强化学习及其应用

Reinforcement Learning and Its Applications

第一章 绪 论

Introduction

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第一章 绪 论

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1.1 Markov 决策过程

1.2 强化学习

1.3 课程内容安排

1.4 小结

Markov Process (MP)

- **MP 定义：**

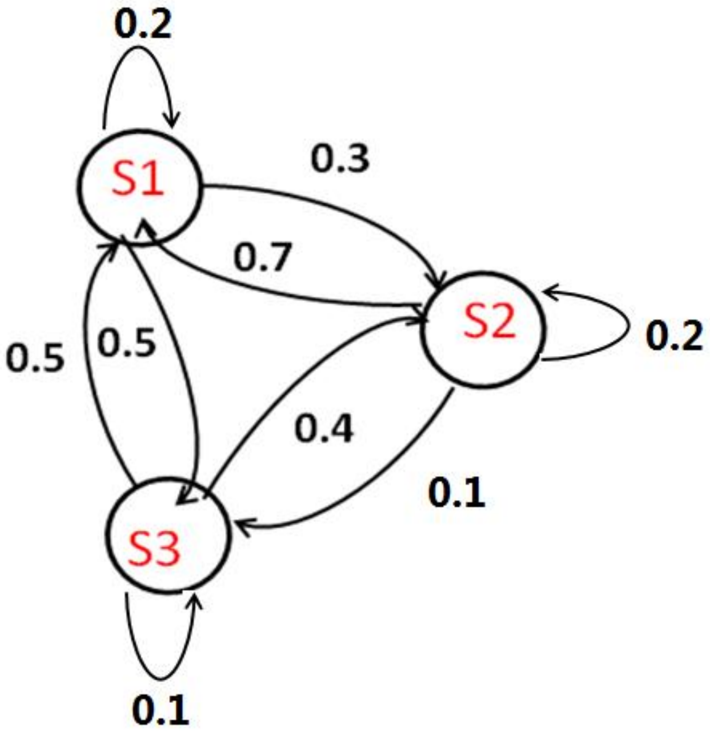
A *Markov Process* (or *Markov Chain*) is a tuple $\langle \mathcal{S}, \mathcal{P} \rangle$

- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$

$$\mathcal{P} = \textit{from} \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{n1} & \dots & \mathcal{P}_{nn} \end{bmatrix}$$

where each row of the matrix sums to 1

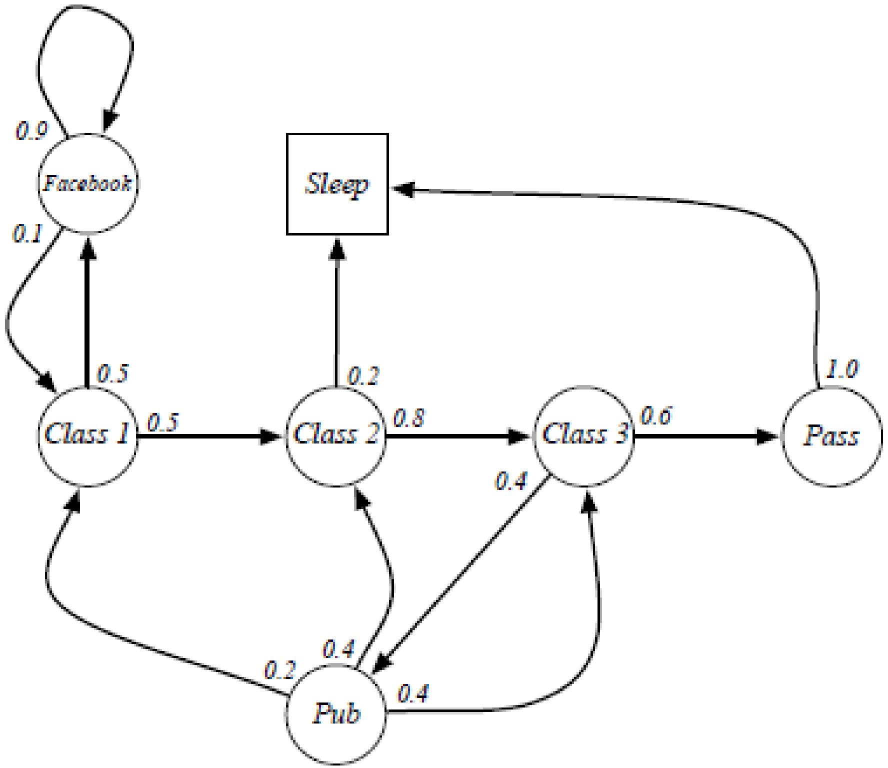
Markov Process (MP)



