarrow (n-x-1<v)$

IFF true.