# Assignment 1 MTF271

Benjamin Elm Jonsson

## P1.1 Analysis

This question revolves around analysing where in a particular domain the reynolds stresses are at their highest. The following equations are to be used in order to analize this.

$$\bar{v}_1 \frac{\partial \bar{v}_1}{\partial x_1} + \bar{v}_2 \frac{\partial \bar{v}_1}{\partial x_2} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_1} + \nu \frac{\partial^2 \bar{v}_1}{\partial x_1^2} - \frac{\partial \overline{v_1'^2}}{\partial x_1} + \nu \frac{\partial^2 \bar{v}_1}{\partial x_2^2} - \frac{\partial \overline{v_1' v_2'}}{\partial x_2}$$
$$\bar{v}_1 \frac{\partial \bar{v}_2}{\partial x_1} + \bar{v}_2 \frac{\partial \bar{v}_2}{\partial x_2} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_2} + \nu \frac{\partial^2 \bar{v}_2}{\partial x_1^2} - \frac{\partial \overline{v_1' v_2'}}{\partial x_1} + \nu \frac{\partial^2 \bar{v}_2}{\partial x_2^2} - \frac{\partial \overline{v_2'^2}}{\partial x_2}$$

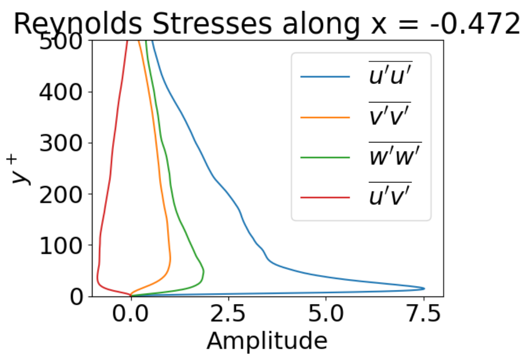
Since the derivative of the reynolds stresses depend on the derivatives of the flow as well as second derivatives, they will be large closer to the wall and smaller farhter away from the wall. This means that the reynolds stresses will quickly become large close to the wall.

## P1.2 The momentum equations

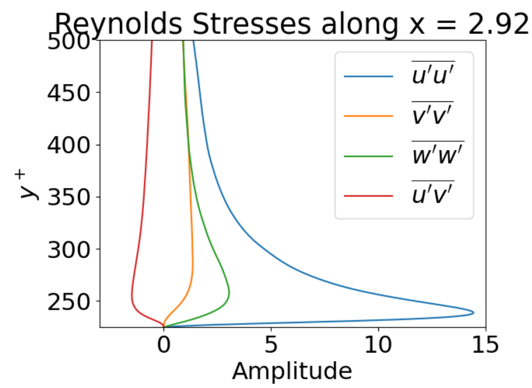
In this part of the report questions 1.1&1.2 will be answered. These questions regard the momentum equations. The case that will be used throughout the report is the *Contraction* case as well as grid lines $x_1 \approx -0.47$ and $x_2 \approx 2.92$. These grid lines are close to the inlet and outlet respectivly. Furthermore, they occur before/after the contraction in the domain respectively. For these reasons they are two good choices and should show interesting differences.

## Assignment 1.1

In the figures below the Reynolds stresses are plotted along two vertical grid lines. These grid lines are (a) $x_1 \approx -0.47$ and (b) $x_2 \approx 2.92$.