Markov 决策过程

Markov Process (MP)

- **例子：Student MP(S, P)**

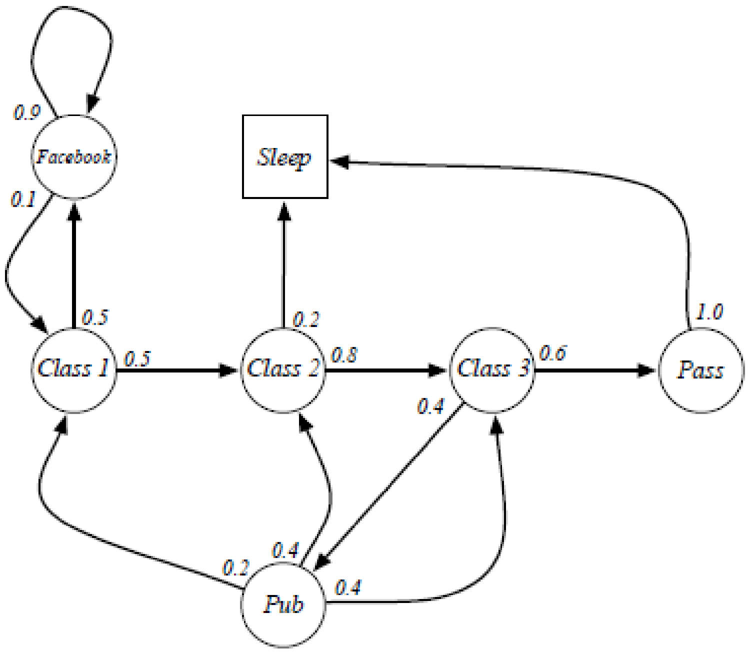


$$\mathcal{P} = \begin{array}{cc} & \begin{array}{ccccccc} C1 & C2 & C3 & Pass & Pub & FB & Sleep \end{array} \\ \begin{array}{c} C1 \\ C2 \\ C3 \\ Pass \\ Pub \\ FB \\ Sleep \end{array} & \left[\begin{array}{ccccccc} & 0.5 & & & & 0.5 & \\ & & 0.8 & & & & 0.2 \\ & & & 0.6 & 0.4 & & \\ & & & & & & 1.0 \\ 0.2 & 0.4 & 0.4 & & & & \\ 0.1 & & & & & 0.9 & \\ & & & & & & 1 \end{array}\right] \end{array}$$

Markov 决策过程

Markov Process (MP)

- **Episodes ~ Student MP <S, P>**



Sample episodes for Student Markov Chain starting from $S_1 = C1$

$$S_1, S_2, ..., S_T$$

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

Markov Reward Process (MRP)

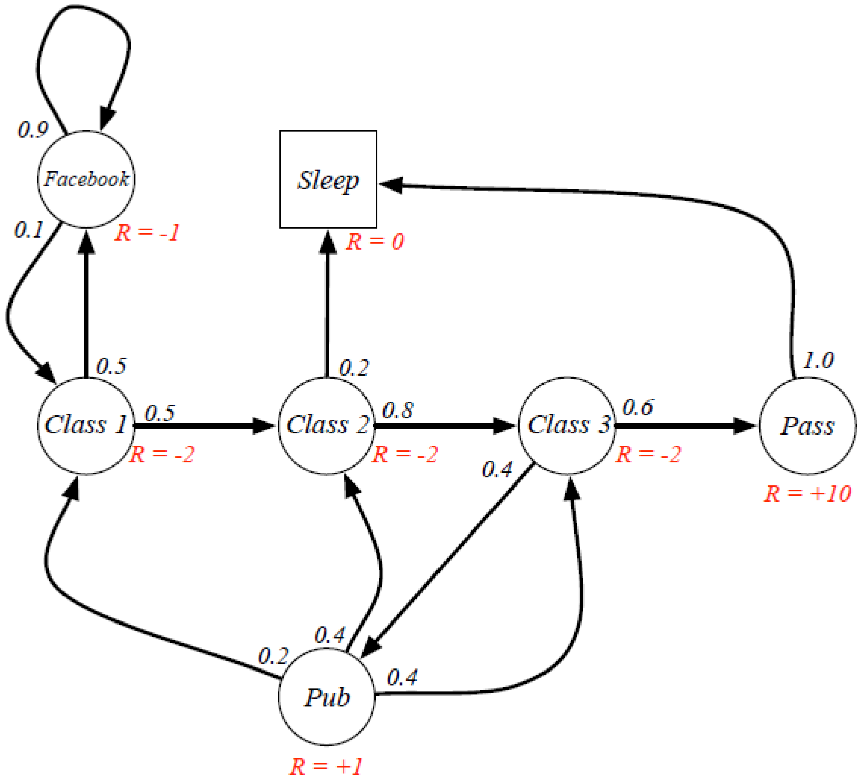
- **MRP 定义：**

A *Markov Reward Process* is a tuple $\langle \mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}_s = \mathbb{E}\left[R_{t+1} \mid S_t = s\right]$
- γ is a discount factor, $\gamma \in [0, 1]$

