$$P(x) = \sum_{x=0}^{\infty} {}^nC_x p^x q^{n-x}$$

8m

$$\text{mean } m = \sum_{x=0}^{\infty} x\,P(x)$$

$$= \sum_{x=0}^{\infty} x.\ {}^nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} x.\ \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} \cancel{x}.\ \frac{n(n-1)!}{((n-1)-(x-1))!\ x.(x-1)!} p.\,p^{(x-1)} q^{((n-1)-(x-1))}$$

$$= \sum_{x=0}^{\infty} {}^{n-1}C_{x-1}\, p^{(x-1)} q^{((n-1)-(x-1))} \qquad \boxed{(a+b)^n = \sum_{x=0}^{\infty} {}^nC_x\, a^{n-x}.b^x}$$

$$= np \sum_{x=0}^{\infty} (\underbrace{p+q}_{1})^{n-1}$$

$$= np.$$

$$\text{variance } \sigma^2 = \sum_{x=0}^{\infty} x^2 P(x) - m^2 \longrightarrow (1)$$

$$\text{consider } \sum_{x=0}^{\infty} x(x-1) + x\,(P(x))$$

$$= \sum_{x=0}^{\infty} x(x-1)\,p(x) + \sum_{x=0}^{\infty} x\,(p(x))$$

$$= \sum_{x=0}^{\infty} x(x-1)\ {}^nC_x p^x q^{n-x} + m$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{n!\ p^x q^{n-x}}{(n-x)!\,.\,x!} + m$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \frac{n(n-1)(n-2)!}{((n-2)-(x-2))!\ \cancel{x(x-1)}(x-2)!} p^2.\,p^{x-2} q^{((n-2)-(x-2))} + m$$

$$= p^2 n(n-1) \sum_{x=2}^{\infty} {}^{n-2}C_{x-2}\, p^{x-2} q^{((n-2)-(x-2))} + m$$

$$= p^2(n-1)n\ (p+q)^{n-2} + np.$$

$$= p^2(n(n-1) + np.$$

from (1).

$$\sigma^2 = p^2(n)(n-1) + np - n^2p^2$$

$$= np^2 + \cancel{n^2p^2} - p^2 n - \cancel{n^2p^2} = np(1-p) = npq$$

$$SD = \sqrt{npq}$$

Exactly $x \Rightarrow x = ?$ and $n \le 30 \rightarrow$ Binomial Distribution

none/not/no $x \Rightarrow x = 0$

Atmost $\Rightarrow x \le$

Atleast $\Rightarrow x \ge$

$n \le 30$, $p << 1$, $n > 30$ } Probability Distribution

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & x < 0 \end{cases}$$

for mean

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\text{mean} = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} 0\, dx + \int_{0}^{\infty} x \alpha e^{-\alpha x}\, dx$$

$$= \alpha \int_0^{\infty} x e^{-\alpha x}\, dx$$

$$= \alpha \left[x \frac{(-)e^{-\alpha x}}{\alpha} - 1 \frac{(-)e^{-\alpha x}}{\alpha^2} \right]_0^{\infty}$$

$$= \alpha \left[\frac{e^{-\alpha x}}{\alpha^2} - \frac{x e^{-\alpha x}}{\alpha} \right]_0^{\infty}$$

$$= \alpha \left[0 - 0 - \frac{1}{\alpha^2} - 0 \right]$$

$$= \cancel{\alpha} \frac{1}{\alpha^{\cancel{2}}} \qquad m = 1/\alpha$$

$$= 1/\alpha$$

$$\text{variance} = \left(\int_{-\infty}^{\infty} x^2 f(x) \right) - m^2$$

$$= \int_{-\infty}^{0} 0\, dx + \int_0^{\infty} x^2 \alpha e^{-\alpha x}\, dx - 1/\alpha^2$$

$$= 0 + \alpha \left[\int_0^{\infty} x^2 e^{-\alpha x}\, dx \right] - 1/\alpha^2$$

$$= \alpha\left[x^2\frac{e^{-\alpha x}}{\alpha} - 2x\frac{e^{-\alpha x}}{\alpha^2} + 2\frac{e^{-\alpha x}}{\alpha^3}\right]_0^\infty - 1/\alpha^2$$

$$= \alpha\left[0 - [2/\alpha^3]\right] - 1/\alpha^2$$

$$= \alpha\left[-\frac{2}{\alpha^3}\right] - \frac{1}{\alpha^2}$$

$$= \frac{-2}{\alpha^2} - \frac{1}{\alpha}$$

$$-\left(\frac{2}{\cancel{\alpha^3}} + \frac{1}{\alpha^2}\right)$$

$$= \frac{-\cancel{2x}-}{\alpha^3} =$$

$$= \frac{+2}{\alpha^2} - \frac{1}{\alpha^2}$$

$$= \frac{1}{\alpha^2}$$

$$\sigma^2 = \frac{1}{\alpha^2}$$

$$SD = \frac{1}{\alpha}.$$

S	R	Q_n	Output Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	X
1	1	1	X

S \ RQ_n	$\bar{R}\bar{Q}_n$	$\bar{R}Q_n$	RQ_n	$R\bar{Q}_n$
$\bar{S}$	0	1 1	3	2
S	1 4	1 5	X 7	X 6

$$Q_{n+1} = S + \bar{R}Q_n$$

mean = ~~$x_i p_i$~~ $x\,p(x)$ (m)

variance = $x^2 p(x) - m^2$ (σ^2)

standard deviation = $\left(\sqrt{\sigma^2}\right)$

binomial distribution

$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$m = np$

$\sigma^2 = \cancel{mnp}\ npq$

$SD = \cancel{\sqrt{mnp}}\ \sqrt{npq}$

poissions pdf

$p(x) = \dfrac{e^{-m} m^{x}}{x!}$

Probability

exponential

$p(x) = \alpha e^{-\alpha x} \quad x > 0$

$\qquad\quad 0 \quad x < 0$

$m = 1/\alpha = SD$

$\sigma^2 = 1/\alpha^2$

continous pdf

$m = \int_{-\infty}^{\infty} x f(x)\,dx$

$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) - m^2$

$SD = \sqrt{\sigma^2}$

Q_{n+1} = next state