(a) Reynolds stresses along $x \approx -0.47$

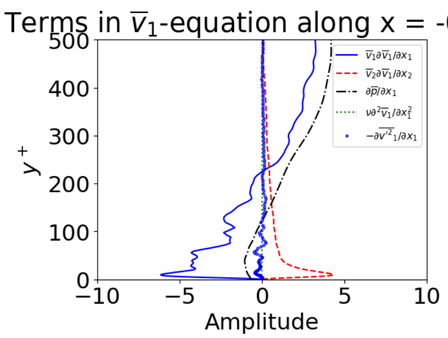



(b) Reynolds stresses along $x \approx 2.9$

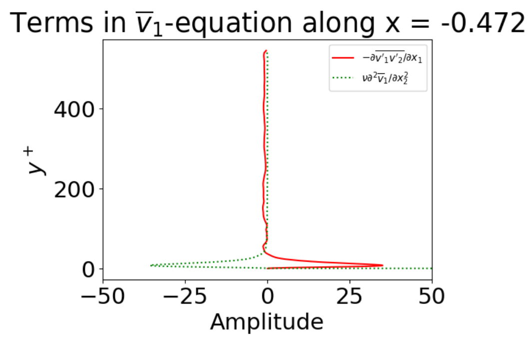
These plots are reasonable, since only $\overline{u'v'}$ is negative.

## Assignment 1.2

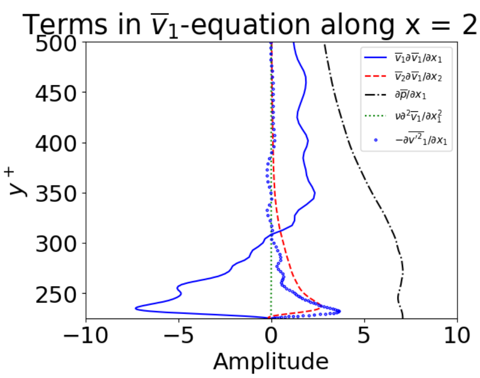
Below all terms in the $\overline{v}_1$ equations along the two same grid lines as previously used. The only negligible term is $\nu\frac{\partial^2\overline{v}_1}{\partial x_1^2}$. This is because the second derivative is small and $\nu$ is also small so the product of these will be very small. This is true for both of the grid lines shown. This term is not negligible where the viscous effects are large.



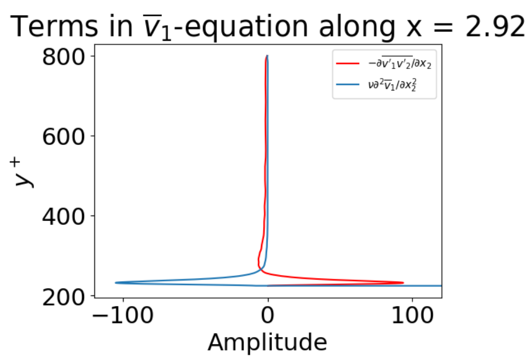

(a) Terms of $\overline{v}_1$-equation along $x \approx -0.47$




(b) Large terms of $\overline{v}_1$-equation along$x \approx -0.47$




(a) Terms of $\overline{v}_1$-equation along $x \approx 2.9$



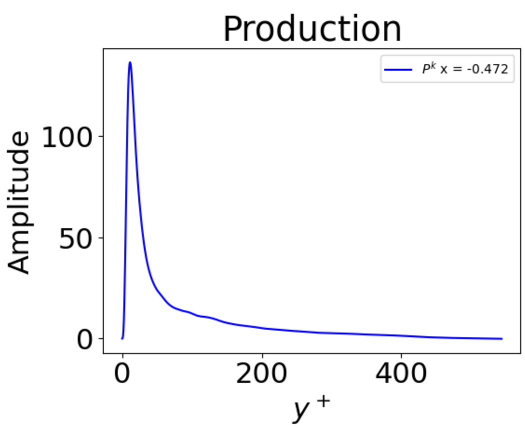

(b) Large terms of $\overline{v}_1$-equation along$x \approx 2.9$

## P1.3 The turbulent kinetic energy equation

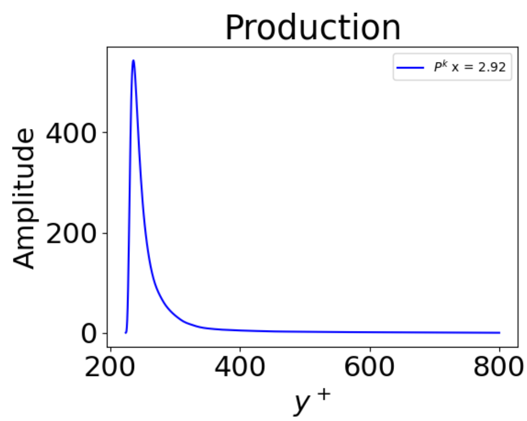
In this part of the report questions 1.3&1.4 will be answered. These questions regard the turbulent kinetic energy equation.