Markov Reward Process (MRP)

- **例子：Student MRP <S, P, R, γ>**



Markov 决策过程

Markov Reward Process (MRP)

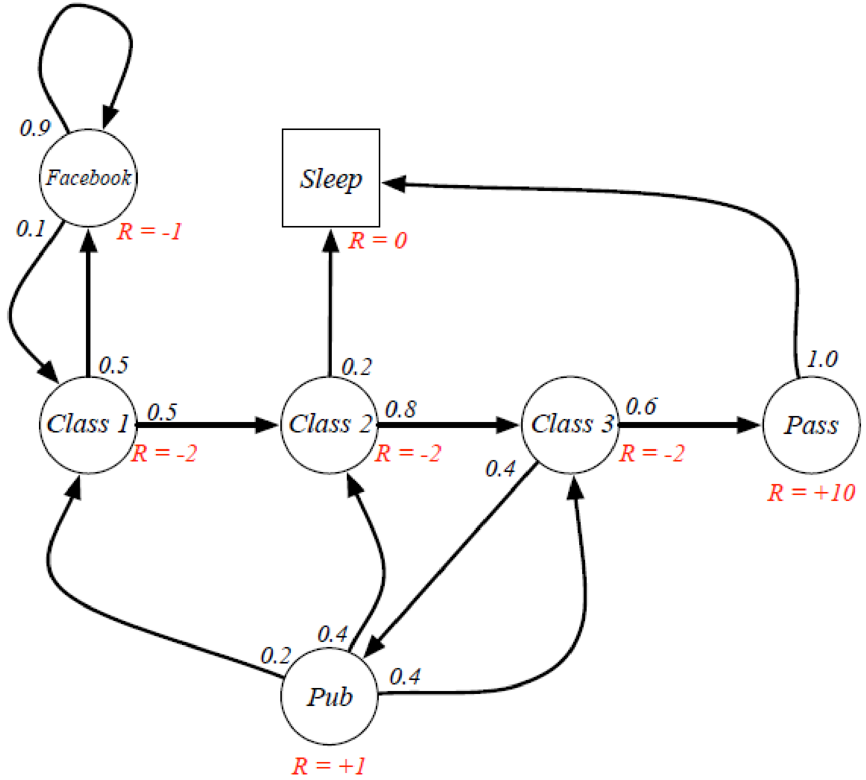
- **Return**

The *return* G_t is the total discounted reward from time-step t.

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Markov Reward Process (MRP)

- **Return of Episode ~Student MRP <S, P, R, γ>**



Sample returns for Student MRP:
Starting from $S_1 = C1$ with $\gamma = \frac{1}{2}$

$$G_1 = R_2 + \gamma R_3 + \ldots + \gamma^{T-2} R_T$$

C1 C2 C3 Pass Sleep	$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 10 * \frac{1}{8}$	$=$	-2.25
C1 FB FB C1 C2 Sleep	$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16}$	$=$	-3.125
C1 C2 C3 Pub C2 C3 Pass Sleep	$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 1 * \frac{1}{8} - 2 * \frac{1}{16} \ldots$	$=$	-3.41
C1 FB FB C1 C2 C3 Pub C1 ...	$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16} \ldots$	$=$	-3.20
FB FB FB C1 C2 C3 Pub C2 Sleep			

Markov Reward Process (MRP)

Value Function (Average Return)

