

## Yoneda : Conversion for Dedukti or Automation for Coq ?

1. Short: This [1] solves some question of Ahrens [2] and Kan-Riehl [3], which is how to program Kelly's <<enriched categories>> and how the inter-dependence of <<naturality>> with <<category>> is cyclic. Also This [4] attempts to clarify the contrast <<categorical algebra>> (ring/locale-presentation and its "internal logic"), from <<categorical logic>> in the style of the <<enriched/encoded/programmed/recursion>> categories of Kelly-Dosen or Lawvere-Lambek and as attempted in [5], for example : the yoneda lemma and most categorical lemmas are no-more-than Gentzen's constructive logic of re-arranging the input-output positions <<modulo naturality>>. Now homotopy/knots/proof-nets may be held as (faithfull or almost-faithfull) semantical techniques (<<descent>>) to do this <<categorical logic>>, and the homotopy itself may be programmed in specialized grammars (for example [6] or HOTT).

2. The common assumption that  $\text{catC}(-, X)$  is dual to  $\text{catC}(Y, -)$  is FALSIFIED. This falsification originates from the description of the composition as some binary form instead of as some functional form which is programmed/encoded/enriched onto the computer. Then get some new thing which is named <<polymorphism>> from which to define <<polymorph category>>. This is the only-ever real description and deduction of the yoneda lemma, which says that the image of  $\text{polyF}$  (which is injective and contained in natural transformations) also contains all natural transformations.

3. Some polymorph category is given by  $\text{polyF}$ , which is commonly  $(\_1 \circ \_2)$ , polymorph in  $V$  and polymorph in  $A$  :

Variable  $\text{obF}$  : Type.

Variable  $\text{polyF00}$  :  $\text{obF} \rightarrow \text{obF} \rightarrow \text{obV}$ .

Notation " $\text{F}[0 \ A1 \ \sim \ A2 \ ]0$ " := ( $\text{polyF00} \ A1 \ A2$ ) (at level 25).

Parameter  $\text{polyF}$  : forall (B :  $\text{obF}$ ), forall (V :  $\text{obV}$ ) (A :  $\text{obF}$ ),

$V(0 \ V \ |- \ \text{F}[0 \ B \ \sim \ A \ ]0 \ )0 \ \rightarrow$

$\text{forall } X : \text{obF}, V(0 \ \text{F}[0 \ A \ \sim \ X \ ]0 \ \ |- \ [0 \ V \ \sim \ \text{F}[0 \ B \ \sim \ X \ ]0 \ ]0 \ )0$ .

4. And to get polymorph functor, instead of describing  $F : \text{catA} \rightarrow \text{catB}$  then (contrast yoneda structures) describe  $\text{catV}[V, \text{catB}[B, F - ]]$  :  $\text{catA} \rightarrow \text{catV}$ , more precisely

Variable  $\text{polyF0}$  :  $\text{obA} \rightarrow \text{obB}$ .

Notation " $\text{F}|0 \ A$ " := ( $\text{polyF0} \ A$ ) (at level 4, right associativity).

Notation " $\text{F}[0 \ B \ \sim \ A \ ]0$ " := ( $\text{B}[0 \ B \ \sim \ \text{F}|0 \ A \ ]0$ ) (at level 25).

Parameter  $\text{polyF}$  : forall (V :  $\text{obV}$ ) (B :  $\text{obB}$ ) (A :  $\text{obA}$ ),

$V(0 \ V \ |- \ \text{F}[0 \ B \ \sim \ A \ ]0 \ )0 \ \rightarrow$

$\text{forall } X : \text{obA}, V(0 \ \text{A}[0 \ A \ \sim \ X \ ]0 \ \ |- \ [0 \ V \ \sim \ \text{F}[0 \ B \ \sim \ X \ ]0 \ ]0 \ )0$ .

5. And to get polymorph transformation, instead of describing  $\phi : G \ A \rightarrow H \ A$  then a-la-dosen (contrast weighted colimiting Kan extension)

describe  $\phi\_f : \text{catV}(V, \text{catB}[B, G \ A]) \rightarrow \text{catV}(V, \text{catB}[B, H \ A])$ , more precisely

Parameter  $\text{poly\_phi}$  : forall (V :  $\text{obV}$ ) (B :  $\text{obB}$ ) (A :  $\text{obA}$ ),

$V(0 \ V \ |- \ \text{F}[0 \ B \ \sim \ A \ ]0 \ )0 \ \rightarrow$

$V(0 \ V \ |- \ G[0 \ B \ \sim \ A \ ]0 \ )0$ .

And finally one shall relate the earlier <<naturality of transformation inside  $\text{catV}$ >> to this new <<polymorphism>> of transformation.

6. The earlier texts referring to MacLane associativity coherence and Dosen semiassociativity coherence and Dosen cut elimination for adjunctions and Chlipala ur/web database programming are all related to this present text which is how to program Borceux logically-enriched categories.

[1] 1337777.000, <https://github.com/1337777/borceux/blob/master/borceuxSolution.v>

[2] Ahrens, [https://github.com/benediktahrens/monads/blob/trunk/CAT/enriched\\_cat.v](https://github.com/benediktahrens/monads/blob/trunk/CAT/enriched_cat.v)

[3] Riehl, <http://www.math.jhu.edu/~eriehl/context.pdf>

[4] 1337777.000, <https://github.com/1337777/borceux/blob/master/chic05.pdf>

[5] 1337777.000, <https://github.com/1337777/dosen/blob/master/itp.pdf>

[6] Ye, <http://katherineye.com/post/129960474471/strange-loops-capturing-knots-with-powerful>