### Assignment 1.3

In the figures below the production term is plotted along the two established grid lines. It is clear that the production term is large close to the wall. This is clear even though the figure $(b)$ is shown from $y^+ = 200$. This is because the case examined is the contraction case and this is simply where the wall is for this grid line. To compare the contributions to the production term the following plot

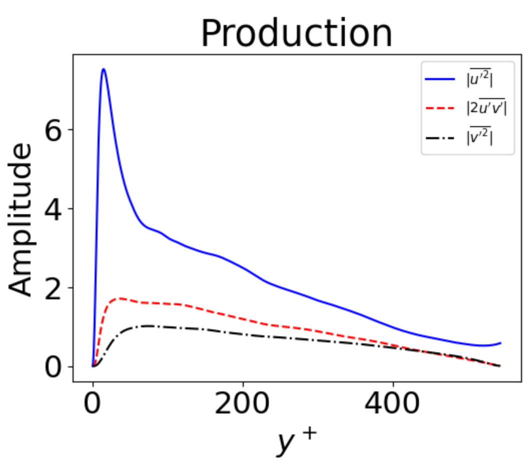



(a) Production term along $x \approx -0.47$

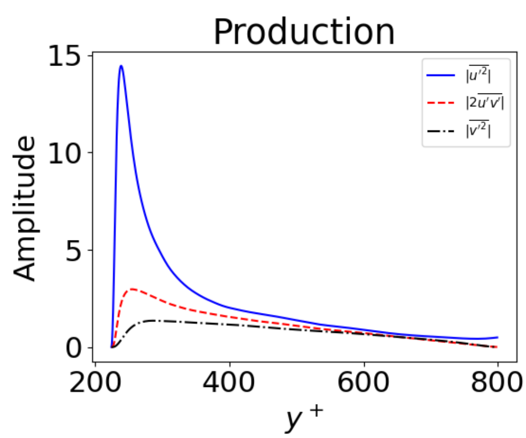



(b) Added dissipation along $x \approx 2.9$

is used. Here it is shown the absolute value of the contributions from $\overline{u'^2}$, $2\overline{u'v'}$ and finally $\overline{v'^2}$. The reason to use the absolute value is to simplífy the comparison.




(a) Contribution to production along $x \approx -0.47$



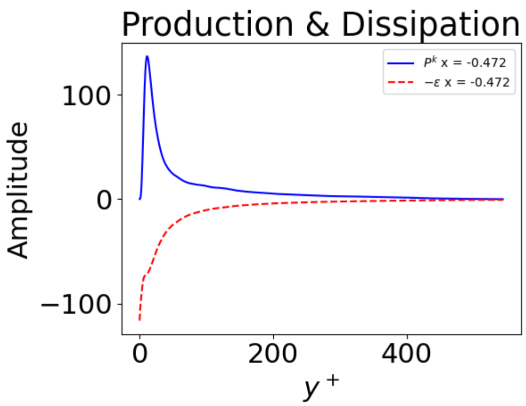

(b) Contribution to production along $x \approx 2.9$

It is clear that $\overline{u'^2}$ contributes the most and in second place comes the mixed term. This is because it occurs twice, so individually the normal stresses contribute the most.