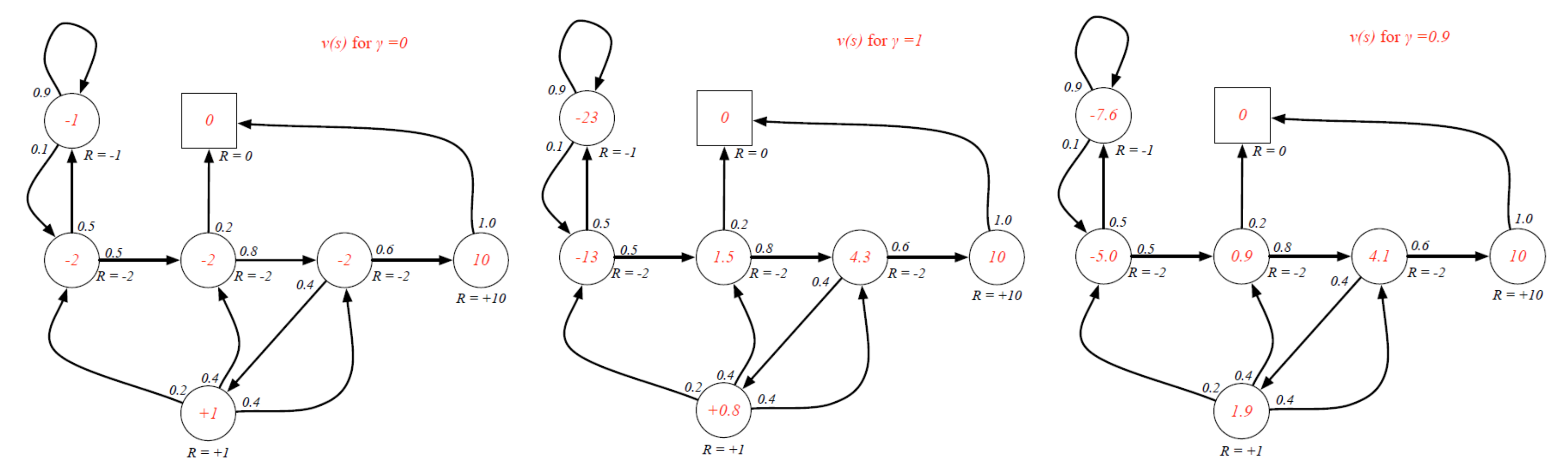
The *state value function* $v(s)$ of an MRP is the expected return starting from state s

$$v(s) = \mathbb{E}[G_t \mid S_t = s]$$

Markov 决策过程

Markov Reward Process (MRP)

- **Value Function for Student MRP <S, P, R, γ >**



Markov Reward Process (MRP)

- **Bellman Equation for MRP**

$$
\begin{aligned}
v(s) &= \mathbb{E}[G_t \mid S_t = s] \\
&= \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \\
&= \mathbb{E}[R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \ldots) \mid S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s]
\end{aligned}
$$

$\Rightarrow$

$$
\begin{aligned}
V(s_t) &= E_{t+1,\sim\ldots}(G_t) \\
&= E_{t+1,\sim\ldots} \\
&= E_{t+1,\sim\ldots} \\
&= E_{t+1}[R_{t+1} + r E_{t+2,\sim\ldots}(G_{t+1})] \\
&= E_{t+1}[R_{t+1} + r\, V(S_{t+1})]
\end{aligned}
$$

$$
v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')
$$

Markov Reward Process (MRP)

- **Bellman Equation for MRP**

$$\begin{aligned}
v(s) &= \mathbb{E}\left[G_t \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma \left(R_{t+2} + \gamma R_{t+3} + \ldots\right) \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s\right]
\end{aligned}$$

$$v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')$$

$$v = \mathcal{R} + \gamma \mathcal{P} v$$

$\Rightarrow$

$$\begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_1 \\ \vdots \\ \mathcal{R}_n \end{bmatrix} + \gamma \begin{bmatrix} \mathcal{P}_{11} & \ldots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{11} & \ldots & \mathcal{P}_{nn} \end{bmatrix} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix}$$

$$v = (I - \gamma \mathcal{P})^{-1} \mathcal{R}$$

Markov Decision Process (MDP)

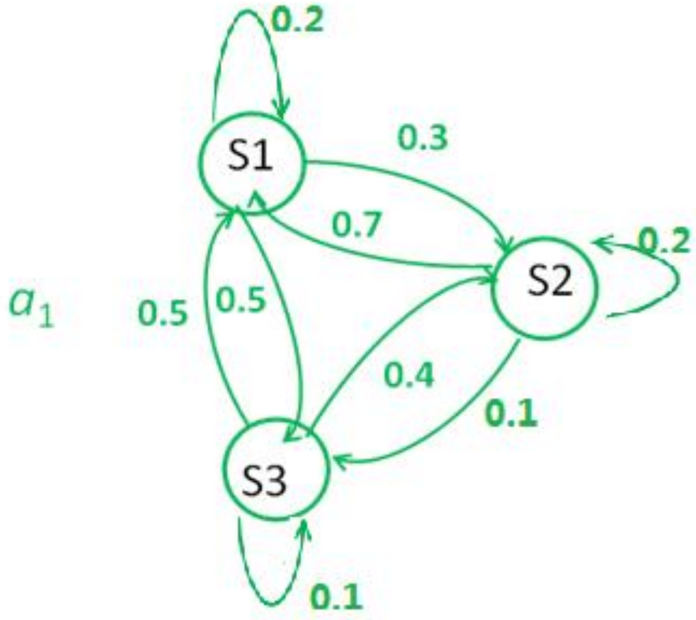
- **MDP 定义：**

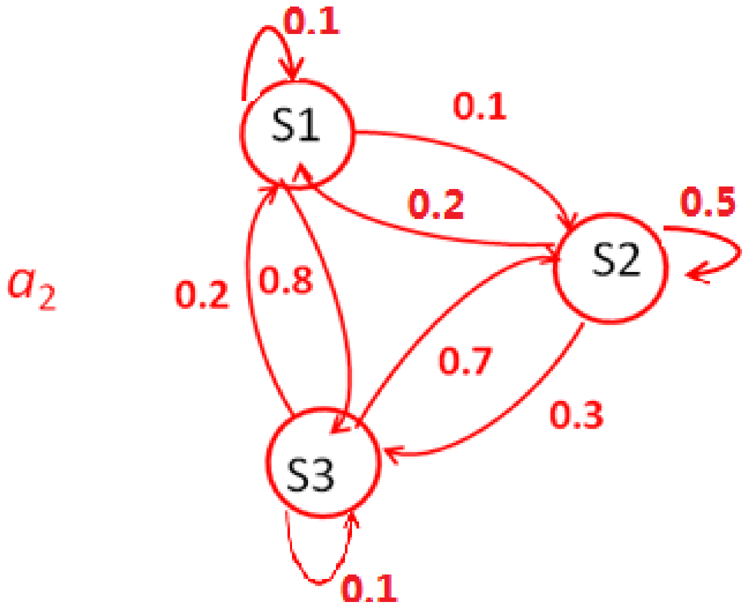
A *Markov Decision Process* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'}^{a} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s, A_t = a\right]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}_s^a = \mathbb{E}\left[R_{t+1} \mid S_t = s, A_t = a\right]$
- γ is a discount factor $\gamma \in [0, 1]$.

形象的解释：MDP 是一个多层的 MRP，每一层对应一个行动 *a*.

Markov Decision Process (MDP)





Markov Decision Process (MDP)

如何 Episodes ~MDP ?

对于状态 *S*， 需要知道如何选择行动 *a* ？

Policy

A *policy* π is a distribution over actions given states

$$\pi(a|s) = \mathbb{P}\left[A_t = a \mid S_t = s\right]$$

i.e. Policies are *stationary* (time-independent),
$A_t \sim \pi(\cdot|S_t), \forall t > 0$

Markov Decision Process (MDP)

Episodes ~MDP by Policy is A MRP

- Given an MDP $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ and a policy π
- The state sequence $S_1, S_2, ...$ is a Markov process $\langle \mathcal{S}, \mathcal{P}^\pi \rangle$
- The state and reward sequence $S_1, R_2, S_2, ...$ is a Markov reward process $\langle \mathcal{S}, \mathcal{P}^\pi, \mathcal{R}^\pi, \gamma \rangle$
- where

$$\mathcal{P}^\pi_{s,s'} = \sum_{a \in \mathcal{A}} \pi(a|s) \mathcal{P}^a_{ss'}$$

$$\mathcal{R}^\pi_s = \sum_{a \in \mathcal{A}} \pi(a|s) \mathcal{R}^a_s$$

Markov 决策过程

Markov Decision Process (MDP)

Value Function

The *state-value function* $v_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

The *action-value function* $q_\pi(s, a)$ is the expected return starting from state s, taking action a, and then following policy π

$$q_\pi(s, a) = \mathbb{E}_\pi \left[G_t \mid S_t = s, A_t = a \right]$$

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s, a)$$

Markov Decision Process (MDP)

- **Bellman Expectation Equation**
  - **State Value Function**

$$v_\pi(s) = \mathbb{E}_\pi\left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s\right]$$

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s)\left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s')\right)$$

$$v_\pi = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi v_\pi$$

Markov Decision Process (MDP)

- **Bellman Expectation Equation**
  - **State Value Function**

$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s \right]$$

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

$$v_\pi = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi v_\pi$$

V.S. MRP:

$$v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')$$

Markov 决策过程

Markov Decision Process (MDP)

- **Action-Value Function**

$$q_\pi(s,a) = \mathbb{E}_\pi\left[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a\right]$$

$$q_\pi(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s')$$

V.S. MRP:

$$v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')$$

Markov 决策过程

Markov Decision Process (MDP)