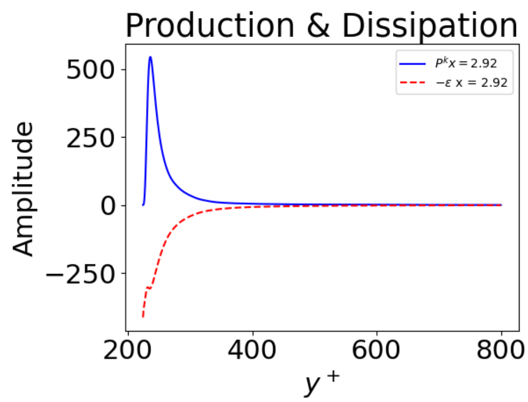
It is clear that the production terms will be large close to the wall since this is where the derivatives will be large. It is also clear that the production will be small far from the wall since the derivatives decrease with the distance from the wall.

## Assignment 1.4

In the plots below the dissipation is added to the plots. This is done for both of the grid lines. As is clear, we have approximate equillibrium between the terms, i.e $P^k + \varepsilon \approx 0$, throughout the domain as well as grid line.




(a) Added dissipation along $x \approx -0.47$



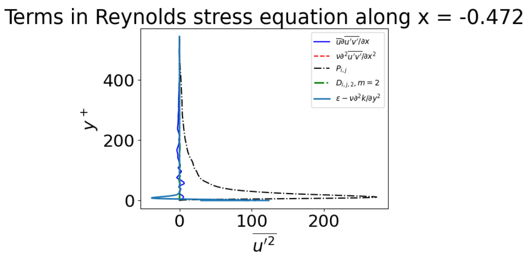

(b) Added dissipation along $x \approx 2.9$

# P1.4 The Reynolds stress equations

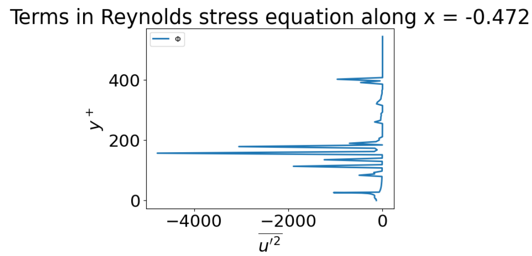
In this part of the report questions 1.5, 1.6, 1.7&1.8 are answered. These questions regard the Reynolds stress equation.

## Assignment 1.5

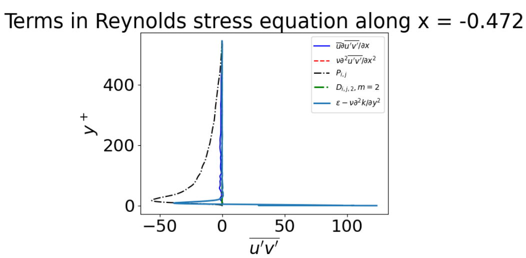
In the figures below the terms in the reynolds stress equations are plotted. This is done for the stresses $\overline{u'^2}$ and $\overline{u'v'}$.