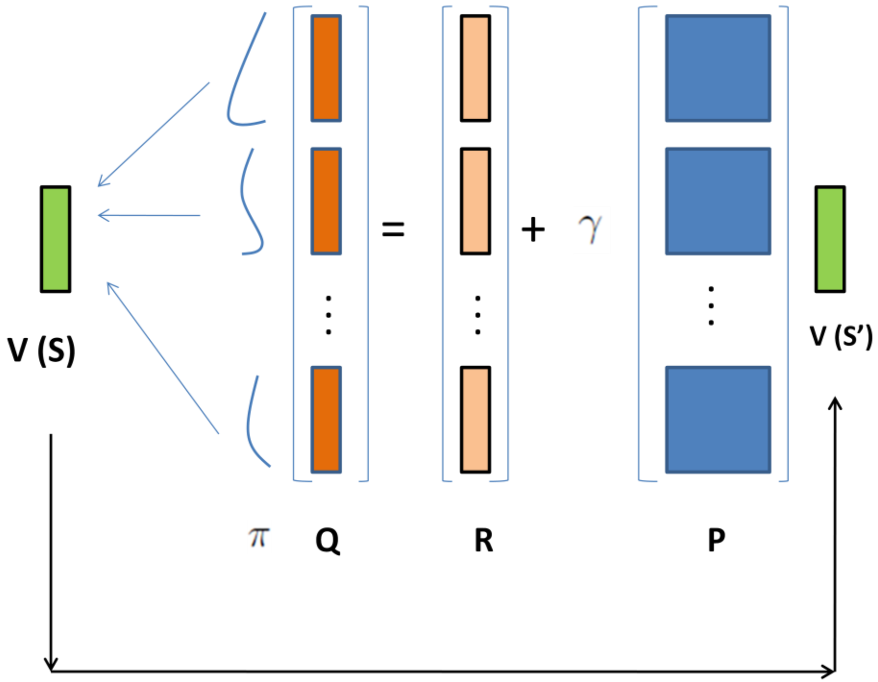
- **Bellman Expectation Equations**

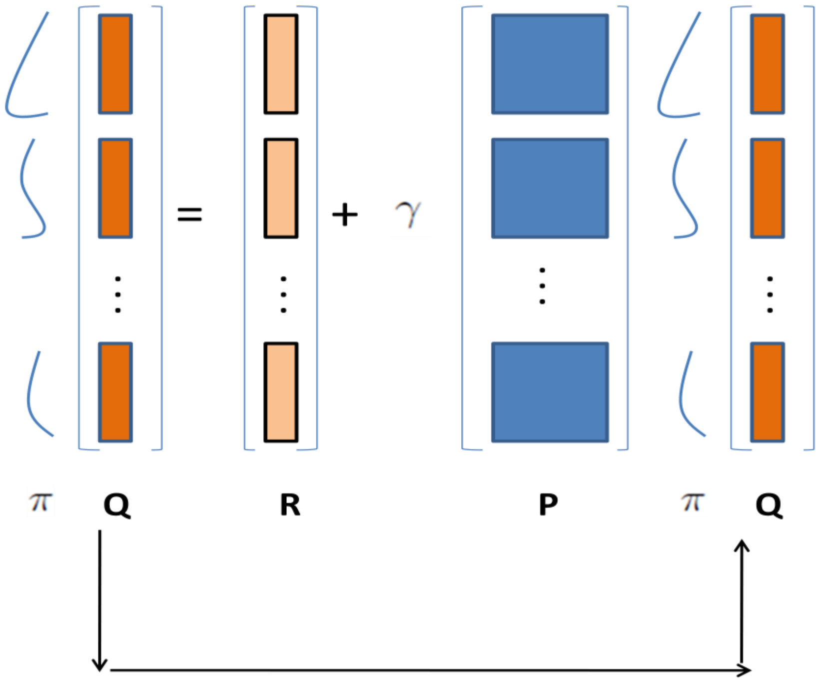
$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

$$q_\pi(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \sum_{a' \in \mathcal{A}} \pi(a'|s') q_\pi(s',a')$$

Markov 决策过程

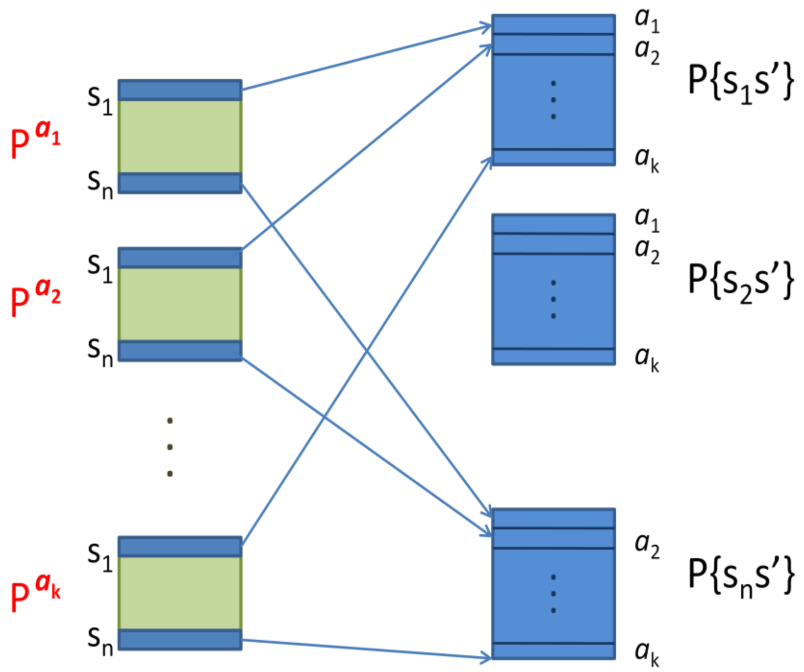
公式的矩阵图形：

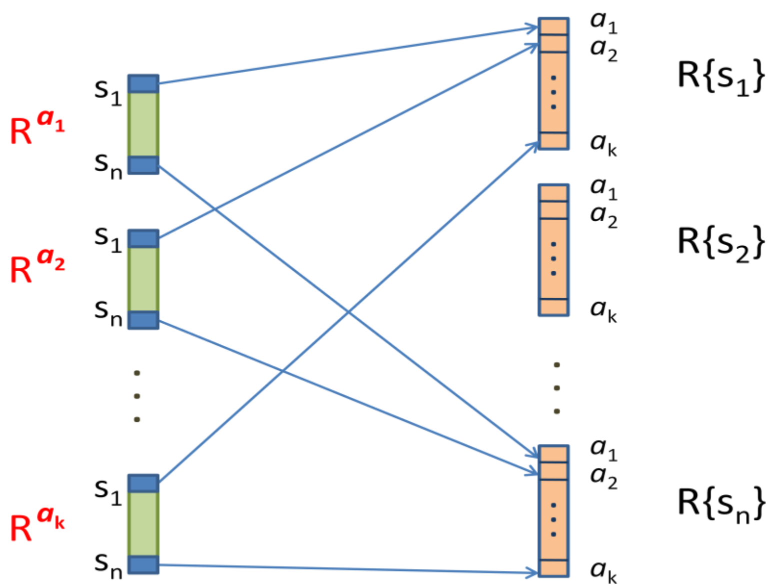




Markov 决策过程

其中





第一章 绪 论

1.1 Markov 决策过程

1.2 强化学习

1.3 课程内容安排

1.4 小结

强化学习问题

问题描述

- **怎样的选择 *a* | S_t，可以使得 Average Return（Value Function）最大？**

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \boxed{\pi(a|s)} \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

Optimal Value Function

The *optimal state-value function* $v_*(s)$ is the maximum value function over all policies

$$v_*(s) = \max_{\pi} v_\pi(s)$$

The *optimal action-value function* $q_*(s, a)$ is the maximum action-value function over all policies

$$q_*(s, a) = \max_{\pi} q_\pi(s, a)$$

Optimal Policy

- **策略的优越性评价：**

$$\pi \geq \pi' \text{ if } v_\pi(s) \geq v_{\pi'}(s), \forall s$$

Theorem

For any Markov Decision Process

- *There exists an optimal policy* π_* *that is better than or equal to all other policies,* $\pi_* \geq \pi, \forall \pi$
- *All optimal policies achieve the optimal value function,* $v_{\pi_*}(s) = v_*(s)$
- *All optimal policies achieve the optimal action-value function,* $q_{\pi_*}(s, a) = q_*(s, a)$

强化学习问题

Optimal Policy

- **Deterministic Optimal Policy**

An optimal policy can be found by maximising over $q_*(s,a)$,

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ q_*(s,a) \\ 0 & otherwise \end{cases}$$

强化学习问题

Bellman Optimality Equation

$$v_*(s) = \max_a q_*(s, a)$$

$$q_*(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

- **Bellman Optimality Equation for v***

$$v_*(s) = \max_a \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

- **Bellman Optimality Equation for Q***

$$q_*(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \max_{a'} q_*(s', a')$$

Solving Bellman Optimality Equation

- Bellman Optimality Equation is non-linear
- No closed form solution (in general)
- Many iterative solution methods
  - Value Iteration
  - Policy Iteration
  - Q-learning
  - Sarsa