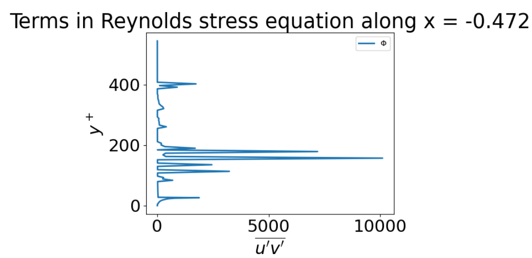

(a) Terms in reynolds stress equation




(b) Terms in reynolds stress equation




(a) Terms in reynolds stress equation




(b) Terms in reynolds stress equation

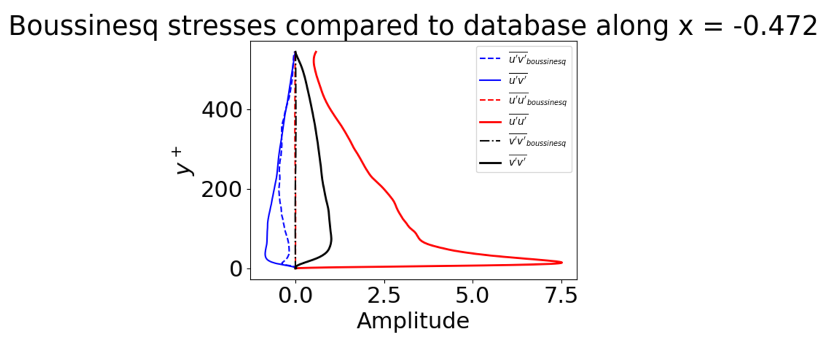
The figure needed to be split up since the pressure strain term is much larger than the others. This is simply for more clarity. The large terms are the production term, the pressure-strain term and the dissipation term. This is accurate for both of the stresses. The rest of them are very small in comparison. The reason for this may be because the large terms contain many summations over indeces, this means that they have more contributions to the overall value.

## Assignment 1.6

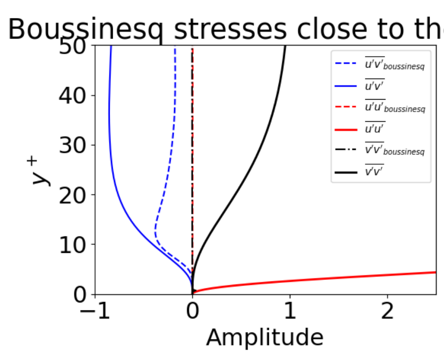
In the plots below the stresses are computed according to the Boussinesq assumption.

$$\overline{v_i'v_j'} = -2\nu_t \hat{s}_{i,j} + \frac{2k}{3}\delta_{i,j} \quad where\ \nu_t = c_\mu \frac{k^2}{\varepsilon}$$

These are plotted alongside the Reynolds stresses in the database. Figure $(b)$ also shows a zoom in close to the wall. It is clear that they all start at the same place and diverge from each other away from the wall they come. From this it is clear to say that the Boussinesq assumption is a good approximation close to the wall but nar from from it. Furthermore, it is also only a good approximation for the shear stresses as $\overline{u'^2}$ divergec very quickly. While the shear stress follows until around $y^+ = 10$.




(a) Stresses calculated accordint to the Boussinesq assumption




(b) Stresses calculated accordint to the Boussinesq assumption

### Assignment 1.7

In the figure below a 3D plot is made of the exact production throughout the whole domain. It is only negative in the contraction and also only very close to the wall. This indicates that the turbulent kinetic energy is transferred from the fluctuations to the mean flow. This is due to circulation, which is reasonable to assume where the flow is contracting.