第一章 绪 论

1.1 Markov 决策过程

1.2 强化学习

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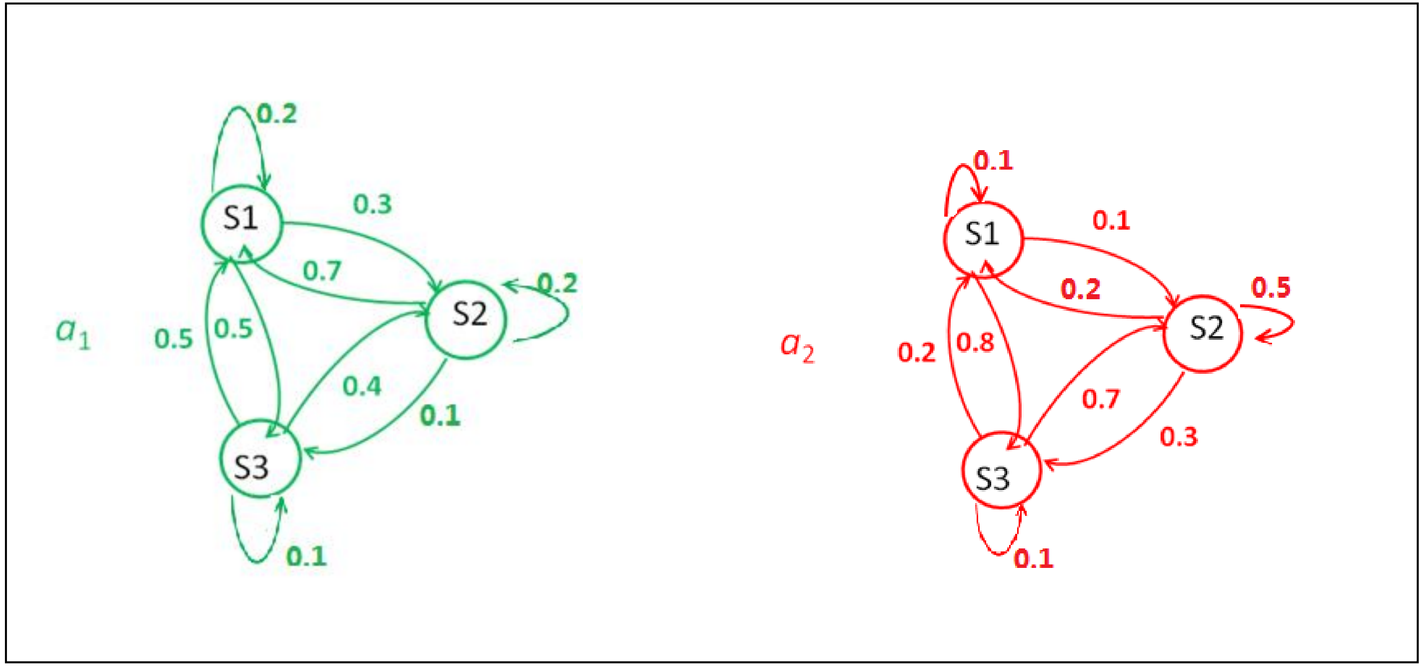
课程内容安排

1　基础知识
2　Bellman最优方程　　标准问题：值估计和策略控制
3　代价值估计
4　策略控制　　随机方法
5　值函数逼近
6　策略梯度方法　　函数拟合方法　　Exploitation
7　模型方法
8　蒙特卡洛树搜索　　环境模拟和探测　　Exploration
9　强化学习应用案例

小结

1. MDP

多个行动 a 的 MRP



小结

2. **Average Return (Value Function)**

MRP：

$$
\begin{aligned}
v(s) &= \mathbb{E}\left[G_t \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma \left(R_{t+2} + \gamma R_{t+3} + \ldots\right) \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right] \\
&= \mathbb{E}\left[R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s\right]
\end{aligned}
$$

$$v(s) = \mathcal{R}_s + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'} v(s')$$

MDP：

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

小结

3. 强化学习问题

怎样的选择 $a | S_t$，可以使得 Average Return (Value Function)最大？

An optimal policy can be found by maximising over $q_*(s, a)$,

$$\pi_*(a|s) = \begin{cases} 1 & \text{if } a = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_*(s, a) \\ 0 & otherwise \end{cases}$$

小结

4. 强化学习本质是求解 Bellman Optimality Equation

$$v_*(s) = \max_a q_*(s,a)$$

$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

Bellman Optimality Equation for v*

$$v_*(s) = \max_a \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

Bellman Optimality Equation for Q*

$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \max_{a'} q_*(s',a')$$

本讲参考文献

1. Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. (Second edition, in progress，draft.
2. David Silver，Slides@《Reinforcement Learning:An Introduction》,2016.