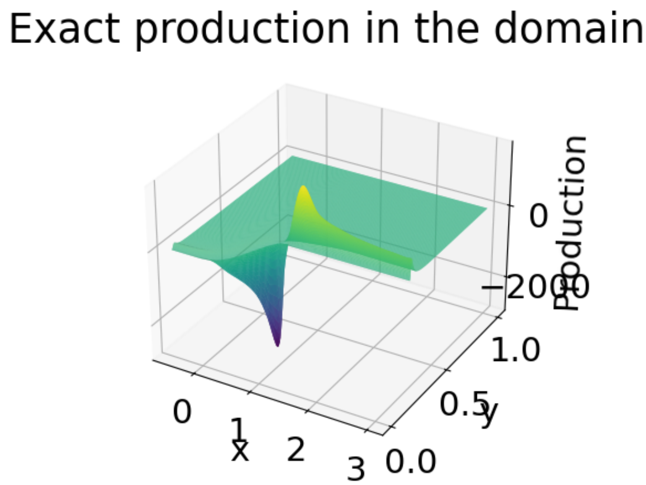


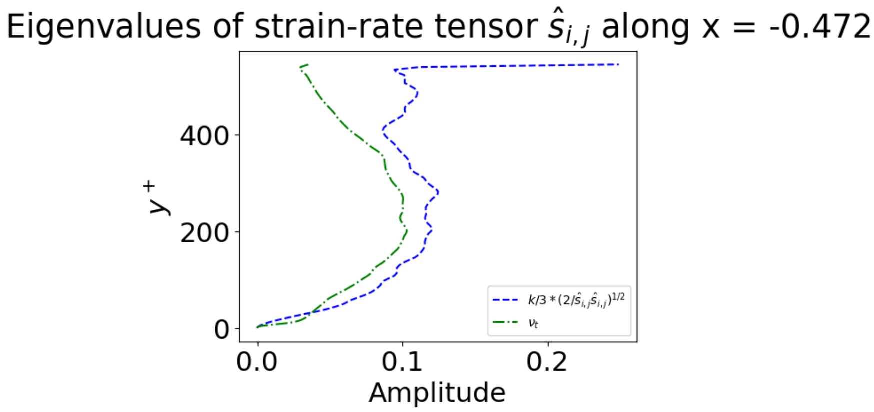

Figure 10: Exact production term in the whole domain

## Assignment 1.8

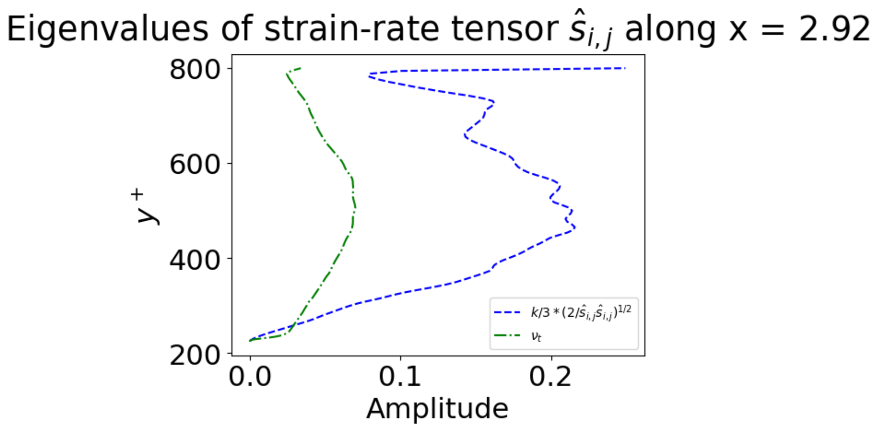
In the figures below the turbulent viscosity is plotted as well as the limiter.

$$\nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\hat{s}_{i,j}\hat{s}_{i,j}}\right)^{1/2}$$

If $\nu_t$ is larger than its limiter than it is limited to be equal to the limiter. This only happens really close to the wall along the two grid lines since the green line for $\nu_t$ is larger than the limiter.

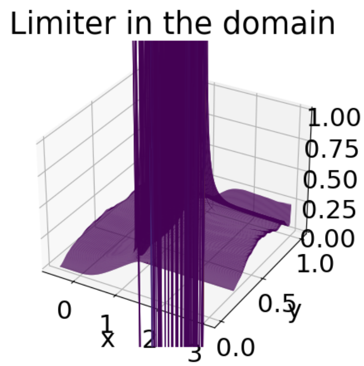



(a) Comparison of limiter and $\nu_t$

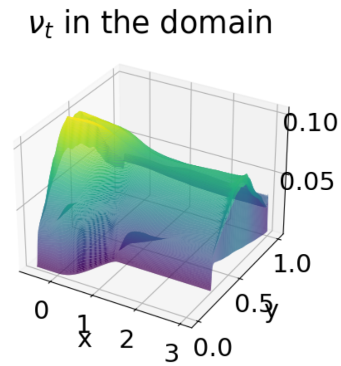



(b) Comparison of limiter and $\nu_t$

Below, the limiter and $\nu_t$ are plotted in the entire domain.




(a) Limiter in the entire domain




(b) $\nu_t$ in the entire domain

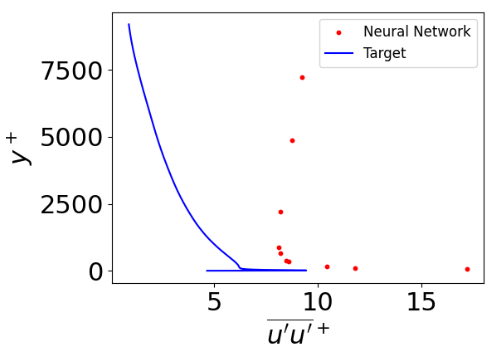
It is clear that it is close to the inlet that there is risk of $\nu_t$ needing to be limited. This is not a stagnation zone since it is affected by recirculation from the contracting region. Since the flow comes in to the domain with speed it is not a stagnation zone.

## P2 Part b: Machine learning

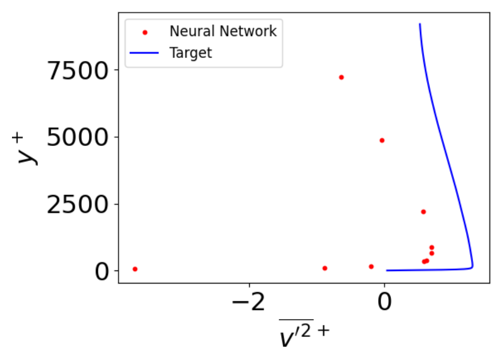
In this part of the report questions 1.9&1.10 will be answered. These questions regard the use of machine learning in turbulence modeling.

### Assignment 1.9

The plots of the predictions for the stresses are shown in the figures below. These perform worse than the predictions that only use DNS data to train on. This is because the data these predictions are trained on are partly already a model, which is worse than true DNS data. Furthermore the data provided has only 50 data points in contrast to the DNS data which has over 900 data points. This limits the data used to 50 data points to have correct dimensions for everything. This is also a big contributor to the poor performance. However, there is clear potential since the predictions loosely follow the direction of the true data. Even though the plot showing the $\overline{u'^2}$ goes the wrong way for large values of $y^+$.




(a) Neural network predictions for $\overline{u'^2}$




(b) Neural network predictions for $\overline{v'^2}$

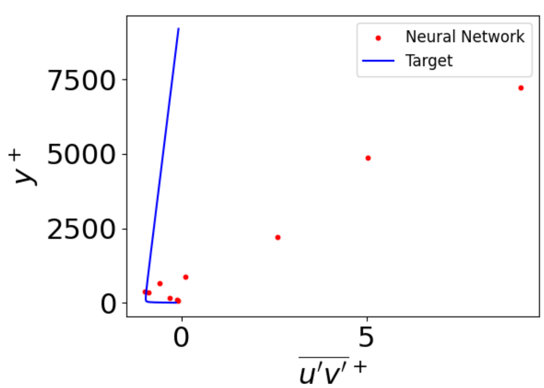



Figure 14: Neural network predictions for $\overline{u'v'}$

## Assignment 1.10

In the figure below the stresses are plotted. There are stresses predicted by a neural network as well as DNS data for the same stresses. The predictions from the neural network are decent for the normal stresses. However it performs poorly when predicting the shear stress $\overline{u'v'}$. However, the figure looks correct.

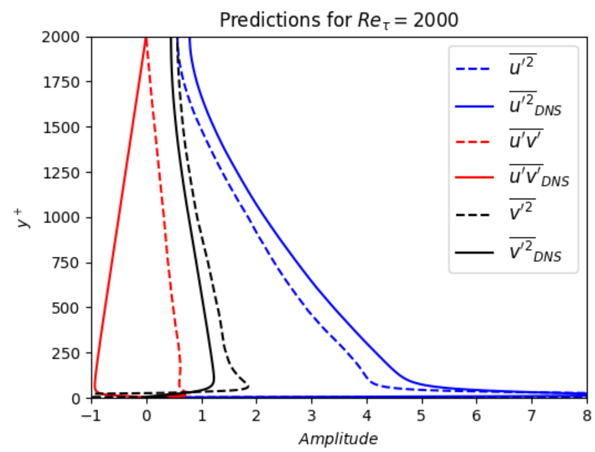



Figure 15: Neural network predictions for $Re_\tau = 2000$